# Answer Set Solving in Practice

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Potassco

# ASP modulo theories: Overview

# Motivation

- Input ASP = DB+KRR+LP+SAT
- Output ASPmT = DB+KRR+LP+S

- ASP solving *ground* | *solve*

  ➥ **logic programs with elusive theory atoms**

- Application areas

  Agents, Assisted Living, Robotics, Planning, Scheduling, Bio- and Cheminformatics, etc

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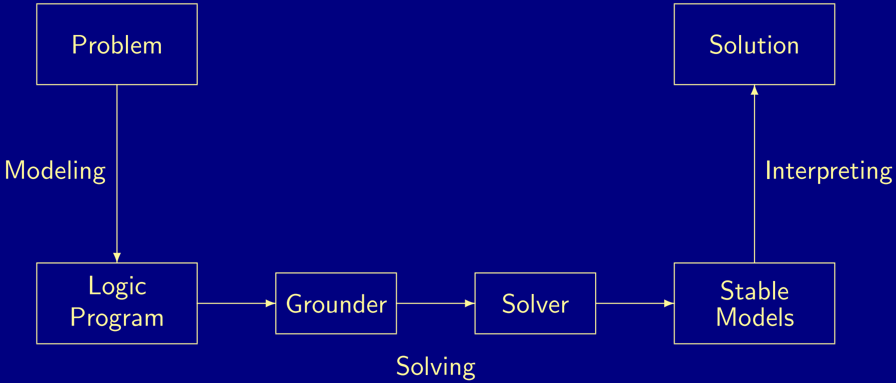
# ASP solving process



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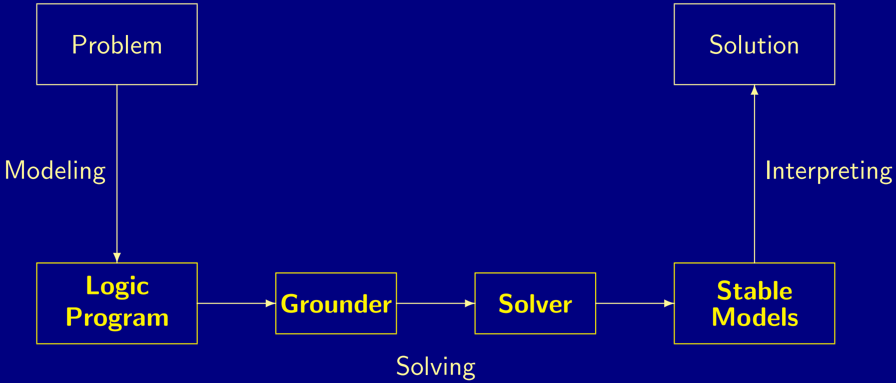

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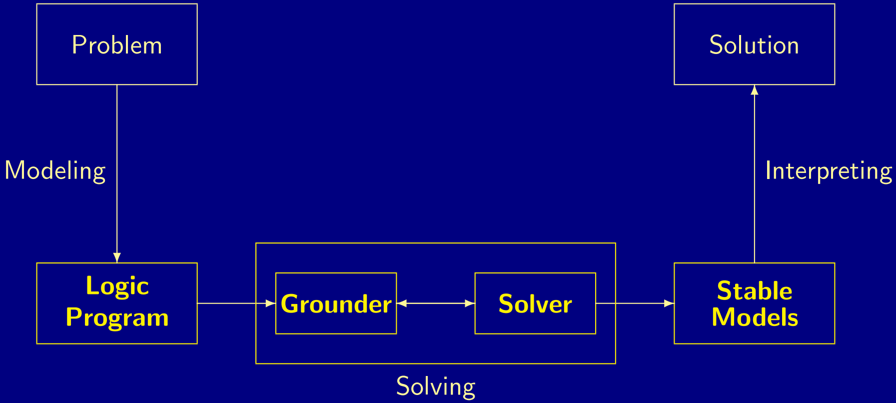
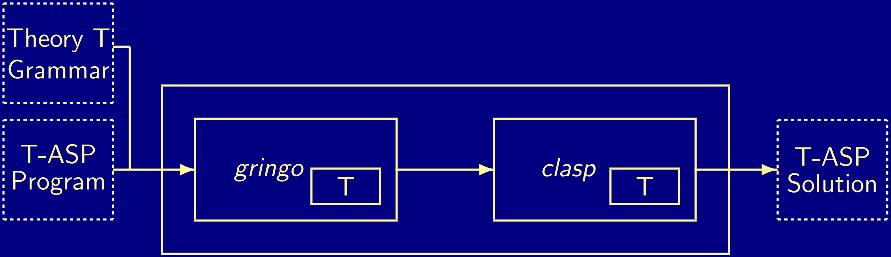
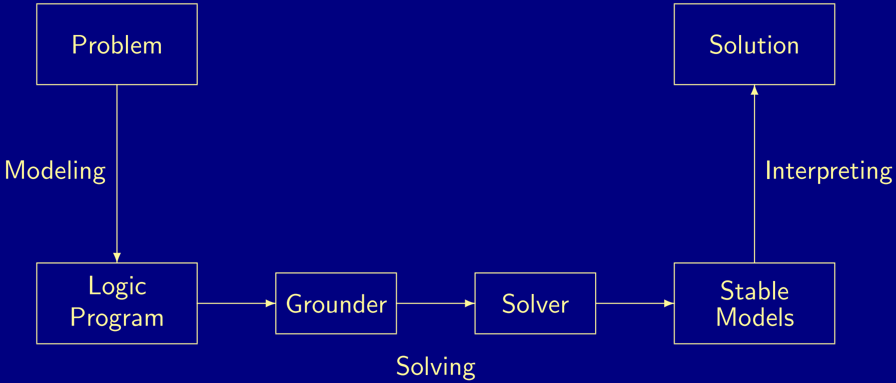


