Answer Set Solving in Practice

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Rough Roadmap

1. Introduction
2. Language
3. Modeling
4. Grounding
5. Foundations
6. Solving
7. Systems
8. Applications

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Resources

- Course material
  - http://www.cs.uni-potsdam.de/wv/lehre
  - http://moodle.cs.uni-potsdam.de
  - http://potassco.sourceforge.net/teaching.html
- Systems
  - clasp http://potassco.sourceforge.net
  - dlv http://www.dlvsystem.com
  - smodels http://www.tcs.hut.fi/Software/smodels
  - gringo http://potassco.sourceforge.net
  - lparse http://www.tcs.hut.fi/Software/smodels
  - clingo http://potassco.sourceforge.net
  - iclingo http://potassco.sourceforge.net
  - oclingo http://potassco.sourceforge.net
  - asparagus http://asparagus.cs.uni-potsdam.de

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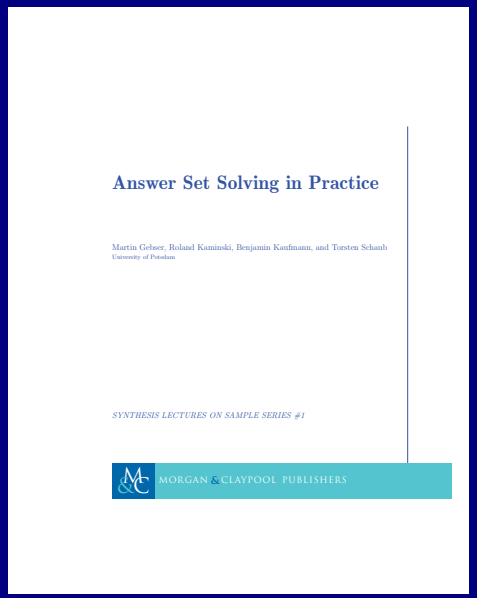
The Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



Resources

- `http://potassco.sourceforge.net/book.html`
- `http://potassco.sourceforge.net/teaching.html`

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Literature

Books	[4], [29], [53]
Surveys	[50], [2], [39], [21], [11]
Articles	[41], [42], [6], [61], [54], [49], [40], etc.

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Constraint Answer Set Programming: Overview

1. Motivation
2. Preliminaries
3. Modeling Language
4. Algorithms

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Outline

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Motivation

- Observation Certain applications are more naturally modeled by mixing Boolean with non-Boolean constructs, eg., accounting for
  - resources,
  - fine timings, or
  - functions over finite domains
- Approach Extend ASP with CP capacities
  - propositional semantics
  - CDCL-style solving capacities
    - following the approach of SMT solving
  - off-the-shelf CP solvers

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Outline

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Constraint Satisfaction Problem

- A constraint satisfaction problem (CSP) consists of
  - a set V of variables,
  - a set D of domains, and
  - a set C of constraints

  such that
  - each variable $v \in V$ has an associated domain $dom(v) \in D$;
  - a constraint c is a pair (S, R) consisting of a k-ary relation R on a vector $S \subseteq V^k$ of variables, called the scope of R

- Note For $S = (v_1, \ldots, v_k)$, we have $R \subseteq dom(v_1) \times \cdots \times dom(v_k)$

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Example

$$\begin{array}{cccccc} & s & e & n & d \\ + & m & o & r & e \\ \hline m & o & n & e & y \end{array}$$

Each letter corresponds exactly to one digit and all variables have to be pairwisely distinct

$$V = \{s, e, n, d, m, o, r, y\}$$

$$D = \{dom(v) = \{0, \ldots, 9\} \mid v \in V\}$$

$$\begin{aligned} C = \{ &(\vec{V}, \quad allDistinct(V)), \\ &(\vec{V}, \quad s \times 1000 + e \times 100 + n \times 10 + d + \\ &\qquad m \times 1000 + o \times 100 + r \times 10 + e == \\ &\qquad m \times 10000 + o \times 1000 + n \times 100 + e \times 10 + y), \\ &((m), m == 1) \} \end{aligned}$$

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Example

	s	e	n	d
+	m	o	r	e
m	o	n	e	y

Each letter corresponds exactly to one digit and all variables have to be pairwisely distinct

	9	5	6	7
+	1	0	8	5
1	0	6	5	2

The example has exactly one solution

$$\{ s \mapsto 9, e \mapsto 5, n \mapsto 6, d \mapsto 7, m \mapsto 1, o \mapsto 0, r \mapsto 8, y \mapsto 2 \}$$

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Constraint satisfaction problem

