ASP foundations and applications

Torsten Schaub

Potassco

Rough Roadmap

1. Foundations
   1. Motivation
   2. Introduction
   3. Modeling
2. Applications
   4. Multi-shot solving
   5. Theory reasoning
   6. Heuristic prgramming
   7. Preferences and optimization
3. Summary

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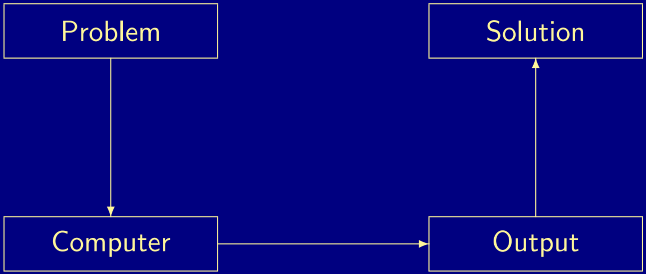
Rough Roadmap

Motivation: Overview

Outline

Informatics

“What is the problem?” versus *“How to solve the problem?”*



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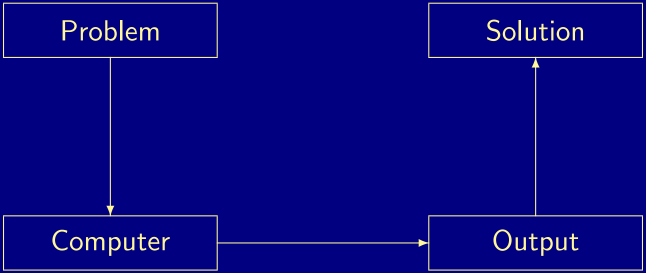

Traditional programming

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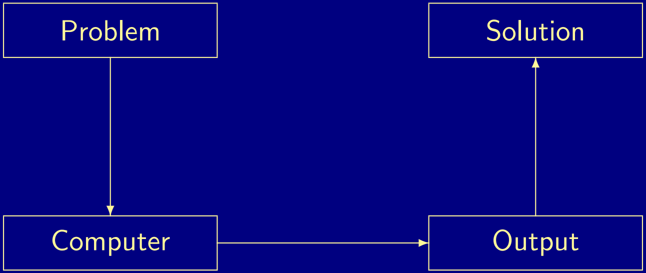

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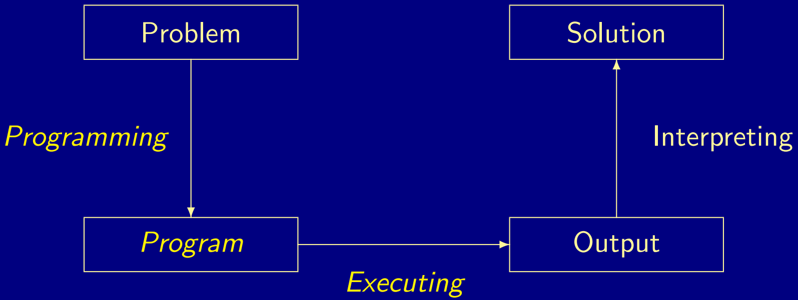

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Declarative problem solving

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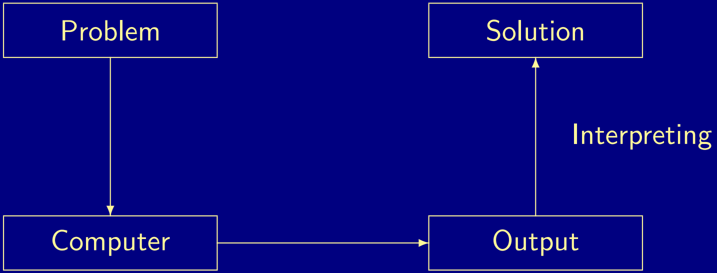

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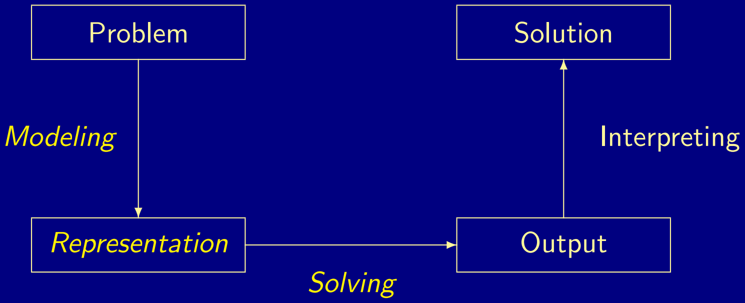

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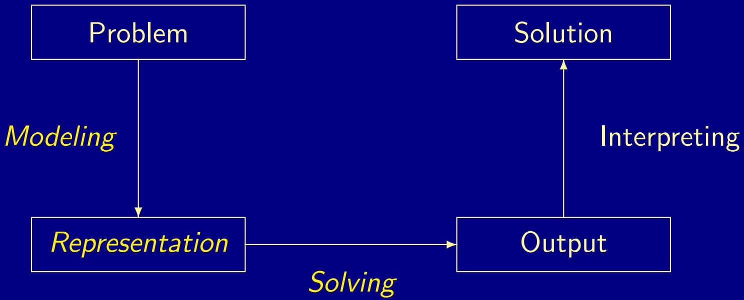

Outline

Answer Set Programming

in a Nutshell

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
- ASP has its roots in
  - (deductive) databases
  - logic programming (with negation)
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - constraint solving (in particular, SATisfiability testing)
- ASP allows for solving all search problems in NP (and NP^{NP}) in a uniform way
- ASP is versatile as reflected by the ASP solver *clasp*, winning first places at ASP, CASC, MISC, PB, and SAT competitions
- ASP embraces many emerging application areas

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  tailored to Knowledge Representation and Reasoning

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$$\textbf{ASP} = \textbf{DB}+\textbf{LP}+\textbf{KR}+\textbf{SAT}$$

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Outline

Some biased moments in time

- '70/'80 Capturing incomplete information

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- '70/'80 Capturing incomplete information
  - Databases Closed world assumption
  - Logic programming Negation as failure
  - Non-monotonic reasoning
    Auto-epistemic and Default logics, Circumscription

Some biased moments in time

- '70/'80 Capturing incomplete information
  - Databases Closed world assumption
    - Axiomatic characterization
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Some biased moments in time

- '70/'80 Capturing incomplete information
  - Databases Closed world assumption
    - Axiomatic characterization
  - Logic programming Negation as failure
    - Herbrand interpretations
    - Fix-point characterizations
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    - Fix-point characterizations
  - Non-monotonic reasoning
    Auto-epistemic and Default logics, Circumscription
    - Extensions of first-order logic
    - Modalities, fix-points, second-order logic

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  - Logic programming semantics
    Well-founded and stable models semantics
  - ASP solving
    "Stable models = Well-founded semantics + Branch"

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    - Stable models semantics derived from non-monotonic logics
    - Alternating fix-point theory
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    "Stable models = Well-founded semantics + Branch"
    - Modeling — Grounding — Solving
    - Icebreakers: `lparse` and `smodels`

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  - Growing dissemination Decision Support for Space Shuttle
  - Constructive logics Equilibrium Logic

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    - Bio-informatics, Linux Package Configuration, Music composition, Robotics, System Design, etc
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    - Roots: Logic of Here-and-There , G3

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Paradigm shift

Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem
2. A solution is given by a derivation of a query

Model Generation based approach (eg. SATisfiability testing)

1. Provide a representation of the problem
2. A solution is given by a model of the representation

Automated planning, Kautz and Selman (ECAI'92)

Represent planning problems as propositional theories so that models not proofs describe solutions

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Model Generation based Problem Solving

Representation	Solution
constraint satisfaction problem	assignment
propositional horn theories	smallest model
propositional theories	models
propositional theories	minimal models
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propositional programs	minimal models
propositional programs	supported models
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first-order theories	models
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first-order theories	Herbrand models
auto-epistemic theories	expansions
default theories	extensions
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LP-style playing with blocks

Prolog program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Prolog queries

```
?- above(a,c).
true.

?- above(c,a).
no.
```

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Prolog queries (testing entailment)

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true.

?- above(c,a).
no.
```

LP-style playing with blocks

Shuffled Prolog program

```
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above(X,Y) :- on(X,Y).
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Prolog queries

```
?- above(a,c).

Fatal Error: local stack overflow.
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above(X,Y) :- on(X,Y).
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Prolog queries (answered via fixed execution)

```
?- above(a,c).

Fatal Error: local stack overflow.
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Formula

$$
\begin{array}{ll}
 & on(a,b) \\
\wedge & on(b,c) \\
\wedge & (on(X,Y) \rightarrow above(X,Y)) \\
\wedge & (on(X,Z) \wedge above(Z,Y) \rightarrow above(X,Y))
\end{array}
$$

Herbrand model

$$
\left\{
\begin{array}{ccccc}
on(a,b), & on(b,c), & on(a,c), & on(b,b), & \\
above(a,b), & above(b,c), & above(a,c), & above(b,b), & above(c,b)
\end{array}
\right\}
$$

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\end{array}
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$$

SAT-style playing with blocks

Formula

$$
\begin{array}{ll}
 & on(a,b) \\
\land & on(b,c) \\
\land & (on(X,Y) \rightarrow above(X,Y)) \\
\land & (on(X,Z) \land above(Z,Y) \rightarrow above(X,Y))
\end{array}
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Herbrand model

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on(a,b), & on(b,c), & on(a,c), & on(b,b), & \\
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Herbrand model (among 426!)

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\left\{
\begin{array}{rrrrr}
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➥ Answer Set Programming (ASP)

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Answer Set Programming *at large*

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Answer Set Programming *commonly*

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Answer Set Programming *in practice*

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Answer Set Programming *in practice*

Representation	Solution
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propositional theories	minimal models
propositional theories	stable models
propositional programs	minimal models
propositional programs	supported models
propositional programs	stable models
first-order theories	models
first-order theories	minimal models
first-order theories	stable models
first-order theories	Herbrand models
auto-epistemic theories	expansions
default theories	extensions
first-order programs	stable Herbrand models

ASP-style playing with blocks

Logic program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Stable Herbrand model

$$\{ \texttt{on}(a,b),\ \texttt{on}(b,c),\ \texttt{above}(b,c),\ \texttt{above}(a,b),\ \texttt{above}(a,c) \}$$

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ASP-style playing with blocks

Logic program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Stable Herbrand model

$$\{\ \mathtt{on}(\mathtt{a},\mathtt{b}),\ \mathtt{on}(\mathtt{b},\mathtt{c}),\ \mathtt{above}(\mathtt{b},\mathtt{c}),\ \mathtt{above}(\mathtt{a},\mathtt{b}),\ \mathtt{above}(\mathtt{a},\mathtt{c})\ \}$$

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ASP-style playing with blocks

Logic program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Stable Herbrand model (and no others)

$\{\ \mathtt{on}(a,b),\ \mathtt{on}(b,c),\ \mathtt{above}(b,c),\ \mathtt{above}(a,b),\ \mathtt{above}(a,c)\ \}$

ASP-style playing with blocks

Logic program

```
on(a,b).
on(b,c).

above(X,Y) :- above(Z,Y), on(X,Z).
above(X,Y) :- on(X,Y).
```

Stable Herbrand model (and no others)

$$\{ \mathtt{on}(a,b),\ \mathtt{on}(b,c),\ \mathtt{above}(b,c),\ \mathtt{above}(a,b),\ \mathtt{above}(a,c) \}$$

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ASP versus LP

ASP	Prolog
Model generation	Query orientation
Bottom-up	Top-down
Modeling language	Programming language
Rule-based format	
Instantiation Flat terms	Unification Nested terms
(Turing +) $NP(^{NP})$	Turing

ASP versus SAT

ASP	SAT
Model generation	
Bottom-up	
Constructive Logic	Classical Logic
Closed (and open) world reasoning	Open world reasoning
Modeling language	—
Complex reasoning modes	Satisfiability testing
Satisfiability	Satisfiability
Enumeration/Projection	—
Intersection/Union	—
Optimization	—
(Turing +) $NP^{(NP)}$	NP

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Outline

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Propositional Normal Logic Programs

- A logic program P is a set of rules of the form

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \dots, b_m, \neg c_1, \dots, \neg c_n}_{\text{body}}$$

  - a and all b_i, c_j are atoms (propositional variables)
  - $\leftarrow$, $,$, $\neg$ denote if, and, and negation
  - intuitive reading: head must be true if body holds

- Semantics given by stable models, informally, models of P justifying each true atom by some rule in P

Logic Programs

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Logic Programs

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  - $\leftarrow$, $,$, $\neg$ denote if, and, and negation
  - intuitive reading: head must be true if body holds

- Semantics given by stable models, informally, models of P justifying each true atom by some rule in P

Normal Logic Programs

- A logic program P is a set of rules of the form

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \dots, b_m, \neg c_1, \dots, \neg c_n}_{\text{body}}$$

  - a and all b_i, c_j are atoms (propositional variables)
  - $\leftarrow$, $,$, $\neg$ denote if, and, and negation
  - intuitive reading: head must be true if body holds

- Semantics given by stable models, informally, models of P justifying each true atom by some rule in P

- Disclaimer The following formalities apply to normal logic programs

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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	
F	**F**	**T**	
F	**T**	**F**	
F	**T**	**T**	
T	**F**	**F**	
T	**F**	**T**	
T	**T**	**F**	
T	**T**	**T**	

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\neg\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$(\neg\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\neg\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
F	**T**	**T**	$(\neg\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	**F**	**F**	$(\neg\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
T	**F**	**T**	$(\neg\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
T	**T**	**F**	$(\neg\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
T	**T**	**T**	$(\neg\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

Potassco

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
F	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
T	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
T	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
T	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
F	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
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T	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

Potassco

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge \mathbf{F}$
F	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
T	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
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T	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge \mathbf{F}$
F	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
T	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
T	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge \mathbf{F}$
T	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

Potassco

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	**F** $\wedge$ **T**
F	**F**	**T**	**F** $\wedge$ **T**
F	**T**	**F**	**T** $\wedge$ **F**
F	**T**	**T**	**T** $\wedge$ **T**
T	**F**	**F**	**T** $\wedge$ **T**
T	**F**	**T**	**T** $\wedge$ **T**
T	**T**	**F**	**T** $\wedge$ **F**
T	**T**	**T**	**T** $\wedge$ **T**

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	**F**
F	**F**	**T**	**F**
F	**T**	**F**	**F**
F	**T**	**T**	**T**
T	**F**	**F**	**T**
T	**F**	**T**	**T**
T	**T**	**F**	**F**
T	**T**	**T**	**T**

- We get four models: $\{b, c\}$, $\{a\}$, $\{a, c\}$, and $\{a, b, c\}$

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Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	
F	**F**	**T**	
F	**T**	**F**	
F	**T**	**T**	
T	**F**	**F**	
T	**F**	**T**	
T	**T**	**F**	
T	**T**	**T**	

Potassco

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\neg \mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**T**	$(\neg \mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**F**	$(\neg \mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**T**	$(\neg \mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**F**	$(\neg \mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**T**	$(\neg \mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**F**	$(\neg \mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**T**	$(\neg \mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**T**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**F**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**T**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c)$
F	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
T	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c)$
F	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
T	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$

Some truth tabling, and now ASP

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F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c)$
F	**T**	**F**	$\mathbf{T} \wedge (b \rightarrow c)$
F	**T**	**T**	$\mathbf{T} \wedge (b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
T	**T**	**F**	$\mathbf{T} \wedge (b \rightarrow c)$
T	**T**	**T**	$\mathbf{T} \wedge (b \rightarrow c)$

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c)$
F	**T**	**F**	$(b \rightarrow c)$
F	**T**	**T**	$(b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
T	**T**	**F**	$(b \rightarrow c)$
T	**T**	**T**	$(b \rightarrow c)$

Reduct

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	$a \wedge (b \rightarrow c)$
F	F	T	$a \wedge (b \rightarrow c)$
F	T	F	$(b \rightarrow c)$
F	T	T	$(b \rightarrow c)$
T	F	F	$a \wedge (b \rightarrow c)$
T	F	T	$a \wedge (b \rightarrow c)$
T	T	F	$(b \rightarrow c)$
T	T	T	$(b \rightarrow c)$

Reduct

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	**F**	**F**	$a \wedge (b \rightarrow c)$	
F	**F**	**T**	$a \wedge (b \rightarrow c)$	
F	**T**	**F**	$(b \rightarrow c)$	
F	**T**	**T**	$(b \rightarrow c)$	$\models$
T	**F**	**F**	$a \wedge (b \rightarrow c)$	$\models a$
T	**F**	**T**	$a \wedge (b \rightarrow c)$	$\models a$
T	**T**	**F**	$(b \rightarrow c)$	
T	**T**	**T**	$(b \rightarrow c)$	$\models$

Reduct

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	**F**	**F**	$a \wedge (b \rightarrow c)$	
F	**F**	**T**	$a \wedge (b \rightarrow c)$	
F	**T**	**F**	$(b \rightarrow c)$	
F	**T**	**T**	$(b \rightarrow c)$	$\models$
T	**F**	**F**	$a \wedge (b \rightarrow c)$	$\models a$
T	**F**	**T**	$a \wedge (b \rightarrow c)$	$\models a$
T	**T**	**F**	$(b \rightarrow c)$	
T	**T**	**T**	$(b \rightarrow c)$	$\models$