# *clingo*'s approach



# Outline

# ASP solving process modulo theories



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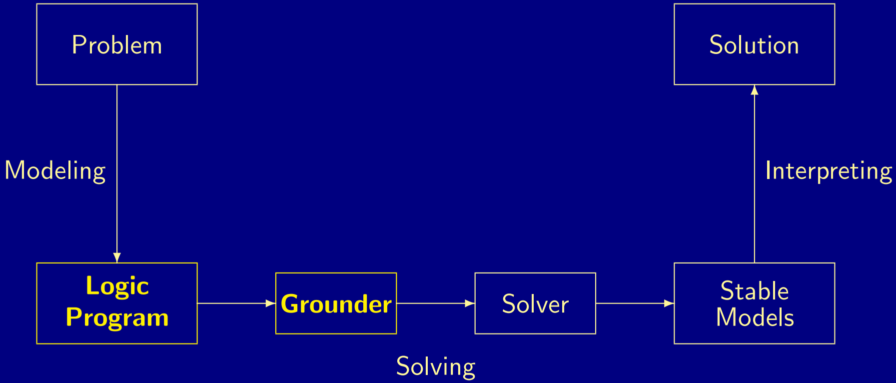

# Linear constraints

```
#theory csp {
    linear_term {
      + : 5, unary;
      - : 5, unary;
      * : 4, binary, left;
      + : 3, binary, left;
      - : 3, binary, left
    };

    dom_term {
      + : 5, unary;
      - : 5, unary;
      .. : 1, binary, left
    };

    show_term {
      / : 1, binary, left
    };

    minimize_term {
      + : 5, unary;
      - : 5, unary;
      * : 4, binary, left;
      + : 3, binary, left;
      - : 3, binary, left;
      @ : 0, binary, left
    };

    &dom/0 : dom_term, {=}, linear_term, any;
    &sum/0 : linear_term, {<=,=,>=,<,>,!=}, linear_term, any;
    &show/0 : show_term, directive;
    &distinct/0 : linear_term, any;
    &minimize/0 : minimize_term, directive
}.
```

## send+more=money

$$\begin{array}{ccccc} & s & e & n & d \\ + & m & o & r & e \\ \hline m & o & n & e & y \end{array}$$

Each letter corresponds exactly to one digit and all variables have to be pairwise distinct

$$\begin{array}{ccccc} & 9 & 5 & 6 & 7 \\ + & 1 & 0 & 8 & 5 \\ \hline 1 & 0 & 6 & 5 & 2 \end{array}$$

The example has exactly one solution

$$\{ s \mapsto 9, e \mapsto 5, n \mapsto 6, d \mapsto 7, m \mapsto 1, o \mapsto 0, r \mapsto 8, y \mapsto 2 \}$$

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| | s | e | n | d |
|---|---|---|---|---|
| + | m | o | r | e |
| m | o | n | e | y |

Each letter corresponds exactly to one digit and all variables have to be pairwise distinct

| | 9 | 5 | 6 | 7 |
|---|---|---|---|---|
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$$\{ s \mapsto 9, e \mapsto 5, n \mapsto 6, d \mapsto 7, m \mapsto 1, o \mapsto 0, r \mapsto 8, y \mapsto 2 \}$$

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```
#include "csp.lp".

digit(1,3,s). digit(2,3,m). digit(sum,4,m).
digit(1,2,e). digit(2,2,o). digit(sum,3,o).
digit(1,1,n). digit(2,1,r). digit(sum,2,n).
digit(1,0,d). digit(2,0,e). digit(sum,1,e).
                            digit(sum,0,y).
base(10).
exp(E) :- digit(_,E,_).

power(1,0).
power(B*P,E) :- base(B), power(P,E-1), exp(E), E>0.

number(N) :- digit(N,_,_), N!= sum.
high(D) :- digit(N,E,D), not digit(N,E+1,_).

&dom {0..9} = X :- digit(_,_,X).

&sum { M*D : digit(N,E,D), power(M,E), number(N);
      -M*D : digit(sum,E,D), power(M,E)        } = 0.


&sum { D } > 0 :- high(D).

&distinct { D : digit(_,_,D) }.

&show { D : digit(_,_,D) }.
```

# send+more=money

```
#include "csp.lp".

digit(1,3,s). digit(2,3,m). digit(sum,4,m).
digit(1,2,e). digit(2,2,o). digit(sum,3,o).
digit(1,1,n). digit(2,1,r). digit(sum,2,n).
digit(1,0,d). digit(2,0,e). digit(sum,1,e).
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```
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```
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digit(1,1,n). digit(2,1,r). digit(sum,2,n).
digit(1,0,d). digit(2,0,e). digit(sum,1,e).
                            digit(sum,0,y).
base(10).
exp(0). exp(1). exp(2). exp(3). exp(4).

power(1,0).
power(10,1). power(100,2). power(1000,3). power(10000,4).

number(1). number(2).
high(s). high(m).

