Answer Set Solving in Practice

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Motivation: Overview

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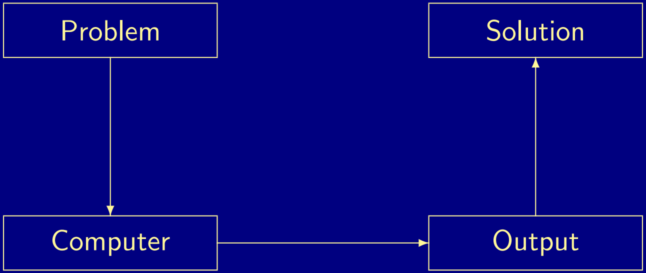
Outline

Informatics

"What is the problem?" versus *"How to solve the problem?"*



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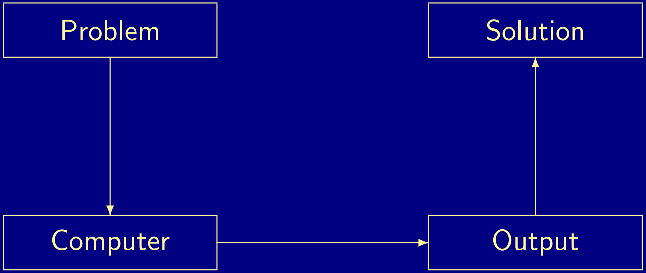

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Traditional programming

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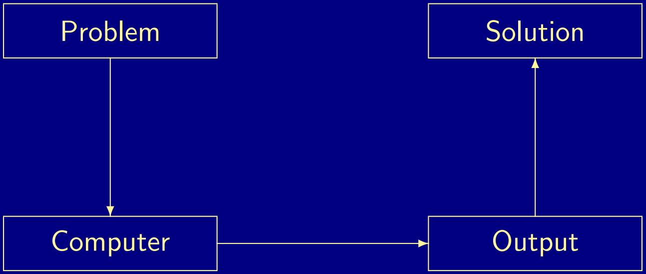

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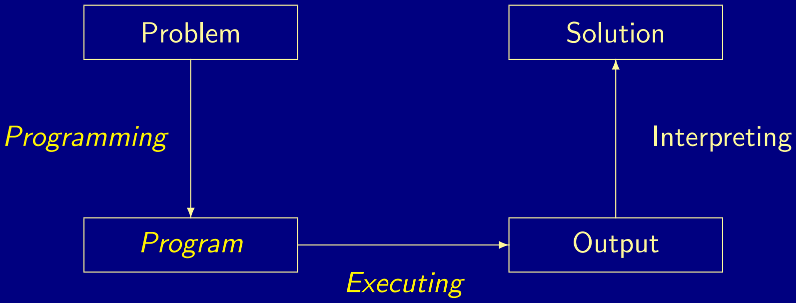

Declarative problem solving

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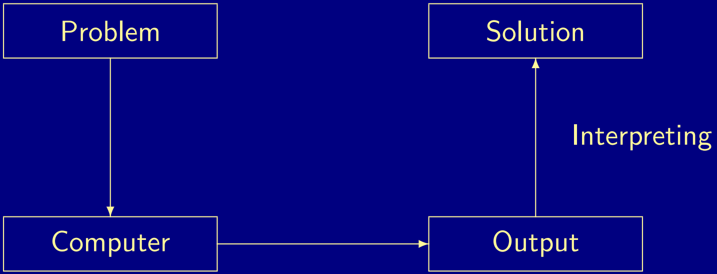

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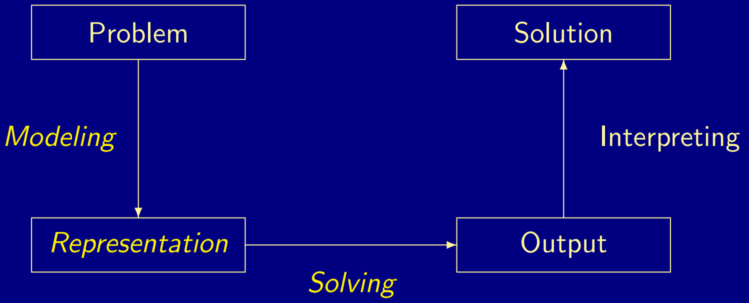