Reduct

Potassco

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	F	F	$a \wedge (b \rightarrow c)$	$\models a$
F	F	T	$a \wedge (b \rightarrow c)$	$\models a$
F	T	F	$(b \rightarrow c)$	$\models$
F	T	T	$(b \rightarrow c)$	$\models$
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$
T	F	T	$a \wedge (b \rightarrow c)$	$\models a$
T	T	F	$(b \rightarrow c)$	$\models$
T	T	T	$(b \rightarrow c)$	$\models$

Reduct

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$		
F	**F**	**F**	$a \wedge (b \rightarrow c)$		
F	**F**	**T**	$a \wedge (b \rightarrow c)$		
F	**T**	**F**	$(b \rightarrow c)$		
F	**T**	**T**	$(b \rightarrow c)$		
T	**F**	**F**	$a \wedge (b \rightarrow c)$	$\models a$	Stable model
T	**F**	**T**	$a \wedge (b \rightarrow c)$		
T	**T**	**F**	$(b \rightarrow c)$		
T	**T**	**T**	$(b \rightarrow c)$		
			Reduct		

- We get one stable model: $\{a\}$

Potassco

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$		
F	**F**	**F**	$a \wedge (b \rightarrow c)$		
F	**F**	**T**	$a \wedge (b \rightarrow c)$		
F	**T**	**F**	$(b \rightarrow c)$		
F	**T**	**T**	$(b \rightarrow c)$		
T	**F**	**F**	$a \wedge (b \rightarrow c)$	$\models a$	Stable model
T	**F**	**T**	$a \wedge (b \rightarrow c)$		
T	**T**	**F**	$(b \rightarrow c)$		
T	**T**	**T**	$(b \rightarrow c)$		

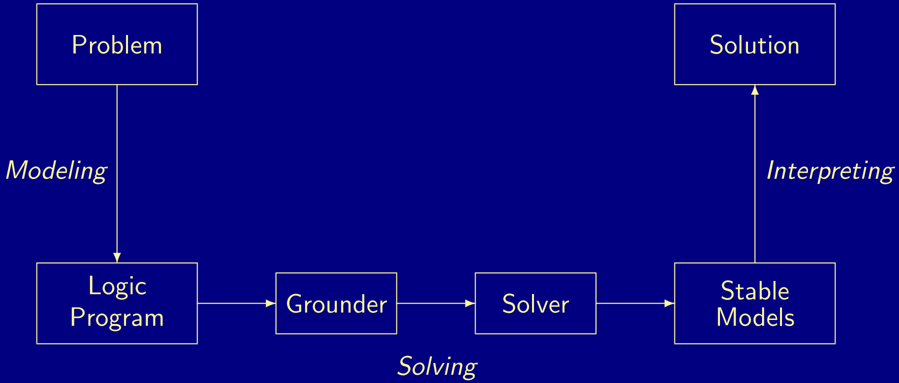
Reduct

- We get one stable model: $\{a\}$
- Stable models $=$ Smallest models of (respective) reducts

Potassco

Outline

ASP modeling, grounding, and solving



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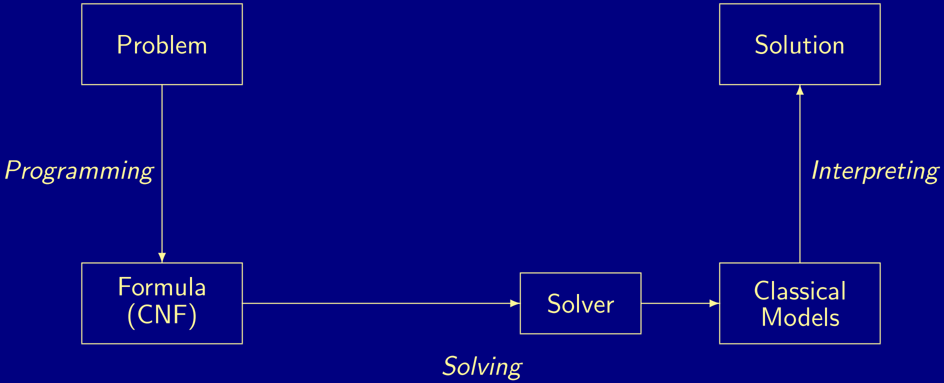
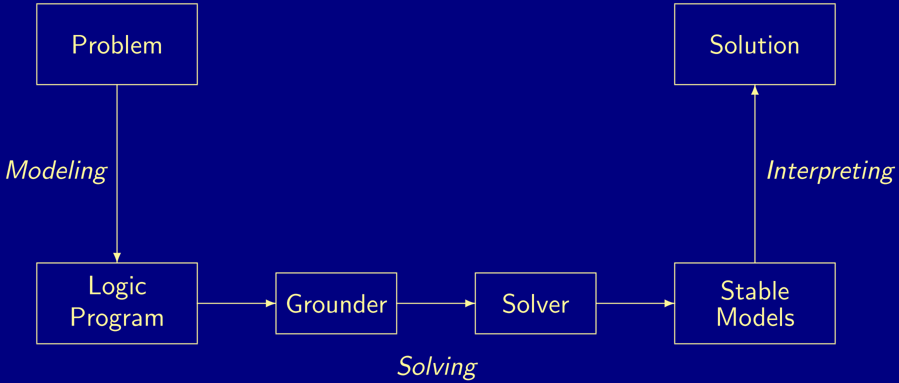
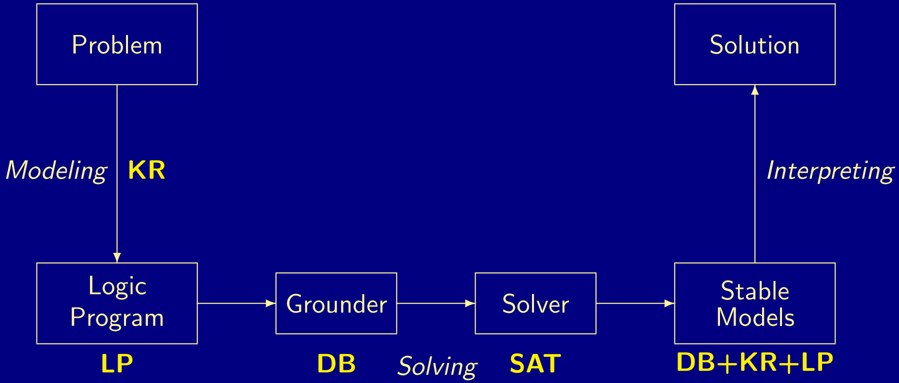
SAT solving


Problem
Solution
Programming
Interpreting
Formula (CNF)
Solver
Classical Models
Solving

Rooting ASP solving

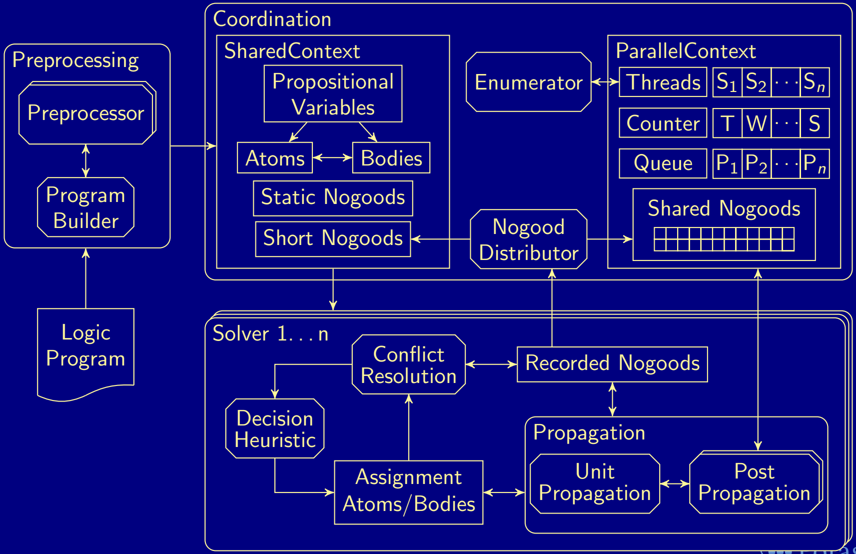


Rooting ASP solving

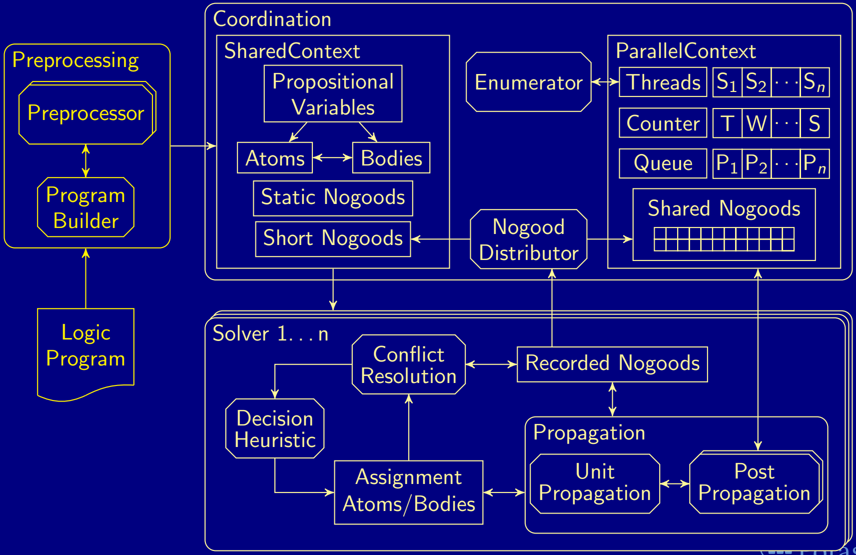


Outline

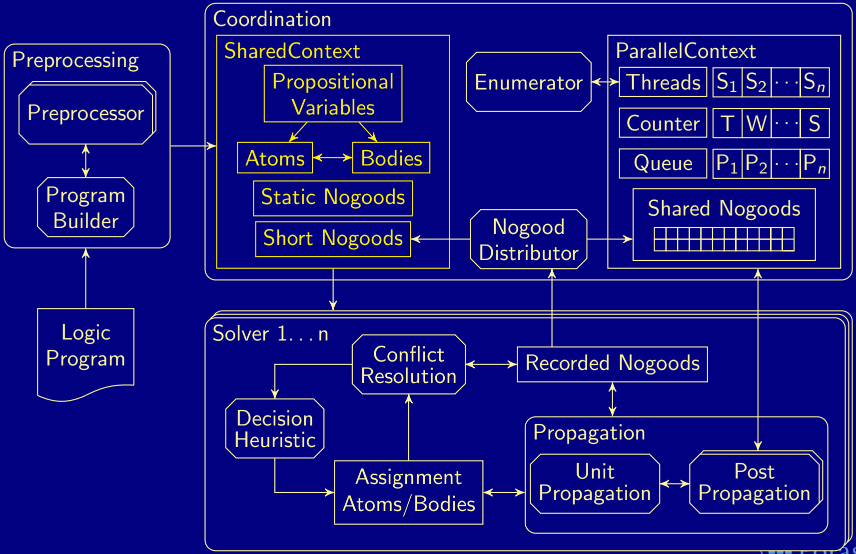
Multi-threaded architecture of *clasp*



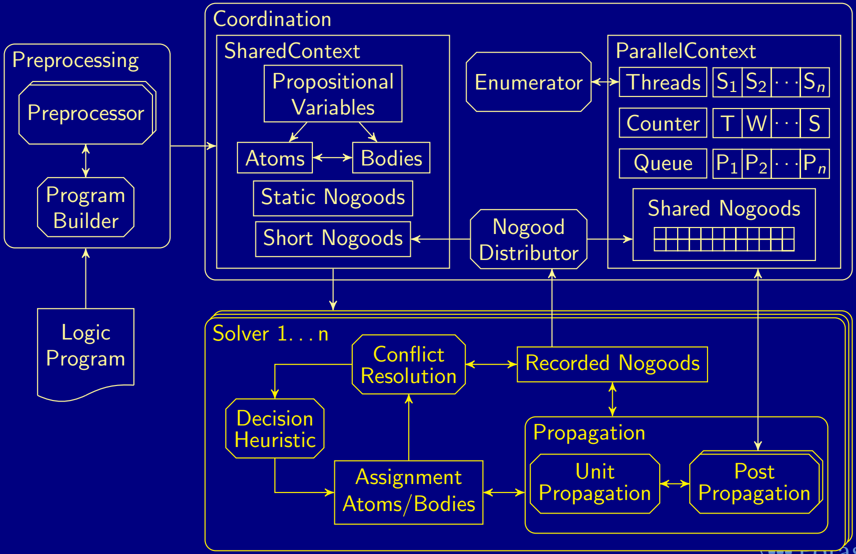
Multi-threaded architecture of *clasp*



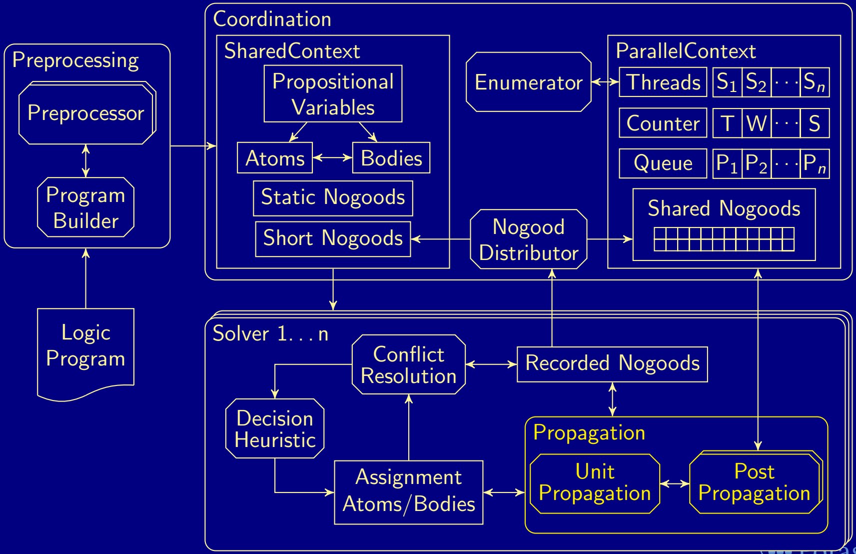
Multi-threaded architecture of *clasp*



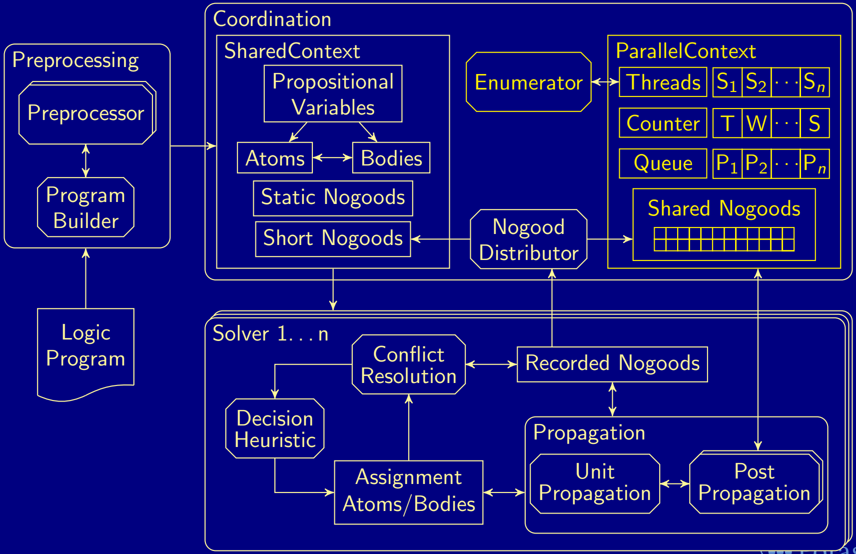
Multi-threaded architecture of *clasp*



Multi-threaded architecture of *clasp*



Multi-threaded architecture of *clasp*



Outline

Two sides of a coin

- ASP as High-level Language
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules

- ASP as Low-level Language
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation

- ASP and Imperative language
  - Control continuously changing logic programs

Two and a half sides of a coin

- ASP as High-level Language
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules
- ASP as Low-level Language
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation
- ASP and Imperative language
  - Control continuously changing logic programs

What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated planning
  - Code optimization
  - Database integration
  - Decision support for NASA shuttle controllers
  - Model checking
  - Music composition
  - Product configuration
  - Robotics
  - System design
  - Systems biology
  - Team-building
  - Timetabling
  - and many many more

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What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated planning
  - Code optimization
  - Database integration
  - Decision support for NASA shuttle controllers
  - Model checking
  - Music composition
  - Product configuration
  - Robotics
  - System design
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What does ASP offer?