&dom{0..9}=s. &dom{0..9}=m. &dom{0..9}=e. &dom{0..9}=o. &dom{0..9}=n. &dom{0..9}=r. &dom{0..9}=d. &dom{0..9}=

&sum{  1000*s;   100*e;   10*n;   1*d;
       1000*m;   100*o;   10*r;   1*e;
    -10000*m; -1000*o; -100*n; -10*e; -1*y } = 0.

&sum{s} > 0. &sum{m} > 0.

&distinct{s; m; e; o; n; r; d; y}.

&show{s; m; e; o; n; r; d; y}.
```
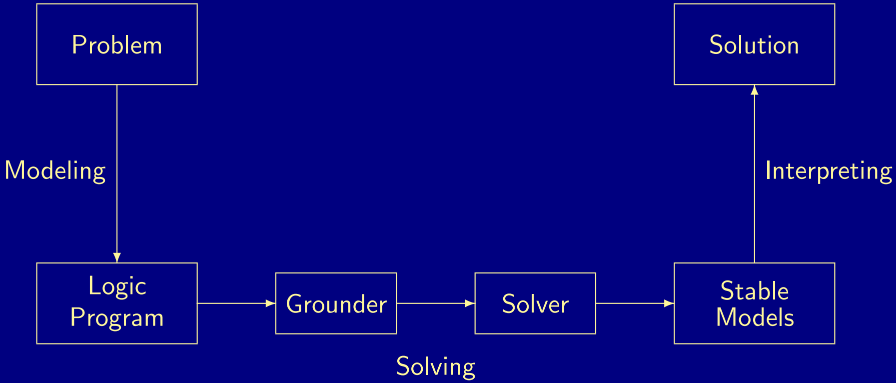

# Outline

# ASP solving process modulo theories



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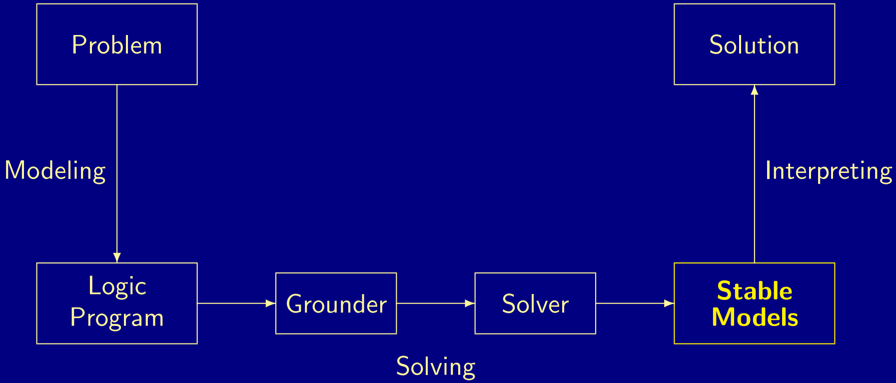

# ASP modulo theories

- We distinguish theory atoms depending upon whether they are
  - defined via rules in the logic program, or
  - external otherwise, or
  - strict being equivalent to the associated constraint, or
  - non-strict only implying the associated constraint.
- Informally, a set $X \subseteq \mathcal{A} \cup \mathcal{T}$ of atoms is a $\mathbb{T}$-stable model of a program $P$ if there is some $\mathbb{T}$-solution $\mathcal{S}$ such that $X$ is a (regular) stable model of the program

$$
\begin{aligned}
P \cup \{a \leftarrow \mid a \in (\mathcal{T}_e \setminus \mathit{head}(P)) \cap \mathcal{S}\}\\
\cup \{\leftarrow \sim a \mid a \in (\mathcal{T}_e \cap \mathit{head}(P)) \cap \mathcal{S}\}\\
\cup \{\{a\} \leftarrow \mid a \in (\mathcal{T}_i \setminus \mathit{head}(P)) \cap \mathcal{S}\}\\
\cup \{\leftarrow a \mid a \in (\mathcal{T} \cap \mathit{head}(P)) \setminus \mathcal{S}\}
\end{aligned}
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- We distinguish theory atoms depending upon whether they are
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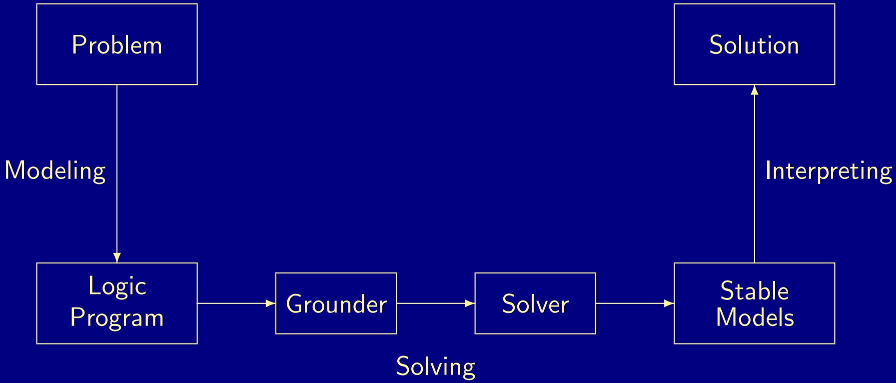
# ASP solving process modulo theories



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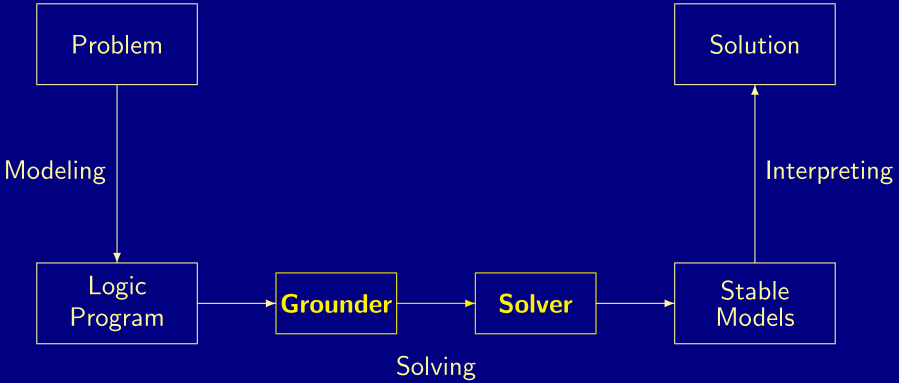

# *aspif* example

