Advanced Data Analysis II

8. Exercise

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Exercise 1 *Graphical models*

We consider the following domain that describes how to start a car engine. We are trying to start the engine of our car. The engine could either start ($engine = 1$) or not ($engine = 0$). There are various reasons for failing to start the engine: the tank could be empty ($tank = 0$), or the starter motor does not rotate ($starter = 0$). The starter requires an intact battery ($battery = 1$) and it must not be defective ($starter\ defect = 0$). We can observe the condition of the tank indirectly through the electric fuel gauge: if the tank is full and the battery provides enough power for the fuel gauge to work, the fuel gauge shows full ($display = 1$), otherwise it shows empty ($display = 0$).

1. Construct a directed graphical model (Bayesian network) on the binary random variables *battery*, *starter defect*, *starter*, *tank*, *display*, and *engine*. Show the graph structure G and the respective (conditional) distributions in tabular form. Set realistic numerical probabilities (note: these are almost never exactly 0 or 1).

2. Check whether the following independences are true based on the D-separation criterion:

   - $starter\ defect \perp engine \mid battery$
   - $battery \perp tank \mid \emptyset$
   - $battery \perp engine \mid starter$
   - $tank \perp starter\ defect \mid engine$

   State for each independency why it applies/does not apply.

3. We observe that the fuel gauge indicating an empty tank ($display = 0$). What is the probability that the tank is really empty ($tank = 0$)?

Exercise 2 *Separating set of nodes*

Let G be the graph structure of a graphical model, and let X be a node in G. We will study the question of which set M of nodes we have to observe such that the node X is independent of all other nodes in G given M. A minimal set M that has this property will be called *separating set*. That is, M is a minimal set with $X \notin M$ and

$$\forall X' \in G \setminus \{X, M\} : X' \perp X | M. \tag{1}$$

Characterize the set M concisely and argue why it is minimal and has the separating property. Hint: D-separation.

Exercise 3 *Acyclic graphs*

Prove the following theorem from graph theory:

A graph G is acyclic if and only if there is an order $\leq_G$ on the nodes of G such that for all $X, X' \in G$ the following condition holds:
$X \to X' \Longrightarrow X \leq_G X'$.

$X \to X'$ means there is a directed edge from node X to node X' in G.