- Notation We use $S(c) = S$ and $R(c) = R$ to access the scope and the relation of a constraint $c = (S, R)$
- For an assignment $A : V \to \bigcup_{v \in V} dom(v)$ and a constraint (S, R) with scope $S = (v_1, \ldots, v_k)$, define

$$sat_C(A) = \{c \in C \mid A(S(c)) \in R(c)\}$$

where $A(S) = (A(v_1), \ldots, A(v_k))$

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Constraint Answer Set Programming

- A constraint logic program P is a logic program over an extended alphabet $\mathcal{A} \cup \mathcal{C}$ where
  - $\mathcal{A}$ is a set of regular atoms and
  - $\mathcal{C}$ is a set of constraint atoms,

  such that $\mathit{head}(r) \in \mathcal{A}$ for each $r \in P$
- Given a set of literals B and some set $\mathcal{B}$ of atoms, we define $B|_{\mathcal{B}} = (B^+ \cap \mathcal{B}) \cup \{\sim a \mid a \in B^- \cap \mathcal{B}\}$

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Constraint Answer Set Programming

- We identify constraint atoms with constraints via a function

$$\gamma : \mathcal{C} \to C$$

- Furthermore, $\gamma(Y) = \{\gamma(c) \mid c \in Y\}$ for any $Y \subseteq \mathcal{C}$
- Note Unlike regular atoms $\mathcal{A}$, constraint atoms $\mathcal{C}$ are not subject to the unique names assumption, eg.

$$\gamma(x < y) \;=\; \gamma(((-y - 1) \leq -(x + 1)) \wedge (x \neq y))$$

- A constraint logic program P is associated with a CSP as follows
  - $C[P] = \gamma(atom(P) \cap \mathcal{C})$,
  - $V[P]$ is obtained from the constraint scopes in $C[P]$,
  - $D[P]$ is provided by a declaration

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Constraint Answer Set Programming

- Let P be a constraint logic program over $\mathcal{A} \cup \mathcal{C}$ and let $A : V[P] \to D[P]$ be an assignment,

  define the constraint reduct of as P wrt A as follows

$$P^A = \{\ head(r) \leftarrow body(r)|_{\mathcal{A}} \mid r \in P, \gamma(body(r)|_{\mathcal{C}}{}^{+}) \subseteq sat_{C[P]}(A), \gamma(body(r)|_{\mathcal{C}}{}^{-}) \cap sat_{C[P]}(A) = \emptyset\ \}$$

- A set $X \subseteq \mathcal{A}$ of (regular) atoms is a constraint answer set of P wrt A, if X is an answer set of P^A.
  - Note: That is, if X is the $\subseteq$-smallest model of $(P^A)^X$

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Outline

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Modeling Language

Note

Although our semantics is propositional, the atoms in $\mathcal{A}$ and $\mathcal{C}$ are constructible from a multi-sorted, first-order signature given by:

- a set $\mathcal{P}_\mathcal{A} \cup \mathcal{P}_\mathcal{C}$ of predicate symbols such that $\mathcal{P}_\mathcal{A} \cap \mathcal{P}_\mathcal{C} = \emptyset$,
- a set $\mathcal{F}_\mathcal{A} \cup \mathcal{F}_\mathcal{C}$ of function symbols (including constant symbols),
- a set $\mathcal{V}_\mathcal{A}$ of regular variable symbols, and
- a set $\mathcal{V}_\mathcal{C} \subseteq \mathcal{T}(\mathcal{F}_\mathcal{A})$ of constraint variable symbols, where $\mathcal{T}(\mathcal{F}_\mathcal{A})$ denotes the set of all ground terms over $\mathcal{F}_\mathcal{A}$

As usual, the atoms in $\mathcal{A} \cup \mathcal{C}$ are obtained by a grounding process

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An Example

$$
\begin{aligned}
& time(0..t_{max}) \\
& bucket(a) \quad bucket(b) \\
1\ \{pour(B,T) : bucket(B)\}\ 1 \ &\leftarrow\ time(T), T < t_{max} \\
1 \leq^{\$} amt(B,T) \ &\leftarrow\ pour(B,T), T < t_{max} \\
amt(B,T) \leq^{\$} 3 \ &\leftarrow\ pour(B,T), T < t_{max} \\
amt(B,T) =^{\$} 0 \ &\leftarrow\ {\sim}pour(B,T), T < t_{max} \\
vol(B,T{+}1) =^{\$} vol(B,T) +^{\$} amt(B,T) \ &\leftarrow\ time(T), T < t_{max} \\
down(B,T) \ &\leftarrow\ vol(C,T) <^{\$} vol(B,T) \\
up(B,T) \ &\leftarrow\ {\sim}down(B,T) \\
vol(a,0) =^{\$} 0 \quad vol(b,0) =^{\$} 1 & \\
&\leftarrow\ up(a,t_{max})
\end{aligned}
$$