```
{a}.
b :- a.
c :- not a.
```

```
asp 1 0 0
1 1 1 1 0 0
1 0 1 2 0 1 1
1 0 1 3 0 1 -1
4 1 a 1 1
4 1 b 1 2
4 1 c 1 3
0
```

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# *aspif* overview

- Rule statements
- Minimize statements
- Projection statements
- Output statements
- External statements
- Assumption statements
- Heuristic statements
- Edge statements
- Theory terms and atoms
- Comments

# *aspif* theory example

```
task(1).
task(2).

duration(1,200).
duration(2,400).

&dom {1..1000} = beg(1).
&dom {1..1000} = end(1).
&dom {1..1000} = beg(2).
&dom {1..1000} = end(2).

&diff{end(1)-beg(1)}<=200.
&diff{end(2)-beg(2)}<=400.

&show{ beg/1; end/1 }.
```

```
asp 1 0 0
1 0 1 1 0 0
1 0 1 2 0 0
1 0 1 3 0 0
1 0 1 4 0 0
1 0 1 5 0 0
1 0 1 6 0 0
4 7 task(1) 0
4 7 task(2) 0
4 15 duration(1,200) 0
4 15 duration(2,400) 0
9 0 1 200
9 0 3 400
9 0 6 1
9 0 11 2
9 1 0 4 diff
9 1 2 2 <=
9 1 4 1 -
9 1 5 3 end
9 1 8 3 beg
9 2 7 5 1 6
9 2 9 8 1 6
9 2 10 4 2 7 9
9 2 12 5 1 11
9 2 13 8 1 11
9 2 14 4 2 12 13
9 4 0 1 10 0
9 4 1 1 14 0
9 6 5 0 1 0 2 1
9 6 6 0 1 1 2 3
0
```

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9 2 12 5 1 11
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9 2 14 4 2 12 13
9 4 0 1 10 0
9 4 1 1 14 0
9 6 5 0 1 0 2 1
9 6 6 0 1 1 2 3
0
```

# *aspif* theory example

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&dom {1..1000} = end(2).

&diff{end(1)-beg(1)}<=200.
&diff{end(2)-beg(2)}<=400.

&show{ beg/1; end/1 }.
```

Only 6 (theory) atoms!