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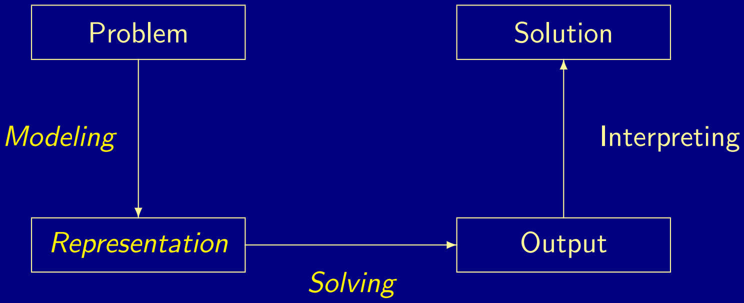

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Answer Set Programming

in a Nutshell

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
- ASP has its roots in
  - (deductive) databases
  - logic programming (with negation)
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - constraint solving (in particular, SATisfiability testing)
- ASP allows for solving all search problems in NP (and NP^{NP}) in a uniform way
- ASP is versatile as reflected by the ASP solver *clasp*, winning first places at ASP, CASC, MISC, PB, and SAT competitions
- ASP embraces many emerging application areas

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  tailored to Knowledge Representation and Reasoning

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$$\textbf{ASP} = \textbf{DB}+\textbf{LP}+\textbf{KR}+\textbf{SAT}$$

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Some biased moments in time

- '70/'80 Capturing incomplete information

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- '70/'80 Capturing incomplete information
  - Databases Closed world assumption
  - Logic programming Negation as failure
  - Non-monotonic reasoning
    Auto-epistemic and Default logics, Circumscription

Some biased moments in time

- '70/'80 Capturing incomplete information
  - Databases Closed world assumption
    - Axiomatic characterization
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Some biased moments in time

- '70/'80 Capturing incomplete information
  - Databases Closed world assumption
    - Axiomatic characterization
  - Logic programming Negation as failure
    - Herbrand interpretations
    - Fix-point characterizations
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    - Axiomatic characterization
  - Logic programming Negation as failure
    - Herbrand interpretations
    - Fix-point characterizations
  - Non-monotonic reasoning
    Auto-epistemic and Default logics, Circumscription
    - Extensions of first-order logic
    - Modalities, fix-points, second-order logic

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  - Logic programming semantics
    Well-founded and stable models semantics
  - ASP solving
    "Stable models = Well-founded semantics + Branch"

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    "Stable models = Well-founded semantics + Branch"
    - Modeling — Grounding — Solving
    - Icebreakers: `lparse` and `smodels`

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- '00 Applications and semantic rediscoveries

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- '00 Applications and semantic rediscoveries
  - Growing dissemination Decision Support for Space Shuttle
  - Constructive logics Equilibrium Logic

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    "Stable models = Well-founded semantics + Branch"
- '00 Applications and semantic rediscoveries
  - Growing dissemination Decision Support for Space Shuttle
    - Bio-informatics, Linux Package Configuration, Music composition, Robotics, System Design, etc
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    - Roots: Logic of Here-and-There , G3

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Paradigm shift

Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem
2. A solution is given by a derivation of a query

Model Generation based approach (eg. SATisfiability testing)

1. Provide a representation of the problem
2. A solution is given by a model of the representation

Automated planning, Kautz and Selman (ECAI'92)

Represent planning problems as propositional theories so that models not proofs describe solutions

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Representation	Solution
constraint satisfaction problem	assignment
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propositional programs	minimal models
propositional programs	supported models
propositional programs	stable models
first-order theories	models
first-order theories	minimal models
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first-order theories	Herbrand models
auto-epistemic theories	expansions
default theories	extensions
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LP-style playing with blocks

Prolog program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Prolog queries

```
?- above(a,c).
true.

?- above(c,a).
no.
```

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Prolog queries (testing entailment)

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?- above(a,c).
true.

