Answer Set Solving in Practice

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Rough Roadmap

1. Introduction
2. Language
3. Modeling
4. Grounding
5. Foundations
6. Solving
7. Systems
8. Applications

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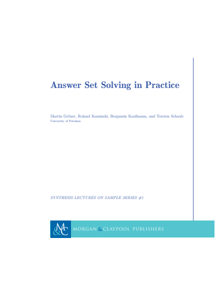
Resources

- Course material
  - http://www.cs.uni-potsdam.de/wv/lehre
  - http://moodle.cs.uni-potsdam.de
  - http://potassco.sourceforge.net/teaching.html
- Systems
  - clasp http://potassco.sourceforge.net
  - dlv http://www.dlvsystem.com
  - smodels http://www.tcs.hut.fi/Software/smodels
  - gringo http://potassco.sourceforge.net
  - lparse http://www.tcs.hut.fi/Software/smodels
  - clingo http://potassco.sourceforge.net
  - iclingo http://potassco.sourceforge.net
  - oclingo http://potassco.sourceforge.net
  - asparagus http://asparagus.cs.uni-potsdam.de

The Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



Resources

- http://potassco.sourceforge.net/book.html
- http://potassco.sourceforge.net/teaching.html

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Literature

Books [4], [29], [53]

Surveys [50], [2], [39], [21], [11]

Articles [41], [42], [6], [61], [54], [49], [40], etc.

Incremental Grounding and Solving: Overview

Outline

Motivation

- Many real-world applications, having exponential state spaces, like
  - bio-informatics,
  - planning,
  - model checking,
  - etc

  have associated PSPACE-decision problems

- Example
  - The plan existence problem of deterministic planning is PSPACE-complete
    the problem of whether there is a plan having a length bounded by a given polynomial is in NP

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- Iterative deepening search

  Consider one problem instance after another by gradually increasing the bound on the solution size

  - Problem This approach
    - is prone to redundancies (in grounding and solving), and
    - cannot harness modern look-back techniques in conflict-driven learning and heuristics

- Incremental approach

  - Idea Avoid redundancy by gradually processing the extensions to a problem rather than repeatedly re-processing the entire extended problem
  - ASP An incremental approach to both grounding and solving is needed

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Incremental program

- An incremental program is a triple (B, P, Q) of logic programs, among which P and Q contain a (single) parameter k ranging over the natural numbers
  - The base program B is meant to describe static knowledge, independent of parameter k
  - The role of P is to capture knowledge accumulating with increasing k
  - The rules in Q are specific for each value of k
- We sometimes denote P and Q by $P[k]$ and $Q[k]$
- Intuitively, we want to decide whether the program

$$R[k/i] = B \cup \bigcup_{1 \leq j \leq i} P[k/j] \cup Q[k/i]$$

has a stable model for some (minimum) integer $i \geq 1$

We write $R[i]$ rather than $R[k/i]$ whenever clear from the context

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Grounding and Solving, iteratively

- Input An incremental program $R[k] = (B, P[k], Q[k])$
- Output A non-empty set of stable models of $R[k/i]$

1. Ground $B \cup P[1] \cup Q[1]$ and Solve $B \cup P[1] \cup Q[1]$ ✗
2. Ground $B \cup P[1] \cup P[2] \cup Q[2]$ and Solve $B \cup P[1] \cup P[2] \cup Q[2]$ ✗
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i etc. until a stable model is obtained ✔

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- A module $\mathbb{P}$ is a triple (P, I, O) consisting of
  - a (ground) program P over $grd(\mathcal{A})$ and
  - sets $I, O \subseteq grd(\mathcal{A})$ such that
    - $I \cap O = \emptyset$,
    - $atom(P) \subseteq I \cup O$, and
    - $head(P) \subseteq O$
- The elements of I and O are called input and output atoms
  - denoted by $I(\mathbb{P})$ and $O(\mathbb{P})$
- Similarly, we refer to (ground) program P by $P(\mathbb{P})$

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Ground Instantiation

- The ground instantiation of a program P is defined as

$$grd(P) = \{r\theta \mid r \in P, \theta : var(r) \rightarrow \mathcal{T}, var(r\theta) = \emptyset\}$$

  where $\mathcal{T}$ is a set of variable-free terms
- Analogously, $grd(\mathcal{A}) = \{a \in \mathcal{A} \mid var(a) = \emptyset\}$ is the set of ground atoms

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Formal Setting

- For a program P over $grd(\mathcal{A})$ and a set $X \subseteq grd(\mathcal{A})$, define

$$P|_X = \{head(r) \leftarrow body(r)^+ \cup L \mid r \in P, \\ body(r)^+ \subseteq X,\ L = \{\sim c \mid c \in body(r)^- \cap X\}\}$$