```
asp 1 0 0
1 0 1 1 0 0
1 0 1 2 0 0
1 0 1 3 0 0
1 0 1 4 0 0
1 0 1 5 0 0
1 0 1 6 0 0
4 7 task(1) 0
4 7 task(2) 0
4 15 duration(1,200) 0
4 15 duration(2,400) 0
9 0 1 200
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9 2 13 8 1 11
9 2 14 4 2 12 13
9 4 0 1 10 0
9 4 1 1 14 0
9 6 5 0 1 0 2 1
9 6 6 0 1 1 2 3
0
```

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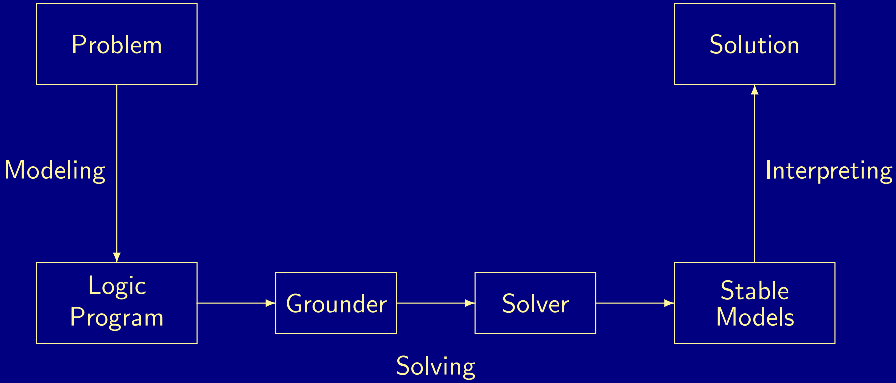

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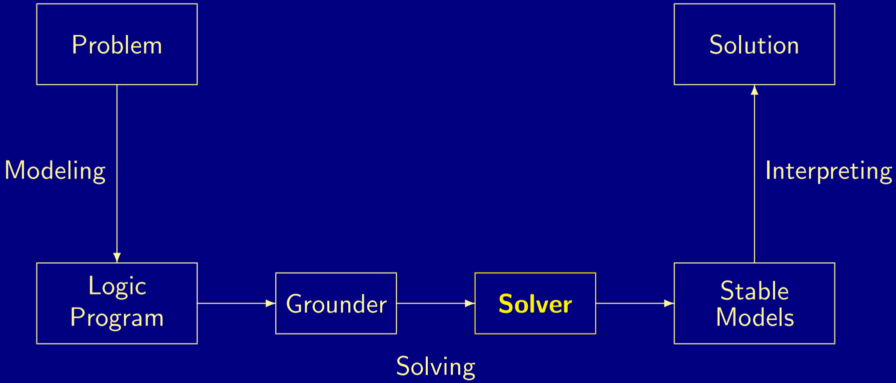

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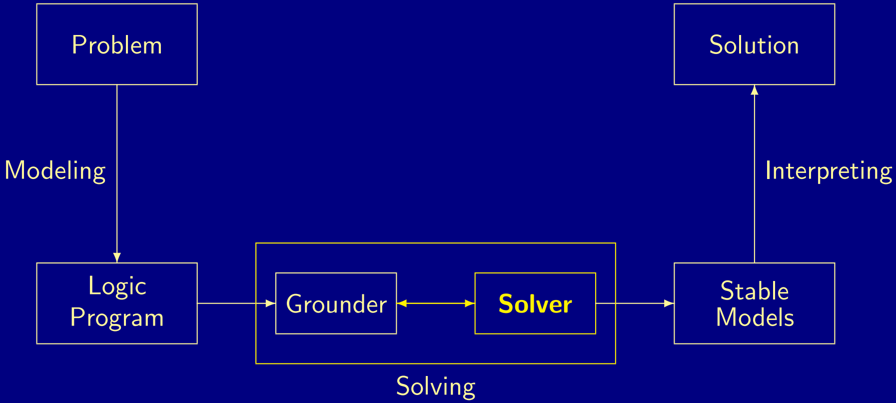
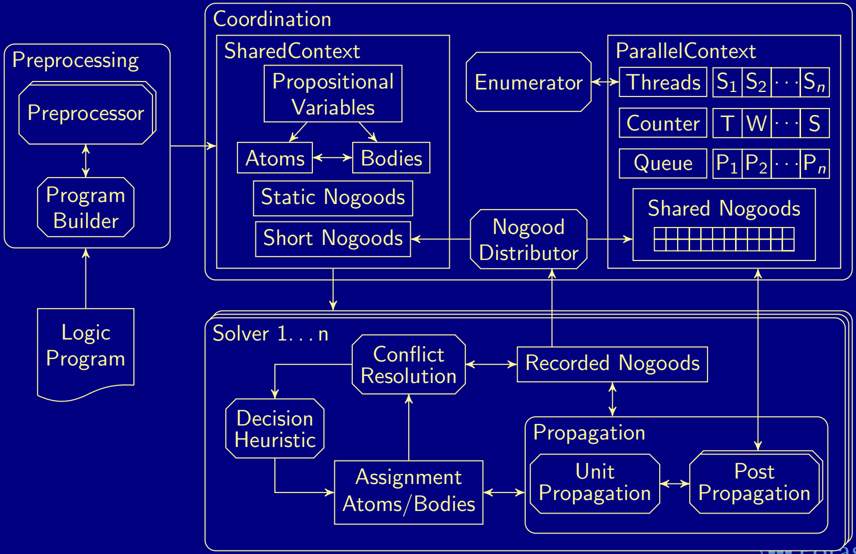


# Architecture of *clasp*



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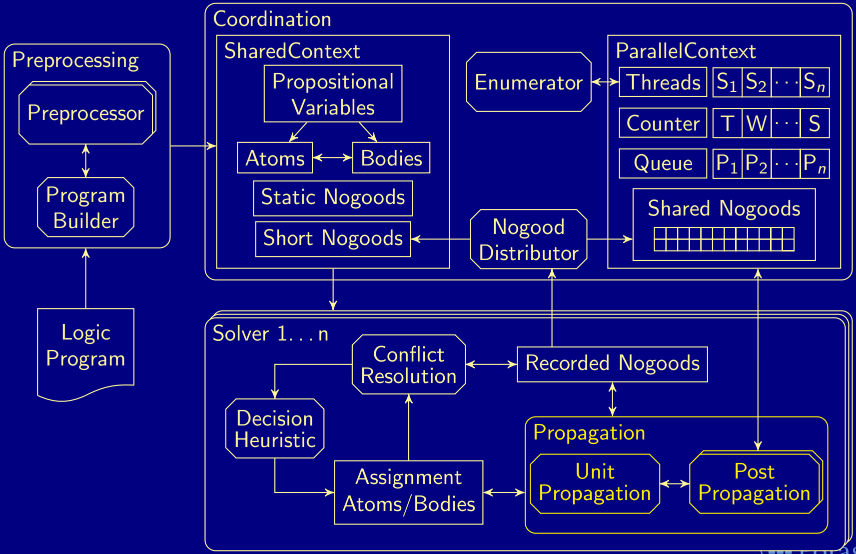
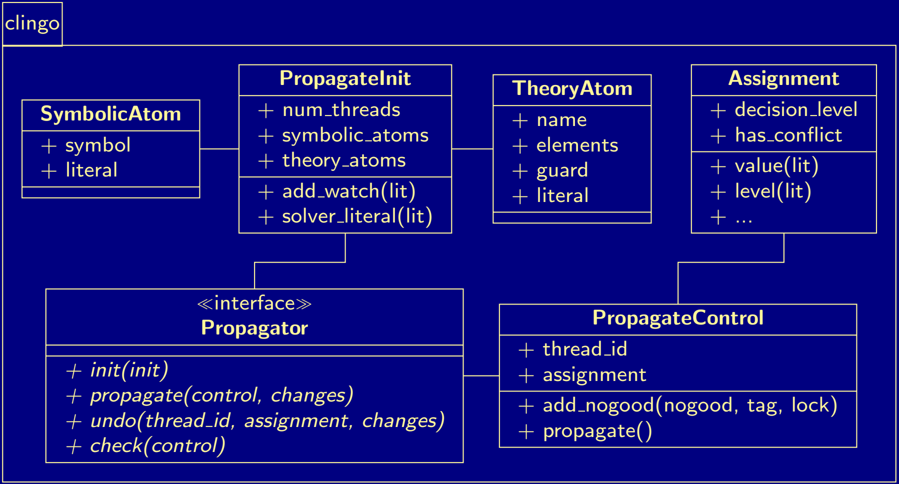


# Conflict-driven constraint learning modulo theories