?- above(c,a).
no.
```

LP-style playing with blocks

Shuffled Prolog program

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?- above(a,c).

Fatal Error: local stack overflow.
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Prolog queries (answered via fixed execution)

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Formula

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\begin{array}{ll}
 & on(a,b) \\
\wedge & on(b,c) \\
\wedge & (on(X,Y) \rightarrow above(X,Y)) \\
\wedge & (on(X,Z) \wedge above(Z,Y) \rightarrow above(X,Y))
\end{array}
$$

Herbrand model

$$
\left\{
\begin{array}{rrrrr}
on(a,b), & on(b,c), & on(a,c), & on(b,b), & \\
above(a,b), & above(b,c), & above(a,c), & above(b,b), & above(c,b)
\end{array}
\right\}
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Herbrand model (among 426!)

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➥ Answer Set Programming (ASP)

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Answer Set Programming *at large*

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Answer Set Programming *commonly*

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Answer Set Programming *in practice*

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ASP-style playing with blocks

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on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Stable Herbrand model

$$\{ \text{on}(a,b), \text{on}(b,c), \text{above}(b,c), \text{above}(a,b), \text{above}(a,c) \}$$

Potassco

ASP-style playing with blocks

Logic program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Stable Herbrand model

$\{ \mathtt{on}(\mathtt{a},\mathtt{b}),\ \mathtt{on}(\mathtt{b},\mathtt{c}),\ \mathtt{above}(\mathtt{b},\mathtt{c}),\ \mathtt{above}(\mathtt{a},\mathtt{b}),\ \mathtt{above}(\mathtt{a},\mathtt{c}) \}$

ASP-style playing with blocks

Logic program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Stable Herbrand model (and no others)

$\{ \texttt{on}(\texttt{a},\texttt{b}),\ \texttt{on}(\texttt{b},\texttt{c}),\ \texttt{above}(\texttt{b},\texttt{c}),\ \texttt{above}(\texttt{a},\texttt{b}),\ \texttt{above}(\texttt{a},\texttt{c}) \}$

ASP-style playing with blocks

Logic program

```
on(a,b).
on(b,c).