- Integration of DB, KR, and SAT techniques
- Succinct, elaboration-tolerant problem representations
  - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
  - including: data, frame axioms, exceptions, defaults, closures, etc

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ASP = DB+LP+KR+SAT

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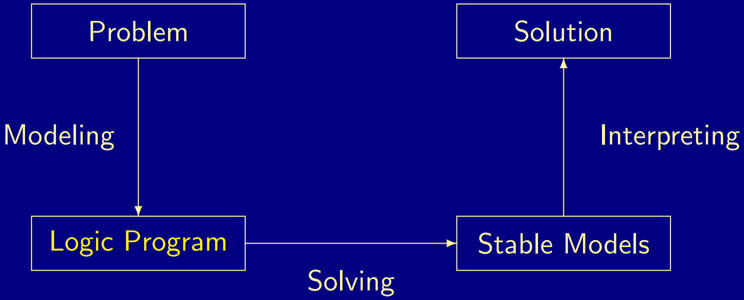
$$\mathbf{ASP} = \mathbf{DB}+\mathbf{LP}+\mathbf{KR}+\mathbf{SMT}^n$$

Introduction: Overview

Outline

Problem solving in ASP: Syntax



Normal logic programs

- A logic program, P, over a set $\mathcal{A}$ of atoms is a finite set of rules
- A (normal) rule, r, is of the form

$$a_0 \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

- Notation

$$\begin{aligned}
head(r) &= a_0 \\
body(r) &= \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \\
body(r)^+ &= \{a_1, \dots, a_m\} \\
body(r)^- &= \{a_{m+1}, \dots, a_n\} \\
atom(P) &= \bigcup_{r \in P} \left(\{head(r)\} \cup body(r)^+ \cup body(r)^-\right) \\
body(P) &= \{body(r) \mid r \in P\}
\end{aligned}$$

- A program P is positive if $body(r)^- = \emptyset$ for all $r \in P$

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$$\begin{array}{rcl} head(r) & = & a_0 \\ body(r) & = & \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \\ body(r)^+ & = & \{a_1, \dots, a_m\} \\ body(r)^- & = & \{a_{m+1}, \dots, a_n\} \\ atom(P) & = & \bigcup_{r \in P} \left(\{head(r)\} \cup body(r)^+ \cup body(r)^-\right) \\ body(P) & = & \{body(r) \mid r \in P\} \end{array}$$

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Rough notational convention

We sometimes use the following notation interchangeably in order to stress the respective view:

	true, false	if	and	or	iff	default negation	classical negation
source code		`:-`	`,`	`;`		`not`	`-`
logic program		$\leftarrow$	,	;		$\sim$	$\neg$
formula	$\bot, \top$	$\rightarrow$	$\wedge$	$\vee$	$\leftrightarrow$	$\sim$	$\neg$

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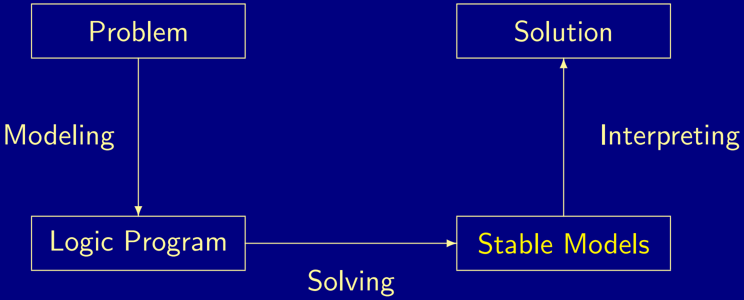
Outline

Problem solving in ASP: Semantics



Formal Definition

Stable models of positive programs

- A set of atoms X is closed under a positive program P iff for any $r \in P$, $head(r) \in X$ whenever $body(r)^+ \subseteq X$
  - X corresponds to a model of P (seen as a formula)
- The smallest set of atoms which is closed under a positive program P is denoted by $Cn(P)$
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of P (ditto)
- The set $Cn(P)$ of atoms is the stable model of a *positive* program P

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Some “logical” remarks

- Positive rules are also referred to as definite clauses
  - Definite clauses are disjunctions with exactly one positive atom:

    $$a_0 \vee \neg a_1 \vee \cdots \vee \neg a_m$$

  - A set of definite clauses has a (unique) smallest model
- Horn clauses are clauses with at most one positive atom
  - Every definite clause is a Horn clause but not vice versa
  - Non-definite Horn clauses can be regarded as integrity constraints
  - A set of Horn clauses has a smallest model or none
- This smallest model is the intended semantics of such sets of clauses
  - Given a positive program P, $Cn(P)$ corresponds to the smallest model of the set of definite clauses corresponding to P

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Basic idea

Consider the logical formula Φ and its three (classical) models:

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

Φ $\boxed{q \wedge (q \wedge \neg r \rightarrow p)}$

Formula Φ has one stable model, often called answer set:

$$\{p, q\}$$

P_Φ

$$\begin{array}{lll} q & \leftarrow & \\ p & \leftarrow & q, \sim r \end{array}$$

Informally, a set X of atoms is a stable model of a logic program P

if X is a (classical) model of P and

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$$P^X = \{head(r) \leftarrow body(r)^+ \mid r \in P \text{ and } body(r)^- \cap X = \emptyset\}$$

- A set X of atoms is a stable model of a program P, if $Cn(P^X) = X$

- Remarks

  $Cn(P^X)$ is the $\subseteq$–smallest (classical) model of P^X

  Each atom in X is justified by an *"applying rule from P"*
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A closer look at P^X

- Alternatively, given a set X of atoms from P,

  P^X is obtained from P by deleting

  1. each rule having $\sim a$ in its body with $a \in X$
     and then
  2. all negative atoms of the form $\sim a$
     in the bodies of the remaining rules

- Note Only negative body literals are evaluated wrt X

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Outline

Example one

$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p\}$	$p \leftarrow p$	$\emptyset$
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p, q\}$	$p \leftarrow p$	$\emptyset$

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Example one

$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$	
$\{\ \ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✘
$\{p\ \}$	$p \leftarrow p$	$\emptyset$	
$\{\ q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✔
$\{p, q\}$	$p \leftarrow p$	$\emptyset$	

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X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✗
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Example one

$P = \{p \leftarrow p, \; q \leftarrow \neg p\}$

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$\{p\quad\}$	$p \leftarrow p$	$\emptyset$	✔
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Example two

$P = \{p \leftarrow \sim q,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$
$\{p\}$	$p \leftarrow$	$\{p\}$
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Example two

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$\{p, q\}$		$\emptyset$	✘

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$P = \{p \leftarrow \neg q, \; q \leftarrow \neg p\}$

X	P^X	$Cn(P^X)$	
$\{ \quad \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$	✘
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Example three

$P = \{p \leftarrow {\sim}p\}$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow$	$\{p\}$
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Some properties

- A logic program may have zero, one, or multiple stable models
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Outline

Programs with variables

Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructible from $\mathcal{T}$

- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \rightarrow \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground Instantiation of P: $ground(P) = \bigcup_{r \in P} ground(r)$

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Programs with variables

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- Let $\mathcal{A}$ be a set of (variable-free) atoms constructible from $\mathcal{T}$ (also called alphabet or Herbrand base)
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An example

$$P = \{\ r(a,b) \leftarrow,\ \ r(b,c) \leftarrow,\ \ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a,b,c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

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Intelligent Grounding aims at reducing the ground instantiation

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Safety

- A normal rule is safe, if each of its variables also occurs in some positive body literal
- A normal program is safe, if all of its rules are safe

Example

$$
\begin{array}{l}
d(a)\\
d(c)\\
d(d)\\
p(a,b)\\
p(b,c)\\
p(c,d)\\
p(X,Z) \leftarrow p(X,Y), p(Y,Z)\\
q(a)\\
q(b)\\
q(X) \leftarrow {\sim}r(X), d(X)\\
r(X) \leftarrow {\sim}q(X), d(X)\\
s(X) \leftarrow {\sim}r(X), p(X,Y), q(Y)
\end{array}
$$

Example

Safe ?

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$d(c)$
$d(d)$

$p(a, b)$
$p(b, c)$
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Example

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$d(a)$	✔
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$p(a,b)$	✔
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$q(a)$	✔
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Stable models of programs with Variables

Let P be a normal logic program with variables

- A set X of (ground) atoms is a stable model of P,
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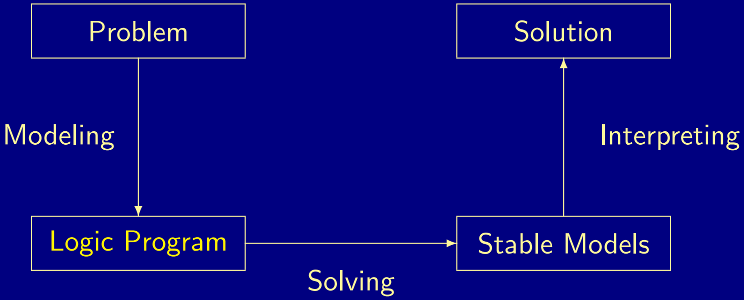
Outline

Problem solving in ASP: Extended Syntax



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- Choice `2 { p(X,Y) : q(X) } 7 :- r(Y)`
- Aggregates `s(Y) :- r(Y), 2 #sum{ X : p(X,Y), q(X) } 7`
- Optimization

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:~ q(X), p(X,C) [C]
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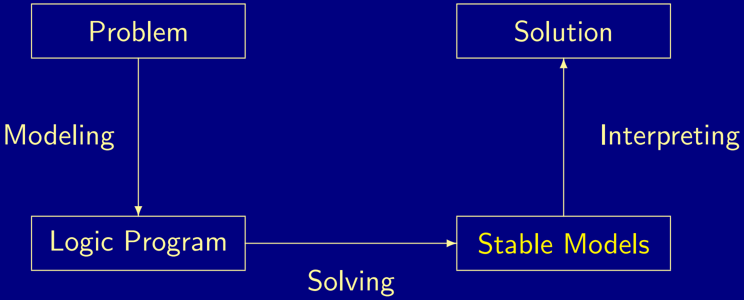
- Multi-objective optimization
  - Weak constraints `:~ q(X), p(X,C) [C@42]`
  - Statements `#minimize { C@42 : q(X), p(X,C) }`

Outline

Problem solving in ASP: Reasoning Modes



Reasoning Modes

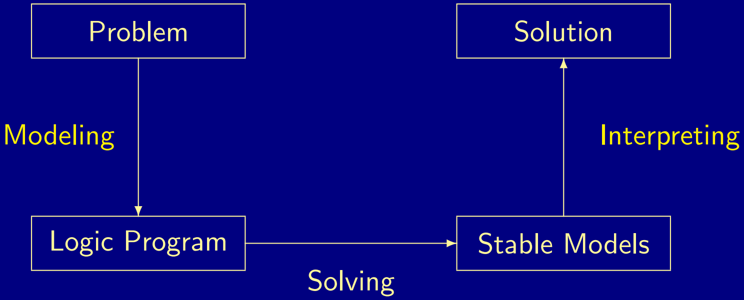
- Satisfiability
- Enumeration[†]
- Projection[†]
- Intersection[‡]
- Union[‡]
- Optimization
- and combinations of them

[†] without solution recording

[‡] without solution enumeration

Basic Modeling: Overview

Modeling and Interpreting



Outline

Guiding principle

- Elaboration Tolerance (McCarthy, 1998)

  "A formalism is elaboration tolerant [if] *it is convenient to modify a set of facts expressed in the formalism to take into account new phenomena or changed circumstances."*

- Uniform problem representation

  For solving a problem instance **I** of a problem class **C**,
  - **I** is represented as a set of facts $P_{\mathbf{I}}$,
  - **C** is represented as a set of rules $P_{\mathbf{C}}$, and
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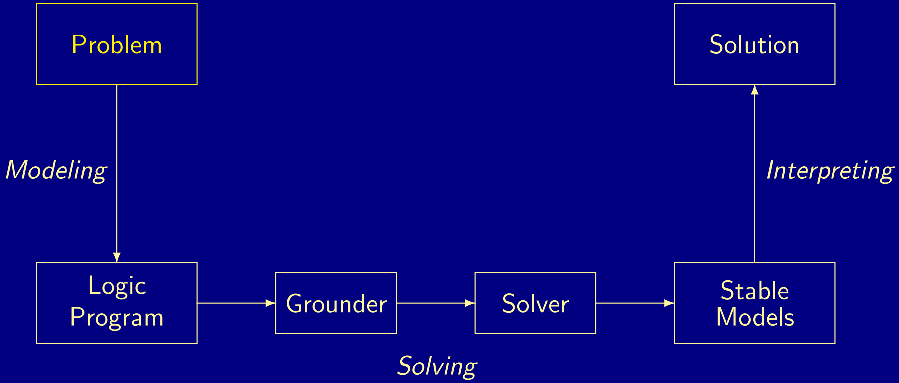

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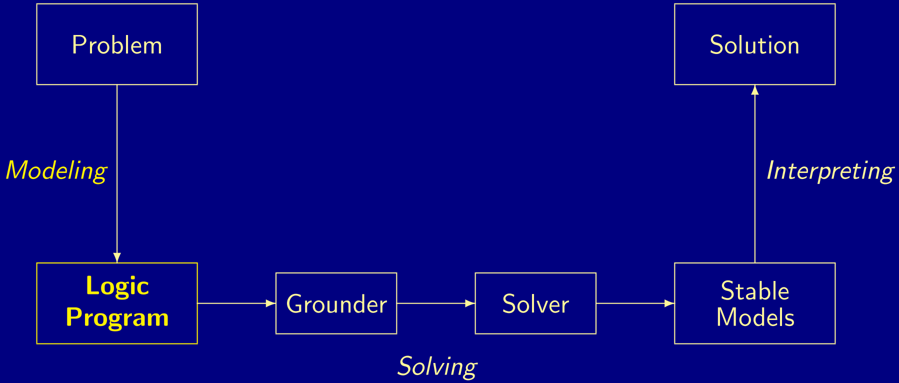

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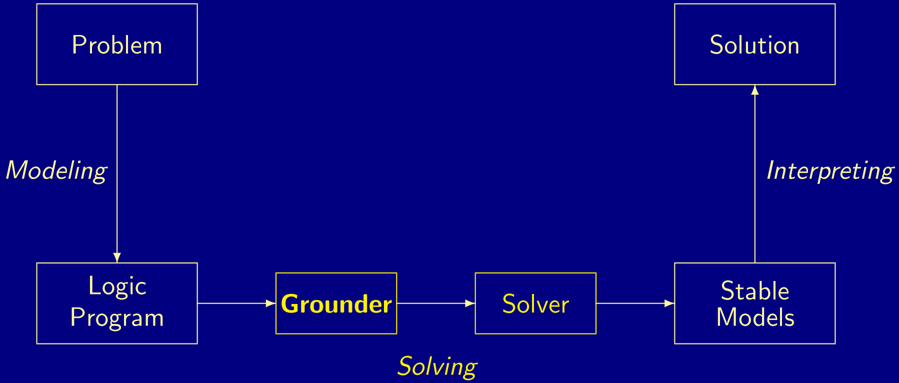

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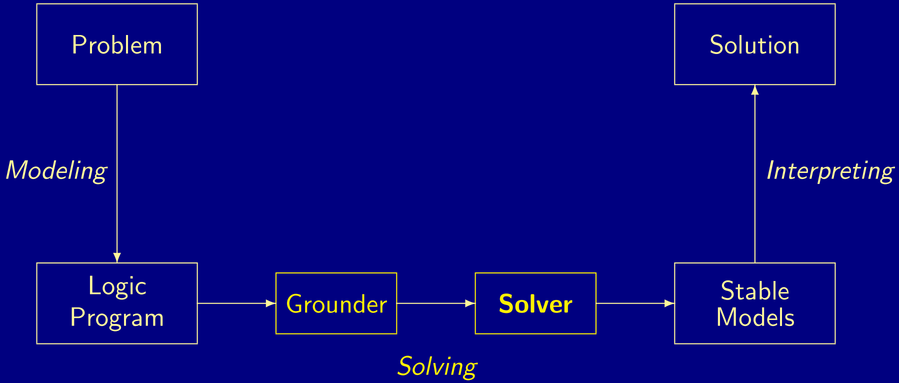

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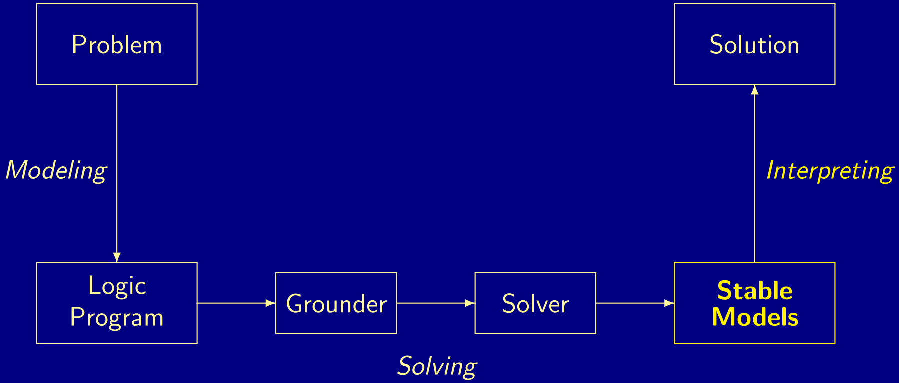

