From SAT to ASP and back!?

Torsten Schaub

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Outline

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Answer Set Programming (ASP)
in a Nutshell

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with effective solving capacities
- tailored to knowledge representation and reasoning
- ASP has its roots in
  - databases
  - logic programming
  - knowledge representation and (non-monotonic) reasoning
  - constraint solving (in particular, SAT)
- ASP allows for solving all search problems in NP (and NP^{NP})
- The versatility of ASP is reflected by the solver *clasp*, winning first places at ASP, CASC, MISC, PB, and SAT
- ASP embraces many emerging application areas

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Some biased moments in time

- '70/'80 Capturing incomplete information

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- '70/'80 Capturing incomplete information
  - Databases Closed world assumption
  - Logic programming Negation as failure
  - Non-monotonic reasoning
    Auto-epistemic and Default logics, Circumscription

Some biased moments in time

- '70/'80 Capturing incomplete information
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    - Axiomatic characterization
  - Logic programming Negation as failure
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Some biased moments in time

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    - Axiomatic characterization
  - Logic programming Negation as failure
    - Herbrand interpretations
    - Fix-point characterizations
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    - Fix-point characterizations
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    - Extensions of first-order logic
    - Modalities, fix-points, second-order logic

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- '90 Amalgamation and computation

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  - Logic programming semantics
    Well-founded and stable models semantics
  - ASP solving
    "Stable models = Well-founded semantics + Branch"

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    - Stable models semantics derived from non-monotonic logics
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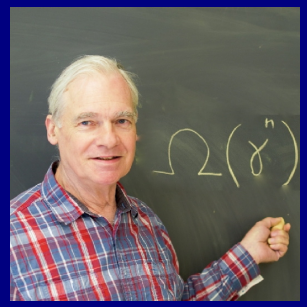

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    "Stable models = Well-founded semantics + Branch"
    - Modeling — Grounding — Solving
    - Icebreakers: `lparse` and `smodels`

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  - Growing dissemination Decision Support for Space Shuttle
  - Constructive logics Equilibrium Logic

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    "Stable models = Well-founded semantics + Branch"
- '00 Applications and semantic rediscoveries
  - Growing dissemination Decision Support for Space Shuttle
    - Bio-informatics, Linux Package Configuration, Music composition, Robotics, System Design, etc
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  - Constructive logics Equilibrium Logic
    - Roots: Logic of Here-and-There (Heyting'30), G3 (Gödel'32)

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Propositional Normal Logic Programs

- A logic program Π is a set of rules of the form

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \ldots, b_m, \neg c_1, \ldots, \neg c_n}_{\text{body}}$$

  - a and all b_i, c_j are atoms (propositional variables)
  - $\leftarrow$, ,, $\neg$ denote if, and, and (default) negation
  - intuitive reading: head must be true if body holds

- Semantics given by stable models, informally,
  models of Π justifying each true atom by some rule in Π

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Logic Programs

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Normal Logic Programs

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- Disclaimer The following formalities apply to normal logic programs

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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

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a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\neg\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
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F	T	F	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
F	T	T	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
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a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	F $\wedge$ T
F	F	T	F $\wedge$ T
F	T	F	T $\wedge$ F
F	T	T	T $\wedge$ T
T	F	F	T $\wedge$ T
T	F	T	T $\wedge$ T
T	T	F	T $\wedge$ F
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T	T	F	F
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- We get four models: $\{b, c\}$, $\{a\}$, $\{a, c\}$, and $\{a, b, c\}$

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T	T	F	$(b \rightarrow c)$	
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F	F	T	$a \wedge (b \rightarrow c)$	
F	T	F	$(b \rightarrow c)$	
F	T	T	$(b \rightarrow c)$	$\models$
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$
T	F	T	$a \wedge (b \rightarrow c)$	$\models a$
T	T	F	$(b \rightarrow c)$	
T	T	T	$(b \rightarrow c)$	$\models$

Reduct

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	F	F	$a \wedge (b \rightarrow c)$	$\models a$
F	F	T	$a \wedge (b \rightarrow c)$	$\models a$
F	T	F	$(b \rightarrow c)$	$\models$
F	T	T	$(b \rightarrow c)$	$\models$
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$
T	F	T	$a \wedge (b \rightarrow c)$	$\models a$
T	T	F	$(b \rightarrow c)$	$\models$
T	T	T	$(b \rightarrow c)$	$\models$

Reduct

Potassco

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$		
F	F	F	$a \wedge (b \rightarrow c)$		
F	F	T	$a \wedge (b \rightarrow c)$		
F	T	F	$(b \rightarrow c)$		
F	T	T	$(b \rightarrow c)$		
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$	Stable model
T	F	T	$a \wedge (b \rightarrow c)$		
T	T	F	$(b \rightarrow c)$		
T	T	T	$(b \rightarrow c)$		

Reduct

- We get one stable model: $\{a\}$

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	F	F	$a \wedge (b \rightarrow c)$	
F	F	T	$a \wedge (b \rightarrow c)$	
F	T	F	$(b \rightarrow c)$	
F	T	T	$(b \rightarrow c)$	
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$ Stable model
T	F	T	$a \wedge (b \rightarrow c)$	
T	T	F	$(b \rightarrow c)$	
T	T	T	$(b \rightarrow c)$	
			Reduct	

- We get one stable model: $\{a\}$
- Stable models = Smallest models of (respective) reducts

Potassco

Outline

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

- Consider the schema: $\neg\neg A \rightarrow A$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg a \rightarrow a) \wedge (\neg\neg b \rightarrow b) \wedge (\neg\neg c \rightarrow c)$
F	**F**	**F**	
F	**F**	**T**	
F	**T**	**F**	
F	**T**	**T**	
T	**F**	**F**	
T	**F**	**T**	
T	**T**	**F**	
T	**T**	**T**	

- Consider the schema: $\neg\neg A \rightarrow A$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg a \rightarrow a) \wedge (\neg\neg b \rightarrow b) \wedge (\neg\neg c \rightarrow c)$
F	**F**	**F**	$(\neg\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg\mathbf{F} \rightarrow a) \wedge (\neg\neg\mathbf{F} \rightarrow b) \wedge (\neg\neg\mathbf{F} \rightarrow c)$
F	**F**	**T**	$(\neg\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg\mathbf{F} \rightarrow a) \wedge (\neg\neg\mathbf{F} \rightarrow b) \wedge (\neg\neg\mathbf{T} \rightarrow c)$
F	**T**	**F**	$(\neg\mathbf{T} \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg\mathbf{F} \rightarrow a) \wedge (\neg\neg\mathbf{T} \rightarrow b) \wedge (\neg\neg\mathbf{F} \rightarrow c)$
F	**T**	**T**	$(\neg\mathbf{T} \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg\mathbf{F} \rightarrow a) \wedge (\neg\neg\mathbf{T} \rightarrow b) \wedge (\neg\neg\mathbf{T} \rightarrow c)$
T	**F**	**F**	$(\neg\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg\mathbf{T} \rightarrow a) \wedge (\neg\neg\mathbf{F} \rightarrow b) \wedge (\neg\neg\mathbf{F} \rightarrow c)$
T	**F**	**T**	$(\neg\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg\mathbf{T} \rightarrow a) \wedge (\neg\neg\mathbf{F} \rightarrow b) \wedge (\neg\neg\mathbf{T} \rightarrow c)$
T	**T**	**F**	$(\neg\mathbf{T} \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg\mathbf{T} \rightarrow a) \wedge (\neg\neg\mathbf{T} \rightarrow b) \wedge (\neg\neg\mathbf{F} \rightarrow c)$
T	**T**	**T**	$(\neg\mathbf{T} \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg\mathbf{T} \rightarrow a) \wedge (\neg\neg\mathbf{T} \rightarrow b) \wedge (\neg\neg\mathbf{T} \rightarrow c)$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg a \rightarrow a) \wedge (\neg\neg b \rightarrow b) \wedge (\neg\neg c \rightarrow c)$
F	**F**	**F**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c) \wedge (\mathbf{F} \rightarrow a) \wedge (\mathbf{F} \rightarrow b) \wedge (\mathbf{F} \rightarrow c)$
F	**F**	**T**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c) \wedge (\mathbf{F} \rightarrow a) \wedge (\mathbf{F} \rightarrow b) \wedge (\mathbf{T} \rightarrow c)$
F	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge (\mathbf{F} \rightarrow a) \wedge (\mathbf{T} \rightarrow b) \wedge (\mathbf{F} \rightarrow c)$
F	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge (\mathbf{F} \rightarrow a) \wedge (\mathbf{T} \rightarrow b) \wedge (\mathbf{T} \rightarrow c)$
T	**F**	**F**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c) \wedge (\mathbf{T} \rightarrow a) \wedge (\mathbf{F} \rightarrow b) \wedge (\mathbf{F} \rightarrow c)$
T	**F**	**T**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c) \wedge (\mathbf{T} \rightarrow a) \wedge (\mathbf{F} \rightarrow b) \wedge (\mathbf{T} \rightarrow c)$
T	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge (\mathbf{T} \rightarrow a) \wedge (\mathbf{T} \rightarrow b) \wedge (\mathbf{F} \rightarrow c)$
T	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge (\mathbf{T} \rightarrow a) \wedge (\mathbf{T} \rightarrow b) \wedge (\mathbf{T} \rightarrow c)$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg a \rightarrow a) \wedge (\neg\neg b \rightarrow b) \wedge (\neg\neg c \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c) \wedge (\mathbf{F} \rightarrow a) \wedge (\mathbf{F} \rightarrow b) \wedge (\mathbf{F} \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c) \wedge (\mathbf{F} \rightarrow a) \wedge (\mathbf{F} \rightarrow b) \wedge c$
F	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge (\mathbf{F} \rightarrow a) \wedge b \wedge (\mathbf{F} \rightarrow c)$
F	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge (\mathbf{F} \rightarrow a) \wedge b \wedge c$
T	**F**	**F**	$a \wedge (b \rightarrow c) \wedge a \wedge (\mathbf{F} \rightarrow b) \wedge (\mathbf{F} \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c) \wedge a \wedge (\mathbf{F} \rightarrow b) \wedge c$
T	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge a \wedge b \wedge (\mathbf{F} \rightarrow c)$
T	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c) \wedge a \wedge b \wedge c$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg a \rightarrow a) \wedge (\neg\neg b \rightarrow b) \wedge (\neg\neg c \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c) \wedge$ **T** $\wedge$ **T** $\wedge$ **T**
F	**F**	**T**	$a \wedge (b \rightarrow c) \wedge$ **T** $\wedge$ **T** $\wedge$ c
F	**T**	**F**	**T** $\wedge (b \rightarrow c) \wedge$ **T** $\wedge$ $b \wedge$ **T**
F	**T**	**T**	**T** $\wedge (b \rightarrow c) \wedge$ **T** $\wedge$ $b \wedge$ c
T	**F**	**F**	$a \wedge (b \rightarrow c) \wedge$ $a \wedge$ **T** $\wedge$ **T**
T	**F**	**T**	$a \wedge (b \rightarrow c) \wedge$ $a \wedge$ **T** $\wedge$ c
T	**T**	**F**	**T** $\wedge (b \rightarrow c) \wedge$ $a \wedge$ $b \wedge$ **T**
T	**T**	**T**	**T** $\wedge (b \rightarrow c) \wedge$ $a \wedge$ $b \wedge$ c

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg a \rightarrow a) \wedge (\neg\neg b \rightarrow b) \wedge (\neg\neg c \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c) \wedge c$
F	**T**	**F**	$(b \rightarrow c) \wedge b$
F	**T**	**T**	$(b \rightarrow c) \wedge b \wedge c$
T	**F**	**F**	$a \wedge (b \rightarrow c) \wedge a$
T	**F**	**T**	$a \wedge (b \rightarrow c) \wedge a \wedge c$
T	**T**	**F**	$(b \rightarrow c) \wedge a \wedge b$
T	**T**	**T**	$(b \rightarrow c) \wedge a \wedge b \wedge c$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge \ulcorner (a \rightarrow a) \urcorner \wedge \ulcorner (b \rightarrow b) \urcorner \wedge \ulcorner (c \rightarrow c) \urcorner$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c) \wedge c$
F	**T**	**F**	$(b \rightarrow c) \wedge b$
F	**T**	**T**	$(b \rightarrow c) \wedge b \wedge c$
T	**F**	**F**	$a \wedge (b \rightarrow c) \wedge a$
T	**F**	**T**	$a \wedge (b \rightarrow c) \wedge a \wedge c$
T	**T**	**F**	$(b \rightarrow c) \wedge a \wedge b$
T	**T**	**T**	$(b \rightarrow c) \wedge a \wedge b \wedge c$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge \ulcorner (\neg a \rightarrow a) \urcorner \wedge \ulcorner (\neg b \rightarrow b) \urcorner \wedge \ulcorner (\neg c \rightarrow c) \urcorner$	
F	**F**	**F**	$a \wedge (b \rightarrow c)$	$\models a$
F	**F**	**T**	$a \wedge (b \rightarrow c) \wedge c$	$\models a, c$
F	**T**	**F**	$(b \rightarrow c) \wedge b$	$\models b, c$
F	**T**	**T**	$(b \rightarrow c) \wedge b \wedge c$	$\models b, c$
T	**F**	**F**	$a \wedge (b \rightarrow c) \wedge a$	$\models a$
T	**F**	**T**	$a \wedge (b \rightarrow c) \wedge a \wedge c$	$\models a, c$
T	**T**	**F**	$(b \rightarrow c) \wedge a \wedge b$	$\models a, b, c$
T	**T**	**T**	$(b \rightarrow c) \wedge a \wedge b \wedge c$	$\models a, b, c$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge \neg(a \rightarrow a) \wedge \neg(b \rightarrow b) \wedge \neg(c \rightarrow c)$	
F	F	F	$a \wedge (b \rightarrow c)$	$\models a$
F	F	T	$a \wedge (b \rightarrow c) \wedge c$	$\models a, c$
F	T	F	$(b \rightarrow c) \wedge b$	$\models b, c$
F	T	T	$(b \rightarrow c) \wedge b \wedge c$	$\models b, c$
T	F	F	$a \wedge (b \rightarrow c) \wedge a$	$\models a$
T	F	T	$a \wedge (b \rightarrow c) \wedge a \wedge c$	$\models a, c$
T	T	F	$(b \rightarrow c) \wedge a \wedge b$	$\models a, b, c$
T	T	T	$(b \rightarrow c) \wedge a \wedge b \wedge c$	$\models a, b, c$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge \sqcap(a \rightarrow a\sqcap) \wedge \sqcap(b \rightarrow b\sqcap) \wedge \sqcap(c \rightarrow c\sqcap)$	
F	**F**	**F**	$a \wedge (b \rightarrow c)$	$\not\models a$
F	**F**	**T**	$a \wedge (b \rightarrow c) \wedge c$	$\not\models a, c$
F	**T**	**F**	$(b \rightarrow c) \wedge b$	$\not\models b, c$
F	**T**	**T**	$(b \rightarrow c) \wedge b \wedge c$	$\models b, c$
T	**F**	**F**	$a \wedge (b \rightarrow c) \wedge a$	$\not\models a$
T	**F**	**T**	$a \wedge (b \rightarrow c) \wedge a \wedge c$	$\not\models a, c$
T	**T**	**F**	$(b \rightarrow c) \wedge a \wedge b$	$\not\models a, b, c$
T	**T**	**T**	$(b \rightarrow c) \wedge a \wedge b \wedge c$	$\models a, b, c$

- We get four stable models: $\{b, c\}$, $\{a\}$, $\{a, c\}$, $\{a, b, c\}$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg a \rightarrow a) \wedge (\neg\neg b \rightarrow b) \wedge (\neg\neg c \rightarrow c)$
F	F	F	$a \wedge (b \rightarrow c)$
F	F	T	$a \wedge (b \rightarrow c) \wedge$ c
F	T	F	$(b \rightarrow c) \wedge$ b
F	T	T	$(b \rightarrow c) \wedge$ $b \wedge c$
T	F	F	$a \wedge (b \rightarrow c) \wedge$ a
T	F	T	$a \wedge (b \rightarrow c) \wedge$ $a \wedge c$
T	T	F	$(b \rightarrow c) \wedge$ $a \wedge b$
T	T	T	$(b \rightarrow c) \wedge$ $a \wedge b \wedge c$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg a \rightarrow a) \wedge (\neg\neg b \rightarrow b) \wedge (\neg\neg c \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c) \wedge c$
F	**T**	**F**	$(b \rightarrow c) \wedge b$
F	**T**	**T**	$(b \rightarrow c) \wedge b \wedge c$
T	**F**	**F**	$a \wedge (b \rightarrow c) \wedge a$
T	**F**	**T**	$a \wedge (b \rightarrow c) \wedge a \wedge c$
T	**T**	**F**	$(b \rightarrow c) \wedge a \wedge b$
T	**T**	**T**	$(b \rightarrow c) \wedge a \wedge b \wedge c$

- Note $\neg\neg A \rightarrow A$ is equivalent to $A \vee \neg A$

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg a \rightarrow a) \wedge (\neg\neg b \rightarrow b) \wedge (\neg\neg c \rightarrow c)$
F	F	F	$a \wedge (b \rightarrow c)$
F	F	T	$a \wedge (b \rightarrow c) \wedge$ c
F	T	F	$(b \rightarrow c) \wedge$ b
F	T	T	$(b \rightarrow c) \wedge$ $b \wedge c$
T	F	F	$a \wedge (b \rightarrow c) \wedge$ a
T	F	T	$a \wedge (b \rightarrow c) \wedge$ $a \wedge c$
T	T	F	$(b \rightarrow c) \wedge$ $a \wedge b$
T	T	T	$(b \rightarrow c) \wedge$ $a \wedge b \wedge c$

- Note $\neg\neg A \rightarrow A$ is equivalent to $A \vee \neg A$
- SAT = ASP + Tertium non datur

Potassco

From ASP to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c) \wedge (\neg\neg a \rightarrow a) \wedge (\neg\neg b \rightarrow b) \wedge (\neg\neg c \rightarrow c)$
F	F	F	$a \wedge (b \rightarrow c)$
F	F	T	$a \wedge (b \rightarrow c) \wedge$ c
F	T	F	$(b \rightarrow c) \wedge$ b
F	T	T	$(b \rightarrow c) \wedge$ $b \wedge c$
T	F	F	$a \wedge (b \rightarrow c) \wedge$ a
T	F	T	$a \wedge (b \rightarrow c) \wedge$ $a \wedge c$
T	T	F	$(b \rightarrow c) \wedge$ $a \wedge b$
T	T	T	$(b \rightarrow c) \wedge$ $a \wedge b \wedge c$

- Note $\neg\neg A \rightarrow A$ is equivalent to $A \vee \neg A$
- SAT = ASP + Tertium non datur (also called choice)

Potassco

Outline

Potassco

From SAT to ASP

$$\Pi = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

$$CF(\Pi) = \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\}$$
$$\cup \{c \leftrightarrow \bot\}$$

$$LF(\Pi) = \{(x \vee y) \rightarrow a \wedge \neg c\}$$

Classical models of $CF(\Pi)$

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

Potassco

From SAT to ASP

$$\Pi = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

$$CF(\Pi) = \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\}$$
$$\cup \{c \leftrightarrow \bot\}$$

$$LF(\Pi) = \{(x \vee y) \rightarrow a \wedge \neg c\}$$

Classical models of Π (only true atoms shown)