above(X,Y) :- above(Z,Y), on(X,Z).
above(X,Y) :- on(X,Y).
```

Stable Herbrand model (and no others)

$\{\ \mathtt{on}(\mathtt{a},\mathtt{b}),\ \mathtt{on}(\mathtt{b},\mathtt{c}),\ \mathtt{above}(\mathtt{b},\mathtt{c}),\ \mathtt{above}(\mathtt{a},\mathtt{b}),\ \mathtt{above}(\mathtt{a},\mathtt{c})\ \}$

ASP versus LP

ASP	Prolog
Model generation	Query orientation
Bottom-up	Top-down
Modeling language	Programming language
Rule-based format	
Instantiation Flat terms	Unification Nested terms
(Turing $+$) $NP(^{NP})$	Turing

ASP versus SAT

ASP	SAT
Model generation	
Bottom-up	
Constructive Logic	Classical Logic
Closed (and open) world reasoning	Open world reasoning
Modeling language	—
Complex reasoning modes	Satisfiability testing
Satisfiability	Satisfiability
Enumeration/Projection	—
Intersection/Union	—
Optimization	—
(Turing +) $NP(^{NP})$	NP

Outline

Propositional Normal Logic Programs

- A logic program P is a set of rules of the form

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \dots, b_m, \neg c_1, \dots, \neg c_n}_{\text{body}}$$

  - a and all b_i, c_j are atoms (propositional variables)
  - $\leftarrow$, $,$, $\neg$ denote if, and, and negation
  - intuitive reading: head must be true if body holds

- Semantics given by stable models, informally,
  models of P justifying each true atom by some rule in P

Logic Programs

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Normal Logic Programs

- A logic program P is a set of rules of the form

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \dots, b_m, \neg c_1, \dots, \neg c_n}_{\text{body}}$$

  - a and all b_i, c_j are atoms (propositional variables)
  - $\leftarrow$, $,$, $\neg$ denote if, and, and negation
  - intuitive reading: head must be true if body holds

- Semantics given by stable models, informally,
  models of P justifying each true atom by some rule in P

- Disclaimer The following formalities apply to normal logic programs