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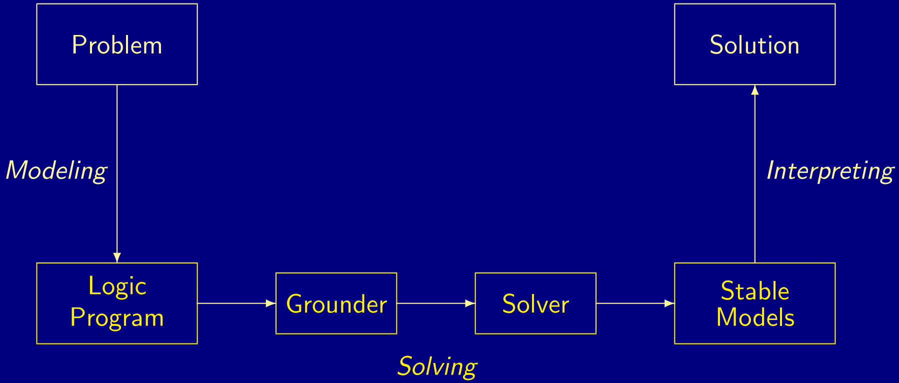

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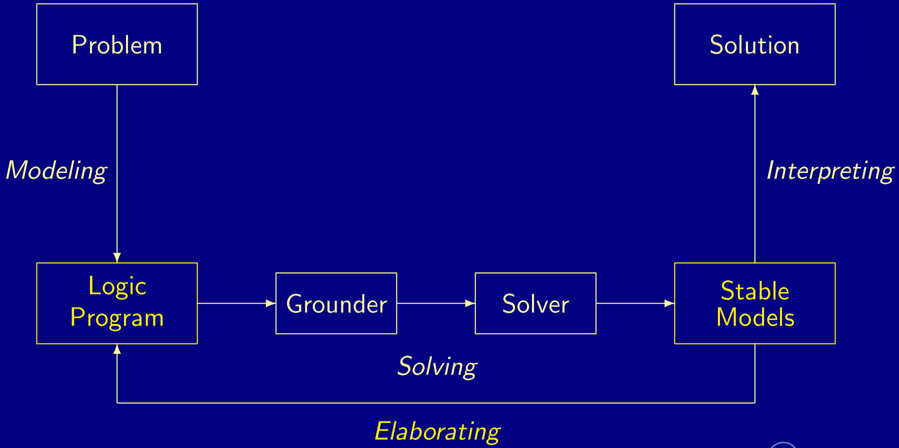
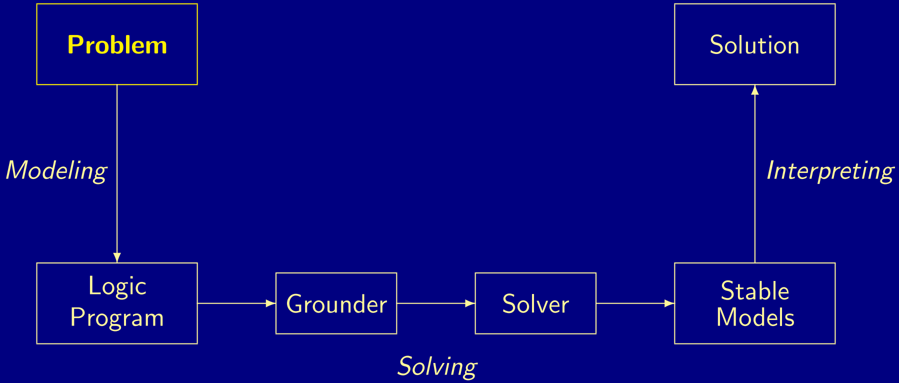


A case-study: Graph coloring



Graph coloring

- Problem instance A graph consisting of nodes and edges

Graph coloring

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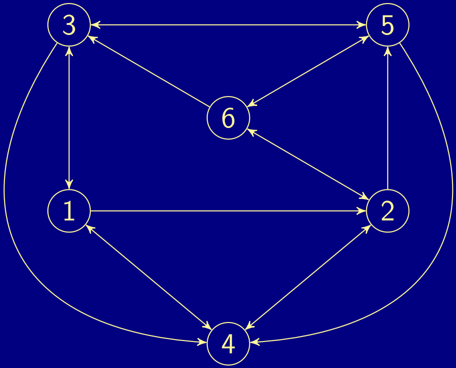

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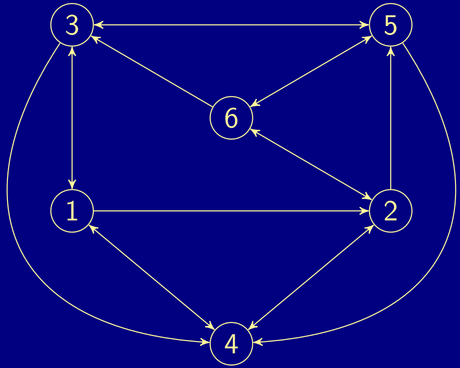

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  - facts formed by predicate `color/1`
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  In other words,

  1. Each node has one color
  2. Two connected nodes must not have the same color

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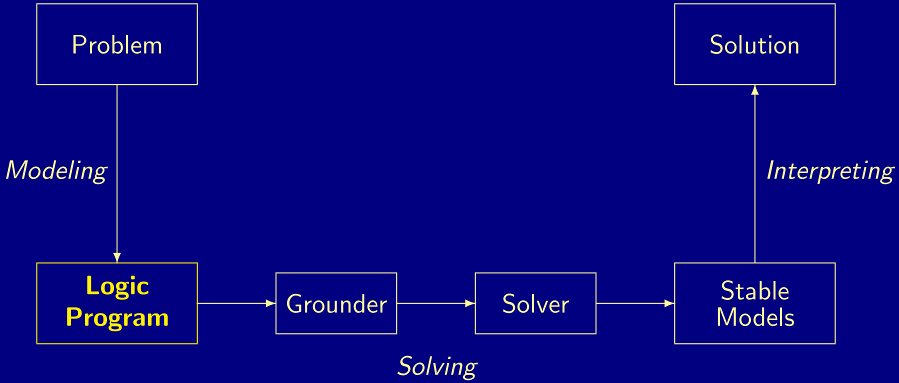
ASP solving process



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Graph coloring

```
node(1..6).

edge(1,2).   edge(1,3).   edge(1,4).
edge(2,4).   edge(2,5).   edge(2,6).
edge(3,1).   edge(3,4).   edge(3,5).
edge(4,1).   edge(4,2).
edge(5,3).   edge(5,4).   edge(5,6).
edge(6,2).   edge(6,3).   edge(6,5).

color(r).    color(b).    color(g).
```

Problem instance

```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
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Problem encoding

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edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).

{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

Problem instance

Problem encoding

Potassco

Graph coloring

```
node(1..6).

edge(1,2).   edge(1,3).   edge(1,4).
edge(2,4).   edge(2,5).   edge(2,6).
edge(3,1).   edge(3,4).   edge(3,5).
edge(4,1).   edge(4,2).
edge(5,3).   edge(5,4).   edge(5,6).
edge(6,2).   edge(6,3).   edge(6,5).

color(r).    color(b).    color(g).
```

Problem instance

```
{ assign(N,C) : color(C) } = 1 :- node(N).

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Graph coloring

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edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
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edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

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```

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Problem encoding

Potassco

Graph coloring

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edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
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Problem instance

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:- edge(N,M), assign(N,C), assign(M,C).
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Problem encoding

Potassco

Graph coloring

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edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

Problem instance

```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

Problem encoding

Potassco

Graph coloring

```
node(1..6).

edge(1,2).   edge(1,3).   edge(1,4).
edge(2,4).   edge(2,5).   edge(2,6).
edge(3,1).   edge(3,4).   edge(3,5).
edge(4,1).   edge(4,2).
edge(5,3).   edge(5,4).   edge(5,6).
edge(6,2).   edge(6,3).   edge(6,5).

color(r).    color(b).    color(g).
```

} Problem instance

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:- edge(N,M), assign(N,C), assign(M,C).
```

} Problem encoding

Graph coloring

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edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

graph.lp

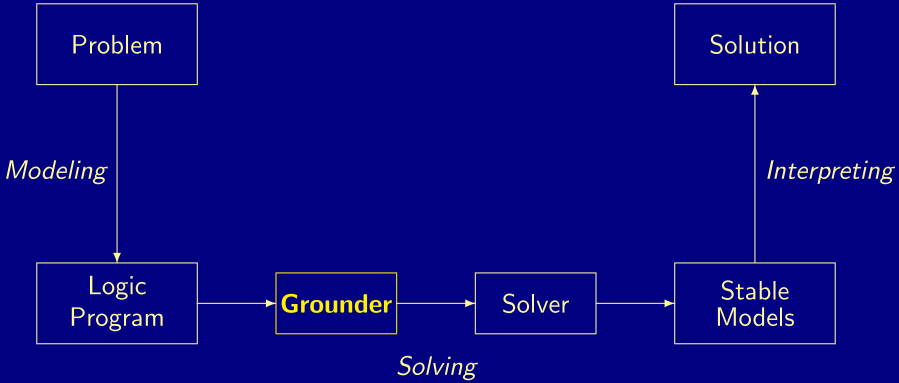
```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

color.lp

Potassco

ASP solving process



Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(2,4).  edge(3,1).  edge(4,1).  edge(5,3).  edge(6,2).
edge(1,3).  edge(2,5).  edge(3,4).  edge(4,2).  edge(5,4).  edge(6,3).
edge(1,4).  edge(2,6).  edge(3,5).              edge(5,6).  edge(6,5).

color(r).  color(b).  color(g).

{assign(1,r),assign(1,b),assign(1,g)} = 1. {assign(4,r),assign(4,b),assign(4,g)} = 1.
{assign(2,r),assign(2,b),assign(2,g)} = 1. {assign(5,r),assign(5,b),assign(5,g)} = 1.
{assign(3,r),assign(3,b),assign(3,g)} = 1. {assign(6,r),assign(6,b),assign(6,g)} = 1.

:- assign(1,r),assign(2,r).  :- assign(2,r),assign(4,r). [...] :- assign(6,r),assign(2,r).
:- assign(1,b),assign(2,b).  :- assign(2,b),assign(4,b).       :- assign(6,b),assign(2,b).
:- assign(1,g),assign(2,g).  :- assign(2,g),assign(4,g).       :- assign(6,g),assign(2,g).
:- assign(1,r),assign(3,r).  :- assign(2,r),assign(5,r).       :- assign(6,r),assign(3,r).
:- assign(1,b),assign(3,b).  :- assign(2,b),assign(5,b).       :- assign(6,b),assign(3,b).
:- assign(1,g),assign(3,g).  :- assign(2,g),assign(5,g).       :- assign(6,g),assign(3,g).
:- assign(1,r),assign(4,r).  :- assign(2,r),assign(6,r).       :- assign(6,r),assign(5,r).
:- assign(1,b),assign(4,b).  :- assign(2,b),assign(6,b).       :- assign(6,b),assign(5,b).
:- assign(1,g),assign(4,g).  :- assign(2,g),assign(6,g).       :- assign(6,g),assign(5,g).
```

Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp

node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(2,4).  edge(3,1).  edge(4,1).  edge(5,3).  edge(6,2).
edge(1,3).  edge(2,5).  edge(3,4).  edge(4,2).  edge(5,4).  edge(6,3).
edge(1,4).  edge(2,6).  edge(3,5).              edge(5,6).  edge(6,5).

color(r).  color(b).  color(g).

{assign(1,r),assign(1,b),assign(1,g)} = 1. {assign(4,r),assign(4,b),assign(4,g)} = 1.
{assign(2,r),assign(2,b),assign(2,g)} = 1. {assign(5,r),assign(5,b),assign(5,g)} = 1.
{assign(3,r),assign(3,b),assign(3,g)} = 1. {assign(6,r),assign(6,b),assign(6,g)} = 1.

:- assign(1,r),assign(2,r).  :- assign(2,r),assign(4,r). [...] :- assign(6,r),assign(2,r).
:- assign(1,b),assign(2,b).  :- assign(2,b),assign(4,b).       :- assign(6,b),assign(2,b).
:- assign(1,g),assign(2,g).  :- assign(2,g),assign(4,g).       :- assign(6,g),assign(2,g).
:- assign(1,r),assign(3,r).  :- assign(2,r),assign(5,r).       :- assign(6,r),assign(3,r).
:- assign(1,b),assign(3,b).  :- assign(2,b),assign(5,b).       :- assign(6,b),assign(3,b).
:- assign(1,g),assign(3,g).  :- assign(2,g),assign(5,g).       :- assign(6,g),assign(3,g).
:- assign(1,r),assign(4,r).  :- assign(2,r),assign(6,r).       :- assign(6,r),assign(5,r).
:- assign(1,b),assign(4,b).  :- assign(2,b),assign(6,b).       :- assign(6,b),assign(5,b).
:- assign(1,g),assign(4,g).  :- assign(2,g),assign(6,g).       :- assign(6,g),assign(5,g).
```

Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp

node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(2,4).  edge(3,1).  edge(4,1).  edge(5,3).  edge(6,2).
edge(1,3).  edge(2,5).  edge(3,4).  edge(4,2).  edge(5,4).  edge(6,3).
edge(1,4).  edge(2,6).  edge(3,5).              edge(5,6).  edge(6,5).

color(r).  color(b).  color(g).

{assign(1,r),assign(1,b),assign(1,g)} = 1. {assign(4,r),assign(4,b),assign(4,g)} = 1.
{assign(2,r),assign(2,b),assign(2,g)} = 1. {assign(5,r),assign(5,b),assign(5,g)} = 1.
{assign(3,r),assign(3,b),assign(3,g)} = 1. {assign(6,r),assign(6,b),assign(6,g)} = 1.

:- assign(1,r),assign(2,r).  :- assign(2,r),assign(4,r). [...] :- assign(6,r),assign(2,r).
:- assign(1,b),assign(2,b).  :- assign(2,b),assign(4,b).       :- assign(6,b),assign(2,b).
:- assign(1,g),assign(2,g).  :- assign(2,g),assign(4,g).       :- assign(6,g),assign(2,g).
:- assign(1,r),assign(3,r).  :- assign(2,r),assign(5,r).       :- assign(6,r),assign(3,r).
:- assign(1,b),assign(3,b).  :- assign(2,b),assign(5,b).       :- assign(6,b),assign(3,b).
:- assign(1,g),assign(3,g).  :- assign(2,g),assign(5,g).       :- assign(6,g),assign(3,g).
:- assign(1,r),assign(4,r).  :- assign(2,r),assign(6,r).       :- assign(6,r),assign(5,r).
:- assign(1,b),assign(4,b).  :- assign(2,b),assign(6,b).       :- assign(6,b),assign(5,b).
:- assign(1,g),assign(4,g).  :- assign(2,g),assign(6,g).       :- assign(6,g),assign(5,g).
```

Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp

node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

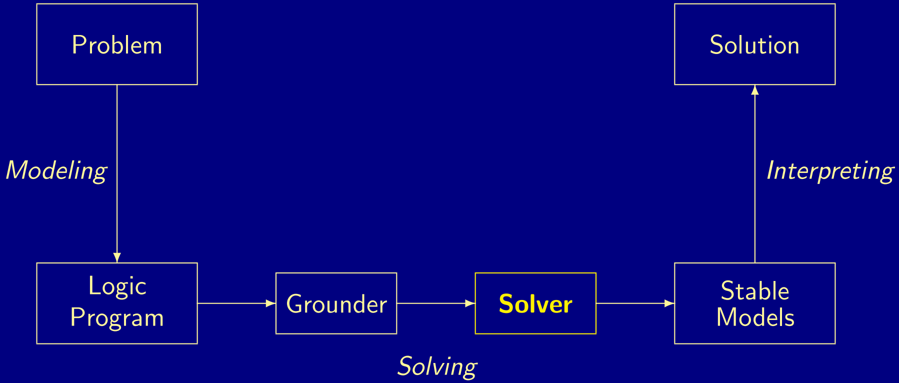
edge(1,2).  edge(2,4).  edge(3,1).  edge(4,1).  edge(5,3).  edge(6,2).
edge(1,3).  edge(2,5).  edge(3,4).  edge(4,2).  edge(5,4).  edge(6,3).
edge(1,4).  edge(2,6).  edge(3,5).              edge(5,6).  edge(6,5).

color(r).  color(b).  color(g).

{assign(1,r),assign(1,b),assign(1,g)} = 1. {assign(4,r),assign(4,b),assign(4,g)} = 1.
{assign(2,r),assign(2,b),assign(2,g)} = 1. {assign(5,r),assign(5,b),assign(5,g)} = 1.
{assign(3,r),assign(3,b),assign(3,g)} = 1. {assign(6,r),assign(6,b),assign(6,g)} = 1.

