Answer Set Solving in Practice

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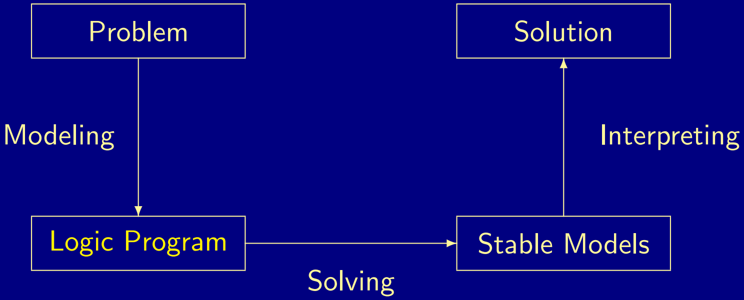
Introduction: Overview

1. Syntax
2. Semantics
3. Examples
4. Reasoning
5. Language
6. Variables

Outline

Syntax


Problem
Solution
Modeling
Interpreting
Logic Program
Stable Models
Solving

Normal logic programs

- A logic program, P, over a set $\mathcal{A}$ of atoms is a finite set of rules
- A (normal) rule, r, is of the form

$$a_0 \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

$$\begin{aligned} h(r) &= a_0 \\ B(r) &= \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \\ B(r)^+ &= \{a_1, \dots, a_m\} \\ B(r)^- &= \{a_{m+1}, \dots, a_n\} \end{aligned}$$

A literal is an atom or a negated atom

A program P is positive if $B(r)^- = \emptyset$ for all $r \in P$

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$$\mathtt{a}_0 \;\mathtt{:-}\; \mathtt{a}_1,\ldots,\mathtt{a}_m,\; \mathtt{not}\; \mathtt{a}_{m+1},\ldots,\; \mathtt{not}\; \mathtt{a}_n.$$

  where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

- Notation

$$\begin{aligned} h(r) &= a_0 \\ B(r) &= \{a_1,\ldots,a_m,{\sim}a_{m+1},\ldots,{\sim}a_n\} \\ B(r)^+ &= \{a_1,\ldots,a_m\} \\ B(r)^- &= \{a_{m+1},\ldots,a_n\} \end{aligned}$$

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h(r) &= a_0 \\
B(r) &= \{a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n\} \\
B(r)^+ &= \{a_1, \dots, a_m\} \\
B(r)^- &= \{a_{m+1}, \dots, a_n\} \\
A(P) &= \bigcup_{r \in P} \left(\{h(r)\} \cup B(r)^+ \cup B(r)^-\right) \\
B(P) &= \{B(r) \mid r \in P\} \\
h(P) &= \{h(r) \mid r \in P\}
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Examples

- Example rules
  - $a \leftarrow b, \sim c$
  - $a \leftarrow \sim c, b$
  - $a \leftarrow$
  - $a \leftarrow b$
  - $a \leftarrow \sim c$
  - $bachelor(joe) \leftarrow male(joe), \sim married(joe)$
- Example literals

  $a, b, c, bachelor(joe), male(joe), married(joe)$

  $\sim c, \sim married(joe)$

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Notational convention

We sometimes use the following notation interchangeably
in order to stress the respective view:

	true, false	if	and	or	iff	default negation	classical negation
source code		`:-`	`,`	`;`		`not`	`-`
logic program		$\leftarrow$	$,$	$;$		$\sim$	$\neg$
formula	$\bot, \top$	$\rightarrow$	$\wedge$	$\vee$	$\leftrightarrow$	$\sim$	$\neg$

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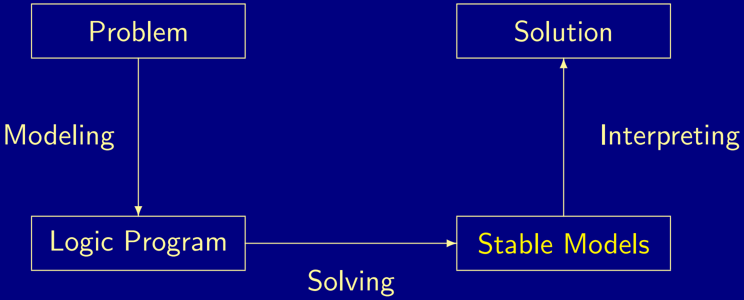
Semantics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Formal Definition