Potassco

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	
F	**F**	**T**	
F	**T**	**F**	
F	**T**	**T**	
T	**F**	**F**	
T	**F**	**T**	
T	**T**	**F**	
T	**T**	**T**	

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\neg \mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$(\neg \mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\neg \mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
F	**T**	**T**	$(\neg \mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	**F**	**F**	$(\neg \mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
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T	**T**	**F**	$(\neg \mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
T	**T**	**T**	$(\neg \mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
F	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
T	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
T	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
T	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

Potassco

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge \mathbf{F}$
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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge \mathbf{F}$
F	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \;\wedge\; (b \rightarrow c)$
F	**F**	**F**	**F** $\wedge$ **T**
F	**F**	**T**	**F** $\wedge$ **T**
F	**T**	**F**	**T** $\wedge$ **F**
F	**T**	**T**	**T** $\wedge$ **T**
T	**F**	**F**	**T** $\wedge$ **T**
T	**F**	**T**	**T** $\wedge$ **T**
T	**T**	**F**	**T** $\wedge$ **F**
T	**T**	**T**	**T** $\wedge$ **T**

Some truth tabling, back to SAT

a	b	c	$(\neg b \to a) \land (b \to c)$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

- We get four models: $\{b, c\}$, $\{a\}$, $\{a, c\}$, and $\{a, b, c\}$

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	
F	**F**	**T**	
F	**T**	**F**	
F	**T**	**T**	
T	**F**	**F**	