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

Potassco

From SAT to ASP

$$\Pi = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

$$CF(\Pi) = \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\} \cup \{c \leftrightarrow \bot\}$$

$$LF(\Pi) = \{(x \vee y) \rightarrow a \wedge \neg c\}$$

Classical models of Π

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- Unsupported atoms
- Unfounded atoms

Potassco

From SAT to ASP

$$\begin{aligned}
\Pi &= \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\} \\
CF(\Pi) &= \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\} \\
&\cup \{c \leftrightarrow \bot\} \\
LF(\Pi) &= \{(x \vee y) \rightarrow a \wedge \neg c\}
\end{aligned}$$

Classical models of Π

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- Unsupported atoms
- Unfounded atoms

Potassco

From SAT to ASP

$$\begin{aligned}
\Pi &= \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\} \\
CF(\Pi) &= \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\} \\
&\cup \{c \leftrightarrow \bot\} \\
LF(\Pi) &= \{(x \vee y) \rightarrow a \wedge \neg c\}
\end{aligned}$$

Classical models of $CF(\Pi)$

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- ~~Unsupported atoms~~ eliminated by completion formulas $CF(\Pi)$
- Unfounded atoms

Potassco

From SAT to ASP

$$
\begin{aligned}
\Pi &= \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\} \\
CF(\Pi) &= \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\} \\
&\cup \{c \leftrightarrow \bot\} \\
LF(\Pi) &= \{(x \vee y) \rightarrow a \wedge \neg c\}
\end{aligned}
$$

Classical models of $CF(\Pi)$

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- ~~Unsupported atoms~~ eliminated by completion formulas $CF(\Pi)$
- Unfounded atoms

Potassco

From SAT to ASP

$$
\begin{aligned}
\Pi &= \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\} \\
CF(\Pi) &= \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\} \\
&\cup \{c \leftrightarrow \bot\} \\
LF(\Pi) &= \{(x \vee y) \rightarrow a \wedge \neg c\}
\end{aligned}
$$

Classical models of $CF(\Pi) \cup LF(\Pi)$

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- ~~Unsupported atoms~~ eliminated by completion formulas $CF(\Pi)$
- ~~Unfounded atoms~~ eliminated by loop formulas $LF(\Pi)$

Potassco

From SAT to ASP

$$
\begin{aligned}
\Pi &= \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\} \\
CF(\Pi) &= \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\} \\
&\cup \{c \leftrightarrow \bot\} \\
LF(\Pi) &= \{(x \vee y) \rightarrow a \wedge \neg c\}
\end{aligned}
$$

Theorem (Lin and Zhao)

X is a stable model of Π iff $X \models CF(\Pi) \cup LF(\Pi)$

Potassco

From SAT to ASP

$$
\begin{aligned}
\Pi &= \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\} \\
CF(\Pi) &= \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\} \\
&\cup \{c \leftrightarrow \bot\} \\
LF(\Pi) &= \{(x \vee y) \rightarrow a \wedge \neg c\}
\end{aligned}
$$

Theorem (Lin and Zhao)

X is a stable model of Π iff $X \models CF(\Pi) \cup LF(\Pi)$

- Size of $CF(\Pi)$ is linear in the size of Π
- Size of $LF(\Pi)$ may be exponential in the size of Π

Potassco

ASP and SAT

- SAT = ASP + Tertium non datur
- ASP = SAT + Completion and Loop formulas

Potassco

Outline

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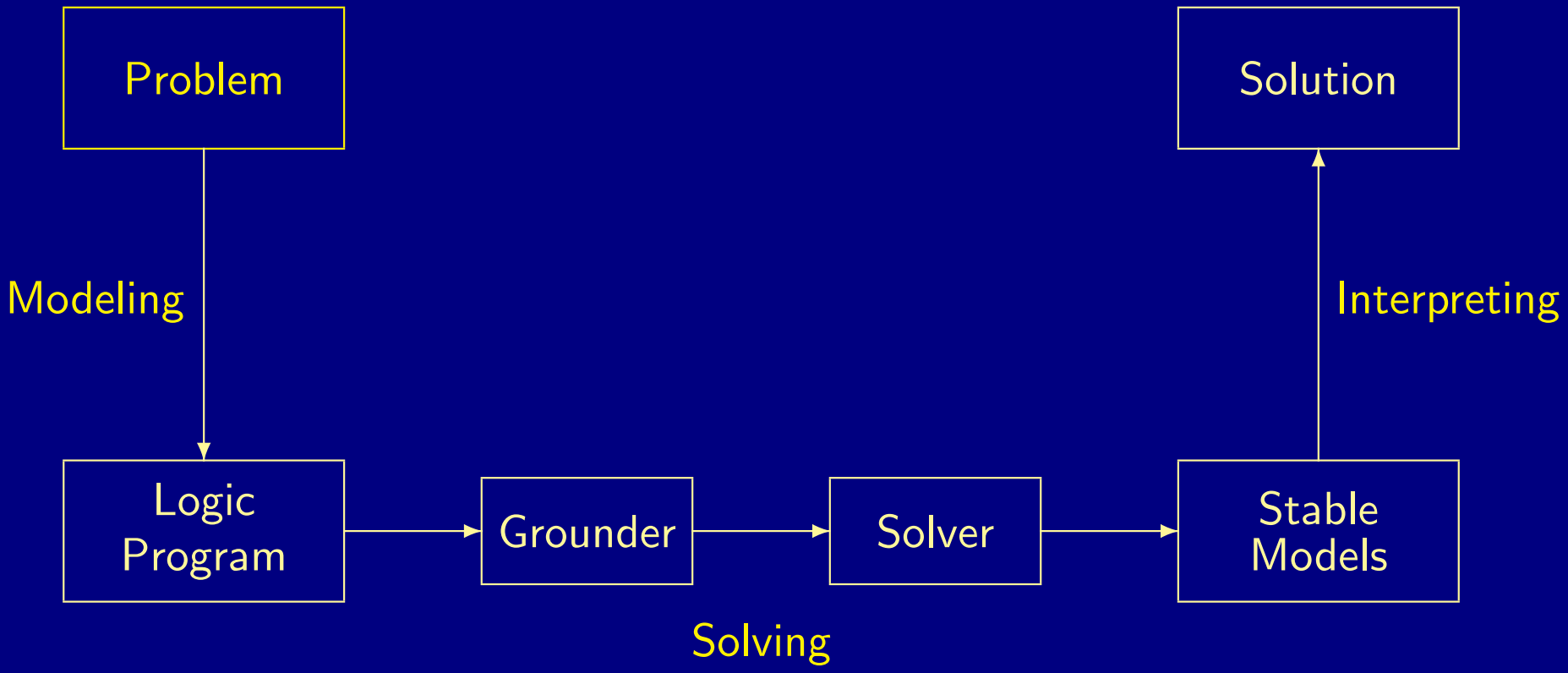
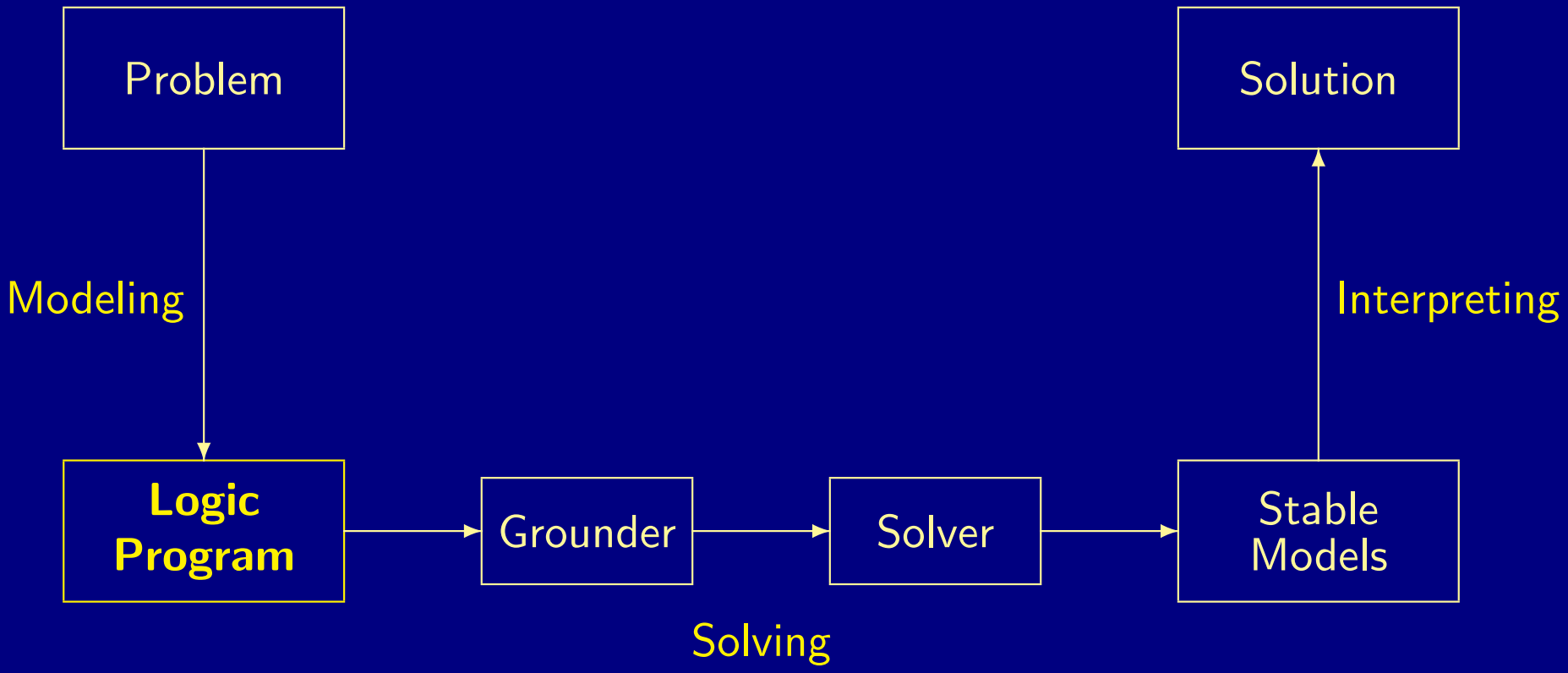
ASP solving process



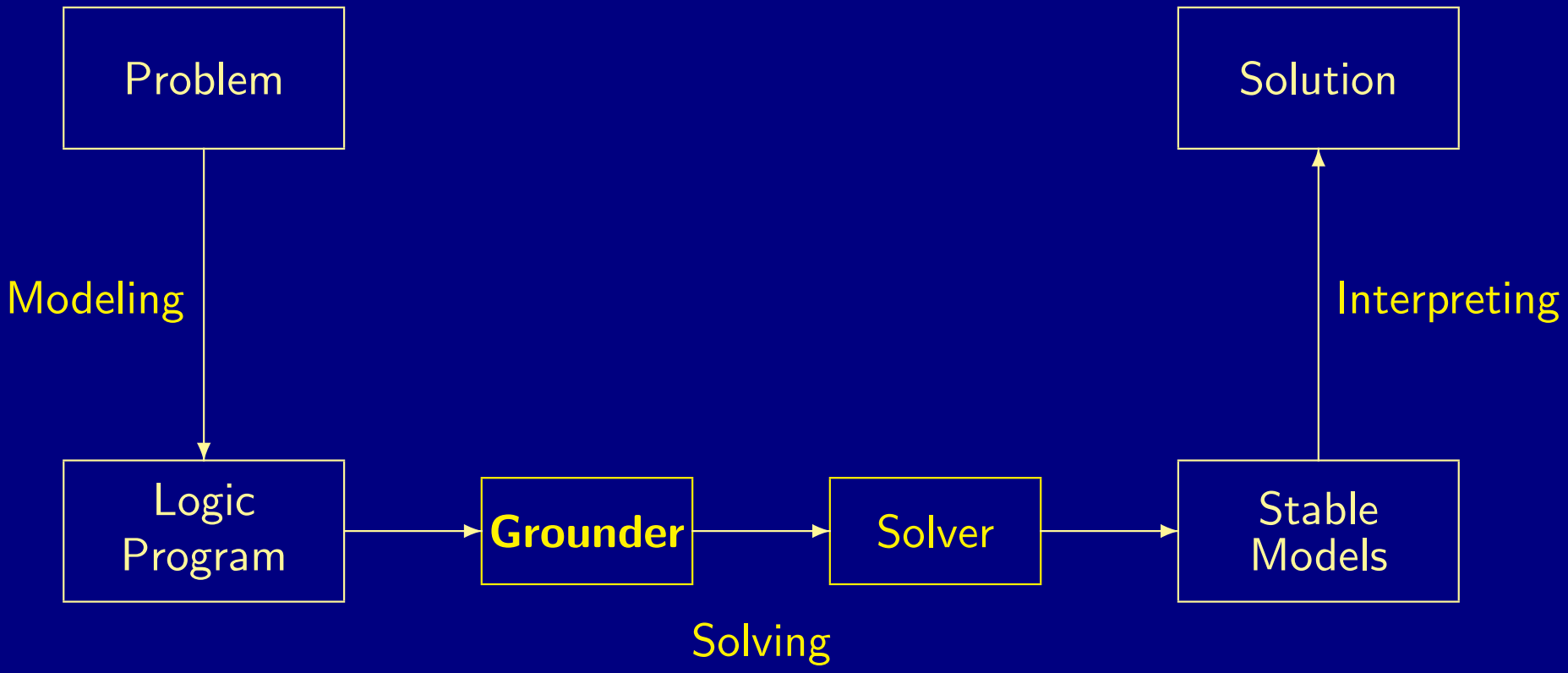
Potassco

ASP solving process



Potassco

ASP solving process



Potassco

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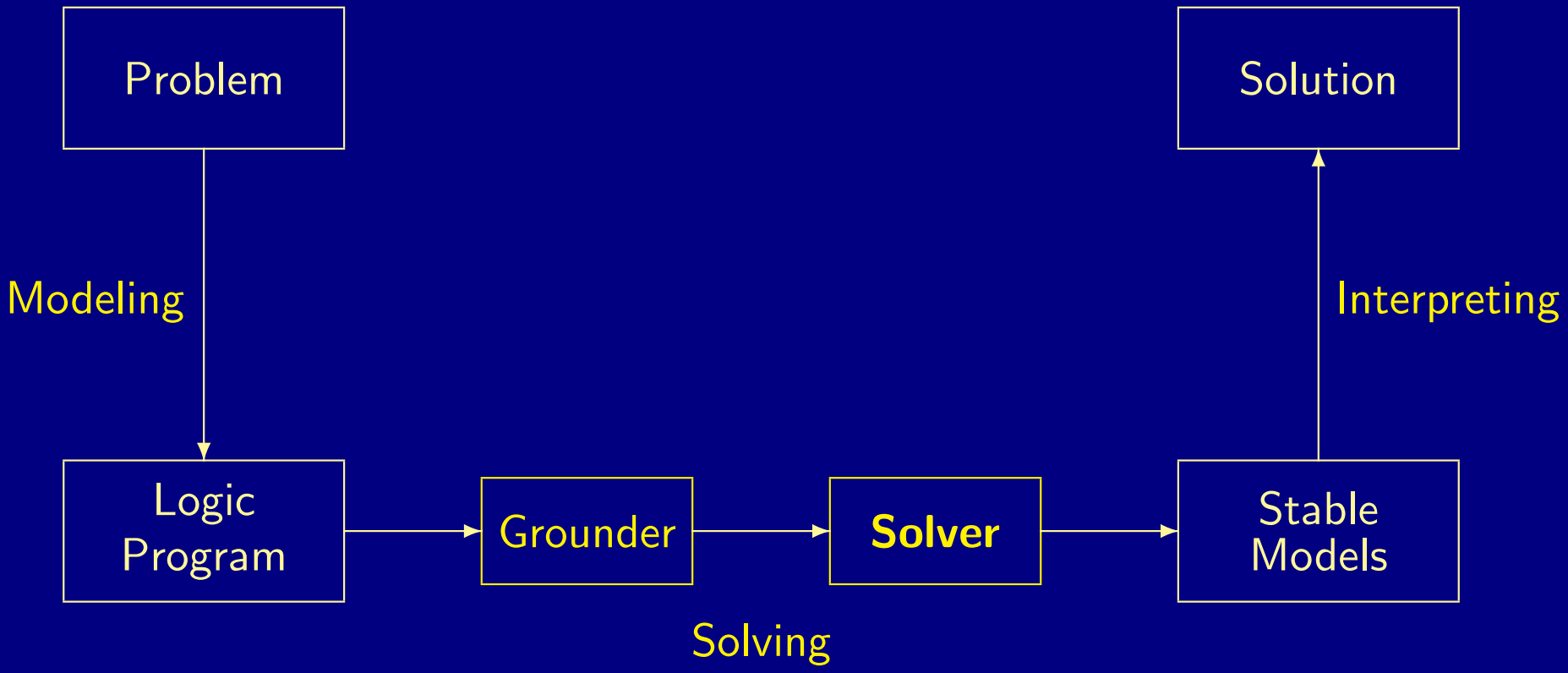