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An Example

- Consider the signature of our exemplary program:

$$\begin{aligned} \{B, C, T\} &\subseteq \mathcal{V}_{\mathcal{A}} \\ \{0, \ldots, t_{max}, +, a, b, amt, vol\} &\subseteq \mathcal{F}_{\mathcal{A}} \\ \{<, time, bucket, pour, up, down\} &\subseteq \mathcal{P}_{\mathcal{A}} \\ \{0, 1, 3, +^{\$}\} &\subseteq \mathcal{F}_{\mathcal{C}} \\ \{=^{\$}, <^{\$}, \leq^{\$}\} &\subseteq \mathcal{P}_{\mathcal{C}} \end{aligned}$$

- With substitution $\{B \mapsto b, T \mapsto 1, t_{max} \mapsto 2\}$, we get:

$$amt(b,1) =^{\$} 0 \leftarrow \sim pour(b,1)$$

$$vol(b,2) =^{\$} vol(b,1) +^{\$} amt(b,1) \leftarrow$$

and, among others, our signature hence contains

$$\begin{aligned} \{amt(b,1), vol(b,1), vol(b,2)\} &\subseteq \mathcal{V}_{\mathcal{C}} \\ \{pour(b,1)\} &\subseteq \mathcal{A} \\ \{amt(b,1) =^{\$} 0,\ vol(b,2) =^{\$} vol(b,1) +^{\$} amt(b,1)\} &\subseteq \mathcal{C} \end{aligned}$$

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An Example

- For $t_{max} = 2$, our program has eleven constraint answer sets, summarized as follows

$up(a,0)$	$pour(a,0)$	$amt(a,0)$	$up(a,1)$	$pour(a,1)$	$amt(a,1)$	$up(a,2)$
T	**T**	1	**T**	**T**	1, 2, 3	**F**
T	**T**	2, 3	**F**	**T**	1, 2, 3	**F**
T	**T**	3	**F**	**F**	0	**F**
T	**F**	0	**T**	**T**	3	**F**

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T	**T**	1	**T**	**T**	$1,2,3$	**F**
T	**T**	$2,3$	**F**	**T**	$1,2,3$	**F**
T	**T**	3	**F**	**F**	0	**F**
T	**F**	0	**T**	**T**	3	**F**

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SAT modulo theories

SAT modulo theories (SMT)

- logical formulas with respect to combinations of background theories
- real numbers, integers, lists, arrays, bit vectors ...

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SAT

$$(x \vee y \vee z) \wedge (\neg x \vee \neg y \vee \neg z)$$

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SAT modulo theories

SAT modulo theories (SMT)

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SMT

$$(\sin(x)^3 = \cos(\log(y) \cdot x) \quad \vee b \quad \vee -x^2 \geq 2.3y)$$

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SAT modulo theories

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- real numbers, integers, lists, arrays, bit vectors ...

SMT

$$(\quad a \quad \vee b \quad \vee -x^2 \geq 2.3y)$$

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SAT modulo theories

SAT modulo theories (SMT)

- logical formulas with respect to combinations of background theories
- real numbers, integers, lists, arrays, bit vectors . . .

SMT

$$(\quad a \quad \vee b \quad \vee \quad c \quad)$$

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CDNL

Main Algorithm

loop

- Propagate
- **if** no conflict **then**
  - **if** partial assignment **then** Decide
    else return solution
- **else if** some decisions made **then**
  - Analyze Conflict
    Record Reason
    Backjump
- **else exit**

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ASP Propagation

Propagation Algorithm

loop

UNIT-PROPAGATION

if conflict **then return**

else if UNFOUNDED-SET **then**

RECORD LOOP-NOGOOD

if conflict **then return**

else return

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Constraint ASP Propagation

Propagation Algorithm

loop

UNIT-PROPAGATION

if conflict **then return**

else if UNFOUNDED-SET **then**

RECORD LOOP-NOGOOD

if conflict **then return**

else if CONSTRAINT-PROPAGATION **then**

if conflict **then return**

else return

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Constraint CDNL

Main Algorithm

loop

- Propagate
- **if** no conflict **then**
  - **if** partial assignment **then** Decide
  - **else if** CSP-Solve **then return** solution
  - **else**
    - Analyze Conflict
    - Record Reason
    - Backjump
- **else if** some decisions made **then**
  - Analyze Conflict
  - Record Reason
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Conflict and reason determination

- Design Goal Use off-the-shelf CP solvers
- Problem Off-the-shelf CP solvers do not provide reasons for conflicts
- Idea Approximate reasons in various ways
  - lazy learning versus early learning
  - use constraint interdependencies or not

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