T	**F**	**T**	
T	**T**	**F**	
T	**T**	**T**	

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\neg \mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**T**	$(\neg \mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
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T	**T**	**F**	$(\neg \mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**T**	$(\neg \mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
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T	**F**	**T**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c)$
F	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
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F	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
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T	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
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F	**T**	**T**	$(b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
T	**T**	**F**	$(b \rightarrow c)$
T	**T**	**T**	$(b \rightarrow c)$

Reduct

Potassco

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c)$
F	**T**	**F**	$(b \rightarrow c)$
F	**T**	**T**	$(b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
T	**T**	**F**	$(b \rightarrow c)$
T	**T**	**T**	$(b \rightarrow c)$

Reduct

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	**F**	**F**	$a \wedge (b \rightarrow c)$	
F	**F**	**T**	$a \wedge (b \rightarrow c)$	
F	**T**	**F**	$(b \rightarrow c)$	
F	**T**	**T**	$(b \rightarrow c)$	$\models$
T	**F**	**F**	$a \wedge (b \rightarrow c)$	$\models a$
T	**F**	**T**	$a \wedge (b \rightarrow c)$	$\models a$
T	**T**	**F**	$(b \rightarrow c)$	
T	**T**	**T**	$(b \rightarrow c)$	$\models$

Reduct

Potassco

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	F	F	$a \wedge (b \rightarrow c)$	
F	F	T	$a \wedge (b \rightarrow c)$	
F	T	F	$(b \rightarrow c)$	
F	T	T	$(b \rightarrow c)$	$\models$