:- assign(1,r),assign(2,r).  :- assign(2,r),assign(4,r). [...] :- assign(6,r),assign(2,r).
:- assign(1,b),assign(2,b).  :- assign(2,b),assign(4,b).       :- assign(6,b),assign(2,b).
:- assign(1,g),assign(2,g).  :- assign(2,g),assign(4,g).       :- assign(6,g),assign(2,g).
:- assign(1,r),assign(3,r).  :- assign(2,r),assign(5,r).       :- assign(6,r),assign(3,r).
:- assign(1,b),assign(3,b).  :- assign(2,b),assign(5,b).       :- assign(6,b),assign(3,b).
:- assign(1,g),assign(3,g).  :- assign(2,g),assign(5,g).       :- assign(6,g),assign(3,g).
:- assign(1,r),assign(4,r).  :- assign(2,r),assign(6,r).       :- assign(6,r),assign(5,r).
:- assign(1,b),assign(4,b).  :- assign(2,b),assign(6,b).       :- assign(6,b),assign(5,b).
:- assign(1,g),assign(4,g).  :- assign(2,g),assign(6,g).       :- assign(6,g),assign(5,g).
```

ASP solving process



Graph coloring: Solving

```
$ gringo graph.lp color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
node(1) [...] assign(6,b) assign(5,g) assign(4,b) assign(3,r) assign(2,r) assign(1,g)
Answer: 2
node(1) [...] assign(6,r) assign(5,g) assign(4,r) assign(3,b) assign(2,b) assign(1,g)
Answer: 3
node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
Answer: 4
node(1) [...] assign(6,r) assign(5,b) assign(4,r) assign(3,g) assign(2,g) assign(1,b)
Answer: 5
node(1) [...] assign(6,g) assign(5,r) assign(4,g) assign(3,b) assign(2,b) assign(1,r)
Answer: 6
node(1) [...] assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

Graph coloring: Solving

```
$ gringo graph.lp color.lp | clasp 0

clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
node(1) [...] assign(6,b) assign(5,g) assign(4,b) assign(3,r) assign(2,r) assign(1,g)
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node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
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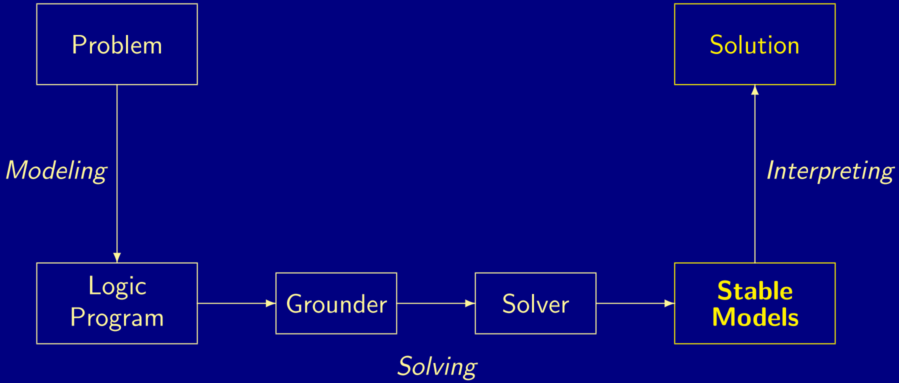
node(1) [...] assign(6,r) assign(5,b) assign(4,r) assign(3,g) assign(2,g) assign(1,b)
Answer: 5
node(1) [...] assign(6,g) assign(5,r) assign(4,g) assign(3,b) assign(2,b) assign(1,r)
Answer: 6
node(1) [...] assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
SATISFIABLE

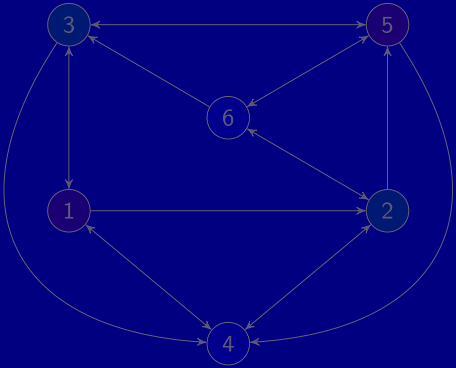
Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

ASP solving process



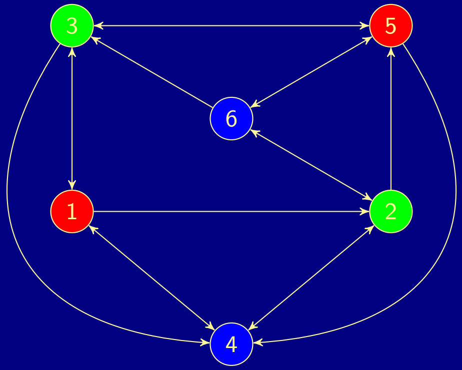
A coloring

```
Answer: 6
node(1)   [...]    \
assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
```



A coloring

```
Answer: 6
node(1)   [...]    \
assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
```



Outline

Potassco

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

- Generator: Generate potential stable model candidates (typically through non-deterministic constructs)
- Tester: Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Basic methodology

Methodology

Generate and Test   (or: Guess and Check)

- Generator — Generate potential stable model candidates (typically through non-deterministic constructs)
- Tester — Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program $=$ Data $+$ Generator $+$ Tester ($+$ Optimizer)

Potassco

Outline

15 Elaboration tolerance

16 ASP solving process

17 Methodology
- Satisfiability
- Queens
- Traveling Salesperson
- Reviewer Assignment
- Planning

Satisfiability testing

- Problem Instance: A propositional formula ϕ in CNF
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program:

Generator	Tester	Stable models
$\{ a \} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
$\{ b \} \leftarrow$	$\leftarrow a, \sim b$	$X_2 = \{\}$

Potassco

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$\{ a \} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
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Satisfiability testing

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Generator	**Tester**	**Stable models**
$\{\, a \,\} \leftarrow$	$\leftarrow \; {\sim}a, b$	$X_1 = \{a, b\}$
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- Logic Program:

Generator	**Tester**	**Stable models**
$\{ a \} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
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Satisfiability testing

- Problem Instance: A propositional formula ϕ in CNF
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

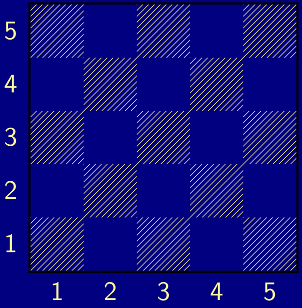
$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program:

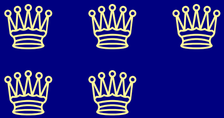
Generator	Tester	Stable models
$\{ a \} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
$\{ b \} \leftarrow$	$\leftarrow a, \sim b$	$X_2 = \{\}$

Outline

The n-Queens Problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another



Defining the Field

queens.lp

```
row(1..n).
col(1..n).
```

- Create file `queens.lp`
- Define the field
  - n rows
  - n columns

Potassco

Defining the Field

Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models      : 1
Time        : 0.000
```

Placing some Queens

queens.lp

```
row(1..n).
col(1..n).
%\alert{\{ queen(I,J) : row(I), col(J) \}.}
```

- Guess a solution candidate
  by placing some queens on the board

Placing some Queens

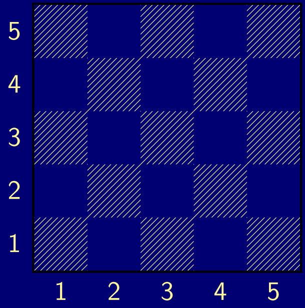
Running ...

```
$ gringo queens.lp --const n=5 | clasp 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) %\alert{queen(1,
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) %\alert{queen(2,
SATISFIABLE

Models      : 3+
```

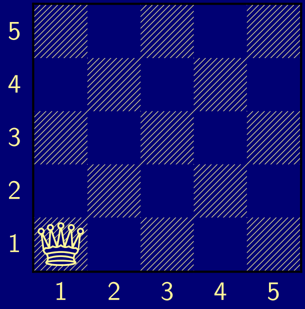
Placing some Queens: Answer 1

Answer 1



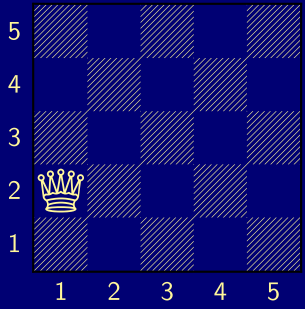
Placing some Queens: Answer 2

Answer 2



Placing some Queens: Answer 3

Answer 3



Placing n Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
%\alert{:-\only<2->{ not} \{ queen(I,J) \} \only<1>
```

- Place exactly n queens on the board

Placing *n* Queens

Running ...

```
$ gringo queens.lp --const n=5 | clasp 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
%\alert{queen(5,1) queen(4,1) queen(3,1) queen(2,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
%\alert{queen(1,2) queen(4,1) queen(3,1) queen(2,1)
```

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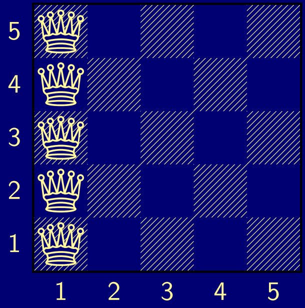
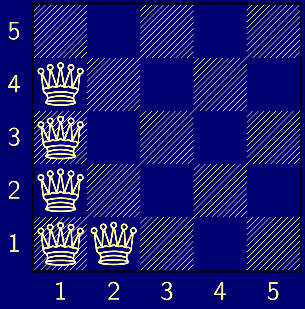
Placing *n* Queens: Answer 1

Answer 1



Placing *n* Queens: Answer 2

Answer 2



Horizontal and Vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
%\alert<1>{:- queen(I,J), queen(I,J'), J != J'.}%
%\uncover<2>{\alert<2>{:- queen(I,J), queen(I',J),
```

- Forbid horizontal attacks
- Forbid vertical attacks

Horizontal and Vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
%\alert<1>{:- queen(I,J), queen(I,J'), J != J'.}%
%\uncover<2>{\alert<2>{:- queen(I,J), queen(I',J),
```

- Forbid horizontal attacks
- Forbid vertical attacks

Horizontal and Vertical Attack

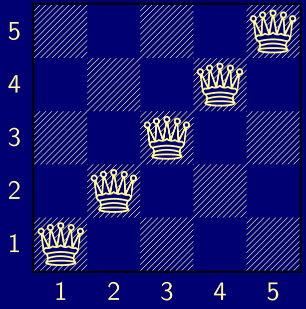
Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
%\alert{queen(5,5) queen(4,4) queen(3,3) queen(2,2)
```

Potassco

Horizontal and Vertical Attack: Answer 1

Answer 1



Diagonal Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
%\alert{:- queen(I,J), queen(I',J'), (I,J) != (I',J
%\alert{:- queen(I,J), queen(I',J'), (I,J) != (I',J
```

- Forbid diagonal attacks

Diagonal Attack

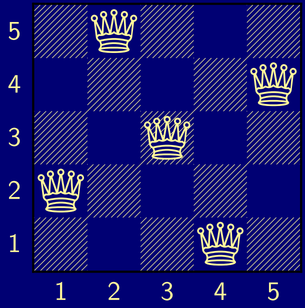
Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
%\alert{queen(4,5) queen(1,4) queen(3,3) queen(5,2)
SATISFIABLE

Models      : 1+
Time        : 0.000
```

Diagonal Attack: Answer 1

Answer 1



Optimizing

queens-opt.lp

```
{ queen(I,1..n) } = 1 :- I = 1..n.
{ queen(1..n,J) } = 1 :- J = 1..n.
 :- { queen(D-J,J) } > 1, D =   2..2*n.
 :- { queen(D+J,J) } > 1, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=2

clingo version 4.1.0
Solving...
SATISFIABLE

Models       : 1+
Time         : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time     : 3758.320s

Choices      : 288594554
Conflicts    : 3442   (Analyzed: 3442)
Restarts     : 17     (Average: 202.47 Last: 3442)
Model-Level  : 7594728.0
Problems     : 1      (Average Length: 0.00 Splits: 0)
Lemmas       : 3442   (Deleted: 0)
  Binary     : 0      (Ratio:   0.00%)
  Ternary    : 0      (Ratio:   0.00%)
  Conflict   : 3442   (Average Length: 229056.5 Ratio: 100.00%)
  Loop       : 0      (Average Length:    0.0 Ratio:   0.00%)
  Other      : 0      (Average Length:    0.0 Ratio:   0.00%)

Atoms        : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules        : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies       : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight        : Yes
Variables    : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664  (Binary: 35.6% Ternary:  0.0% Other: 64.4%)

Backjumps    : 3442   (Average: 681.19 Max: 169512 Sum: 2344658)
  Executed   : 3442   (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
  Bounded    : 0      (Average:  0.00 Max:      0 Sum:       0 Ratio:   0.00%)
```

And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=2
```

```
clingo version 4.1.0
Solving...
SATISFIABLE

Models       : 1+
Time         : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time     : 3758.320s

Choices      : 288594554
Conflicts    : 3442   (Analyzed: 3442)
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Problems     : 1      (Average Length: 0.00 Splits: 0)
Lemmas       : 3442   (Deleted: 0)
  Binary     : 0      (Ratio:   0.00%)
  Ternary    : 0      (Ratio:   0.00%)
  Conflict   : 3442   (Average Length: 229056.5 Ratio: 100.00%)
  Loop       : 0      (Average Length:    0.0 Ratio:   0.00%)
  Other      : 0      (Average Length:    0.0 Ratio:   0.00%)

Atoms        : 75084857 (Original: 75069989 Auxiliary: 14868)
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Constraints  : 66664  (Binary: 35.6% Ternary:  0.0% Other: 64.4%)

Backjumps    : 3442   (Average: 681.19 Max: 169512 Sum: 2344658)
  Executed   : 3442   (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
  Bounded    : 0      (Average:  0.00 Max:    0 Sum:       0 Ratio:   0.00%)
```

Outline

Potassco

Traveling Salesperson

```
node(1..6).

edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Traveling Salesperson

```
node(1..6).

edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Potassco

Traveling Salesperson

```
node(1..6).

edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Potassco

Traveling Salesperson

```
node(1..6).

edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).

edge(X,Y) :- cost(X,Y,_).
```

Traveling Salesperson

```
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(X).
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

Potassco

Traveling Salesperson

```
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(X).
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

Traveling Salesperson

```
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(X).
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

Traveling Salesperson

```
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(X).
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

Outline

Reviewer Assignment

by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Potassco

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Potassco

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
[...]

#count { P,R : assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 <= #count { P,R : assigned(P,R), paper(P) } <= 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 <= #count { P,R : assignedB(P,R), paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
[...]

#count { P,R : assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 <= #count { P,R : assigned(P,R), paper(P) } <= 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 <= #count { P,R : assignedB(P,R), paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Outline

Simplistic STRIPS Planning

```
time(1..k).

fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).    query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

{ occ(A,T) : action(A) } = 1 :- time(T).
:- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- occ(A,T), add(A,F).
holds(F,T) :- holds(F,T-1), time(T), not occ(A,T) : del(A,F).

:- query(F), not holds(F,k).
```

Simplistic STRIPS Planning

```
time(1..k).

fluent(p).     action(a).     action(b).      init(p).
fluent(q).       pre(a,p).      pre(b,q).
fluent(r).       add(a,q).      add(b,r).     query(r).
                 del(a,p).      del(b,q).

holds(P,0) :- init(P).

{ occ(A,T) : action(A) } = 1 :- time(T).
:- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- occ(A,T), add(A,F).
holds(F,T) :- holds(F,T-1), time(T), not occ(A,T) : del(A,F).

:- query(F), not holds(F,k).
```

Potassco

Simplistic STRIPS Planning

```
time(1..k).

fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).     query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

{ occ(A,T) : action(A) } = 1 :- time(T).
:- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- occ(A,T), add(A,F).
holds(F,T) :- holds(F,T-1), time(T), not occ(A,T) : del(A,F).

:- query(F), not holds(F,k).
```

Potassco

Simplistic STRIPS Planning

```
time(1..k).

fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).    query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

{ occ(A,T) : action(A) } = 1 :- time(T).
:- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- occ(A,T), add(A,F).
holds(F,T) :- holds(F,T-1), time(T), not occ(A,T) : del(A,F).

:- query(F), not holds(F,k).
```

Potassco

Multi-shot ASP Solving: Overview

Outline

Potassco

Motivation

- Claim ASP is an under-the-hood technology

  That is, in practice, it mainly serves as a solving engine within an encompassing software environment

- Single-shot solving: *ground* | *solve*

  Multi-shot solving: *ground* | *solve*

  ➥ **continuously changing logic programs**

  Agents, Assisted Living, Robotics, Planning, Query-answering, etc

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- Single-shot solving: *ground* | *solve*
- Multi-shot solving: *ground* | *solve*

  ➥ **continuously changing logic programs**

- Application areas

  Agents, Assisted Living, Robotics, Planning, Query-answering, etc

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- Single-shot solving: *ground* | *solve*
- Multi-shot solving: *ground** | *solve**

  ➥ **continuously changing logic programs**

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  Agents, Assisted Living, Robotics, Planning, Query-answering, etc

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- Claim ASP is an under-the-hood technology

  That is, in practice, it mainly serves as a solving engine within an encompassing software environment

- Single-shot solving: *ground* | *solve*
- Multi-shot solving: (*ground** | *solve**)*

  ➥ **continuously changing logic programs**

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  Agents, Assisted Living, Robotics, Planning, Query-answering, etc

Potassco

Motivation

- Claim ASP is an under-the-hood technology

  That is, in practice, it mainly serves as a solving engine within an encompassing software environment

- Single-shot solving: *ground* | *solve*
- Multi-shot solving: (*input* | *ground** | *solve**)*

  ➥ **continuously changing logic programs**

- Application areas

  Agents, Assisted Living, Robotics, Planning, Query-answering, etc

Potassco

Motivation

- Claim ASP is an under-the-hood technology

  That is, in practice, it mainly serves as a solving engine within an encompassing software environment

- Single-shot solving: *ground* | *solve*
- Multi-shot solving: (*input* | *ground** | *solve** | *theory*)*

  ➥ **continuously changing logic programs**

- Application areas

  Agents, Assisted Living, Robotics, Planning, Query-answering, etc

Potassco

Motivation

- Claim ASP is an under-the-hood technology

  That is, in practice, it mainly serves as a solving engine within an encompassing software environment

- Single-shot solving: *ground* | *solve*
- Multi-shot solving: (*input* | *ground** | *solve**| *theory* | ...)*
  - ➥ **continuously changing logic programs**
- Application areas

  Agents, Assisted Living, Robotics, Planning, Query-answering, etc

Potassco

Motivation

- Claim ASP is an under-the-hood technology

  That is, in practice, it mainly serves as a solving engine within an encompassing software environment

- Single-shot solving: *ground* | *solve*
- Multi-shot solving: (*input* | *ground** | *solve**| *theory* | . . .)*
  - ➥ **continuously changing logic programs**

- Application areas

  Agents, Assisted Living, Robotics, Planning, Query-answering, etc

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Clingo = ASP + Control

- ASP

  `#program <name> [ (<parameters>) ]`

  `#program play(t).`

  `#external <atom> [ : <body> ]`

  `#external mark(X,Y,P,t) : field(X,Y), player(P).`

- Control

  Python (`www.python.org`)

  `prg.solve(), prg.ground(parts), ...`

  C, Lua, and Prolog embeddings are available too

- Integration

  in ASP: embedded scripting language (`#script`)

  in Python: library import (`import clingo`)

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Vanilla *clingo*

```
#script (python)
def main(prg):
    parts = []
    parts.append(("base", []))
    prg.ground(parts)
    prg.solve()
#end.
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Hello world!

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#script (python)
def main(prg):
    print("Hello world!")
#end.
```

```
$ clingo hello.lp
clingo version 4.5.0
Reading from hello.lp
Hello world!
UNKNOWN

Models      : 0+
Calls       : 1
Time        : 0.009s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
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Preview on incremental solving

```
#program base.

p(0).