- Note $P|_X$ projects the bodies of rules in P to the atoms of X

- For a program P over $\mathcal{A}$ and $I \subseteq grd(\mathcal{A})$, define $\mathbb{P}(I)$ as the module

$$(\, grd(P)|_Y, I, head(grd(P)|_X)\,)$$

where $X = I \cup head(grd(P))$ and $Y = I \cup head(grd(P)|_X)$

- For $\mathbb{P}(I) = (P', I, O)$, we have $O \subseteq grd(\mathcal{A})$ and $atom(P') \subseteq I \cup O$

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A Simple Example

- Consider

$$P[k] = \{p(k) \leftarrow p(Y), {\sim}p(2) \qquad p(k) \leftarrow p(2)\}$$

and note that $grd(P[1])$ is infinite !

- For $P[1]$ and $I = \{p(0)\}$, we get the module

$$(\, grd(P[1])|_{\{p(0),p(1)\}}\, ,\, \{p(0)\}\, ,\, \{p(1)\}\,)$$

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Modular Incremental Program

- Define the join, $\mathbb{P} \sqcup \mathbb{Q}$, of two modules $\mathbb{P}$ and $\mathbb{Q}$ as the module

$$(\; P(\mathbb{P}) \cup P(\mathbb{Q}) \, , \; I(\mathbb{P}) \cup (I(\mathbb{Q}) \setminus O(\mathbb{P})) \, , \; O(\mathbb{P}) \cup O(\mathbb{Q}) \;) \; ,$$

  provided that $(I(\mathbb{P}) \cup O(\mathbb{P})) \cap O(\mathbb{Q}) = \emptyset$

- Note
  - Recursion between two modules to be joined is disallowed
  - Recursion is allowed within each module

- An incremental program $(B, P[k], Q[k])$ is modular, if the modules

$$\mathbb{P}_i = \mathbb{P}_{i-1} \sqcup \mathbb{P}[i](O(\mathbb{P}_{i-1})) \quad \text{and} \quad \mathbb{Q}_i = \mathbb{P}_i \sqcup \mathbb{Q}[i](O(\mathbb{P}_i))$$

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A pragmatic approach

- An incremental program $(B, P[k], Q[k])$ is modular, if
  - atoms defined in B comprise dedicated predicates or 0 as argument,
  - atoms defined in $P[k]$ comprise k as argument, and
  - atoms defined in $Q[k]$ comprise dedicated predicates and k as argument

- The above conditions can be formalized as follows:
  - $atom(grd(B)) \cap \left(\bigcup_{1 \leq i} head(grd(P[i] \cup Q[i]))\right) = \emptyset$,
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Outline

Incremental ASP Solving (made very easy)

- Grounding For a program P over $\mathcal{A}$ and $I \subseteq grd(\mathcal{A})$, an incremental grounder is a partial function

  $$\textsc{Ground} : (P, I) \mapsto (P', O) \ ,$$

  where P' is a program over $grd(\mathcal{A})$ and $O \subseteq grd(\mathcal{A})$
- Solving For programs R, R' over $grd(\mathcal{A})$ and a set L of literals over $grd(\mathcal{A})$, an incremental solver is a pair of total functions

  $$\textsc{Add} : R \mapsto R' \quad \text{and} \quad \textsc{Solve} : L \mapsto \chi,$$

  where χ is a subset of the power set of $grd(\mathcal{A})$

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Making rules volatile

- For a program Q over $grd(\mathcal{A})$ and a new atom $\alpha \notin grd(\mathcal{A})$, define

$$Q(\alpha) = \{head(r) \leftarrow body(r) \cup \{\alpha\} \mid r \in Q\}$$

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Algorithm 1: iSolve

Input : An incremental program $(B, P[k], Q[k])$
Output : A nonempty set of stable models
Internal : A grounder Grounder
Internal : A solver Solver

$i \leftarrow 0$
$(P_0, O) \leftarrow$ Grounder.$\textsc{Ground}(B, \emptyset)$
Solver.$\textsc{Add}(P_0)$
loop
  $i \leftarrow i + 1$
  $(P_i, O_i) \leftarrow$ Grounder.$\textsc{Ground}(P[i], O)$
  Solver.$\textsc{Add}(P_i)$
  $O \leftarrow O \cup O_i$
  $(Q_i, O_i') \leftarrow$ Grounder.$\textsc{Ground}(Q[i], O)$
  Solver.$\textsc{Add}(Q_i(\alpha_i) \cup \{\{\alpha_i\} \leftarrow\} \cup \{\leftarrow \alpha_{i-1}\})$
  $\chi \leftarrow$ Solver.$\textsc{Solve}(\{\alpha_i\})$
  if $\chi \neq \emptyset$ **then return** $\{X \setminus \{\alpha_i\} \mid X \in \chi\}$

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