Potassco

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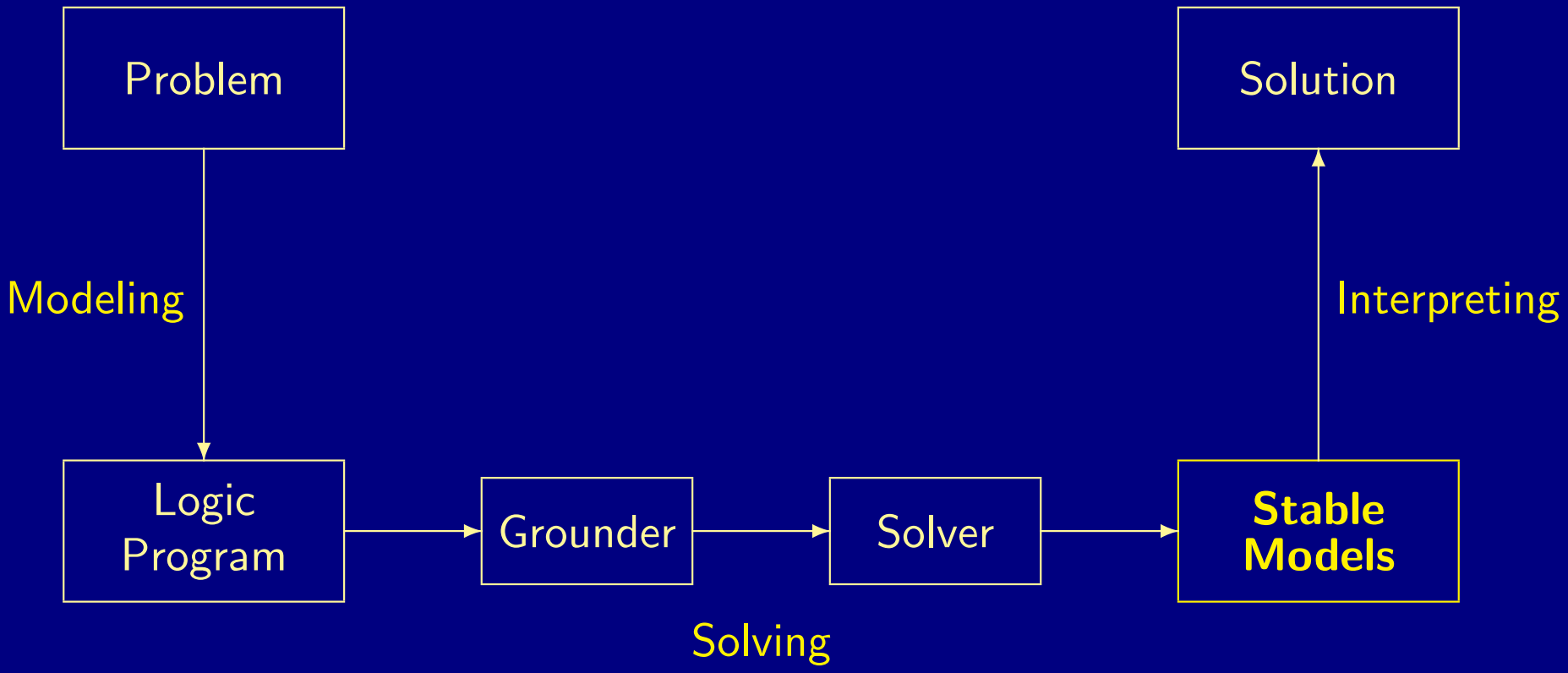
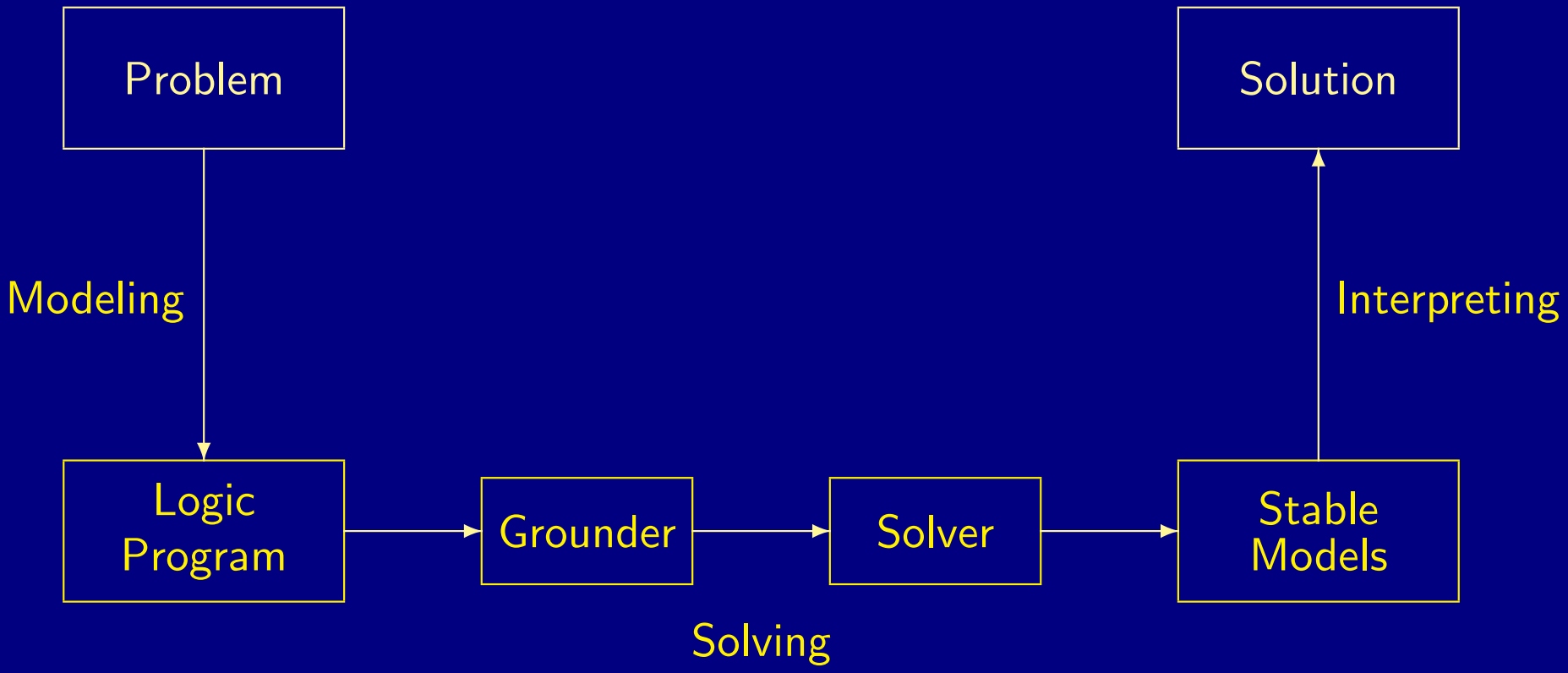


Potassco

ASP solving process



Potassco

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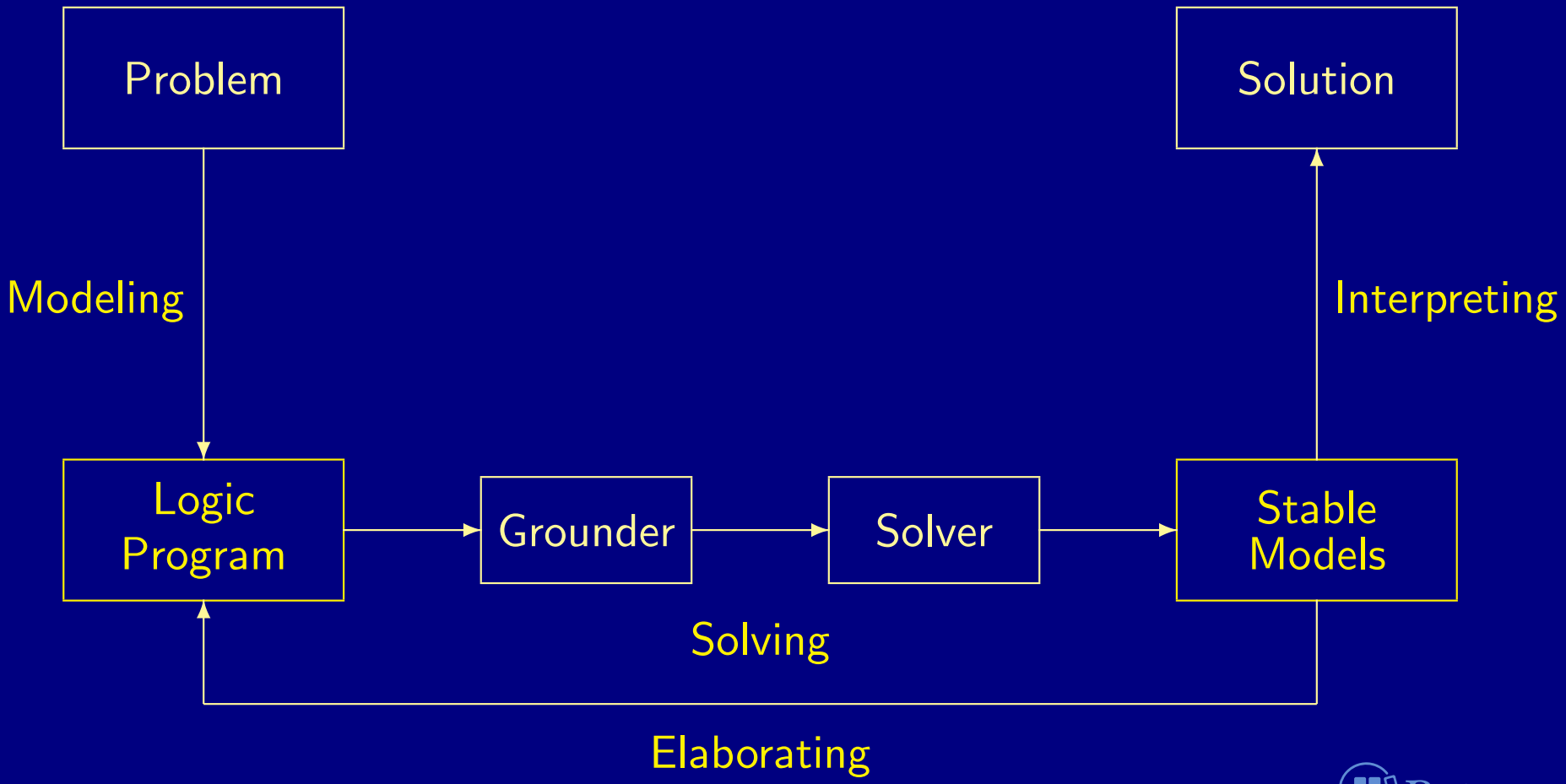

Potassco

Outline

Potassco

Guiding principle

- Elaboration Tolerance (McCarthy, 1998)

  "A formalism is elaboration tolerant [if] *it is convenient to modify a set of facts expressed in the formalism to take into account new phenomena or changed circumstances."*

- Uniform problem representation

  For solving a problem instance **I** of a problem class **C**,
  - **I** is represented as a set of facts $\Pi_{\mathbf{I}}$,
  - **C** is represented as a set of rules $\Pi_{\mathbf{C}}$, and
  - $\Pi_{\mathbf{C}}$ can be used to solve all problem instances in **C**

Potassco

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Potassco

Language constructs

- Variables
- Conditional literals
- Disjunction
- Integrity constraints
- Choice
- Aggregates
- Optimization

```
p(X) :- q(X)
p :- q(X) : r(X)
p(X) ; q(X) :- r(X)
:- q(X), p(X)
2 { p(X,Y) : q(X) } 7 :- r(Y)
s(Y) :- r(Y), 2 #sum{ X : p(X,Y), q(X) } 7
```

```
:~ q(X), p(X,C) [C]
#minimize { C : q(X), p(X,C) }
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Potassco

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Potassco

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- Choice `2 { p(X,Y) : q(X) } 7 :- r(Y)`
- Aggregates `s(Y) :- r(Y), 2 #sum{ X : p(X,Y), q(X) } 7`
- Optimization
  - Weak constraints `:~ q(X), p(X,C) [C]`
  - Minimize statements `#minimize { C : q(X), p(X,C) }`

Potassco

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Potassco

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- Aggregates `s(Y) :- r(Y), 2 #sum{ X : p(X,Y), q(X) } 7`
- Optimization
  - Weak constraints `:~ q(X), p(X,C) [C]`
  - Statements `#minimize { C : q(X), p(X,C) }`

Potassco

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- Integrity constraints `:- q(X), p(X)`
- Choice `2 { p(X,Y) : q(X) } 7 :- r(Y)`
- Aggregates `s(Y) :- r(Y), 2 #sum{ X : p(X,Y), q(X) } 7`
- Multi-objective optimization
  - Weak constraints `:~ q(X), p(X,C) [C@42]`
  - Statements `#minimize { C@42 : q(X), p(X,C) }`

Potassco

Satisfiability testing

$(a \leftrightarrow b) \wedge c$

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```
{ a ; b ; c }.

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- Note $\{A\}$ is an abbreviation for $A \vee \neg A$

Potassco

(Lexico) Maximum satisfiability testing

$(a \leftrightarrow b) \wedge c$

```
{ a ; b ; c }.

 :- not a, b.
 :- a, not b.
 :- not c.
```

```
{ a ; b ; c }.

 :- not a, b.
 :~ a, not b. [10@2]
 :~ not c.    [100@1]
```

- Note $\{A\}$ is an abbreviation for $A \vee \neg A$

Potassco

Traveling salesperson

Basic encoding

```
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(X).
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(Y).

reached(X) :- X = #min { Y : node(Y) }.
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.

node(X) :- edge(X,_).        node(X) :- edge(_,X).

edge(X,Y) :- cost(X,Y,_).