Stable models of positive programs

- A set of atoms X is closed under a positive program P iff for any $r \in P$, $h(r) \in X$ whenever $B(r)^+ \subseteq X$
  - X corresponds to a model of P (seen as a formula)
- The smallest set of atoms which is closed under a positive program P is denoted by $Cn(P)$
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of P (ditto)
- The set $Cn(P)$ of atoms is the stable model of a *positive* program P

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Some "logical" remarks

- Positive rules are also referred to as definite clauses
  - Definite clauses are disjunctions with exactly one positive atom:

    $$a_0 \vee \neg a_1 \vee \cdots \vee \neg a_m$$

  - A set of definite clauses has a (unique) smallest model
- Horn clauses are clauses with at most one positive atom
  - Every definite clause is a Horn clause but not vice versa
  - Non-definite Horn clauses can be regarded as integrity constraints
  - A set of Horn clauses has a smallest model or none
- This smallest model is the intended semantics of such sets of clauses
  - Given a positive program P, $Cn(P)$ corresponds to the smallest model of the set of definite clauses corresponding to P

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Basic idea

Consider the logical formula Φ and its three (classical) models:

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

Φ $\boxed{q \wedge (q \wedge \neg r \rightarrow p)}$

Formula Φ has one stable model, often called answer set:

$$\{p, q\}$$

P_Φ $\boxed{\begin{array}{lll} q & \leftarrow & \\ p & \leftarrow & q, \ \sim r \end{array}}$

Informally, a set X of atoms is a stable model of a logic program P

- if X is a (classical) model of P and
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- The reduct, P^X, of a program P relative to a set X of atoms is defined by

$$P^X = \{h(r) \leftarrow B(r)^+ \mid r \in P \text{ and } B(r)^- \cap X = \emptyset\}$$

- A set X of atoms is a stable model of a program P, if $Cn(P^X) = X$

- Remarks

  $Cn(P^X)$ is the $\subseteq$–smallest (classical) model of P^X

  Each atom in X is justified by an *"applying rule from P"*
  Set X is stable under *"applying rules from P"*

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A closer look at P^X

- Alternatively, given a set X of atoms from P,

  P^X is obtained from P by deleting

  1. each rule having $\sim a$ in its body with $a \in X$
     and then
  2. all negative atoms of the form $\sim a$
     in the bodies of the remaining rules

- Note Only negative body literals are evaluated

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Outline

Example one

$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p\}$	$p \leftarrow p$	$\emptyset$
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p, q\}$	$p \leftarrow p$	$\emptyset$

Example one

$P = \{p \leftarrow p, \;\; q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$	
$\{\quad\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✗
$\{p\quad\}$	$p \leftarrow p$	$\emptyset$	
$\{\quad q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✔
$\{p, q\}$	$p \leftarrow p$	$\emptyset$	

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$P = \{p \leftarrow p,\ q \leftarrow \neg p\}$

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Example two

$P = \{p \leftarrow {\sim}q,\ q \leftarrow {\sim}p\}$

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$$P = \{p \leftarrow {\sim}q,\ q \leftarrow {\sim}p\}$$

X	P^X	$Cn(P^X)$	
$\{\ \ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$	✘
$\{p\ \}$	$p \leftarrow$	$\{p\}$	✔
$\{\ q\}$	$q \leftarrow$	$\{q\}$	✔
$\{p, q\}$		$\emptyset$	✘

Example two

$$P = \{p \leftarrow {\sim}q,\ q \leftarrow {\sim}p\}$$

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$\{\quad\}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$	✘
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$\{\quad q\}$	$q \leftarrow$	$\{q\}$	✔
$\{p, q\}$		$\emptyset$	✘

Example two

$$P = \{p \leftarrow \neg q,\ q \leftarrow \neg p\}$$

X	P^X	$Cn(P^X)$	
$\{\ \ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$	✘
$\{p\ \}$	$p \leftarrow$	$\{p\}$	✔
$\{\ q\}$	$q \leftarrow$	$\{q\}$	✔
$\{p, q\}$		$\emptyset$	✔

Example three

$$P = \{p \leftarrow \sim p\}$$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow$	$\{p\}$
$\{p\}$		$\emptyset$