```
(I)  initialize                          // register theory propagators and initialize watches
     loop
        propagate completion, loop, and recorded nogoods   // deterministically assign literals
        if no conflict then
           if all variables assigned then
(C)           if some δ ∈ Δ_T is violated for T ∈ 𝕋 then record δ   // theory propagator's check
              else return variable assignment                       // 𝕋-stable model found
           else
(P)           propagate theories T ∈ 𝕋     // theory propagators may record theory nogoods
              if no nogood recorded then decide   // non-deterministically assign some literal
        else
           if top-level conflict then return unsatisfiable
           else
              analyze                // resolve conflict and record a conflict constraint
(U)           backjump               // undo assignments until conflict constraint is unit
```

# Propagator interface



# The *dot* propagator

```
#script (python)

import sys
import time

class Propagator:
    def init(self, init):
        self.sleep = .1
        for atom in init.symbolic_atoms:
            init.add_watch(init.solver_literal(atom.literal))

    def propagate(self, ctl, changes):
        for l in changes:
            sys.stdout.write(".")
            sys.stdout.flush()
            time.sleep(self.sleep)
        return True

    def undo(self, solver_id, assign, undo):
        for l in undo:
            sys.stdout.write("\b \b")
            sys.stdout.flush()
            time.sleep(self.sleep)

def main(prg):
    prg.register_propagator(Propagator())
    prg.ground([("base", [])])
    prg.solve()
    sys.stdout.write("\n")