cost(1,2,6).  cost(1,3,2).  cost(1,4,9).
cost(2,4,4).  cost(2,5,5).  cost(2,6,6).  [...]
```

Potassco

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cost(2,4,4).  cost(2,5,5).  cost(2,6,6).  [...]
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Potassco

Outline

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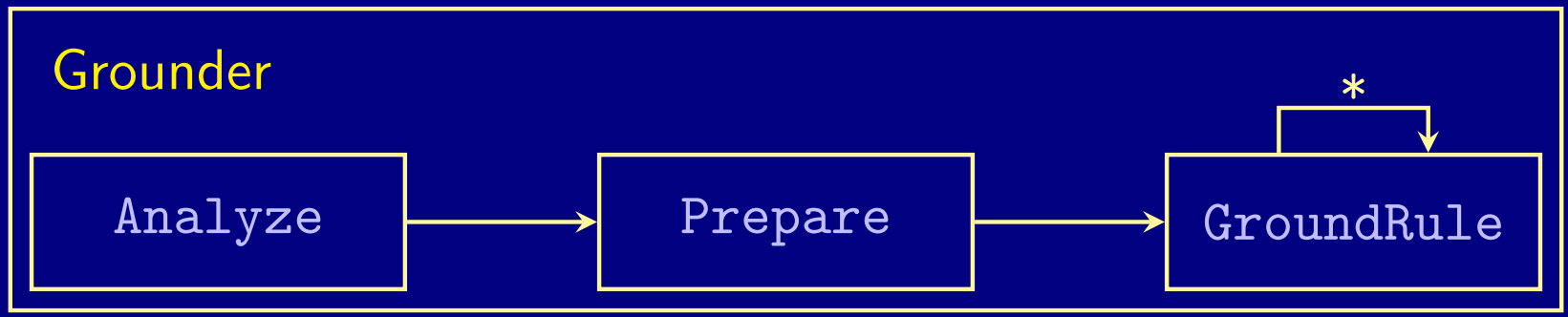
Grounding Logic Programs



- Grounding algorithm uses three functions

`Analyze`	groups rules into components and determines recursive predicates
`Prepare`	rewrites rules based on recursive predicates
`GroundRule`	instantiates rules iteratively following semi-naive database evaluation

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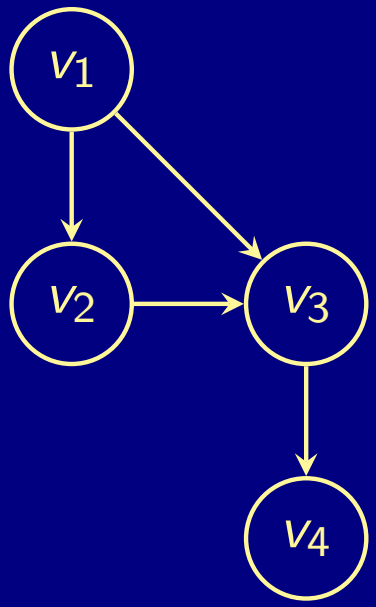
Reachability Problem



$\texttt{vertex}(\texttt{v}_1).$ $\texttt{edge}(\texttt{v}_1, \texttt{v}_2).$
$\texttt{vertex}(\texttt{v}_2).$ $\texttt{edge}(\texttt{v}_1, \texttt{v}_3).$
$\texttt{vertex}(\texttt{v}_3).$ $\texttt{edge}(\texttt{v}_2, \texttt{v}_3).$
$\texttt{vertex}(\texttt{v}_4).$ $\texttt{edge}(\texttt{v}_3, \texttt{v}_4).$

$$\texttt{reach}(\texttt{X}, \texttt{Y}) \texttt{ :- } \texttt{edge}(\texttt{X}, \texttt{Y}). \quad (1)$$
$$\texttt{reach}(\texttt{X}, \texttt{Y}) \texttt{ :- } \texttt{reach}(\texttt{X}, \texttt{Z}), \texttt{edge}(\texttt{Z}, \texttt{Y}). \quad (2)$$

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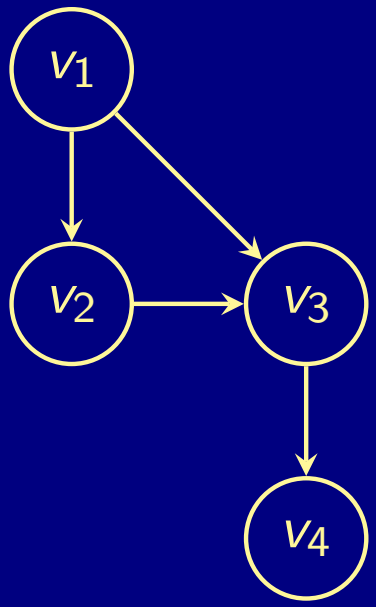
Reachability Problem



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Potassco

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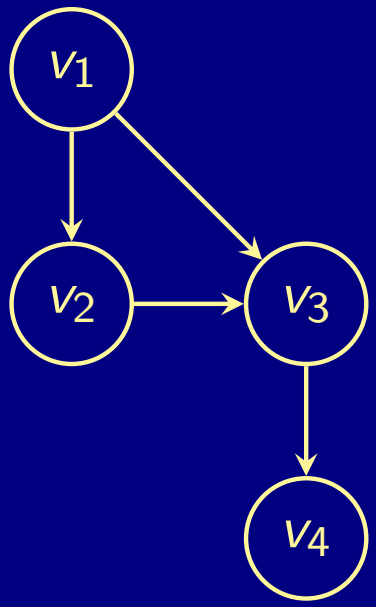

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$\texttt{vertex}(\texttt{v}_3).$ $\texttt{edge}(\texttt{v}_2, \texttt{v}_3).$
$\texttt{vertex}(\texttt{v}_4).$ $\texttt{edge}(\texttt{v}_3, \texttt{v}_4).$

$\texttt{reach}(\texttt{X}, \texttt{Y}) \texttt{ :- } \texttt{edge}(\texttt{X}, \texttt{Y}).$ $\rightsquigarrow$ 16 rules (1)

$\texttt{reach}(\texttt{X}, \texttt{Y}) \texttt{ :- } \texttt{reach}(\texttt{X}, \texttt{Z}), \texttt{edge}(\texttt{Z}, \texttt{Y}).$ $\rightsquigarrow$ 64 rules (2)

Potassco

Grounding

$0 \quad \texttt{edge}(\texttt{v}_1, \texttt{v}_2), \texttt{edge}(\texttt{v}_1, \texttt{v}_3), \texttt{edge}(\texttt{v}_2, \texttt{v}_3), \texttt{edge}(\texttt{v}_3, \texttt{v}_4), \ldots$

Potassco

Grounding

0	$\texttt{edge}(\texttt{v}_1,\texttt{v}_2),\texttt{edge}(\texttt{v}_1,\texttt{v}_3),\texttt{edge}(\texttt{v}_2,\texttt{v}_3),\texttt{edge}(\texttt{v}_3,\texttt{v}_4),\ldots$	
1	$\texttt{reach}(\texttt{v}_1,\texttt{v}_2)$:- $\texttt{edge}(\texttt{v}_1,\texttt{v}_2)$	(1)
	$\texttt{reach}(\texttt{v}_1,\texttt{v}_3)$:- $\texttt{edge}(\texttt{v}_1,\texttt{v}_3)$	(1)
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Potassco

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3	$\texttt{reach}(\texttt{v}_1,\texttt{v}_4)$:- $\texttt{edge}(\texttt{v}_3,\texttt{v}_4), \texttt{reach}(\texttt{v}_1,\texttt{v}_3)$	(2)

Potassco

Simplifying while Grounding

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	$\texttt{reach}(\texttt{v}_1,\texttt{v}_3)$:- $\texttt{edge}(\texttt{v}_1,\texttt{v}_3)$	(1)
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	$\texttt{reach}(\texttt{v}_3,\texttt{v}_4)$:- $\texttt{edge}(\texttt{v}_3,\texttt{v}_4)$	(1)
2	$\texttt{reach}(\texttt{v}_1,\texttt{v}_3)$:- $\texttt{edge}(\texttt{v}_2,\texttt{v}_3), \texttt{reach}(\texttt{v}_1,\texttt{v}_2)$	(2)
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Potassco

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	$\texttt{reach}(\texttt{v}_3,\texttt{v}_4)$	(1)
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Potassco

Simplifying while Grounding

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1	$\texttt{reach}(\texttt{v}_1,\texttt{v}_2)$	(1)
	$\texttt{reach}(\texttt{v}_1,\texttt{v}_3)$	(1)
	$\texttt{reach}(\texttt{v}_2,\texttt{v}_3)$	(1)
	$\texttt{reach}(\texttt{v}_3,\texttt{v}_4)$	(1)
2	$\texttt{reach}(\texttt{v}_1,\texttt{v}_3)$:- $\texttt{reach}(\texttt{v}_1,\texttt{v}_2)$	(2)