#program step (t).

p(t) :- p(t-1).

#program check (t).
#external plug(t).

:- not p(42), plug(t).
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Outline

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#program declaration

- A program declaration is of form

  `#program` $n\,(p_1,\dots,p_k)$

  where $n, p_1, \dots, p_k$ are non-integer constants
- We call n the name of the declaration and $p_1, \dots, p_k$ its parameters
- Convention Different occurrences of program declarations with the same name share the same parameters
- Example

```
#program acid(k).
b(k).
c(X,k) :- a(X).
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Scope of #program declarations

- The scope of an occurrence of a program declaration in a list of rules and declarations consists of the set of all rules and non-program declarations appearing between the occurrence and the next occurrence of a program declaration or the end of the list
- Rules and non-program declarations outside the scope of any program declaration are implicitly preceded by a base program declaration
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Scope of #program declarations

- Given a list R of (non-ground) rules and declarations and a name n, we define $R(n)$ as the set of all rules and non-program declarations in the scope of all occurrences of program declarations with name n
- We often refer to $R(n)$ as a subprogram of R
- Example
  - $R(\texttt{base}) = \{a(1), a(2)\}$
  - $R(\texttt{acid}) = \{b(k), c(X,k) \leftarrow a(X)\}$
- Given a name n with associated parameters $(p_1, \dots, p_k)$, the instantiation of $R(n)$ with a term tuple $(t_1, \dots, t_k)$ results in the set

  $$R(n)[p_1/t_1, \dots, p_k/t_k]$$

  obtained by replacing in $R(n)$ each occurrence of p_i by t_i

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Contextual grounding

- Rules are grounded relative to a set of atoms, called atom base
- Given a set R of (non-ground) rules and two sets C, D of ground atoms, we define an instantiation of R relative to C as a ground program $ground_C(R)$ over D subject to the following conditions:

$$C \subseteq D \subseteq C \cup head(ground_C(R))$$

$$ground_C(R) \subseteq \{head(r) \leftarrow body(r)^+ \cup \{\sim a \mid a \in body(r)^- \cap D\} \mid r \in ground(R), body(r)^+ \subseteq D\}$$

- Example Given $R = \{\ a(X) \leftarrow f(X), e(X);\ \ b(X) \leftarrow f(X), \sim e(X)\ \}$ and $C = \{f(1), f(2), e(1)\}$, we obtain

$$ground_C(R) = \left\{ \begin{array}{ll} a(1) \leftarrow f(1), e(1); & b(1) \leftarrow f(1), \sim e(1) \\ & b(2) \leftarrow f(2) \end{array} \right\}$$

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#external declaration

- An external declaration is of form

  $\texttt{\#external}\ a : B$

  where a is an atom and B a rule body
- A logic program with external declarations is said to be extensible
- Example

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#external e(X) : f(X), X < 2.
f(1..2).
a(X) :- f(X), e(X).
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Grounding extensible logic programs

- Given an extensible program R, we define

$$Q = \{a \leftarrow B, \varepsilon \mid (\texttt{\#external}\ a : B) \in R\}$$
$$R' = \{a \leftarrow B \in R\}$$

- Note An external declaration is treated as a rule $a \leftarrow B, \varepsilon$ where ε is a ground marking atom
- Given an atom base C, the ground instantiation of an extensible logic program R is defined as a (ground) logic program P with externals E where

$$P = \{r \in ground_{C \cup \{\varepsilon\}}(R' \cup Q) \mid \varepsilon \notin body(r)\}$$
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Example

- Extensible program

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Atom base $\{$g(1)$\} \cup \{\varepsilon\}$

- Ground program

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with externals $\{$e(1)$\}$

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- Extensible program

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Atom base $\{$g(1)$\} \cup \{\varepsilon\}$

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Outline

Module

- The assembly of subprograms can be characterized by means of modules:
- A module $\mathbb{P}$ is a triple (P, I, O) consisting of
  - a (ground) program P over $ground(\mathcal{A})$ and
  - sets $I, O \subseteq ground(\mathcal{A})$ such that
    - $I \cap O = \emptyset$,
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- The elements of I and O are called input and output atoms
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Composing modules

- Two modules $\mathbb{P}$ and $\mathbb{Q}$ are compositional, if
  $O(\mathbb{P}) \cap O(\mathbb{Q}) = \emptyset$ and
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  Recursion between two modules to be joined is disallowed

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The join, $\mathbb{P} \sqcup \mathbb{Q}$, of two modules $\mathbb{P}$ and $\mathbb{Q}$ is defined as the module

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Composing logic programs with externals

- Idea Each `ground` instruction induces a module to be joined with the module representing the current program state

- Given an atom base C, a (non-ground) extensible program R induces the module

$$\mathbb{R}(C) = (P, (C \cup E) \setminus head(P), head(P))$$

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Example

- Atom base $C = \{\texttt{g(1)}\}$
- Extensible program R

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#external e(X) : f(X), g(X)
f(1). f(2).
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$$= \left(\left\{ \begin{array}{l} f(1),\ f(2), \\ a(1) \leftarrow f(1), e(1), \\ b(1) \leftarrow f(1), {\sim}e(1), \\ b(2) \leftarrow f(2) \end{array} \right\}, \left\{ \begin{array}{l} g(1), \\ e(1) \end{array} \right\}, \left\{ \begin{array}{l} f(1),\ f(2), \\ a(1), \\ b(1), b(2) \end{array} \right\} \right)$$

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Capturing program states by modules

- Each program state is captured by a module
  - The input and output atoms of each module provide the atom base
- The initial program state is given by the empty module

$$\mathbb{P}_0 = (\emptyset, \emptyset, \emptyset)$$

- The program state succeeding $\mathbb{P}_i$ is captured by the module

$$\mathbb{P}_{i+1} = \mathbb{P}_i \sqcup \mathbb{R}_{i+1}(I(\mathbb{P}_i) \cup O(\mathbb{P}_i))$$

  where $\mathbb{R}_{i+1}(I(\mathbb{P}_i) \cup O(\mathbb{P}_i))$ captures the result of grounding an extensible program R relative to atom base $I(\mathbb{P}_i) \cup O(\mathbb{P}_i)$

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Outline

Clingo state

- A *clingo* state is a triple

$$(\boldsymbol{R}, \mathbb{P}, V)$$

  where

  - $\boldsymbol{R}$ is a collection of extensible (non-ground) logic programs
  - $\mathbb{P}$ is a module
  - V is a three-valued assignment over $I(\mathbb{P})$

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- Note Input atoms in $I(\mathbb{P})$ are taken to be false by default

create

- $create(R) : \mapsto (\boldsymbol{R}, \mathbb{P}, V)$

  for a list R of (non-ground) rules and declarations where

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- $\mathit{ground}((n, \boldsymbol{p}_n)_{n \in N}) : (\boldsymbol{R}, \mathbb{P}_1, V_1) \mapsto (\boldsymbol{R}, \mathbb{P}_2, V_2)$

  for a collection $(n, \boldsymbol{p}_n)_{n \in N}$ such that $N \subseteq \mathcal{C}$ and $\boldsymbol{p}_n \in \mathcal{T}^k$ for some k where

  - $\mathbb{P}_2 = \mathbb{P}_1 \sqcup \mathbb{R}(I(\mathbb{P}_1) \cup O(\mathbb{P}_1))$
    and $\mathbb{R}(I(\mathbb{P}_1) \cup O(\mathbb{P}_1))$ is the module obtained from
    - extensible program $\bigcup_{n \in N} R_n[\boldsymbol{p}/\boldsymbol{p}_n]$ and
    - atom base $I(\mathbb{P}_1) \cup O(\mathbb{P}_1)$

    for $(R_c)_{c \in \mathcal{C}} = \boldsymbol{R}$
  - $V_2^t = \{a \in I(\mathbb{P}_2) \mid V_1(a) = t\}$
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ground

- Notes
  - The external status of an atom is eliminated once it becomes defined by a rule in some added program
    This is accomplished by module composition, namely, the elimination of output atoms from input atoms
  - Jointly grounded subprograms are treated as a single subprogram
  - $\mathit{ground}((n, \boldsymbol{p}), (n, \boldsymbol{p}))(s) = \mathit{ground}((n, \boldsymbol{p}))(s)$ while $\mathit{ground}((n, \boldsymbol{p}))(\mathit{ground}((n, \boldsymbol{p}))(s))$ leads to two non-compositional modules whenever $\mathit{head}(R_n) \neq \emptyset$
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assignExternal

- $\mathit{assignExternal}(a, v) : (\boldsymbol{R}, \mathbb{P}, V_1) \mapsto (\boldsymbol{R}, \mathbb{P}, V_2)$

  for a ground atom a and $v \in \{t, u, f\}$ where

  - if $v = t$
    - $V_2^t = V_1^t \cup \{a\}$ if $a \in I(\mathbb{P})$, and $V_2^t = V_1^t$ otherwise
    - $V_2^u = V_1^u \setminus \{a\}$
  - if $v = u$
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    - $V_2^u = V_1^u \cup \{a\}$ if $a \in I(\mathbb{P})$, and $V_2^u = V_1^u$ otherwise
  - if $v = f$
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Only input atoms, that is, non-overwritten externals are affected

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Potassco

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Potassco

solve

- $solve((A^t, A^f)) : (\boldsymbol{R}, \mathbb{P}, V) \mapsto (\boldsymbol{R}, \mathbb{P}, V)$ prints the set

$$\{X \mid X \text{ is a stable model of } \mathbb{P} \text{ wrt } V \text{ st } A^t \subseteq X \text{ and } A^f \cap X = \emptyset\}$$

where the stable models of a module $\mathbb{P}$ wrt an assignment V are given by the stable models of the program

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#script declaration

- A script declaration is of form

  `#script(python)` *P* `#end`

  where *P* is a Python program
- Analogously for Lua
- `main` routine exercises control (from within *clingo*, not from Python)

```
#script(python)
def main(prg):
    prg.ground([("base",[])])
    prg.solve()
#end.
```

```
#script(python)
def main(prg):
    prg.ground([("acid",[42])])
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Potassco

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Potassco

Example

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#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
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Potassco

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Potassco

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#end.
```

Potassco

Extensible programs

- Initial *clingo* state

$$(\boldsymbol{R}_0, \mathbb{P}_0, V_0) = ((R(\texttt{base}), R(\texttt{succ})), (\emptyset, \emptyset, \emptyset), (\emptyset, \emptyset))$$

where

$$R(\texttt{base}) = \left\{ \begin{array}{ll} \texttt{\#external}\ p(1) & p(0) \leftarrow p(3) \\ \texttt{\#external}\ p(2) & p(0) \leftarrow {\sim}p(0) \\ \texttt{\#external}\ p(3) & \end{array} \right\}$$

$$R(\texttt{succ}) = \left\{ \begin{array}{l} \texttt{\#external}\ p(n+3) \\ p(n) \leftarrow p(n+3) \\ p(n) \leftarrow {\sim}p(n+1), {\sim}p(n+2) \end{array} \right\}$$

- Initial atom base $I(\mathbb{P}_0) \cup O(\mathbb{P}_0) = \emptyset$

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Potassco

Extensible programs

- Initial *clingo* state, or more precisely, state of *clingo* object 'prg'

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- Initial atom base $I(\mathbb{P}_0) \cup O(\mathbb{P}_0) = \emptyset$

Extensible programs

- Initial *clingo* state, or more precisely, state of *clingo* object 'prg'

$$create(R) = ((R(\texttt{base}), R(\texttt{succ})), (\emptyset, \emptyset, \emptyset), (\emptyset, \emptyset))$$

where R is the list of rules and declarations in Line 1-8 and

$$R(\texttt{base}) = \left\{ \begin{array}{ll} \texttt{\#external}\ p(1) & p(0) \leftarrow p(3) \\ \texttt{\#external}\ p(2) & p(0) \leftarrow {\sim}p(0) \\ \texttt{\#external}\ p(3) & \end{array} \right\}$$

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- Initial atom base $I(\mathbb{P}_0) \cup O(\mathbb{P}_0) = \emptyset$
- Note *create*(R) is invoked implicitly to create *clingo* object 'prg'

Example

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#external p(1;2;3).
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```

Potassco

prg.ground([("base", [])])

- Global *clingo* state $(\boldsymbol{R}_0, \mathbb{P}_0, V_0)$, including atom base $\emptyset$
- Input Extensible program $R(\texttt{base})$
- Output Module

$$\mathbb{R}_1(\emptyset) = (P_1, E_1, \{p(0)\}) \qquad \text{where}$$
$$P_1 = \{p(0) \leftarrow p(3);\ p(0) \leftarrow {\sim}p(0)\}$$
$$E_1 = \{p(1), p(2), p(3)\}$$

- Result *clingo* state

$$(\boldsymbol{R}_1, \mathbb{P}_1, V_1) = (\boldsymbol{R}_0, \mathbb{P}_0 \sqcup \mathbb{R}_1(\emptyset), V_0)$$

where

$$\mathbb{P}_1 = \mathbb{P}_0 \sqcup \mathbb{R}_1(\emptyset) = (\emptyset, \emptyset, \emptyset) \sqcup (P_1, E_1, \{p(0)\})$$
$$= (\{p(0) \leftarrow p(3);\ p(0) \leftarrow {\sim}p(0)\}, \{p(1), p(2), p(3)\}, \{p(0)\})$$

`prg.ground([("base", [])])`

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$$\mathbb{R}_1(\emptyset) = (P_1, E_1, \{p(0)\}) \qquad \text{where}$$
$$P_1 = \{p(0) \leftarrow p(3);\ p(0) \leftarrow {\sim}p(0)\}$$
$$E_1 = \{p(1), p(2), p(3)\}$$

- Result *clingo* state

$$(\boldsymbol{R}_1, \mathbb{P}_1, V_1) = (\boldsymbol{R}_0, \mathbb{P}_0 \sqcup \mathbb{R}_1(\emptyset), V_0)$$

where

$$\mathbb{P}_1 = \mathbb{P}_0 \sqcup \mathbb{R}_1(\emptyset) = (\emptyset, \emptyset, \emptyset) \sqcup (P_1, E_1, \{p(0)\})$$
$$= (\{p(0) \leftarrow p(3);\ p(0) \leftarrow {\sim}p(0)\}, \{p(1), p(2), p(3)\}, \{p(0)\})$$

`prg.ground([("base", [])])`

- Global *clingo* state $(\boldsymbol{R}_0, \mathbb{P}_0, V_0)$, including atom base $\emptyset$
- Input Extensible program $R(\texttt{base})$
- Output Module

$$\mathbb{R}_1(\emptyset) = (P_1, E_1, \{p(0)\}) \qquad \text{where}$$
$$P_1 = \{p(0) \leftarrow p(3);\ p(0) \leftarrow {\sim}p(0)\}$$
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$$= (\{p(0) \leftarrow p(3);\ p(0) \leftarrow {\sim}p(0)\}, \{p(1), p(2), p(3)\}, \{p(0)\})$$

prg.ground([("base", [])])

- Global *clingo* state $(\boldsymbol{R}_0, \mathbb{P}_0, V_0)$, including atom base $\emptyset$
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$$\mathbb{R}_1(\emptyset) = (P_1, E_1, \{p(0)\}) \qquad \text{where}$$
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where

$$\mathbb{P}_1 = \mathbb{P}_0 \sqcup \mathbb{R}_1(\emptyset) = (\emptyset, \emptyset, \emptyset) \sqcup (P_1, E_1, \{p(0)\})$$
$$= (\{p(0) \leftarrow p(3);\ p(0) \leftarrow {\sim}p(0)\}, \{p(1), p(2), p(3)\}, \{p(0)\})$$

Potassco

prg.ground([("base", [])])

- Global *clingo* state $(\boldsymbol{R}_0, \mathbb{P}_0, V_0)$, including atom base $\emptyset$
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$$\mathbb{P}_1 = \mathbb{P}_0 \sqcup \mathbb{R}_1(\emptyset) = (\emptyset, \emptyset, \emptyset) \sqcup (P_1, E_1, \{p(0)\})$$
$$= (\{p(0) \leftarrow p(3);\ p(0) \leftarrow {\sim}p(0)\}, \{p(1), p(2), p(3)\}, \{p(0)\})$$

Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from clingo import Fun
def main(prg):
>>    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Potassco

Example