Example three

$P = \{p \leftarrow {\sim}p\}$

X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow$	$\{p\}$	✗
$\{p\}$		$\emptyset$	

Example three

$P = \{p \leftarrow {\sim}p\}$

X	P^X	$Cn(P^X)$
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$\{\ \}$	$p \leftarrow$	$\{p\}$	✗
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Example three

$P = \{p \leftarrow {\sim}p\}$

X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow$	$\{p\}$	✘
$\{p\}$		$\emptyset$	✘

Example three

$P = \{p \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow$	$\{p\}$	✘
$\{p\}$		$\emptyset$	✘

Example three

$P = \{p \leftarrow \neg p\}$

X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow$	$\{p\}$	✘
$\{p\}$		$\emptyset$	✔

Some properties

- A logic program may have zero, one, or multiple stable models
- If X is a stable model of a logic program P,
  then $X \subseteq h(P)$
- If X is a stable model of a logic program P,
  then X is a (classical) model of P
- If X and Y are stable models of a *normal* program P,
  then $X \not\subset Y$

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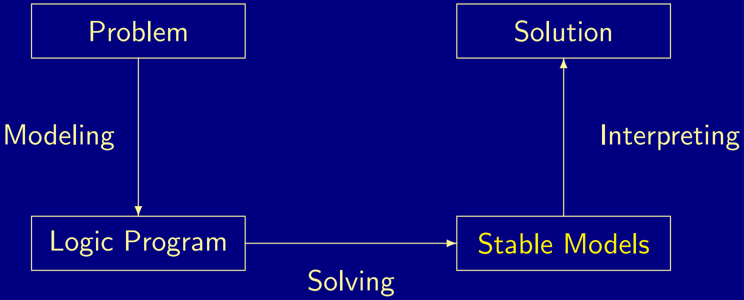
Exemplars

Logic program	Answer sets
`a.`	{a}
`a :- b.`	{}
`a :- b.    b.`	{a,b}
`a :- b.    b :- a.`	{}
`a :- not c.`	{a}
`a :- not c.    c.`	{c}
`a :- not c.    c :- not a.`	{a}, {c}
`a :- not a.`	

Outline

Reasoning modes



Reasoning modes

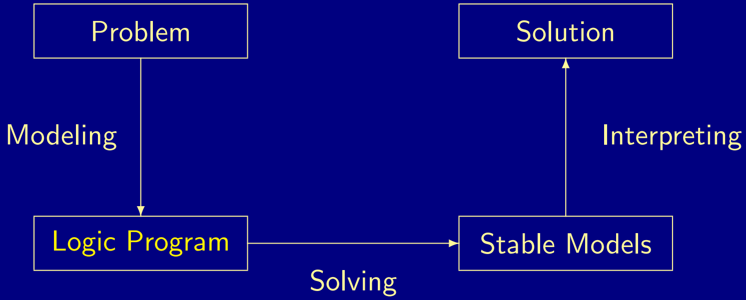
- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization
- and combinations of them