	$\texttt{reach}(\texttt{v}_1,\texttt{v}_4)$:- $\texttt{reach}(\texttt{v}_1,\texttt{v}_3)$	(2)
	$\texttt{reach}(\texttt{v}_2,\texttt{v}_4)$:- $\texttt{reach}(\texttt{v}_2,\texttt{v}_3)$	(2)
3	$\texttt{reach}(\texttt{v}_1,\texttt{v}_4)$:- $\texttt{edge}(\texttt{v}_3,\texttt{v}_4), \texttt{reach}(\texttt{v}_1,\texttt{v}_3)$	(2)

Potassco

Simplifying while Grounding

0	$\texttt{edge}(\texttt{v}_1,\texttt{v}_2), \texttt{edge}(\texttt{v}_1,\texttt{v}_3), \texttt{edge}(\texttt{v}_2,\texttt{v}_3), \texttt{edge}(\texttt{v}_3,\texttt{v}_4), \ldots$	
1	$\texttt{reach}(\texttt{v}_1,\texttt{v}_2)$	(1)
	$\texttt{reach}(\texttt{v}_1,\texttt{v}_3)$	(1)
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3	$\texttt{reach}(\texttt{v}_1,\texttt{v}_4) \texttt{ :- } \texttt{reach}(\texttt{v}_1,\texttt{v}_3)$	(2)

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- Similar simplifications are done with negative information

Potassco

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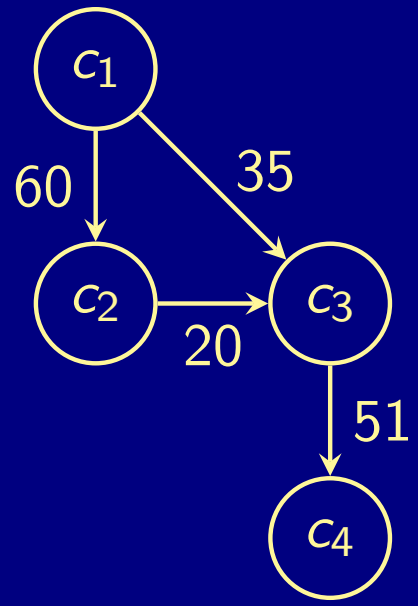
- Similar simplifications are done with negative information
- Grounding is sufficient for solving this reachability problem

Potassco

Company Controls Problem



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$$\texttt{controls}(\texttt{X}, \texttt{Y}) \texttt{ :-}$$
$$\#\texttt{sum}\,\{\texttt{S} : \texttt{owns}(\texttt{X}, \texttt{Y}, \texttt{S});$$
$$\texttt{S}, \texttt{Z} : \texttt{controls}(\texttt{X}, \texttt{Z}), \texttt{owns}(\texttt{Z}, \texttt{Y}, \texttt{S})\} > 50,$$
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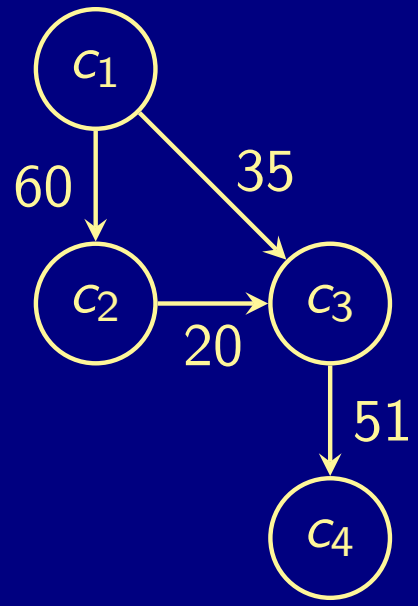

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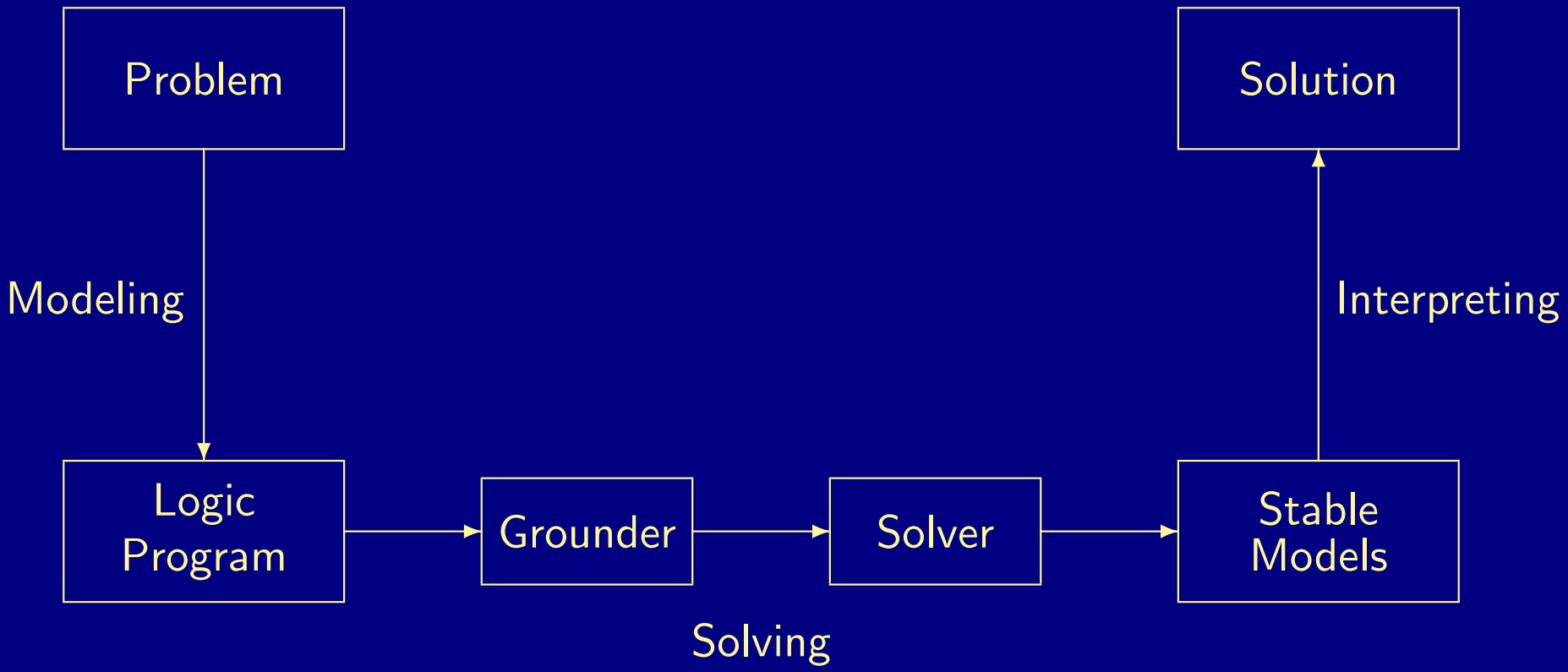
ASP Grounding



- `satgrnd` is available at `http://research.ics.aalto.fi/software/sat/satgrnd`

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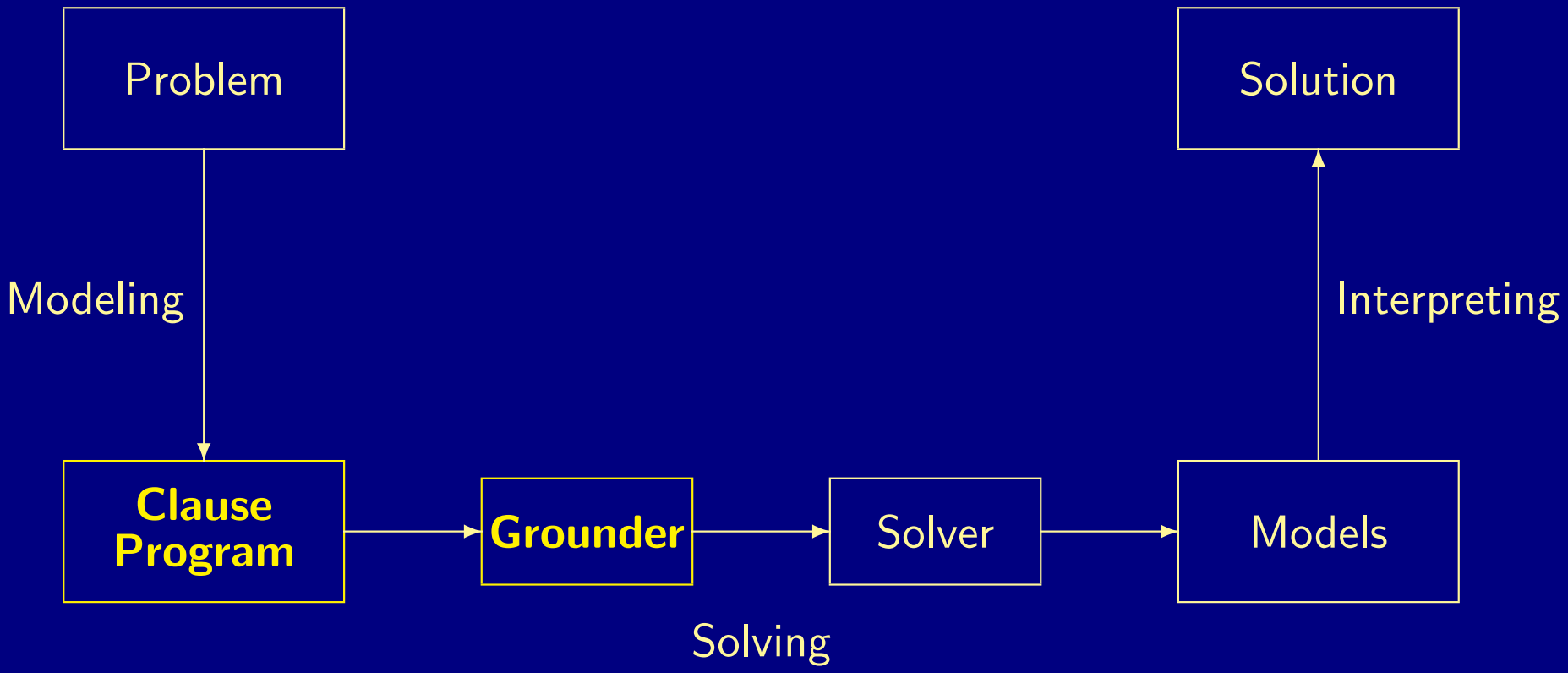
ASP Grounding for SAT



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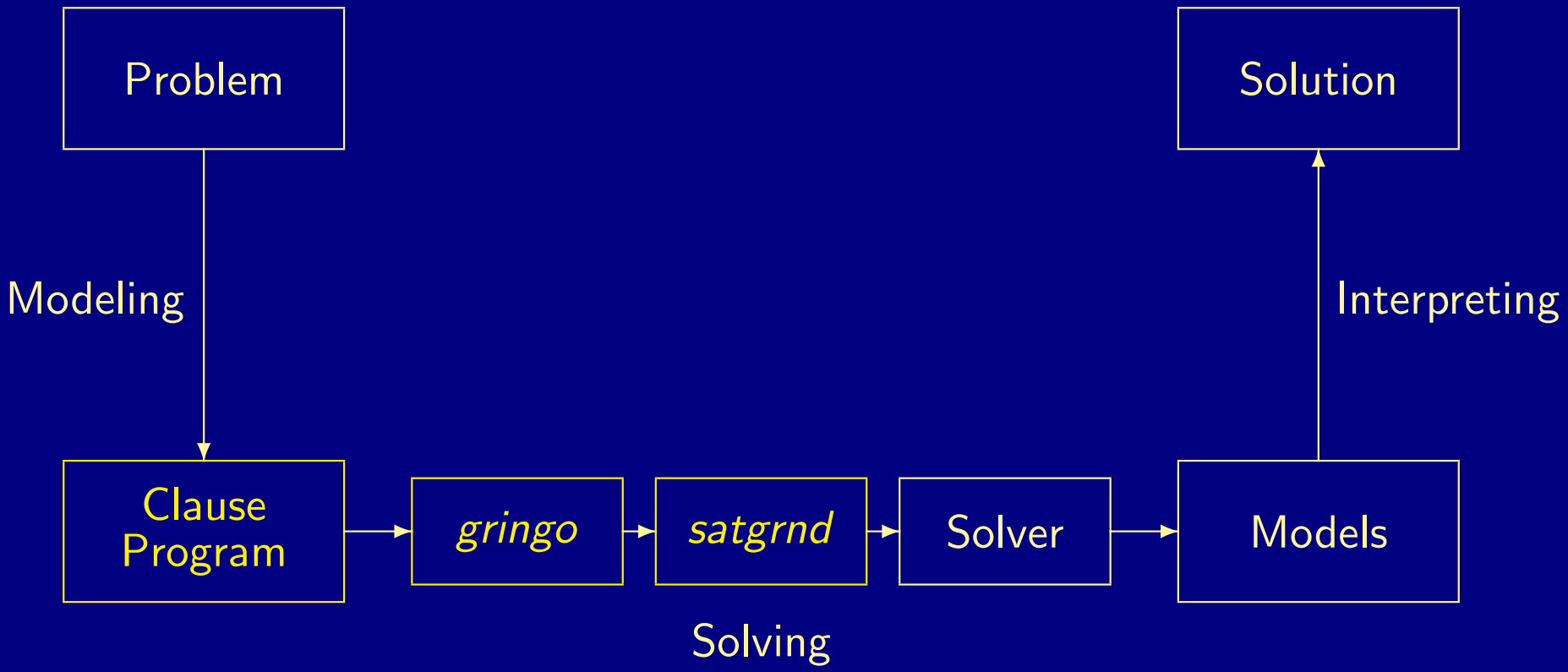

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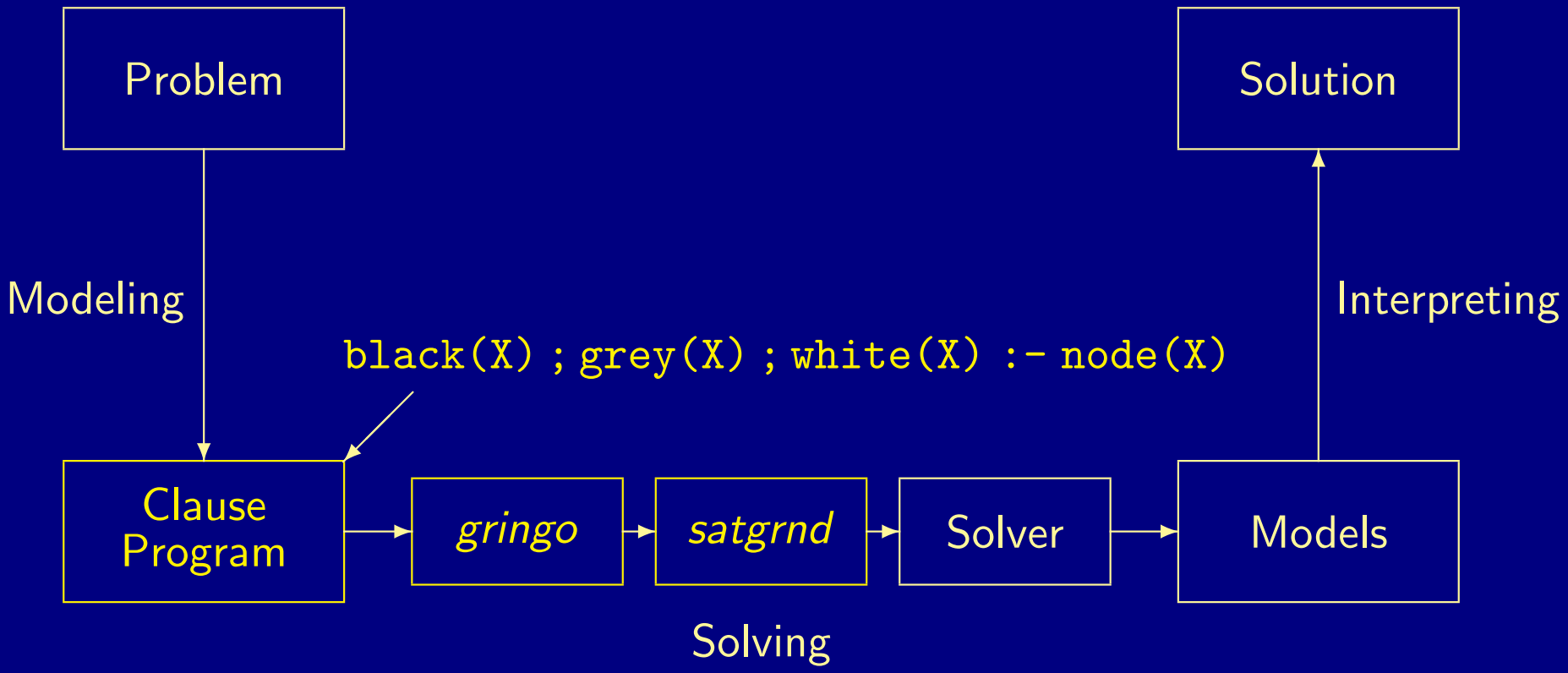

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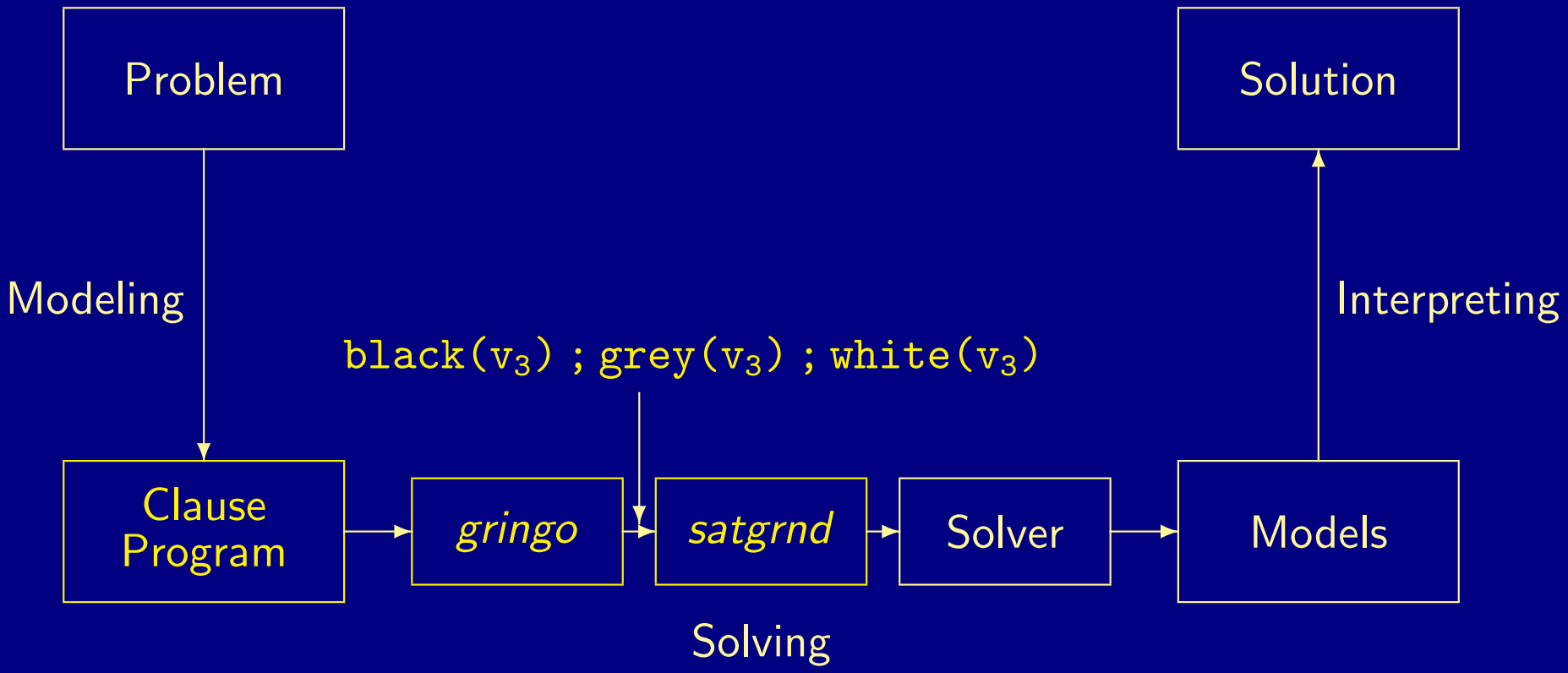

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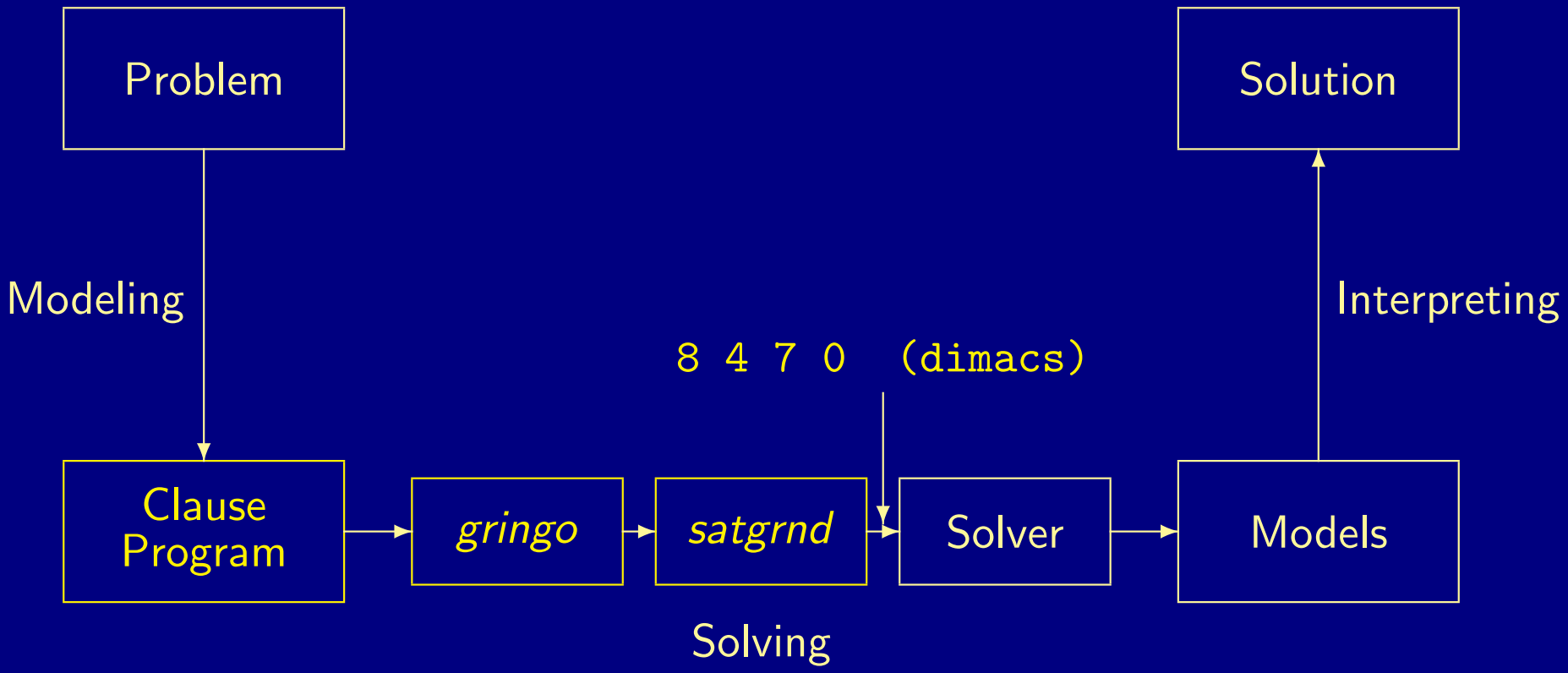

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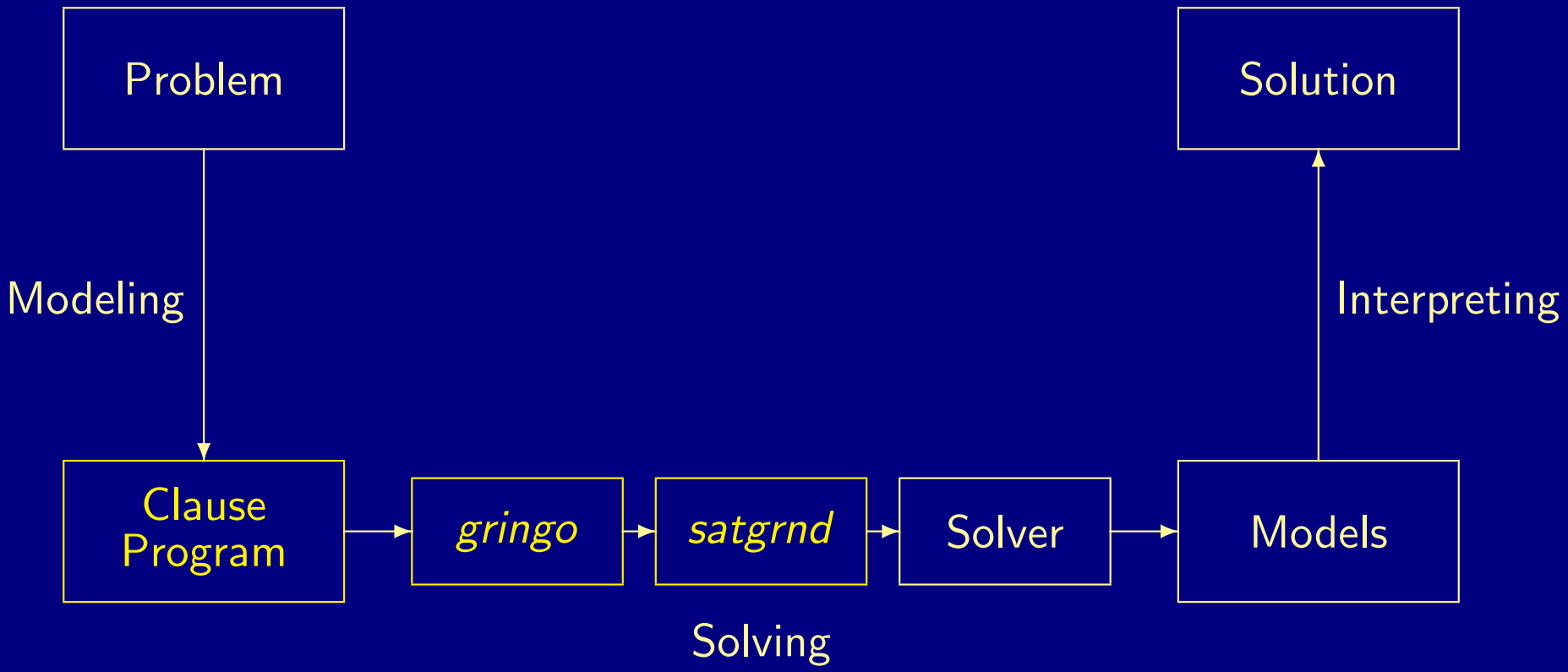

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Outline

Potassco

The solver *clasp*

- Beyond deciding (stable) model existence, *clasp* allows for
  - Enumeration (without solution recording)
  - Projective enumeration (without solution recording)
  - Intersection and Union (linear solving process)
  - Multi-objective Optimization
  - and combinations thereof
- *clasp* allows for
  - ASP solving (*aspif* and *smodels* format)
  - SAT and MaxSAT solving (extended *dimacs* format)
  - PB solving (*opb* and *wbo* format)
- *clasp* pursues a coarse-grained, task-parallel approach to parallel search via shared memory multi-threading

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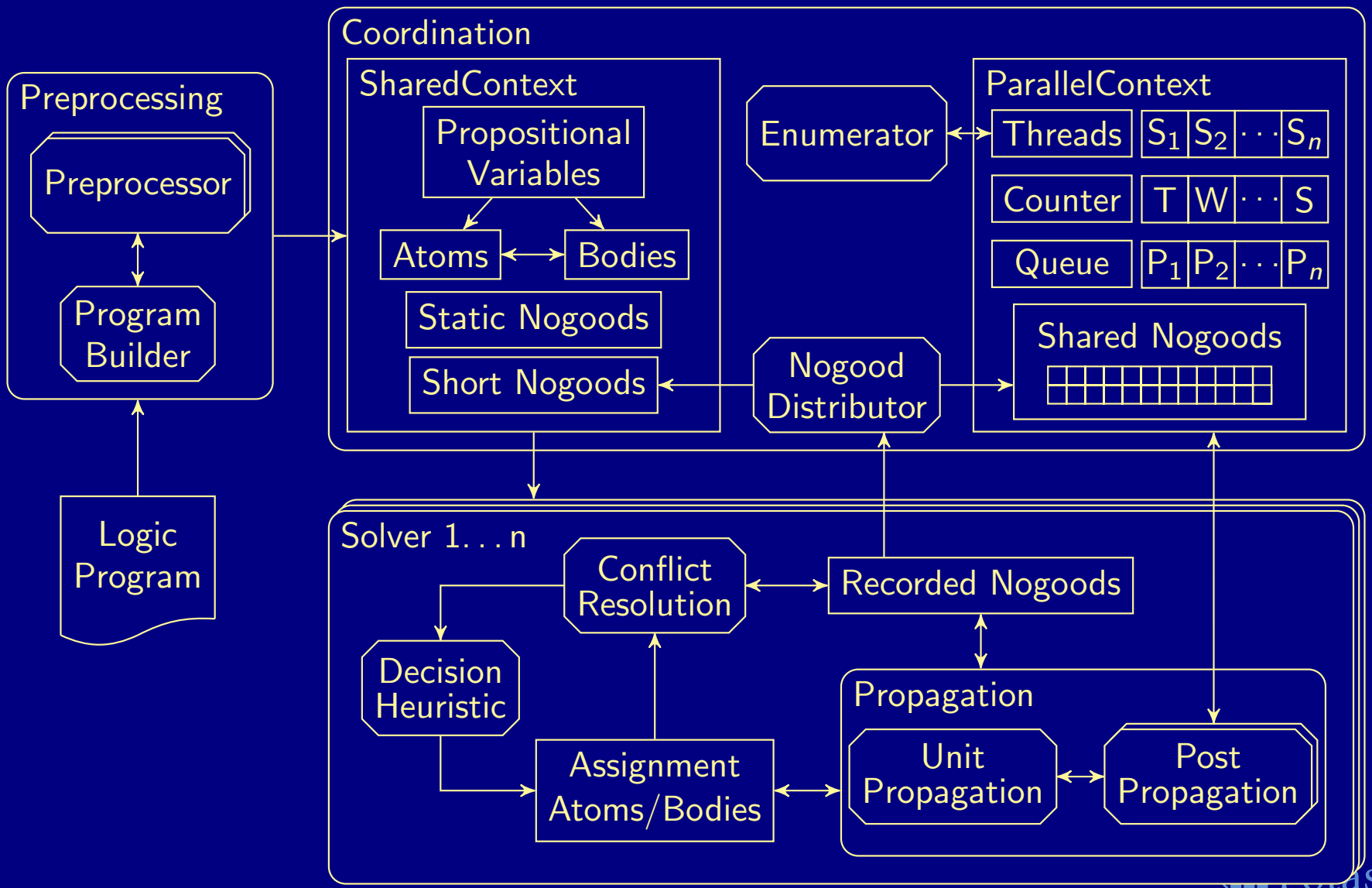
Multi-threaded architecture of *clasp*



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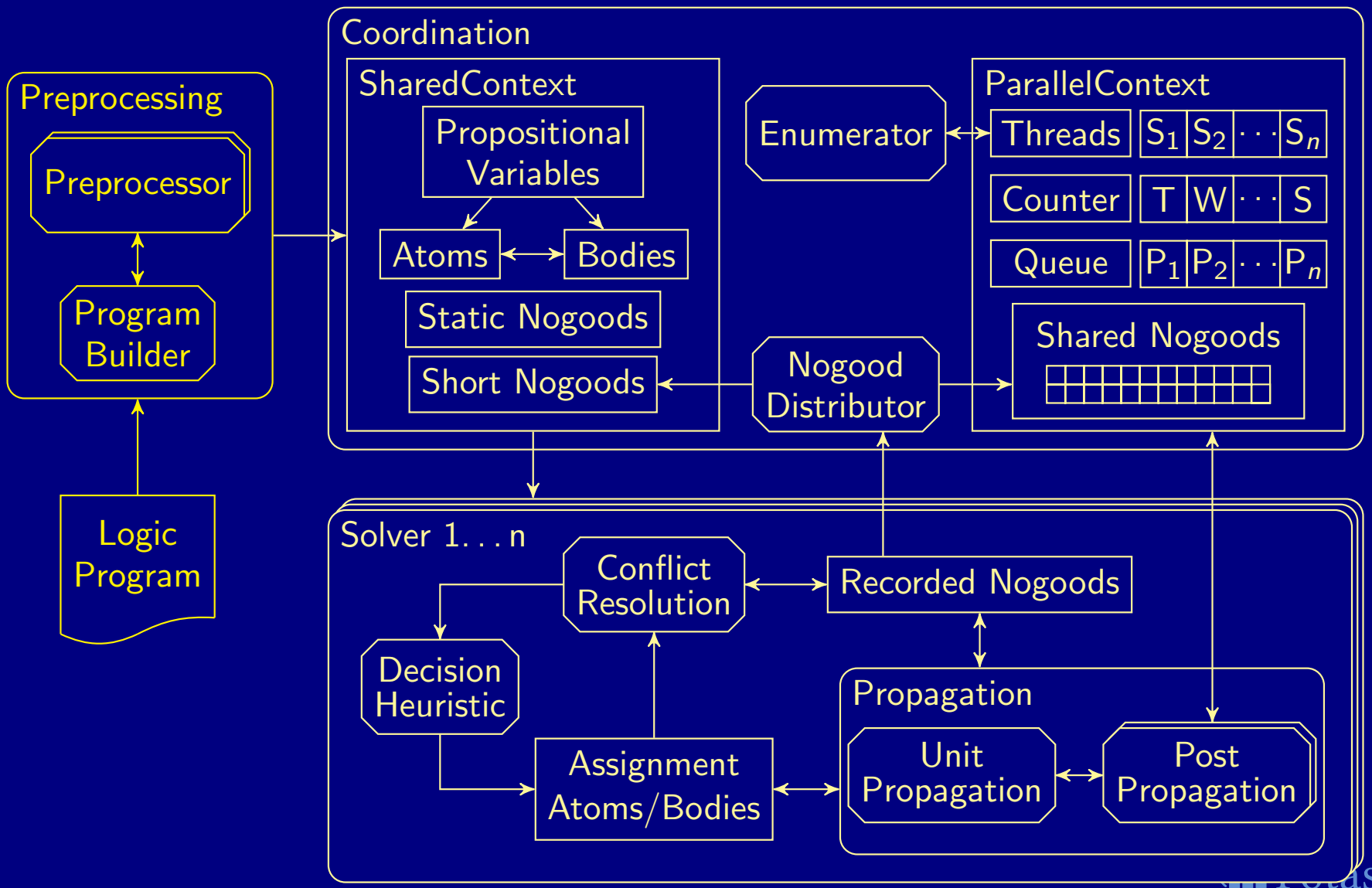

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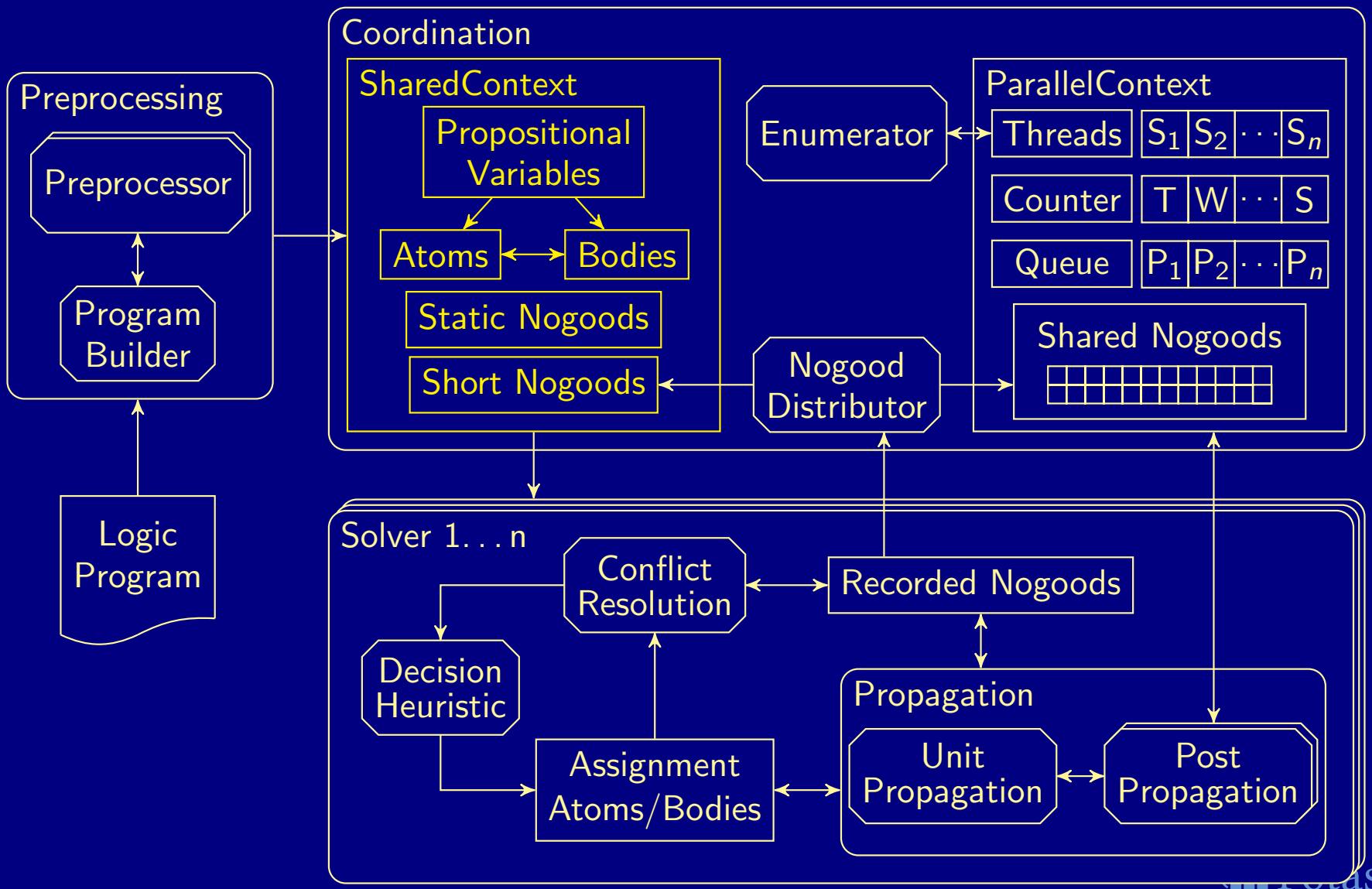

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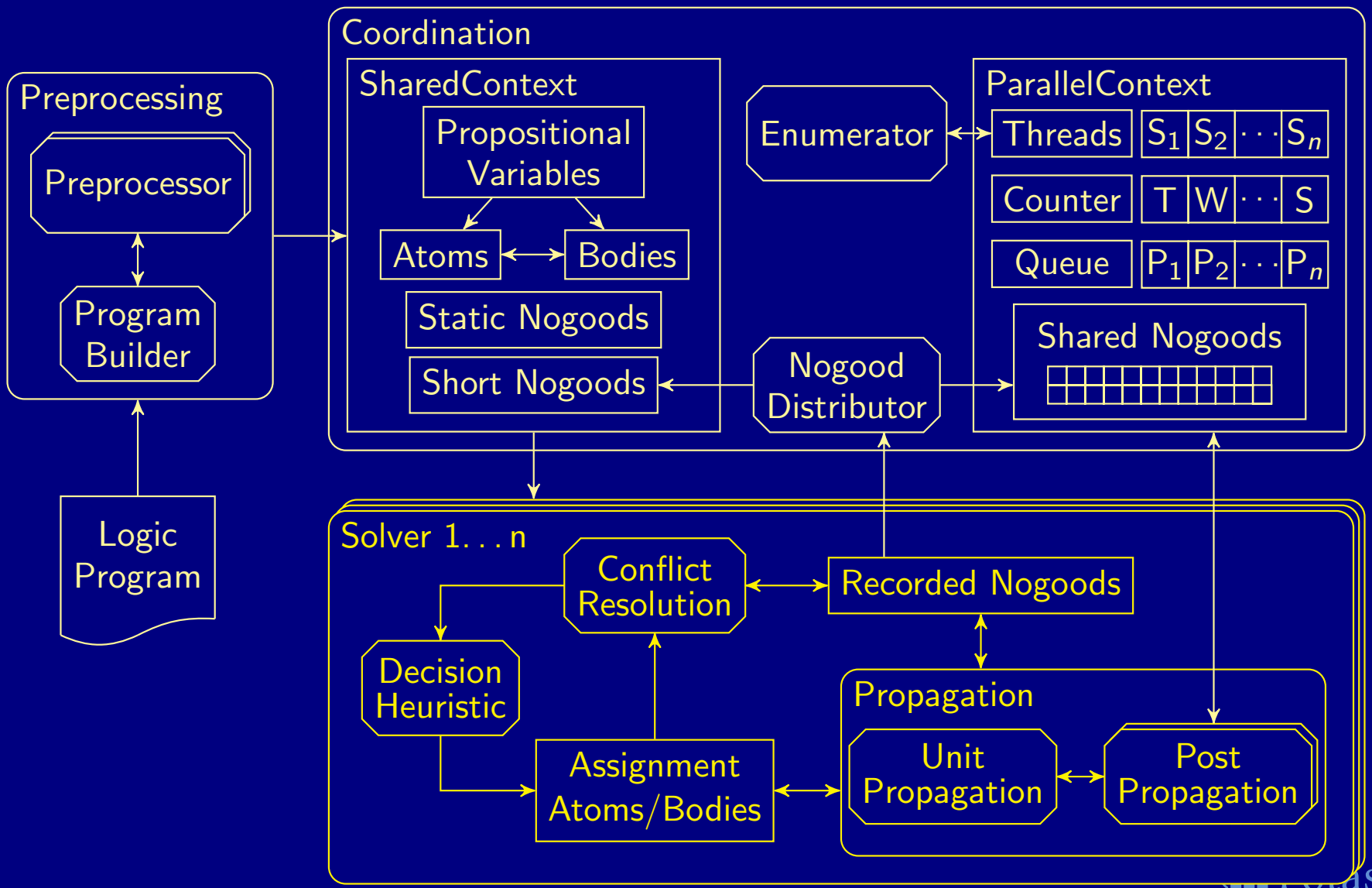

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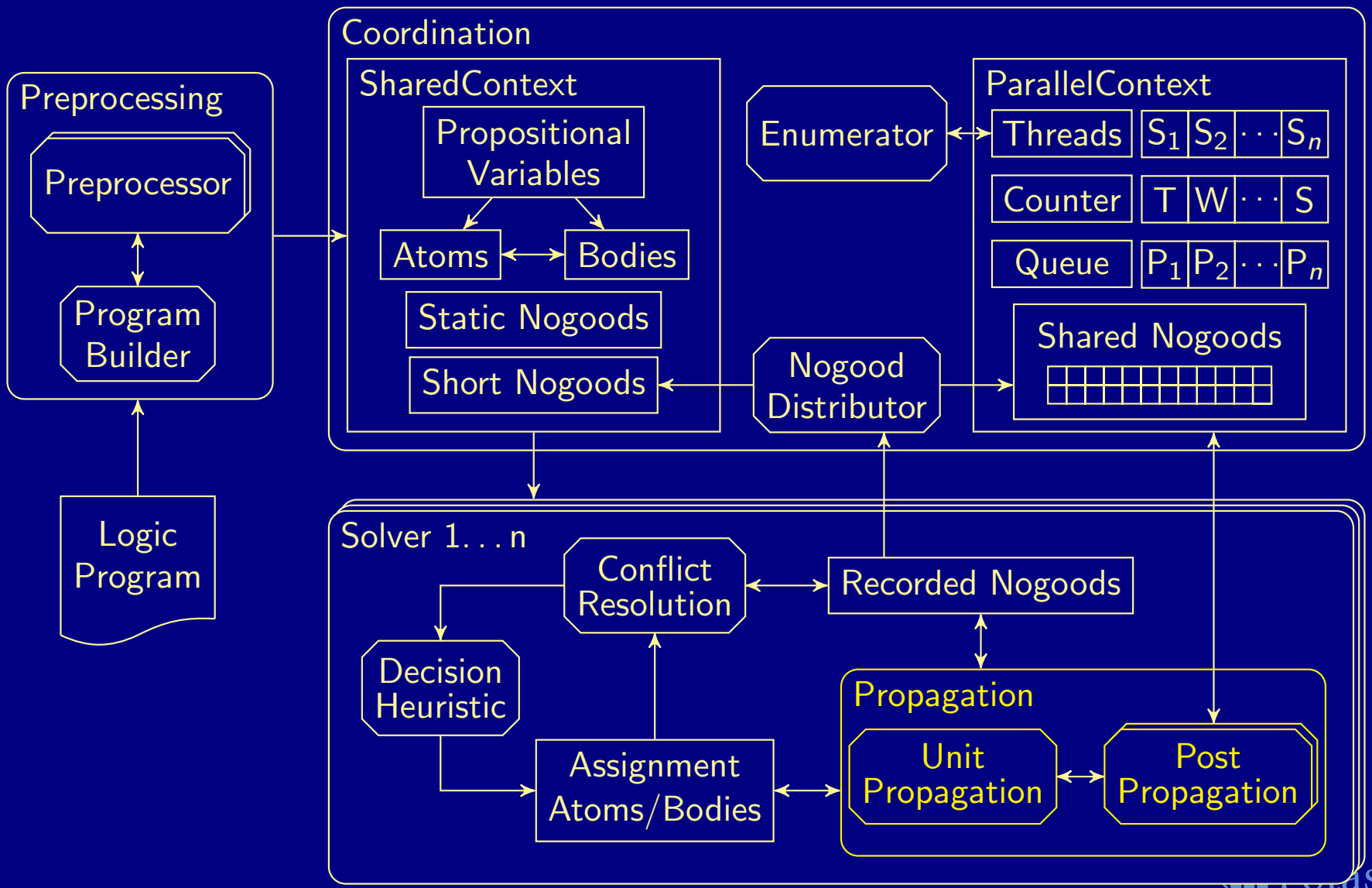

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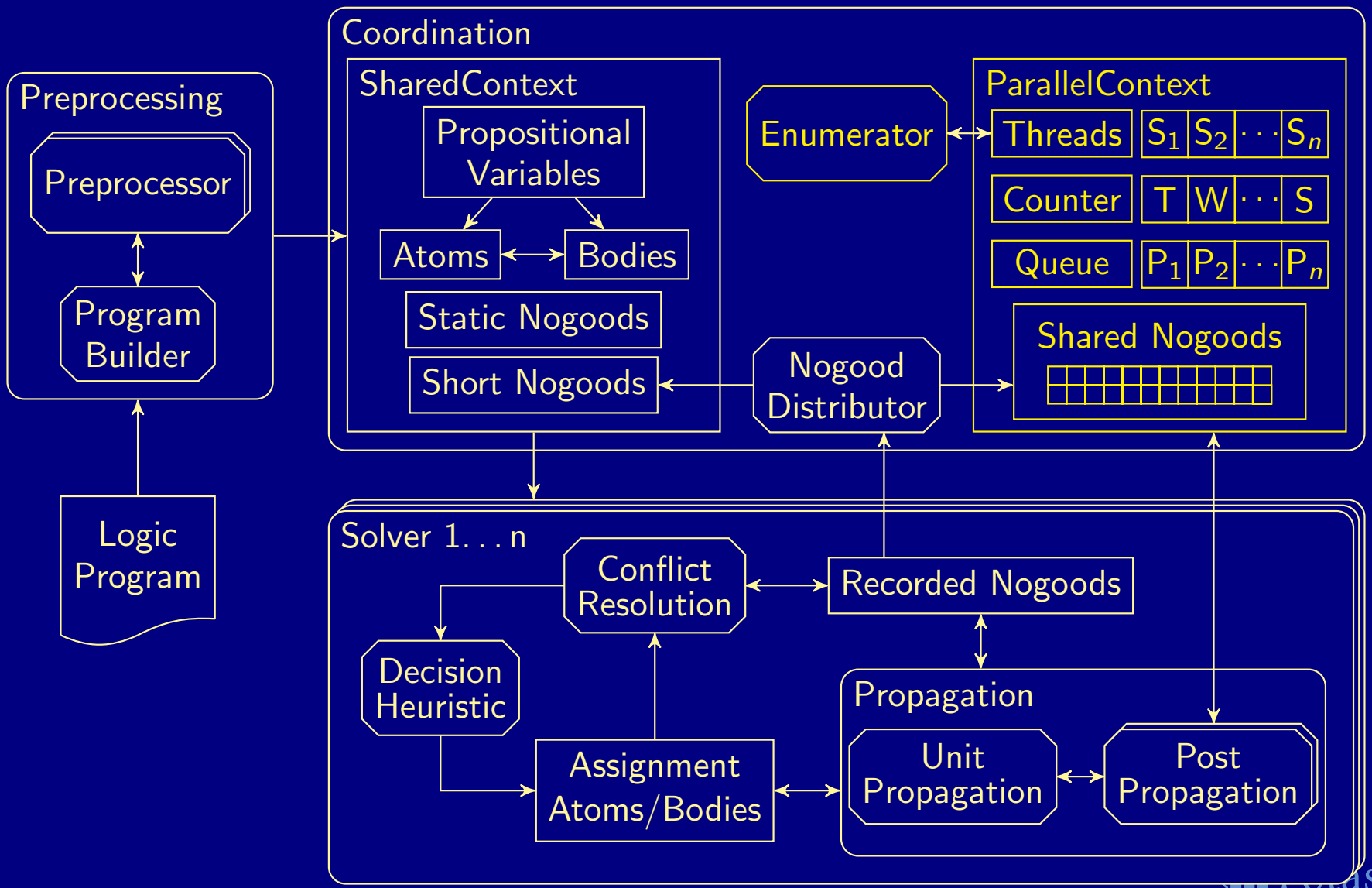

Outline

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Heuristic programming

- Heuristic directives

  $$\texttt{\#heuristic}\ a\ :\ l_1, \ldots, l_n.\ [k@p, m]$$

  where
  - a is an atom, and $l_1, \ldots, l_n$ are literals
  - k and p are integers
  - m is a heuristic modifier among `init`, `factor`, `level`, `sign`, `true`, or `false`
- Implementation bias on the `vsids` heuristic in *clasp*
- Examples

  Boosting convergence when optimizing