#end.
```

# Outline

# Difference logic propagation

| Problem | # | ASP | | ASP modulo *DL* (stateless) | | | | ASP modulo *DL* (stateful) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | defined | | external | | defined | | external | |
| | | T | TO | T | TO | T | TO | T | TO | T | TO |
| Flow shop | 120 | 569 | 110 | 283 | 40 | 382 | 70 | **177** | **30** | 281 | 50 |
| Job shop | 80 | 600 | 80 | 600 | 80 | 600 | 80 | **37** | **0** | 43 | **0** |
| Open shop | 60 | 405 | 40 | 214 | 20 | 213 | 20 | **2** | **0** | **2** | **0** |
| Total | 260 | 525 | 230 | 366 | 140 | 398 | 170 | **72** | **30** | 109 | 50 |

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# Difference logic propagation

| Problem | # | ASP | | ASP modulo *DL* (stateless) | | | | ASP modulo *DL* (stateful) | | | |
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# Outline

## Builtin acyclicity checking

- Edge statement

$$\#\texttt{edge}\ (u,v)\ :\ l_1,\dots,l_n. \tag{3}$$

- A set $X$ of atoms is an acyclic stable of a logic program $P$, if
  1. $X$ is a stable model of $P$ and
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# Constraint Satisfaction Problem

- A constraint satisfaction problem (CSP) consists of
  - a set $V$ of variables,
  - a set $D$ of domains, and
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  such that
  - each variable $v \in V$ has an associated domain $dom(v) \in D$;
  - a constraint $c$ is a pair $(S, R)$ consisting of a $k$-ary relation $R$ on a vector $S \subseteq V^k$ of variables, called the scope of $R$

- Note For $S = (v_1, \ldots, v_k)$, we have $R \subseteq dom(v_1) \times \cdots \times dom(v_k)$

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# Example

$$\begin{array}{ccccc} & s & e & n & d \\ + & m & o & r & e \\ \hline m & o & n & e & y \end{array}$$

Each letter corresponds exactly to one digit and all variables have to be pairwise distinct

$$V = \{s, e, n, d, m, o, r, y\}$$

$$D = \{dom(v) = \{0, \ldots, 9\} \mid v \in V\}$$

$$\begin{array}{rll} C = \{ & (\vec{V}, & allDistinct(V)), \\ & (\vec{V}, & s \times 1000 + e \times 100 + n \times 10 + d + \\ & & m \times 1000 + o \times 100 + r \times 10 + e == \\ & & m \times 10000 + o \times 1000 + n \times 100 + e \times 10 + y), \\ & ((m), m == 1) \} & \end{array}$$

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| | | | | |