† without solution recording

‡ without solution enumeration

Outline

Extended syntax



Language constructs

- Variables `p(X) :- q(X)`
- Conditional literals `p :- q(X) : r(X)`
- Disjunction `p(X) ; q(X) :- r(X)`
- Integrity constraints `:- q(X), p(X)`
- Choice `2 { p(X,Y) : q(X) } 7 :- r(Y)`
- Aggregates `s(Y) :- r(Y), 2 #sum{ X : p(X,Y), q(X) } 7`
- Optimization

```
:~ q(X), p(X,C) [C]
#minimize { C : q(X), p(X,C) }
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  - Weak constraints `:~ q(X), p(X,C) [C]`
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- Choice `2 { p(X,Y) : q(X) } 7 :- r(Y)`
- Aggregates `s(Y) :- r(Y), 2 #sum{ X : p(X,Y), q(X) } 7`
- Multi-objective optimization
  - Weak constraints `:~ q(X), p(X,C) [C@42]`
  - Statements `#minimize { C@42 : q(X), p(X,C) }`

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Outline

Example

$$d(a)$$
$$d(c)$$
$$d(d)$$
$$p(a, b)$$
$$p(b, c)$$
$$p(c, d)$$
$$p(X, Z) \leftarrow p(X, Y), p(Y, Z)$$
$$q(a)$$
$$q(b)$$
$$q(X) \leftarrow \sim r(X), d(X)$$
$$r(X) \leftarrow \sim q(X), d(X)$$
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Example

$$
\begin{array}{l}
d(a) \\
d(c) \\
d(d) \\
p(a,b) \\
p(b,c) \\
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\end{array}
$$

Grounding instantiation

Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructible from $\mathcal{T}$
- A variable-free atom is also called ground
- Ground instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \rightarrow \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground instantiation of P: $ground(P) = \bigcup_{r \in P} ground(r)$

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Grounding instantiation

Let P be a logic program

- Let $\mathcal{T}$ be a set of variable-free terms (also called Herbrand universe)
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructible from $\mathcal{T}$ (also called alphabet or Herbrand base)
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An example

$$P = \{\ r(a,b) \leftarrow,\ \ r(b,c) \leftarrow,\ \ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a,b,c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

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Grounding aims at reducing the ground instantiation

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- Grounding aims at reducing the ground instantiation

Safety

- A normal rule is safe, if each of its variables also occurs in some positive body literal
- A normal program is safe, if all of its rules are safe

Example

$d(a)$
$d(c)$
$d(d)$

$p(a,b)$
$p(b,c)$
$p(c,d)$
$p(X,Z) \leftarrow p(X,Y), p(Y,Z)$

$q(a)$
$q(b)$
$q(X) \leftarrow {\sim}r(X)$

$r(X) \leftarrow {\sim}q(X), d(X)$
$s(X) \leftarrow {\sim}r(X), p(X,Y), q(Y)$

Example

Safe ?

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$d(c)$
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Example

	Safe ?
$d(a)$	✔
$d(c)$	✔
$d(d)$	✔
$p(a, b)$	✔
$p(b, c)$	✔
$p(c, d)$	✔
$p(X, Z) \leftarrow p(X, Y), p(Y, Z)$	
$q(a)$	✔
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$r(X) \leftarrow \sim q(X), d(X)$	
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Example

Rule	**Safe ?**
$d(a)$	✔
$d(c)$	✔
$d(d)$	✔
$p(a,b)$	✔
$p(b,c)$	✔
$p(c,d)$	✔
$p(X,Z) \leftarrow p(X,Y), p(Y,Z)$	
$q(a)$	✔
$q(b)$	✔
$q(X) \leftarrow \sim r(X)$	
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Example

	Safe ?
$d(a)$	✔
$d(c)$	✔
$d(d)$	✔
$p(a,b)$	✔
$p(b,c)$	✔
$p(c,d)$	✔
$p(X,Z) \leftarrow p(X,Y), p(Y,Z)$	✔
$q(a)$	✔
$q(b)$	✔
$q(X) \leftarrow \sim r(X)$	
$r(X) \leftarrow \sim q(X), d(X)$	
$s(X) \leftarrow \sim r(X), p(X,Y), q(Y)$	

Example

	Safe ?
$d(a)$	✔
$d(c)$	✔
$d(d)$	✔
$p(a,b)$	✔
$p(b,c)$	✔
$p(c,d)$	✔
$p(X,Z) \leftarrow p(X,Y), p(Y,Z)$	✔
$q(a)$	✔
$q(b)$	✔
$q(X) \leftarrow \sim r(X)$	✘
$r(X) \leftarrow \sim q(X), d(X)$	
$s(X) \leftarrow \sim r(X), p(X,Y), q(Y)$	

Example

	Safe ?
$d(a)$	✔
$d(c)$	✔
$d(d)$	✔
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$p(b,c)$	✔
$p(c,d)$	✔
$p(X,Z) \leftarrow p(X,Y), p(Y,Z)$	✔
$q(a)$	✔
$q(b)$	✔
$q(X) \leftarrow {\sim}r(X), d(X)$	✔
$r(X) \leftarrow {\sim}q(X), d(X)$	
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$q(a)$	✔
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$q(X) \leftarrow \sim r(X), d(X)$	✔
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Stable models of programs with Variables

Let P be a normal logic program with variables

- A set X of (ground) atoms is a stable model of P, if $Cn(ground(P)^X) = X$

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