  ```
  #heuristic cycle(X,Y) : edge(X,Y). [1,false]
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  Backward search when planning

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Potassco

Outline

Potassco

Motivation

- Multi-shot solving
  is about solving continuously changing logic programs
  in an operative way

- Application areas

  Agents, Assisted Living, Robotics, Planning, Query-answering, etc

- Why

  ASP is non-monotonic
  ASP involves both grounding and solving

- How

  clingo 4 (*clingo* = *gringo* + *clasp*)

Potassco

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Potassco

Clingo = ASP + Control

- ASP

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#program <name> [ (<parameters>) ]
          #program play(t).
#external <atom> [ :  <body> ]
          #external mark(X,Y,P,t) : field(X,Y), player(P).
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- Control

Lua (www.lua.org)

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prg:solve(), prg:ground(parts), ...
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Python (www.python.org)

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C and Prolog embeddings are available soon

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in ASP: embedded scripting language (#script)
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    - Example `#external mark(X,Y,P,t) : field(X,Y), player(P).`
- Control
  - Lua (`www.lua.org`)
    - Example `prg:solve(), prg:ground(parts), ...`
  - Python (`www.python.org`)
    - Example `prg.solve(), prg.ground(parts), ...`
  - C and Prolog embeddings are available soon
- Integration
  - in ASP: embedded scripting language (`#script`)
  - in Lua/Python: library import (`import gringo`)

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Vanilla *Clingo*

```
#script (python)
def main(prg):
    parts = []
    parts.append(("base", []))
    prg.ground(parts)
    prg.solve()
#end.
```

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```

Incremental solving

```
#program base.

p(0).

#program step (t).

p(t) :- p(t-1).

#program check (t).
#external plug(t).

:- not p(42), plug(t).
```

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```

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Controlling incremental solving

```
from sys import stdout
from gringo import SolveResult, Fun, Control

prg = Control()
prg.load("inc.lp")

ret, parts, i = SolveResult.UNSAT, [], 1
parts.append(("base", []))
while ret == SolveResult.UNSAT:
    parts.append(("step", [i]))
    parts.append(("check", [i]))
    prg.ground(parts)
    prg.release_external(Fun("plug", [i-1]))
    prg.assign_external(Fun("plug", [i]), True)
    f = lambda m: stdout.write(str(m))
    ret, parts, i = prg.solve(on_model=f), [], i+1
```

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Outline

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Motivation

- Preference handling
  involves the combination of qualitative and quantitative preferences
- ASP systems provide optimization statements representing (lexicographically ordered) objective functions using summation
- Goal a framework for handling preferences among the (stable) models of logic programs
  - capturing existing approaches and
  - allowing for an easy implementation of new ones
- Implementation *asprin*

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Potassco

Example

Your vacation (logic) program ...