```
#external p(1;2;3).
p(0) :- p(3).
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    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

`prg.assign_external(Fun("p",[3]),True)`

- Global *clingo* state $(\boldsymbol{R}_1, \mathbb{P}_1, V_1)$
- Input assignment $p(3) \mapsto t$
- Result *clingo* state

$$(\boldsymbol{R}_2, \mathbb{P}_2, V_2) = (\boldsymbol{R}_0, \mathbb{P}_1, (\{p(3)\}, \emptyset))$$

Potassco

`prg.assign_external(Fun("p",[3]),True)`

- Global *clingo* state $(\boldsymbol{R}_1, \mathbb{P}_1, V_1)$
- Input assignment $p(3) \mapsto t$
- Result *clingo* state

$$(\boldsymbol{R}_2, \mathbb{P}_2, V_2) = (\boldsymbol{R}_0, \mathbb{P}_1, (\{p(3)\}, \emptyset))$$

Potassco

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    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Potassco

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    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Potassco

`prg.solve()`

- Global *clingo* state $(\boldsymbol{R}_2, \mathbb{P}_2, V_2)$
- Input empty assignment
- Result *clingo* state

$$(\boldsymbol{R}_2, \mathbb{P}_2, V_2) = (\boldsymbol{R}_0, \mathbb{P}_1, (\{p(3)\}, \emptyset))$$

stable model $\{p(0), p(3)\}$ of $\mathbb{P}_2$ wrt V_2

prg.solve()

- Global *clingo* state $(\boldsymbol{R}_2, \mathbb{P}_2, V_2)$
- Input empty assignment
- Result *clingo* state

$$(\boldsymbol{R}_2, \mathbb{P}_2, V_2) = (\boldsymbol{R}_0, \mathbb{P}_1, (\{p(3)\}, \emptyset))$$

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$$(\boldsymbol{R}_2, \mathbb{P}_2, V_2) = (\boldsymbol{R}_0, \mathbb{P}_1, (\{p(3)\}, \emptyset))$$

- Print stable model $\{p(0), p(3)\}$ of $\mathbb{P}_2$ wrt V_2

Potassco

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Example

```
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#script(python)
from clingo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
>>  prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Potassco

Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

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    prg.solve()
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    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

prg.assign_external(Fun("p",[3]),False)

- Global *clingo* state $(\boldsymbol{R}_2, \mathbb{P}_2, V_2)$
- Input assignment $p(3) \mapsto f$
- Result *clingo* state

$$(\boldsymbol{R}_3, \mathbb{P}_3, V_3) = (\boldsymbol{R}_0, \mathbb{P}_1, (\emptyset, \emptyset))$$

Potassco

`prg.assign_external(Fun("p",[3]),False)`

- Global *clingo* state $(\boldsymbol{R}_2, \mathbb{P}_2, V_2)$
- Input assignment $p(3) \mapsto f$
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Potassco

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    prg.ground([("succ", [1]),("succ", [2])])
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    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

`prg.solve()`

- Global *clingo* state $(\boldsymbol{R}_3, \mathbb{P}_3, V_3)$
- Input empty assignment
- Result *clingo* state

$$(\boldsymbol{R}_3, \mathbb{P}_3, V_3) = (\boldsymbol{R}_0, \mathbb{P}_1, (\emptyset, \emptyset))$$

- Print no stable model of $\mathbb{P}_3$ wrt V_3

Potassco

prg.solve()

- Global *clingo* state $(\boldsymbol{R}_3, \mathbb{P}_3, V_3)$
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- Result *clingo* state

$$(\boldsymbol{R}_3, \mathbb{P}_3, V_3) = (\boldsymbol{R}_0, \mathbb{P}_1, (\emptyset, \emptyset))$$

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    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Example

```
   #external p(1;2;3).
   p(0) :- p(3).
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   p(n) :- p(n+3).
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   #script(python)
   from clingo import Fun
   def main(prg):
       prg.ground([("base", [])])
       prg.assign_external(Fun("p", [3]), True)
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       prg.solve()
>>     prg.ground([("succ", [1]),("succ", [2])])
       prg.solve()
       prg.ground([("succ", [3])])
       prg.solve()
   #end.
```

prg.ground([("succ",[1]),("succ",[2])])

- Global *clingo* state $(\boldsymbol{R}_3, \mathbb{P}_3, V_3)$, including atom base
  $I(\mathbb{P}_3) \cup O(\mathbb{P}_3) = \{p(0), p(1), p(2), p(3)\}$
- Input Extensible program $R(\mathtt{succ})[\mathtt{n}/1] \cup R(\mathtt{succ})[\mathtt{n}/2]$
- Output Module

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(P_4, \left\{\begin{array}{l} p(0), p(4), \\ p(3), p(5) \end{array}\right\}, \left\{\begin{array}{l} p(1), \\ p(2) \end{array}\right\}\right) \quad \text{where}$$

$$P_4 = \left\{\begin{array}{ll} p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{array}\right\}$$

$$E_4 = \{p(4), p(5)\}$$

- Result *clingo* state

$$(\boldsymbol{R}_4, \mathbb{P}_4, V_4) = (\boldsymbol{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

`prg.ground([("succ",[1]),("succ",[2])])`

- Global *clingo* state $(\boldsymbol{R}_3, \mathbb{P}_3, V_3)$, including atom base
  $$I(\mathbb{P}_3) \cup O(\mathbb{P}_3) = \{p(0), p(1), p(2), p(3)\}$$
- Input Extensible program $R(\mathtt{succ})[\mathtt{n}/1] \cup R(\mathtt{succ})[\mathtt{n}/2]$
- Output Module

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(P_4, \left\{\begin{matrix} p(0), p(4), \\ p(3), p(5) \end{matrix}\right\}, \left\{\begin{matrix} p(1), \\ p(2) \end{matrix}\right\}\right) \quad \text{where}$$

$$P_4 = \left\{\begin{matrix} p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{matrix}\right\}$$

$$E_4 = \{p(4), p(5)\}$$

- Result *clingo* state

$$(\boldsymbol{R}_4, \mathbb{P}_4, V_4) = (\boldsymbol{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

`prg.ground([("succ",[1]),("succ",[2])])`

- Result *clingo* state

$$(\boldsymbol{R}_4, \mathbb{P}_4, V_4) = (\boldsymbol{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\left\{ \begin{array}{lll} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(0) \leftarrow {\sim}p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{array} \right\}, \left\{ \begin{array}{r} p(4), \\ p(3), p(5) \end{array} \right\}, \left\{ \begin{array}{r} p(0), p(1), \\ p(2) \end{array} \right\} \right)$$

$$\mathbb{P}_3 = \left(\left\{ \begin{array}{l} p(0) \leftarrow p(3); \\ p(0) \leftarrow {\sim}p(0) \end{array} \right\}, \{p(1), p(2), p(3)\}, \{p(0)\} \right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\left\{ \begin{array}{ll} p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{array} \right\}, \left\{ \begin{array}{l} p(0), p(4), \\ p(3), p(5) \end{array} \right\}, \left\{ \begin{array}{l} p(1), \\ p(2) \end{array} \right\} \right)$$

prg.ground([("succ",[1]),("succ",[2])])

- Result *clingo* state

$$(\boldsymbol{R}_4, \mathbb{P}_4, V_4) = (\boldsymbol{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\begin{Bmatrix} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(0) \leftarrow {\sim}p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{Bmatrix}, \begin{Bmatrix} p(4), \\ p(3), p(5) \end{Bmatrix}, \begin{Bmatrix} p(0), p(1), \\ p(2) \end{Bmatrix} \right)$$

$$\mathbb{P}_3 = \left(\begin{Bmatrix} p(0) \leftarrow p(3); \\ p(0) \leftarrow {\sim}p(0) \end{Bmatrix}, \{p(1), p(2), p(3)\}, \{p(0)\} \right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\begin{Bmatrix} p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{Bmatrix}, \begin{Bmatrix} p(0), p(4), \\ p(3), p(5) \end{Bmatrix}, \begin{Bmatrix} p(1), \\ p(2) \end{Bmatrix} \right)$$

prg.ground([("succ",[1]),("succ",[2])])

- Result *clingo* state

$$(\boldsymbol{R}_4, \mathbb{P}_4, V_4) = (\boldsymbol{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\left\{ \begin{array}{lll} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(0) \leftarrow \sim p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{array} \right\}, \left\{ \begin{array}{r} p(4), \\ p(3), p(5) \end{array} \right\}, \left\{ \begin{array}{r} p(0), p(1), \\ p(2) \end{array} \right\} \right)$$

$$\mathbb{P}_3 = \left(\left\{ \begin{array}{l} p(0) \leftarrow p(3); \\ p(0) \leftarrow \sim p(0) \end{array} \right\}, \{p(1), p(2), p(3)\}, \{p(0)\} \right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\left\{ \begin{array}{ll} p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{array} \right\}, \left\{ \begin{array}{l} p(0), p(4), \\ p(3), p(5) \end{array} \right\}, \left\{ \begin{array}{l} p(1), \\ p(2) \end{array} \right\} \right)$$

Potassco

prg.ground([("succ",[1]),("succ",[2])])

- Result *clingo* state

$$(\boldsymbol{R}_4, \mathbb{P}_4, V_4) = (\boldsymbol{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\left\{ \begin{matrix} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(0) \leftarrow \sim p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{matrix} \right\}, \left\{ \begin{matrix} p(4), \\ p(3), p(5) \end{matrix} \right\}, \left\{ \begin{matrix} p(0), p(1), \\ p(2) \end{matrix} \right\} \right)$$

$$\mathbb{P}_3 = \left(\left\{ \begin{matrix} p(0) \leftarrow p(3); \\ p(0) \leftarrow \sim p(0) \end{matrix} \right\}, \{p(1), p(2), p(3)\}, \{p(0)\} \right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\left\{ \begin{matrix} p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{matrix} \right\}, \left\{ \begin{matrix} p(0), p(4), \\ p(3), p(5) \end{matrix} \right\}, \left\{ \begin{matrix} p(1), \\ p(2) \end{matrix} \right\} \right)$$

`prg.ground([("succ",[1]),("succ",[2])])`

- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\begin{Bmatrix} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(0) \leftarrow \sim p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{Bmatrix}, \begin{Bmatrix} p(4), \\ p(3), p(5) \end{Bmatrix}, \begin{Bmatrix} p(0), p(1), \\ p(2) \end{Bmatrix}\right)$$

$$\mathbb{P}_3 = \left(\begin{Bmatrix} p(0) \leftarrow p(3); \\ p(0) \leftarrow \sim p(0) \end{Bmatrix}, \{p(1), p(2), p(3)\}, \{p(0)\}\right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\begin{Bmatrix} p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{Bmatrix}, \begin{Bmatrix} p(0), p(4), \\ p(3), p(5) \end{Bmatrix}, \begin{Bmatrix} p(1), \\ p(2) \end{Bmatrix}\right)$$

prg.ground([("succ",[1]),("succ",[2])])

- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\left\{ \begin{array}{lll} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(0) \leftarrow \sim p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{array} \right\}, \left\{ \begin{array}{r} p(4), \\ p(3), p(5) \end{array} \right\}, \left\{ \begin{array}{r} p(0), p(1), \\ p(2) \end{array} \right\} \right)$$

$$\mathbb{P}_3 = \left(\left\{ \begin{array}{l} p(0) \leftarrow p(3); \\ p(0) \leftarrow \sim p(0) \end{array} \right\}, \{p(1), p(2), p(3)\}, \{p(0)\} \right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\left\{ \begin{array}{ll} p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{array} \right\}, \left\{ \begin{array}{l} p(0), p(4), \\ p(3), p(5) \end{array} \right\}, \left\{ \begin{array}{l} p(1), \\ p(2) \end{array} \right\} \right)$$

prg.ground([("succ",[1]),("succ",[2])])

- Result *clingo* state

$$(\boldsymbol{R}_4, \mathbb{P}_4, V_4) = (\boldsymbol{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\left\{ \begin{array}{lll} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(0) \leftarrow {\sim}p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{array} \right\}, \left\{ \begin{array}{r} p(4), \\ p(3), p(5) \end{array} \right\}, \left\{ \begin{array}{r} p(0), p(1), \\ p(2) \end{array} \right\} \right)$$

$$\mathbb{P}_3 = \left(\left\{ \begin{array}{l} p(0) \leftarrow p(3); \\ p(0) \leftarrow {\sim}p(0) \end{array} \right\}, \{p(1), p(2), p(3)\}, \{p(0)\} \right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\left\{ \begin{array}{ll} p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{array} \right\}, \left\{ \begin{array}{l} p(0), p(4), \\ p(3), p(5) \end{array} \right\}, \left\{ \begin{array}{l} p(1), \\ p(2) \end{array} \right\} \right)$$

prg.ground([("succ",[1]),("succ",[2])])

- Result *clingo* state

$$(\boldsymbol{R}_4, \mathbb{P}_4, V_4) = (\boldsymbol{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\left\{ \begin{matrix} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(0) \leftarrow {\sim}p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{matrix} \right\}, \left\{ \begin{matrix} p(4), \\ p(3), p(5) \end{matrix} \right\}, \left\{ \begin{matrix} p(0), p(1), \\ p(2) \end{matrix} \right\} \right)$$

$$\mathbb{P}_3 = \left(\left\{ \begin{matrix} p(0) \leftarrow p(3); \\ p(0) \leftarrow {\sim}p(0) \end{matrix} \right\}, \{p(1), p(2), p(3)\}, \{p(0)\} \right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\left\{ \begin{matrix} p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{matrix} \right\}, \left\{ \begin{matrix} p(0), p(4), \\ p(3), p(5) \end{matrix} \right\}, \left\{ \begin{matrix} p(1), \\ p(2) \end{matrix} \right\} \right)$$

prg.ground([("succ",[1]),("succ",[2])])

- Result *clingo* state

$$(\boldsymbol{R}_4, \mathbb{P}_4, V_4) = (\boldsymbol{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\left\{\begin{matrix} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(0) \leftarrow {\sim}p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{matrix}\right\}, \left\{\begin{matrix} p(4), \\ p(3), p(5) \end{matrix}\right\}, \left\{\begin{matrix} p(0), p(1), \\ p(2) \end{matrix}\right\}\right)$$

$$\mathbb{P}_3 = \left(\left\{\begin{matrix} p(0) \leftarrow p(3); \\ p(0) \leftarrow {\sim}p(0) \end{matrix}\right\}, \{p(1), p(2), p(3)\}, \{p(0)\}\right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\left\{\begin{matrix} p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{matrix}\right\}, \left\{\begin{matrix} p(0), p(4), \\ p(3), p(5) \end{matrix}\right\}, \left\{\begin{matrix} p(1), \\ p(2) \end{matrix}\right\}\right)$$

Example

```
   #external p(1;2;3).
   p(0) :- p(3).
   p(0) :- not p(0).

   #program succ(n).
   #external p(n+3).
   p(n) :- p(n+3).
   p(n) :- not p(n+1), not p(n+2).