|---|---|---|---|---|
| | 9 | 5 | 6 | 7 |
| + | 1 | 0 | 8 | 5 |
| 1 | 0 | 6 | 5 | 2 |

The example has exactly one solution

$$\{\ s \mapsto 9, e \mapsto 5, n \mapsto 6, d \mapsto 7, m \mapsto 1, o \mapsto 0, r \mapsto 8, y \mapsto 2\ \}$$

## Constraint satisfaction problem

- Notation We use $S(c) = S$ and $R(c) = R$ to access the scope and the relation of a constraint $c = (S, R)$
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# Constraint Answer Set Programming

- A constraint logic program $P$ is a logic program over an extended alphabet $\mathcal{A} \cup \mathcal{C}$ where
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$$\gamma(x < y) \ = \ \gamma(((-y-1) \leq -(x+1)) \wedge (x \neq y))$$

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- *adsolver*
  - extension of ASP solver *smodels*
- *clingcon*
  - extension of ASP system *clingo* (viz. *gringo* and *clasp*)
  - lazy approach
- *aspartame*
  - translational approach (independent of ASP system)
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- *aspmt*, *dlvhex*, *ezcsp*, *gasp*, *inca*, ...

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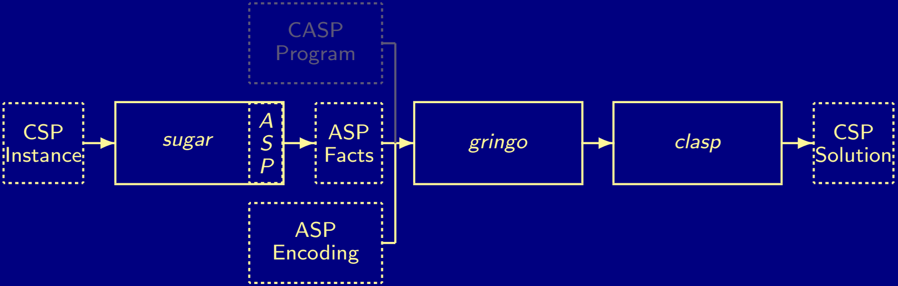


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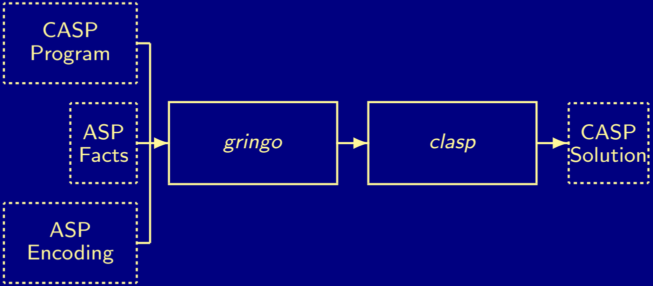


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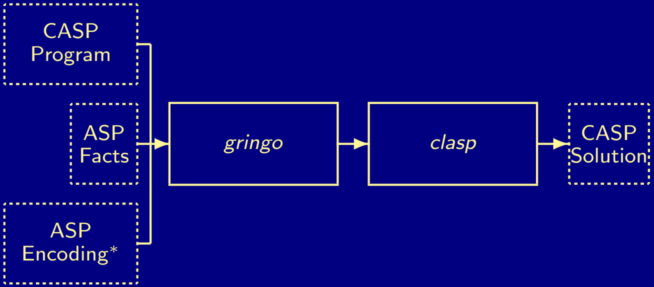


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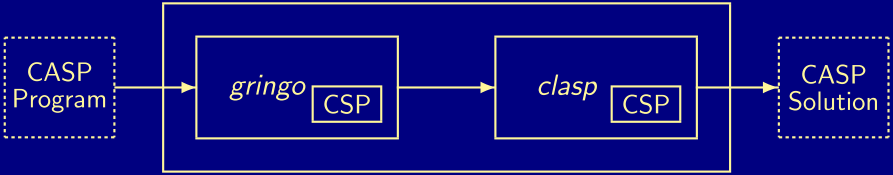


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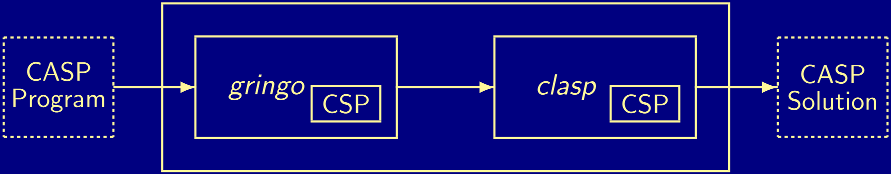


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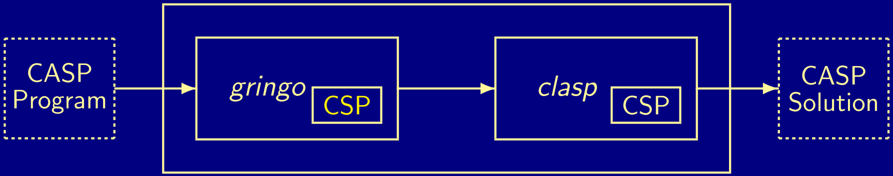


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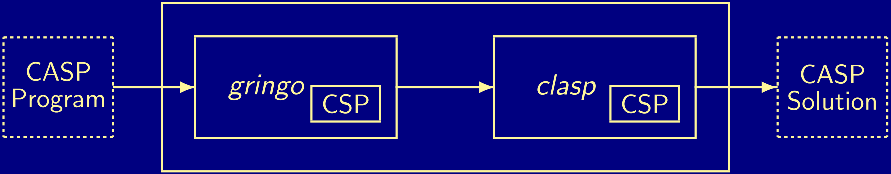


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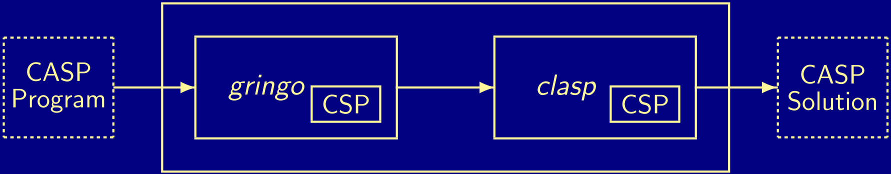


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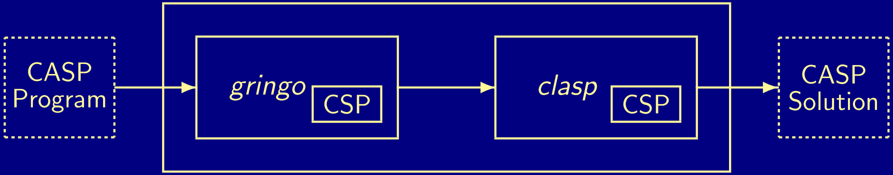


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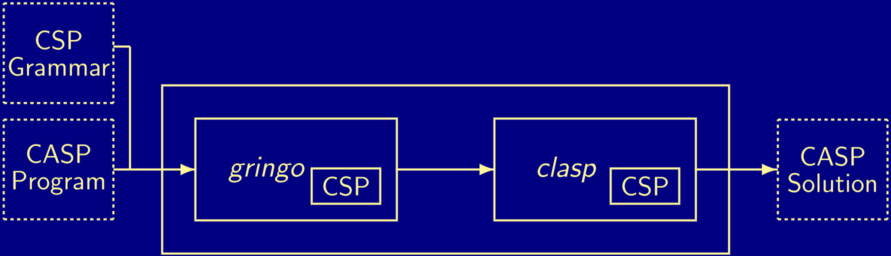


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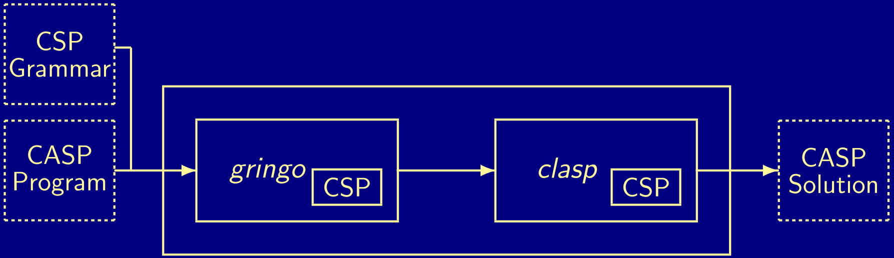


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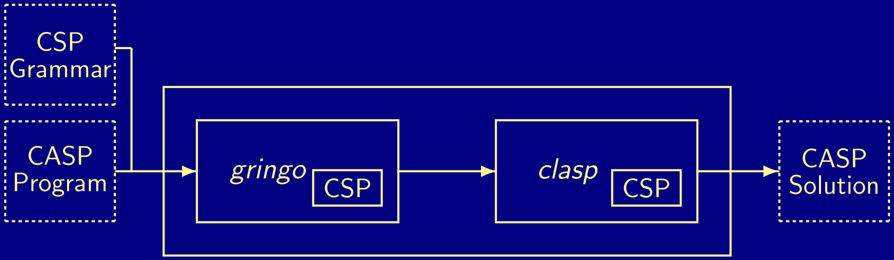
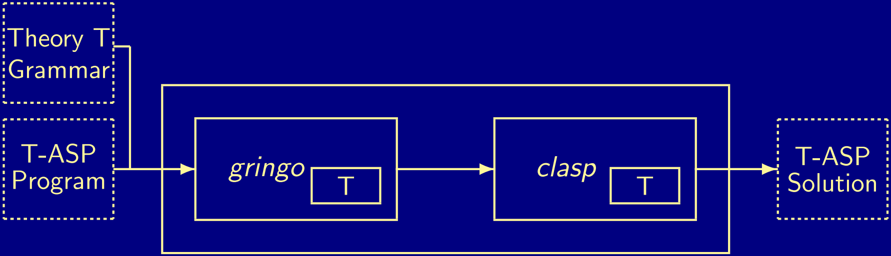


# *clingcon* instantiates *clingo*



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