... talking about *sauna*, *dive*, *hike*, *bungee*, *hot*, etc

$\#preference(bucks, less(weight))\{40 : sauna, 70 : dive\}$

$\#preference(fun, superset)\{sauna, dive, hike, \neg bungee\}$

$\#preference(temps, aso)\{dive > sauna \,\|\, hot, sauna > dive \,\|\, \neg hot\}$

$\#preference(all, pareto)\{name(bucks), name(fun), name(temps)\}$

$\#optimize(all)$

Potassco

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Outline

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Motivation

- Confession ASP is not a silver bullet
- ASP modulo theories
  is about integrating dedicated reasoning procedure
- Application areas
  Agents, Assisted Living, Robotics, Planning, Scheduling, Bio- and Cheminformatics
- Caveats
  - ASP is non-monotonic
  - ASP involves both grounding and solving
- Implementation *clingo* 5 (and *clingcon* for CSP)

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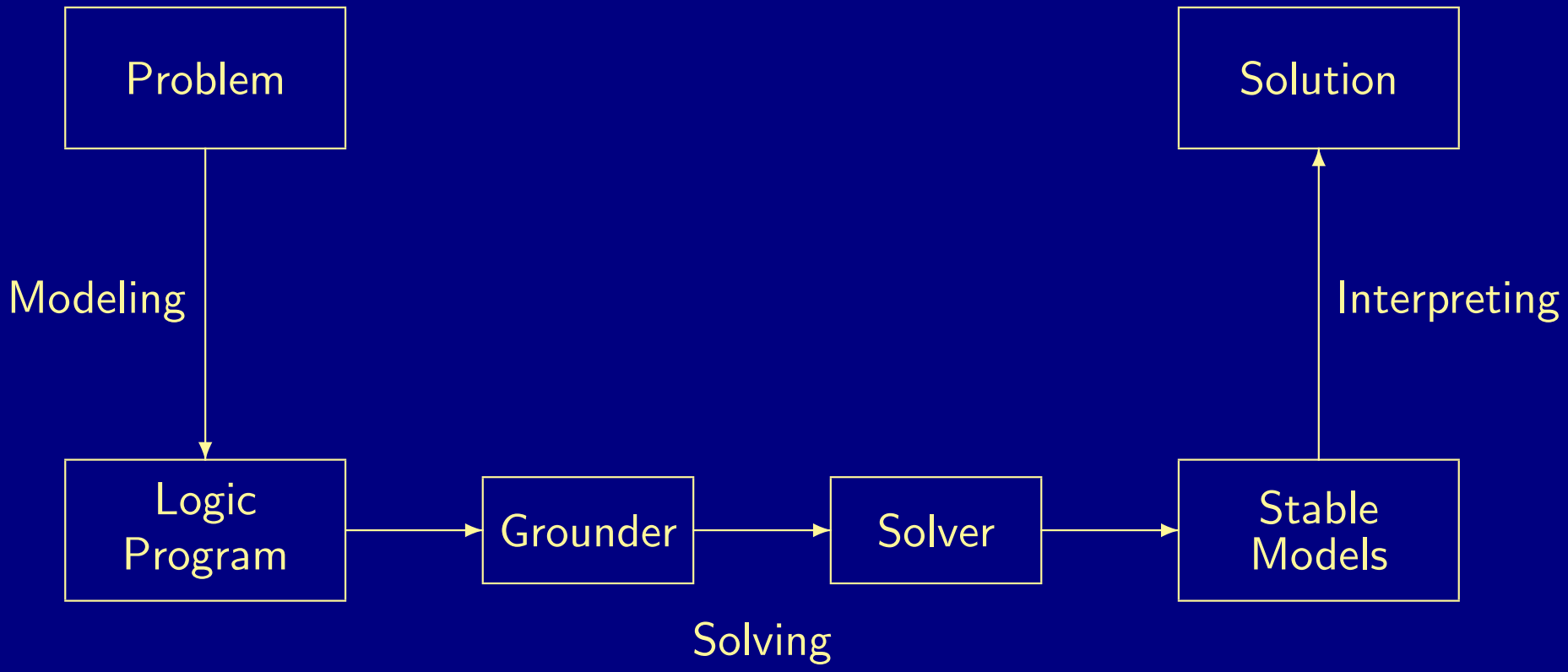
ASP solving process modulo theories



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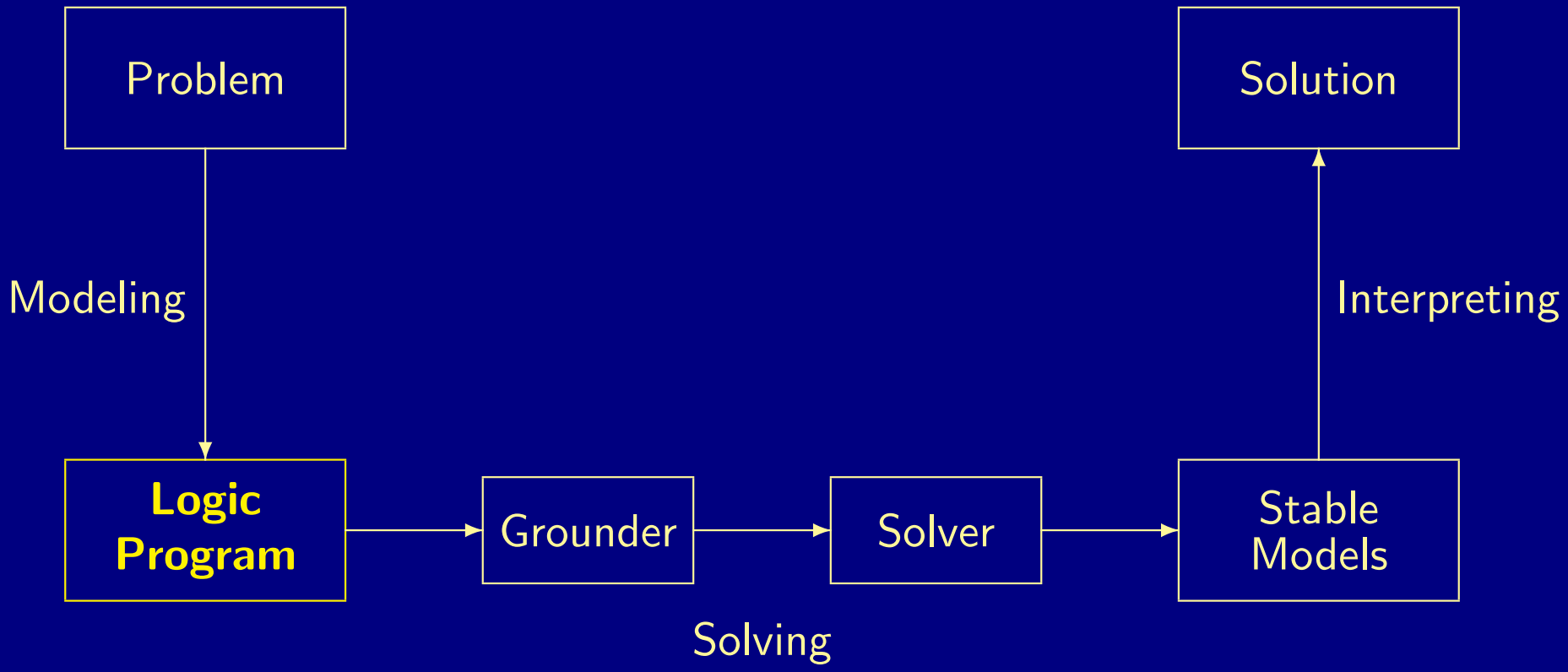

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Linear constraints

```
#theory csp {
    linear_term {
      + : 5, unary;
      - : 5, unary;
      * : 4, binary, left;
      + : 3, binary, left;
      - : 3, binary, left
    };

    dom_term {
      +  : 5, unary;
      -  : 5, unary;
      .. : 1, binary, left
    };

    &dom/0 : dom_term, {=}, linear_term, any;
    &sum/0 : linear_term, {<=,=,>=,<,>,!=}, linear_term, any;
    &show/0 : show_term, directive;
    &distinct/0 : linear_term, any;
    &minimize/0 : minimize_term, directive
}.
```

```
show_term {
  / : 1, binary, left
};

minimize_term {
  + : 5, unary;
  - : 5, unary;
  * : 4, binary, left;
  + : 3, binary, left;
  - : 3, binary, left;
  @ : 0, binary, left
};
```

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send+more=money

```
#include "csp.lp".

digit(1,3,s).   digit(2,3,m).   digit(sum,4,m).
digit(1,2,e).   digit(2,2,o).   digit(sum,3,o).
digit(1,1,n).   digit(2,1,r).   digit(sum,2,n).
digit(1,0,d).   digit(2,0,e).   digit(sum,1,e).
                                digit(sum,0,y).

base(10).
exp(E) :- digit(_,E,_).

power(1,0).
power(B*P,E) :- base(B), power(P,E-1), exp(E), E>0.

summand(N) :- digit(N,_,_), N!= sum.
high(D) :- digit(N,E,D), not digit(N,E+1,_).

&dom {0..9} = X :- digit(_,_,X).

&sum { M*D : digit(N,E,D),   power(M,E), summand(N);
      -M*D : digit(sum,E,D), power(M,E)       } = 0.

&sum { D } > 0 :- high(D).

&distinct { D : digit(_,_,D) }.

&show { D : digit(_,_,D) }.
```

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```

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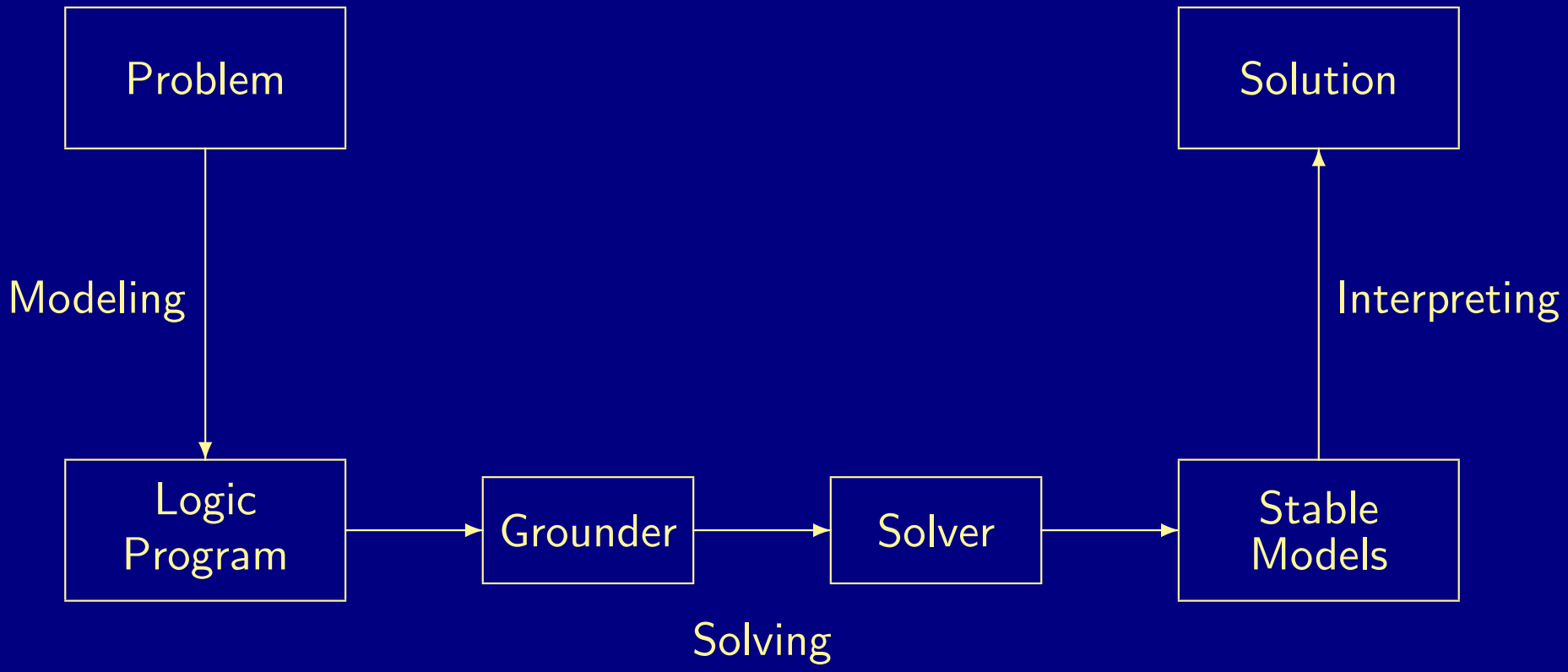
ASP solving process modulo theories



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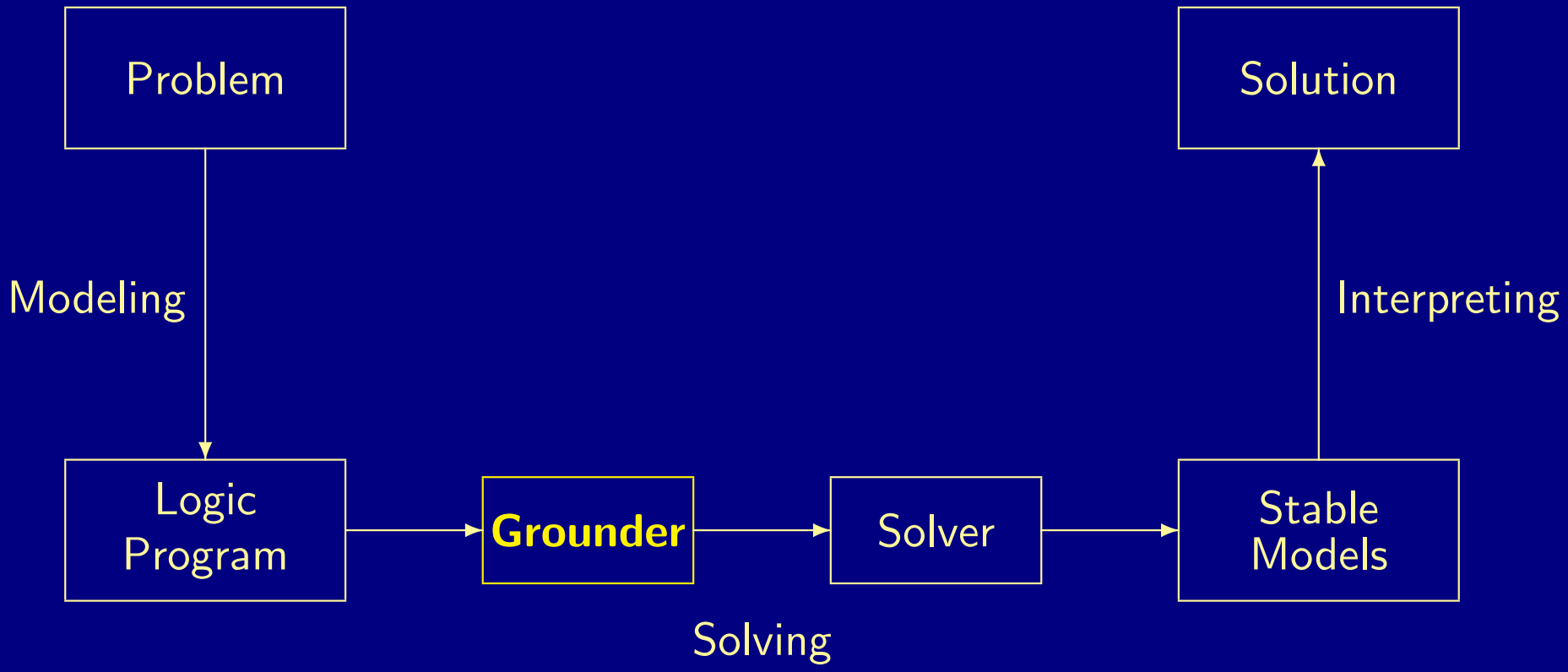

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send+more=money

```
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digit(1,2,e).   digit(2,2,o).   digit(sum,3,o).
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digit(1,0,d).   digit(2,0,e).   digit(sum,1,e).
                                digit(sum,0,y).

base(10).
exp(0).  exp(1).  exp(2).  exp(3).  exp(4).

power(1,0).
power(10,1).  power(100,2).  power(1000,3).  power(10000,4).

summand(1).  summand(2).
high(s).  high(m).

&dom{0..9}=s. &dom{0..9}=m. &dom{0..9}=e. &dom{0..9}=o. &dom{0..9}=n. &dom{0..9}=r. &do

&sum{           1000*s;  100*e;  10*n;  1*d;
                1000*m;  100*o;  10*r;  1*e;
     -10000*m; -1000*o; -100*n; -10*e; -1*y  } = 0.

&sum{s} > 0.  &sum{m} > 0.

&distinct{s; m; e; o; n; r; d; y}.

&show{s; m; e; o; n; r; d; y}.
```
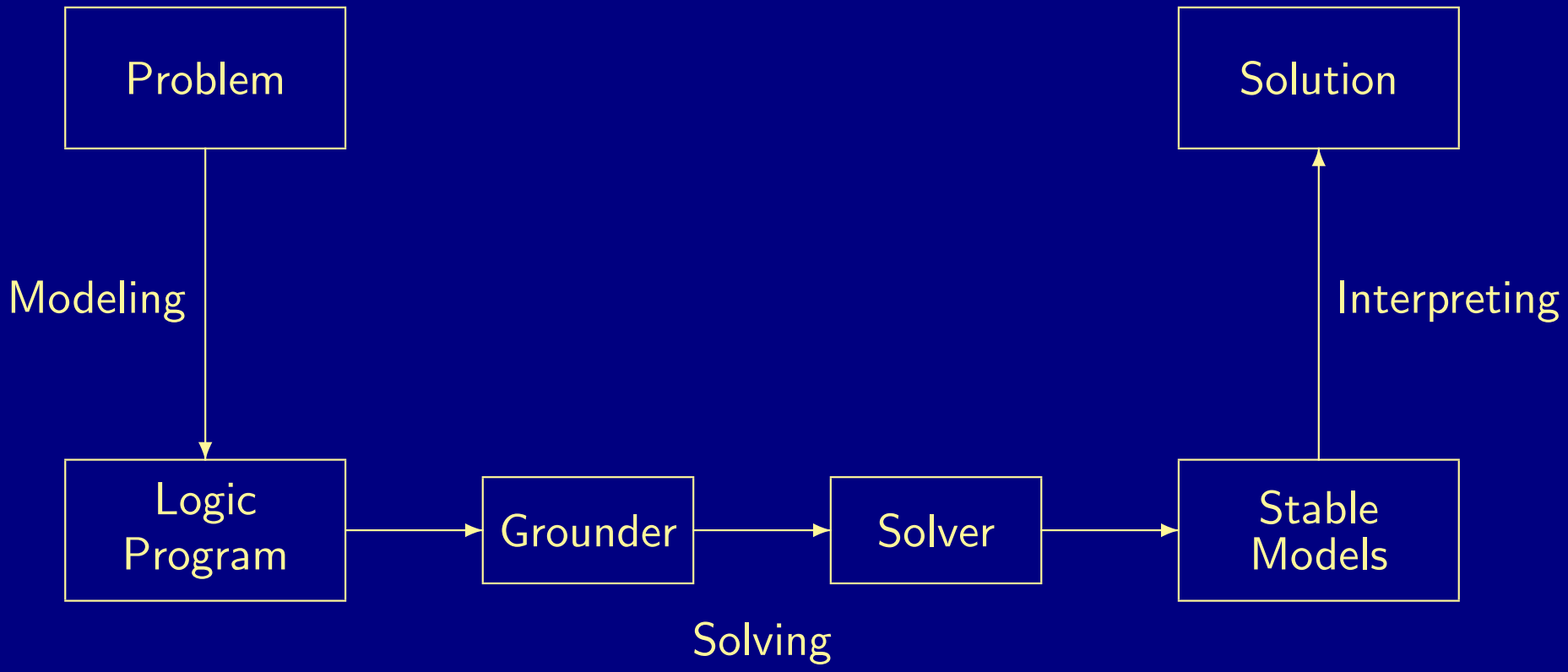