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$
T	F	T	$a \wedge (b \rightarrow c)$	$\models a$
T	T	F	$(b \rightarrow c)$	
T	T	T	$(b \rightarrow c)$	$\models$

Reduct

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	F	F	$a \wedge (b \rightarrow c)$	$\models a$
F	F	T	$a \wedge (b \rightarrow c)$	$\models a$
F	T	F	$(b \rightarrow c)$	$\models$
F	T	T	$(b \rightarrow c)$	$\models$
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$
T	F	T	$a \wedge (b \rightarrow c)$	$\models a$
T	T	F	$(b \rightarrow c)$	$\models$
T	T	T	$(b \rightarrow c)$	$\models$

Reduct

Potassco

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$		
F	**F**	**F**	$a \wedge (b \rightarrow c)$		
F	**F**	**T**	$a \wedge (b \rightarrow c)$		
F	**T**	**F**	$(b \rightarrow c)$		
F	**T**	**T**	$(b \rightarrow c)$		
T	**F**	**F**	$a \wedge (b \rightarrow c)$	$\models a$	Stable model
T	**F**	**T**	$a \wedge (b \rightarrow c)$		
T	**T**	**F**	$(b \rightarrow c)$		
T	**T**	**T**	$(b \rightarrow c)$		
			Reduct		

- We get one stable model: $\{a\}$

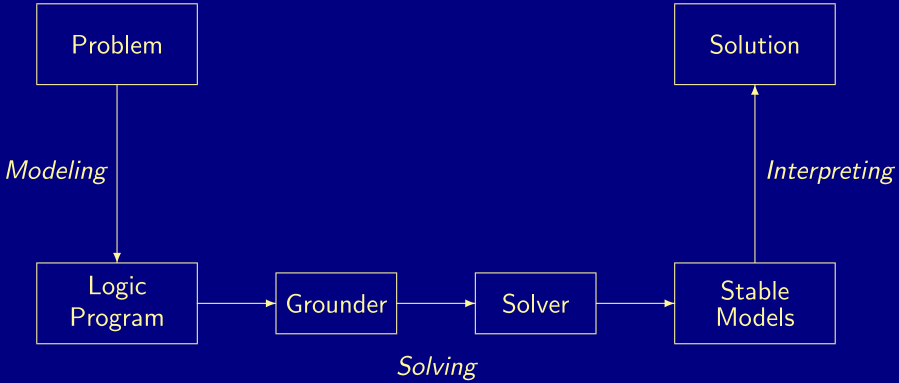
Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$		
F	**F**	**F**	$a \wedge (b \rightarrow c)$		
F	**F**	**T**	$a \wedge (b \rightarrow c)$		
F	**T**	**F**	$(b \rightarrow c)$		
F	**T**	**T**	$(b \rightarrow c)$		
T	**F**	**F**	$a \wedge (b \rightarrow c)$	$\models a$	Stable model
T	**F**	**T**	$a \wedge (b \rightarrow c)$		
T	**T**	**F**	$(b \rightarrow c)$		
T	**T**	**T**	$(b \rightarrow c)$		
			Reduct		

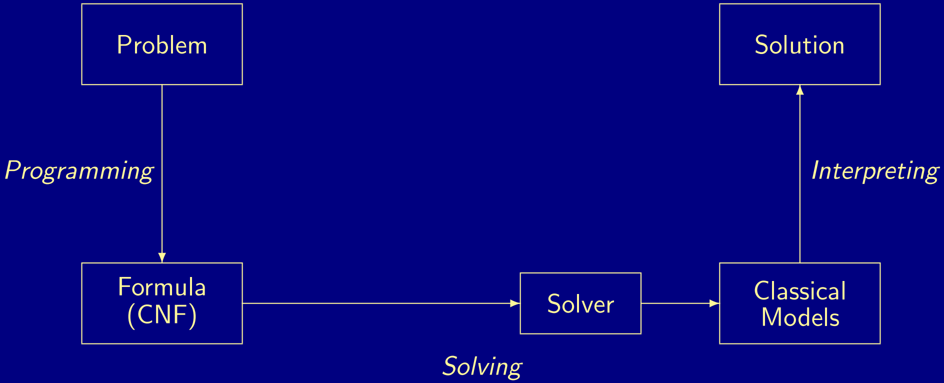
- We get one stable model: $\{a\}$
- Stable models $=$ Smallest models of (respective) reducts

Outline

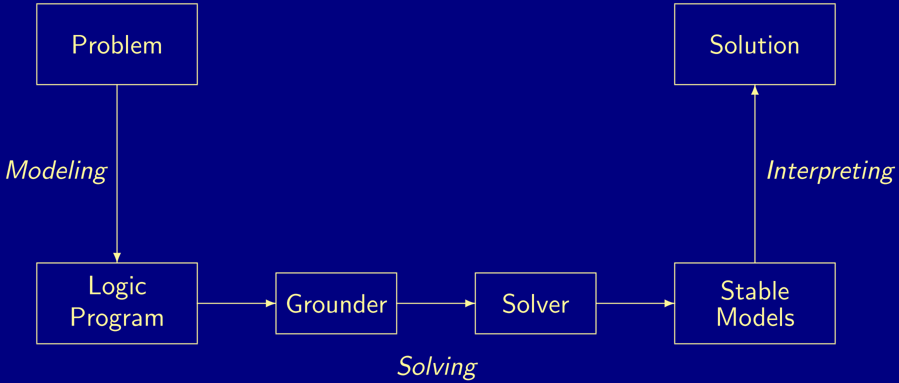
ASP modeling, grounding, and solving



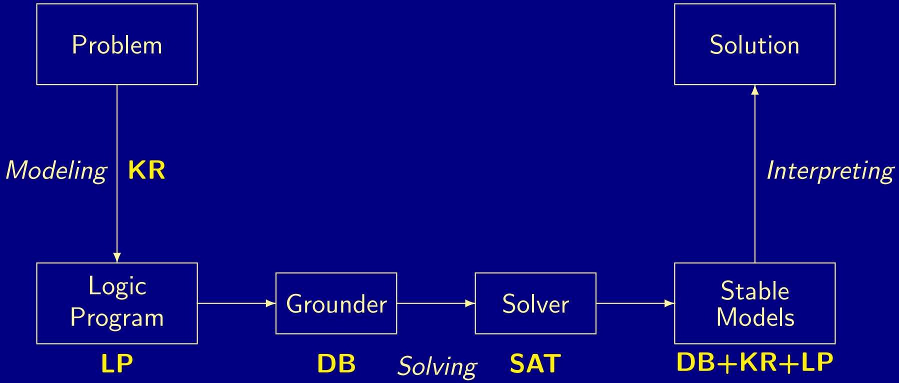
SAT solving


Problem
Programming
Formula (CNF)
Solver
Solving
Classical Models
Interpreting
Solution

Rooting ASP solving

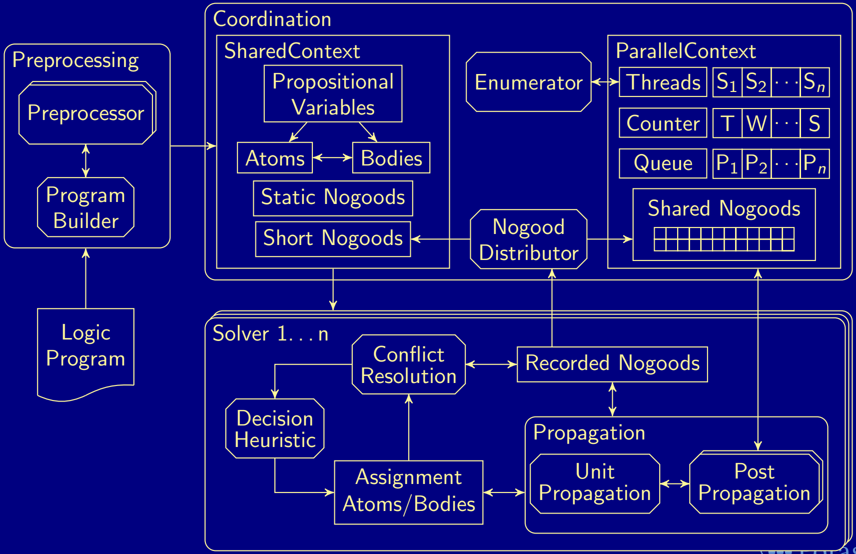