Potassco

ASP solving process modulo theories



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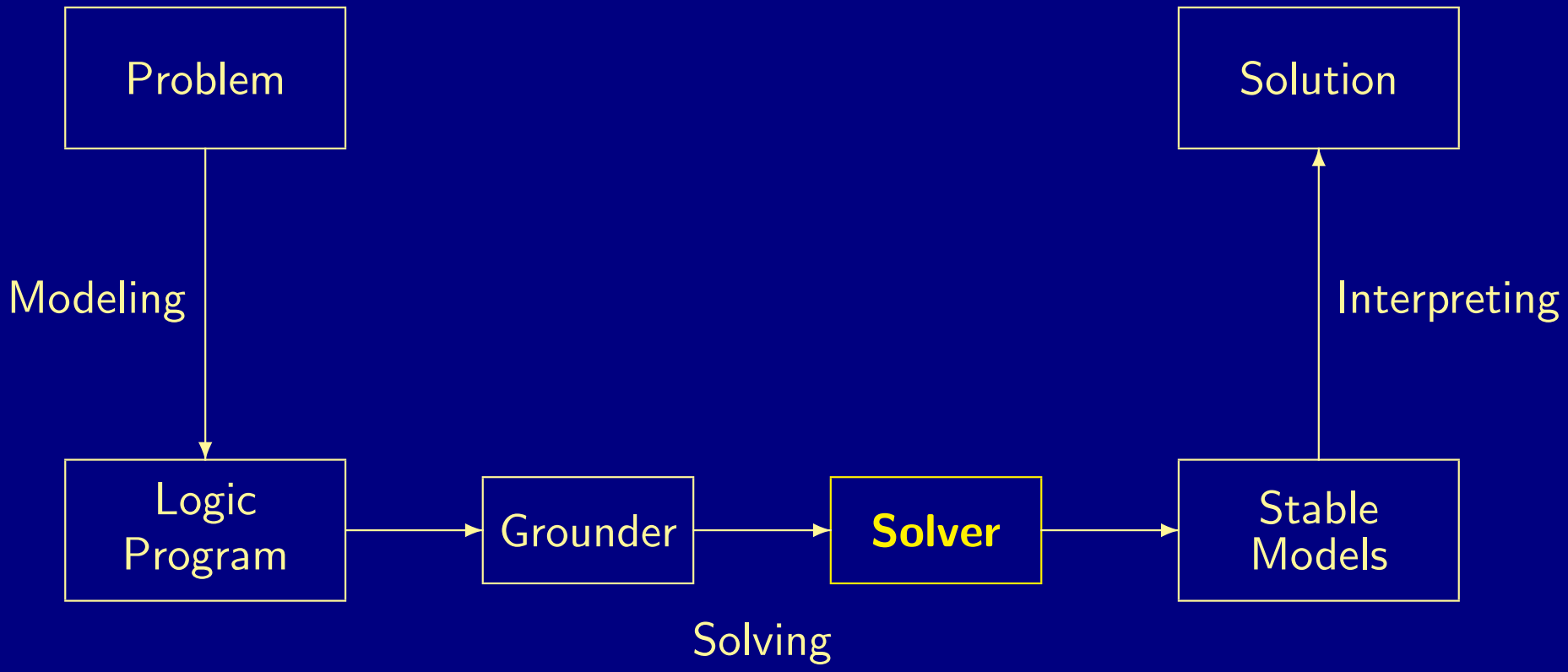

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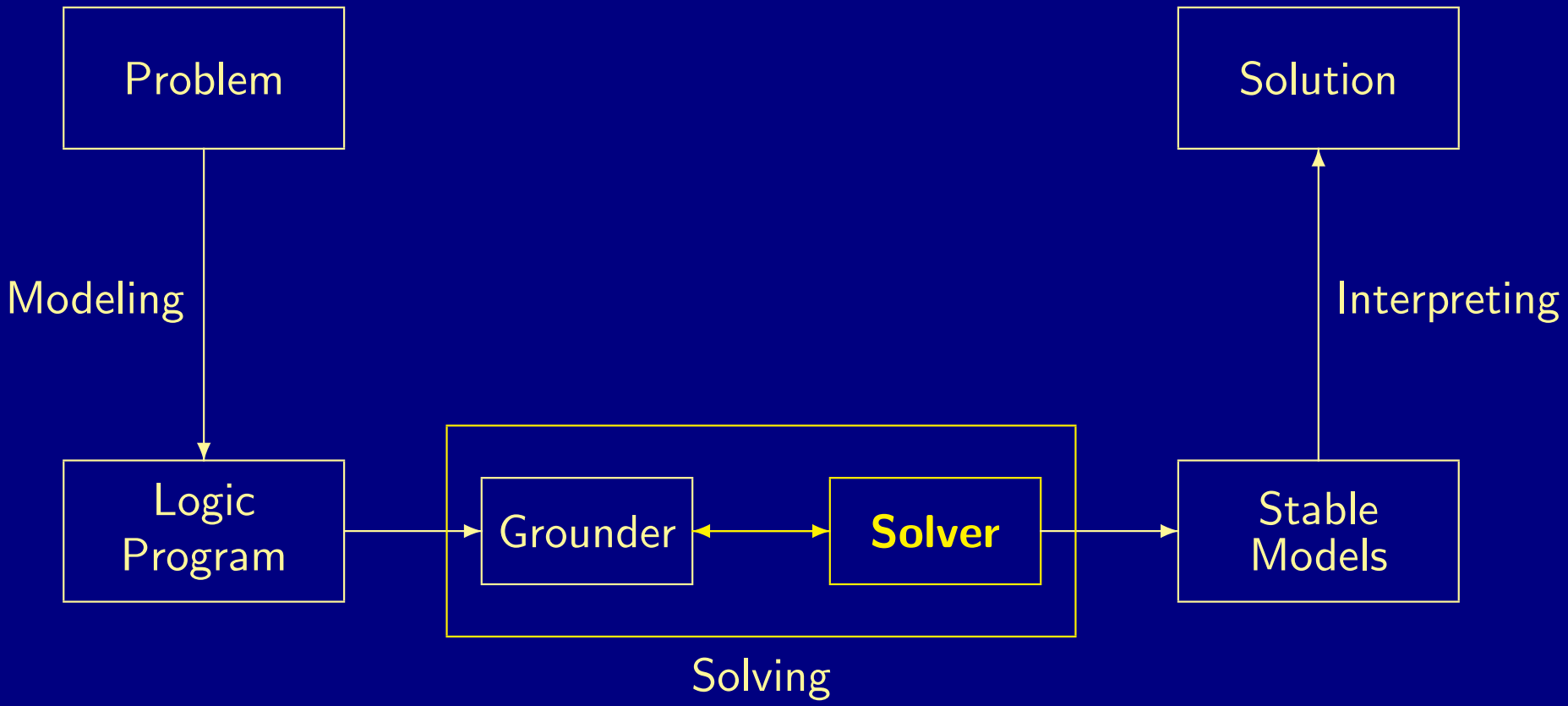

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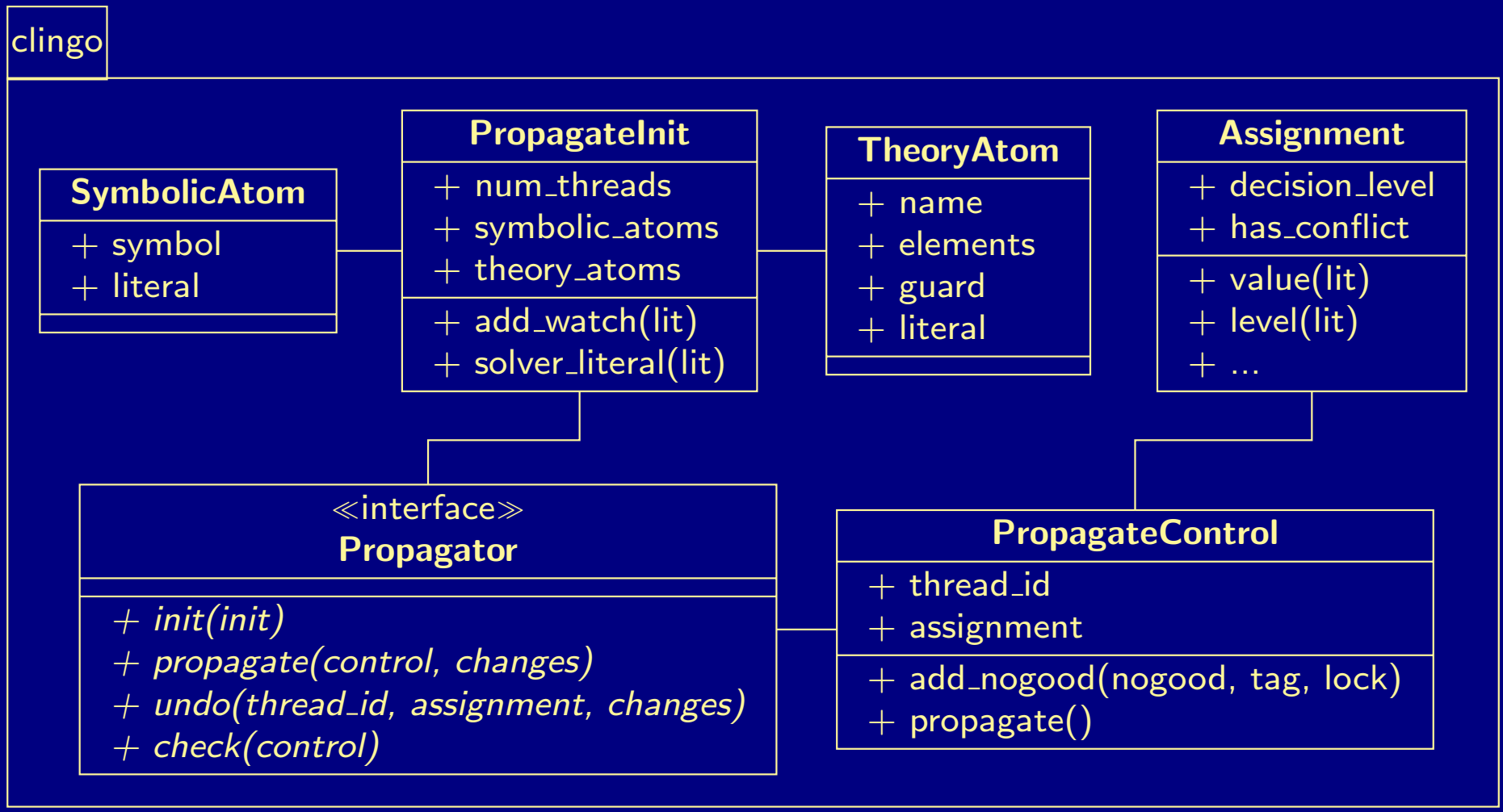
Propagator interface



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The *dot* propagator

```
#script (python)

import sys
import time

class Propagator:
    def init(self, init):
        self.sleep = .1
        for atom in init.symbolic_atoms:
            init.add_watch(init.solver_literal(atom.literal))

    def propagate(self, ctl, changes):
        for l in changes:
            sys.stdout.write(".")
            sys.stdout.flush()
            time.sleep(self.sleep)
        return True

    def undo(self, solver_id, assign, undo):
        for l in undo:
            sys.stdout.write("\b \b")
            sys.stdout.flush()
            time.sleep(self.sleep)

def main(prg):
    prg.register_propagator(Propagator())
    prg.ground([("base", [])])
    prg.solve()
    sys.stdout.write("\n")

#end.
```

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Outline

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Take home message

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Take home message

ASP = DB+LP+KR+SAT

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Take home message

$$\textbf{ASP} = \textbf{DB}+\textbf{LP}+\textbf{KR}+\textbf{SMT}^{n}$$

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Take home message

ASP = DB+LP+KR+SMT^n

http://potassco.sourceforge.net

Potassco

Epilogue

After all, it's all about Tweety!

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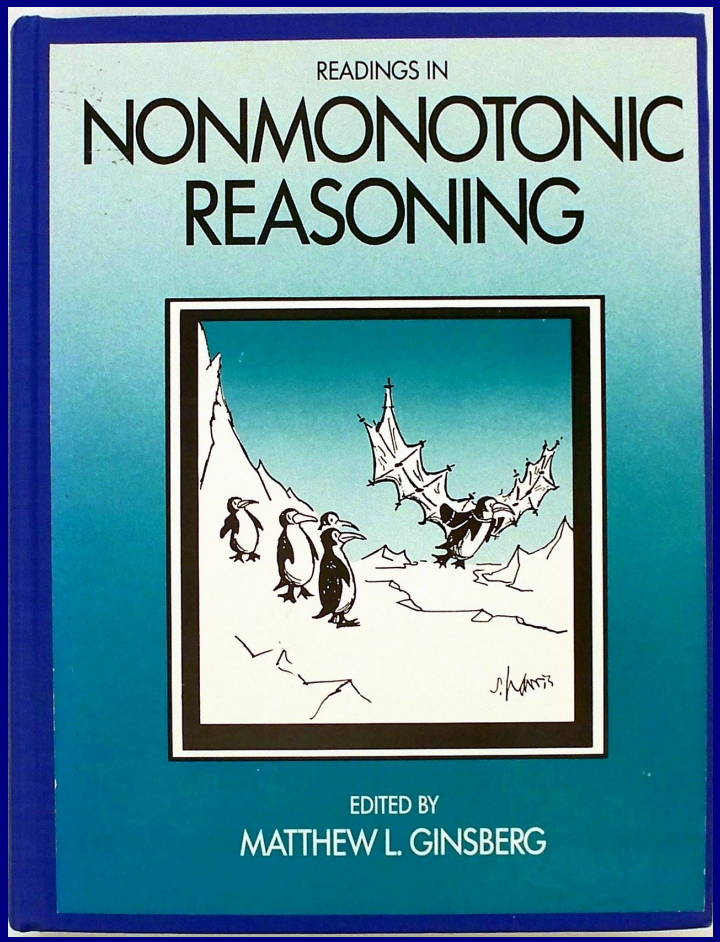

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Epilogue

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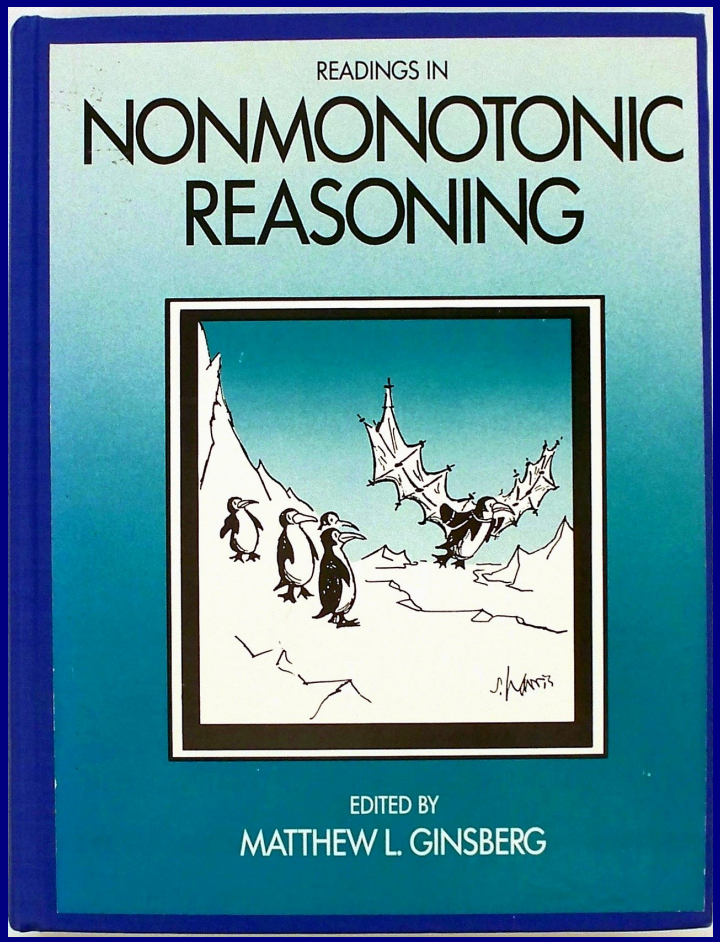

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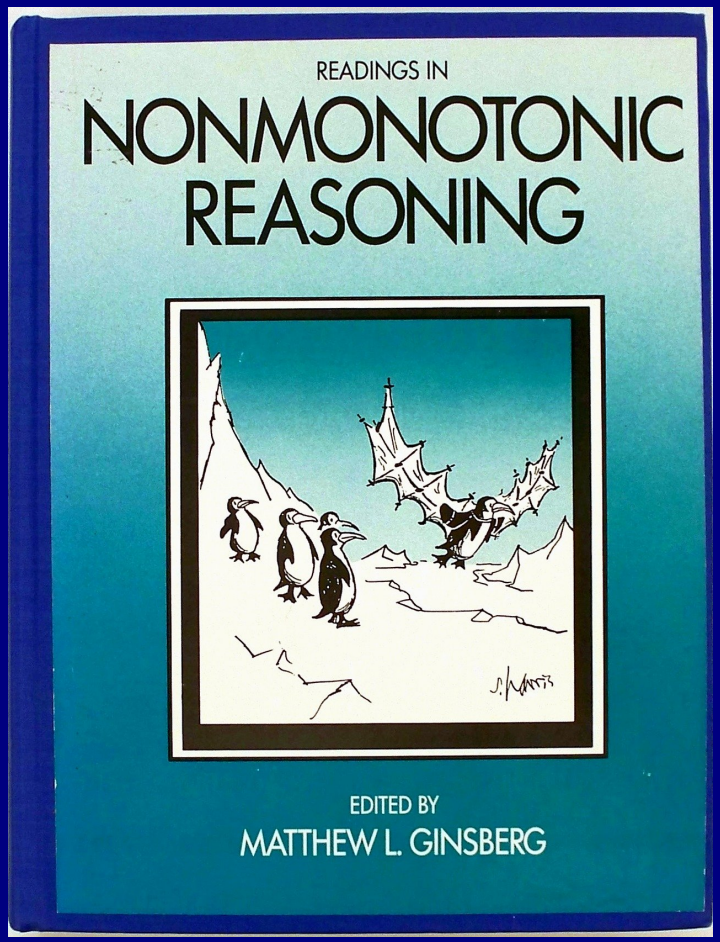

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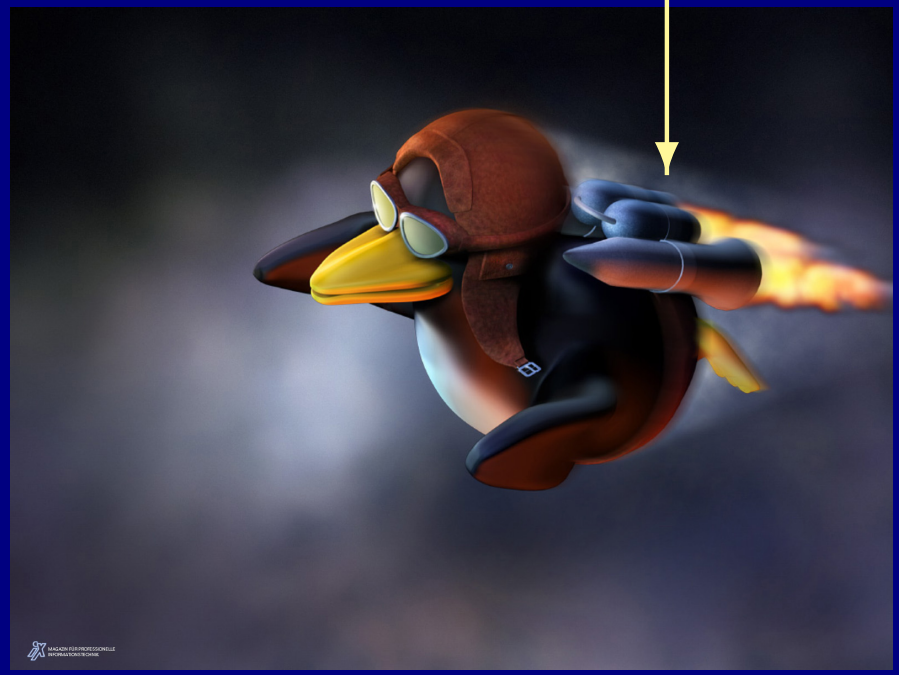
SAT



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