Rooting ASP solving

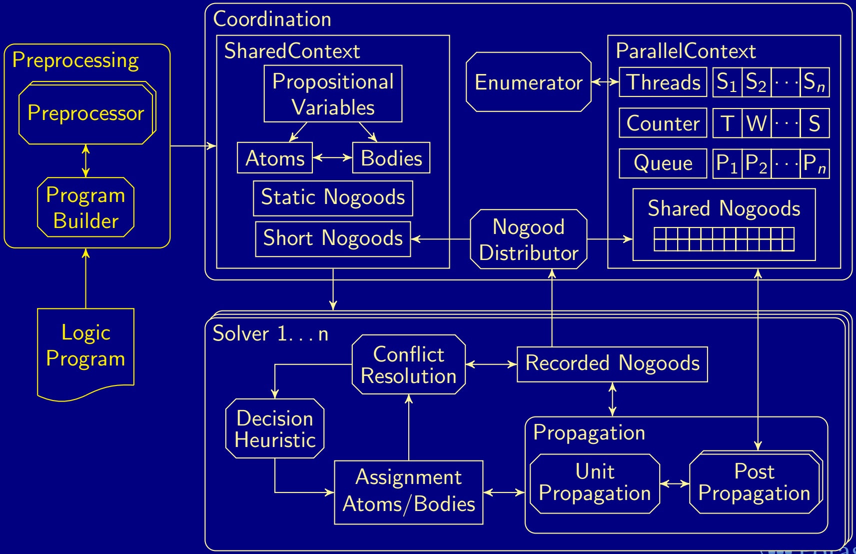


Outline

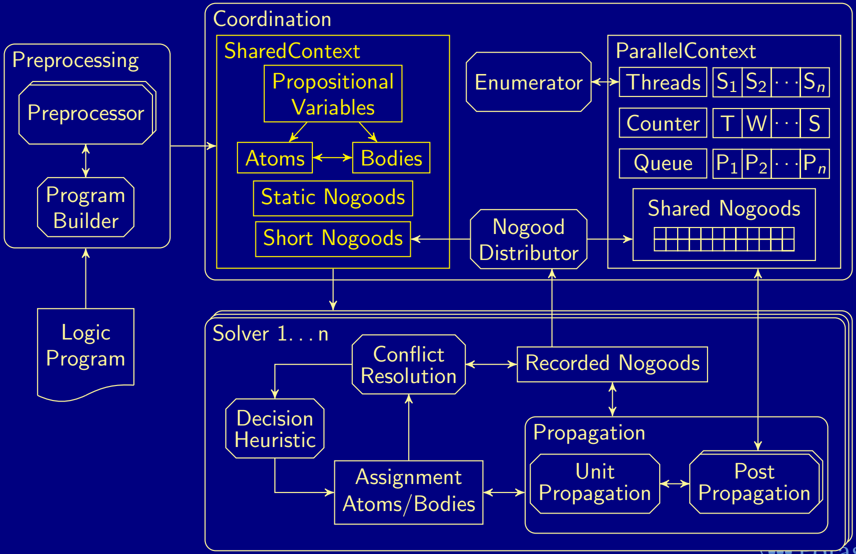
Multi-threaded architecture of *clasp*



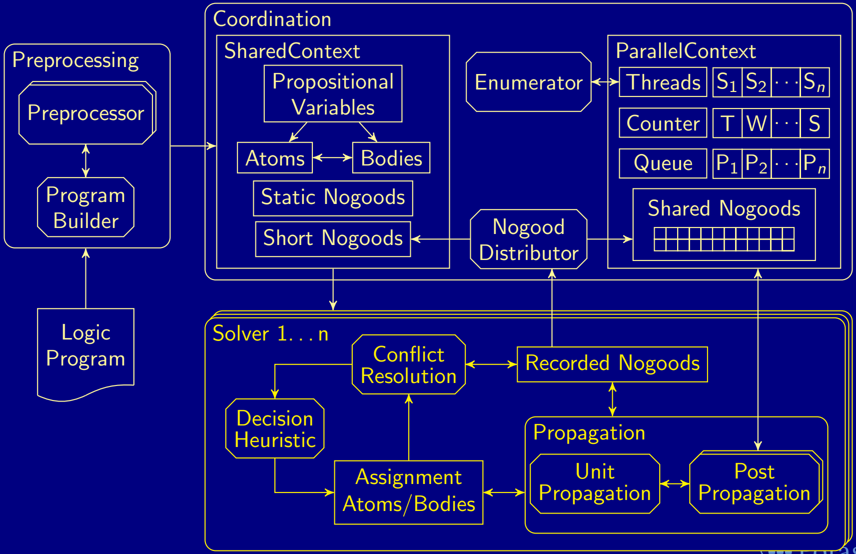
Multi-threaded architecture of *clasp*



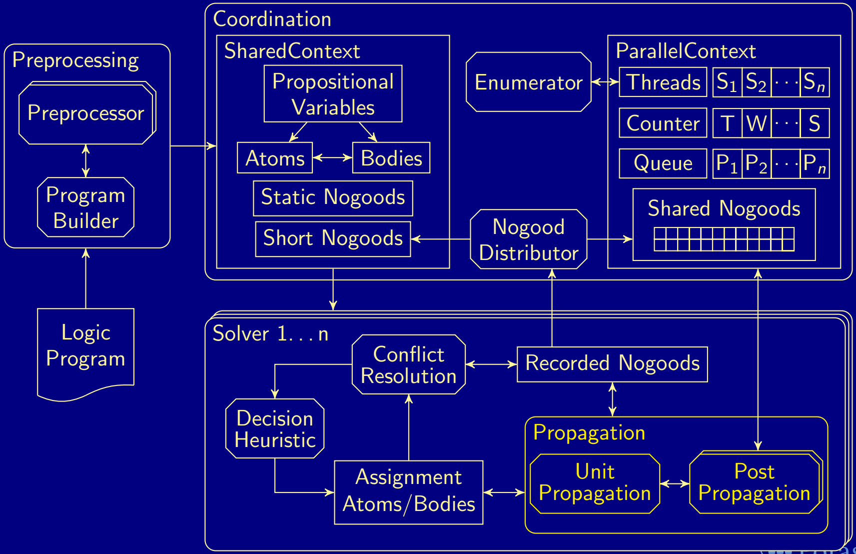
Multi-threaded architecture of *clasp*



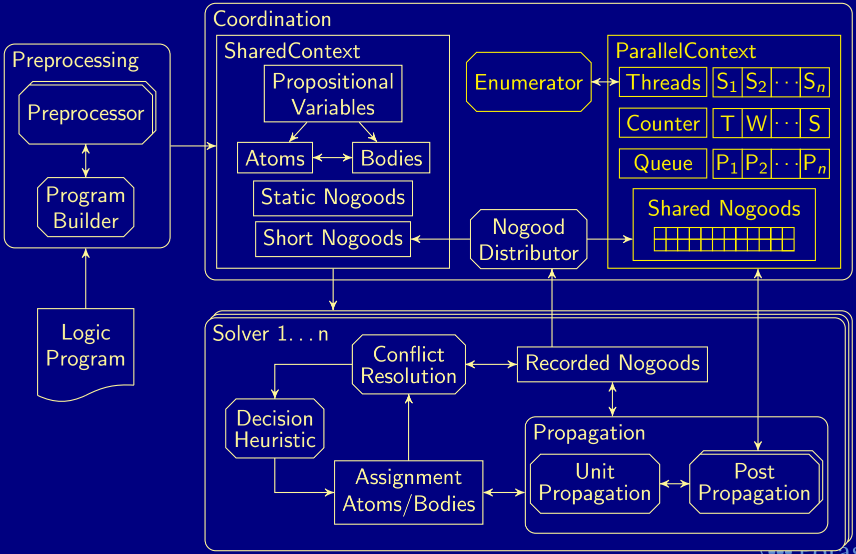
Multi-threaded architecture of *clasp*



Multi-threaded architecture of *clasp*



Multi-threaded architecture of *clasp*



Outline

Two sides of a coin

- ASP as High-level Language
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules
- ASP as Low-level Language
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation
- ASP and Imperative language
  - Control continuously changing logic programs

Two and a half sides of a coin

- ASP as High-level Language
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules
- ASP as Low-level Language
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation
- ASP and Imperative language
  - Control continuously changing logic programs

What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated planning
  - Code optimization
  - Database integration
  - Decision support for NASA shuttle controllers
  - Model checking
  - Music composition
  - Product configuration
  - Robotics
  - System design
  - Systems biology
  - Team-building
  - Timetabling
  - and many many more

What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated planning
  - Code optimization
  - Database integration
  - Decision support for NASA shuttle controllers
  - Model checking
  - Music composition
  - Product configuration
  - Robotics
  - System design
  - Systems biology
  - Team-building
  - Timetabling
  - and many many more

What does ASP offer?

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  - Rapid application development tool
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