   #script(python)
   from clingo import Fun
   def main(prg):
       prg.ground([("base", [])])
       prg.assign_external(Fun("p", [3]), True)
       prg.solve()
       prg.assign_external(Fun("p", [3]), False)
       prg.solve()
>>     prg.ground([("succ", [1]),("succ", [2])])
       prg.solve()
       prg.ground([("succ", [3])])
       prg.solve()
   #end.
```

Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from clingo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
>>  prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Potassco

`prg.solve()`

- Global *clingo* state $(\boldsymbol{R}_4, \mathbb{P}_4, V_4)$
- Input empty assignment
- Result *clingo* state

$$(\boldsymbol{R}_4, \mathbb{P}_4, V_4) = (\boldsymbol{R}_0, \mathbb{P}_4, V_3)$$

- Print no stable model of $\mathbb{P}_4$ wrt V_4

`prg.solve()`

- Global *clingo* state $(\boldsymbol{R}_4, \mathbb{P}_4, V_4)$
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prg.solve()

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Example

```
#external p(1;2;3).
p(0) :- p(3).
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#script(python)
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def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
>>  prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from clingo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
>>  prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Potassco

prg.ground([("succ", [3])])

- Global *clingo* state $(\boldsymbol{R}_4, \mathbb{P}_4, V_4)$, including atom base
  $I(\mathbb{P}_4) \cup O(\mathbb{P}_4) = \{p(0), p(1), p(2), p(3), p(4), p(5)\}$
- Input Extensible program $R(\mathtt{succ})[\mathtt{n}/3]$
- Output Module

$$\mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)) = \left(P_5, \left\{\begin{array}{l} p(0), p(1), p(2), \\ p(4), p(5), p(6) \end{array}\right\}, \{p(3)\}\right)$$

where $P_5 = \{p(3) \leftarrow p(6);\ p(3) \leftarrow {\sim}p(4), {\sim}p(5)\}$

$E_5 = \{p(6)\}$

- Result *clingo* state

$$(\boldsymbol{R}_5, \mathbb{P}_5, V_5) = (\boldsymbol{R}_0, \mathbb{P}_4 \sqcup \mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)), V_3)$$

`prg.ground([("succ", [3])])`

- Global *clingo* state $(\boldsymbol{R}_4, \mathbb{P}_4, V_4)$, including atom base
  $$I(\mathbb{P}_4) \cup O(\mathbb{P}_4) = \{p(0), p(1), p(2), p(3), p(4), p(5)\}$$
- Input Extensible program $R(\mathtt{succ})[\mathtt{n}/3]$
- Output Module

$$\mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)) = \left(P_5, \left\{\begin{array}{l} p(0), p(1), p(2), \\ p(4), p(5), p(6) \end{array}\right\}, \{p(3)\}\right)$$

$$\text{where } P_5 = \{p(3) \leftarrow p(6);\ p(3) \leftarrow \sim p(4), \sim p(5)\}$$

$$E_5 = \{p(6)\}$$

- Result *clingo* state

$$(\boldsymbol{R}_5, \mathbb{P}_5, V_5) = (\boldsymbol{R}_0, \mathbb{P}_4 \sqcup \mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)), V_3)$$

Potassco

`prg.ground([("succ", [3])])`

- Global *clingo* state $(\boldsymbol{R}_4, \mathbb{P}_4, V_4)$, including atom base
  $$I(\mathbb{P}_4) \cup O(\mathbb{P}_4) = \{p(0), p(1), p(2), p(3), p(4), p(5)\}$$
- Input Extensible program $R(\mathtt{succ})[\mathtt{n}/3]$
- Output Module

$$\mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)) = \left(P_5, \left\{\begin{array}{l} p(0), p(1), p(2), \\ p(4), p(5), p(6) \end{array}\right\}, \{p(3)\}\right)$$

$$\text{where } P_5 = \{p(3) \leftarrow p(6);\ p(3) \leftarrow {\sim}p(4), {\sim}p(5)\}$$

$$E_5 = \{p(6)\}$$

- Result *clingo* state

$$(\boldsymbol{R}_5, \mathbb{P}_5, V_5) = (\boldsymbol{R}_0, \mathbb{P}_4 \sqcup \mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)), V_3)$$

prg.ground([("succ", [3])])

- Result *clingo* state

$$(\boldsymbol{R}_5, \mathbb{P}_5, V_5) = (\boldsymbol{R}_0, \mathbb{P}_4 \sqcup \mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)), V_3)$$

where

$$\boldsymbol{R}_5 = (R(\texttt{base}), R(\texttt{succ}))$$

$$P(\mathbb{P}_5) = \left\{ \begin{array}{lll} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(0) \leftarrow {\sim}p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4); \\ & p(3) \leftarrow p(6); & p(3) \leftarrow {\sim}p(4), {\sim}p(5) \end{array} \right\}$$

$$I(\mathbb{P}_5) = \{p(4), p(5), p(6)\}$$

$$O(\mathbb{P}_5) = \{p(0), p(1), p(2), p(3)\}$$

$$V_5 = (\emptyset, \emptyset)$$

Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from clingo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
>>  prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from clingo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
>>  prg.solve()
#end.
```

prg.solve()

- Global *clingo* state $(\boldsymbol{R}_5, \mathbb{P}_5, V_5)$
- Input empty assignment
- Result *clingo* state

$$(\boldsymbol{R}_5, \mathbb{P}_5, V_5) = (\boldsymbol{R}_0, \mathbb{P}_5, V_3)$$

- Print stable model $\{p(0), p(3)\}$ of $\mathbb{P}_5$ wrt V_5

prg.solve()

- Global *clingo* state $(\boldsymbol{R}_5, \mathbb{P}_5, V_5)$
- Input empty assignment
- Result *clingo* state

$$(\boldsymbol{R}_5, \mathbb{P}_5, V_5) = (\boldsymbol{R}_0, \mathbb{P}_5, V_3)$$

- Print stable model $\{p(0), p(3)\}$ of $\mathbb{P}_5$ wrt V_5

prg.solve()

- Global *clingo* state $(\boldsymbol{R}_5, \mathbb{P}_5, V_5)$
- Input empty assignment
- Result *clingo* state

$$(\boldsymbol{R}_5, \mathbb{P}_5, V_5) = (\boldsymbol{R}_0, \mathbb{P}_5, V_3)$$

- Print stable model $\{p(0), p(3)\}$ of $\mathbb{P}_5$ wrt V_5

simple.lp

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from clingo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Potassco

Clingo on the run

```
$ clingo simple.lp
clingo version 4.5.0
Reading from simple.lp
Solving...
Answer: 1
p(3) p(0)
Solving...
Solving...
Solving...
Answer: 1
p(3) p(0)
SATISFIABLE

Models       : 2+
Calls        : 4
Time         : 0.019s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.010s
```

Clingo on the run

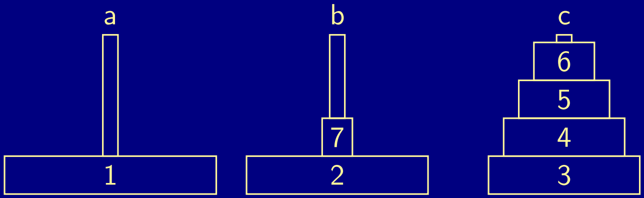
```
$ clingo simple.lp
clingo version 4.5.0
Reading from simple.lp
Solving...
Answer: 1
p(3) p(0)
Solving...
Solving...
Solving...
Answer: 1
p(3) p(0)
SATISFIABLE

Models       : 2+
Calls        : 4
Time         : 0.019s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.010s
```

Potassco

Outline

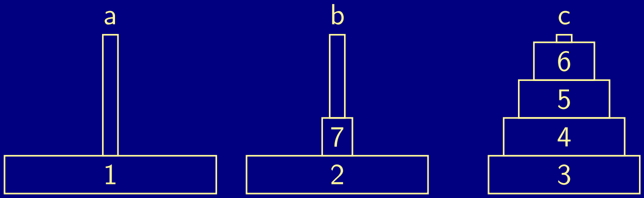
Towers of Hanoi Instance




```
peg(a;b;c).   disk(1..7).

    init_on(1,a).  init_on((2;7),b).  init_on((3;4;5;6),c).
goal_on((3;4),a).                    goal_on((1;2;5;6;7),c).
```

Towers of Hanoi Instance




```
peg(a;b;c).   disk(1..7).

    init_on(1,a).  init_on((2;7),b).  init_on((3;4;5;6),c).
goal_on((3;4),a).                   goal_on((1;2;5;6;7),c).
```

Towers of Hanoi Encoding

```
#program base.

on(D,P,0) :- init_on(D,P).
```

Towers of Hanoi Encoding

```
#program step(t).

1 { move(D,P,t) : disk(D), peg(P) } 1.

moved(D,t) :- move(D,_,t).
blocked(D,P,t) :- on(D+1,P,t-1), disk(D+1).
blocked(D,P,t) :- blocked(D+1,P,t), disk(D+1).
:- move(D,P,t), blocked(D-1,P,t).
:- moved(D,t), on(D,P,t-1), blocked(D,P,t).

on(D,P,t) :- on(D,P,t-1), not moved(D,t).
on(D,P,t) :- move(D,P,t).
:- not 1 { on(D,P,t) : peg(P) } 1, disk(D).
```

Potassco

Towers of Hanoi Encoding

```
#program check(t).
#external query(t).

:- goal_on(D,P), not on(D,P,t), query(t).
```

Incremental Solving (ASP)

```
#script (python)

from clingo import SolveResult, Fun

def main(prg):
    ret, parts, step = SolveResult.UNSAT, [], 1
    parts.append(("base", []))
    while ret == SolveResult.UNSAT:
        parts.append(("step", [step]))
        parts.append(("check", [step]))
        prg.ground(parts)
        prg.release_external(Fun("query", [step-1]))
        prg.assign_external(Fun("query", [step]), True)
        ret, parts, step = prg.solve(), [], step+1

#end.
```

Potassco

Incremental Solving (tohCtrl.lp)

```
#script (python)

from clingo import SolveResult, Fun

def main(prg):
    ret, parts, step = SolveResult.UNSAT, [], 1
    parts.append(("base", []))
    while ret == SolveResult.UNSAT:
        parts.append(("step", [step]))
        parts.append(("check", [step]))
        prg.ground(parts)
        prg.release_external(Fun("query", [step-1]))
        prg.assign_external(Fun("query", [step]), True)
        ret, parts, step = prg.solve(), [], step+1

#end.
```

Potassco

Incremental Solving

```
$ clingo toh.lp tohCtrl.lp
clingo version 4.5.0
Reading from toh.lp ...
Solving...
Solving...
[...]
Solving...
Answer: 1
move(7,a,1)  move(6,b,2)  move(7,b,3)  move(5,a,4)  move(7,c,5)  move(6,a,6)  \
move(7,a,7)  move(4,b,8)  move(7,b,9)  move(6,c,10) move(7,c,11) move(5,b,12) \
move(1,c,13) move(7,a,14) move(6,b,15) move(7,b,16) move(3,a,17) move(7,c,18) \
move(6,a,19) move(7,a,20) move(5,c,21) move(7,b,22) move(6,c,23) move(7,c,24) \
move(4,a,25) move(7,a,26) move(6,b,27) move(7,b,28) move(5,a,29) move(7,c,30) \
move(6,a,31) move(7,a,32) move(2,c,33) move(7,c,34) move(6,b,35) move(7,b,36) \
move(5,c,37) move(7,a,38) move(6,c,39) move(7,c,40)
SATISFIABLE

Models       : 1+
Calls        : 40
Time         : 0.312s (Solving: 0.22s 1st Model: 0.01s Unsat: 0.21s)
CPU Time     : 0.300s
```

Incremental Solving

```
$ clingo toh.lp tohCtrl.lp
clingo version 4.5.0
Reading from toh.lp ...
Solving...
Solving...
[...]
Solving...
Answer: 1
move(7,a,1)  move(6,b,2)  move(7,b,3)  move(5,a,4)  move(7,c,5)  move(6,a,6)  \
move(7,a,7)  move(4,b,8)  move(7,b,9)  move(6,c,10) move(7,c,11) move(5,b,12) \
move(1,c,13) move(7,a,14) move(6,b,15) move(7,b,16) move(3,a,17) move(7,c,18) \
move(6,a,19) move(7,a,20) move(5,c,21) move(7,b,22) move(6,c,23) move(7,c,24) \
move(4,a,25) move(7,a,26) move(6,b,27) move(7,b,28) move(5,a,29) move(7,c,30) \
move(6,a,31) move(7,a,32) move(2,c,33) move(7,c,34) move(6,b,35) move(7,b,36) \
move(5,c,37) move(7,a,38) move(6,c,39) move(7,c,40)
SATISFIABLE

Models       : 1+
Calls        : 40
Time         : 0.312s (Solving: 0.22s 1st Model: 0.01s Unsat: 0.21s)
CPU Time     : 0.300s
```

Incremental Solving (Python)

```
from sys import stdout
from clingo import SolveResult, Fun, Control

prg = Control()
prg.load("toh.lp")

ret, parts, step = SolveResult.UNSAT, [], 1
parts.append(("base", []))
while ret == SolveResult.UNSAT:
    parts.append(("step", [step]))
    parts.append(("check", [step]))
    prg.ground(parts)
    prg.release_external(Fun("query", [step-1]))
    prg.assign_external(Fun("query", [step]), True)
    f = lambda m: stdout.write(str(m))
    ret, parts, step = prg.solve(on_model=f), [], step+1
```

Incremental Solving (tohCtrl.py)

```
from sys import stdout
from clingo import SolveResult, Fun, Control

prg = Control()
prg.load("toh.lp")

ret, parts, step = SolveResult.UNSAT, [], 1
parts.append(("base", []))
while ret == SolveResult.UNSAT:
    parts.append(("step", [step]))
    parts.append(("check", [step]))
    prg.ground(parts)
    prg.release_external(Fun("query", [step-1]))
    prg.assign_external(Fun("query", [step]), True)
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```

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    prg.assign_external(Fun("query", [step]), True)
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```

Incremental Solving (tohCtrl.py)

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```

Incremental Solving (tohCtrl.py)

```
from sys import stdout
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prg = Control()
prg.load("toh.lp")

ret, parts, step = SolveResult.UNSAT, [], 1
parts.append(("base", []))
while ret == SolveResult.UNSAT:
    parts.append(("step", [step]))
    parts.append(("check", [step]))
    prg.ground(parts)
    prg.release_external(Fun("query", [step-1]))
    prg.assign_external(Fun("query", [step]), True)
    f = lambda m: stdout.write(str(m))
    ret, parts, step = prg.solve(on_model=f), [], step+1
```

Potassco

Incremental Solving (Python)

```
$ python tohCtrl.py
move(7,c,40) move(7,a,20) move(7,c,18) move(6,a,31) move(6,b,15) move(7,b,36) \
move(7,c,24) move(7,c,11) move(3,a,17) move(6,a,19) move(7,b,3)  move(7,c,5)  \
move(7,a,1)  move(6,b,35) move(6,c,10) move(6,a,6)  move(6,b,2)  move(7,b,9)  \
move(7,a,7)  move(4,b,8)  move(7,a,38) move(7,b,16) move(5,a,29) move(7,b,22) \
move(6,c,39) move(6,c,23) move(5,b,12) move(4,a,25) move(1,c,13) move(5,a,4)  \
move(7,a,14) move(7,a,26) move(6,b,27) move(7,a,32) move(7,b,28) move(7,c,30) \
move(2,c,33) move(5,c,21) move(7,c,34) move(5,c,37)
```

Potassco

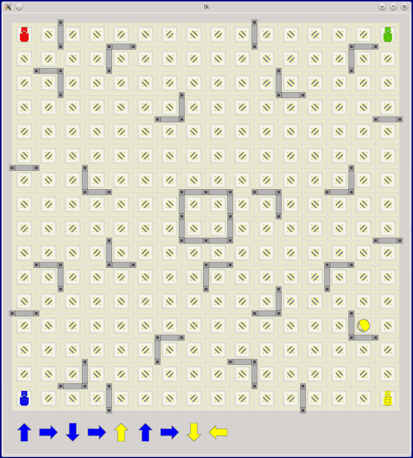
Incremental Solving (Python)

```
$ python tohCtrl.py
move(7,c,40) move(7,a,20) move(7,c,18) move(6,a,31) move(6,b,15) move(7,b,36) \
move(7,c,24) move(7,c,11) move(3,a,17) move(6,a,19) move(7,b,3)  move(7,c,5)  \
move(7,a,1)  move(6,b,35) move(6,c,10) move(6,a,6)  move(6,b,2)  move(7,b,9)  \
move(7,a,7)  move(4,b,8)  move(7,a,38) move(7,b,16) move(5,a,29) move(7,b,22) \
move(6,c,39) move(6,c,23) move(5,b,12) move(4,a,25) move(1,c,13) move(5,a,4)  \
move(7,a,14) move(7,a,26) move(6,b,27) move(7,a,32) move(7,b,28) move(7,c,30) \
move(2,c,33) move(5,c,21) move(7,c,34) move(5,c,37)
```

Outline

Alex Rudolph's Ricochet Robots

Solving goal(13) from cornered robots

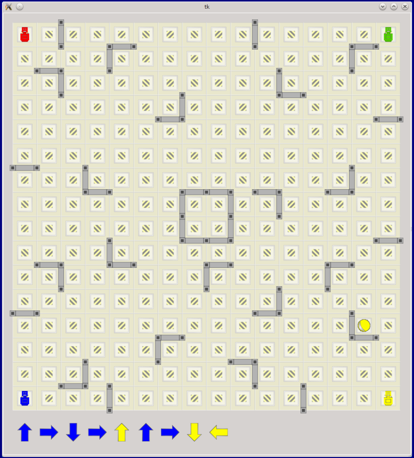


- Four robots roaming
  - horizontally
  - vertically

  up to blocking objects, ricocheting (optionally)
- Goal Robot on target (sharing same color)

Alex Rudolph's Ricochet Robots

Solving goal(13) from cornered robots



- Four robots roaming
  - horizontally
  - vertically

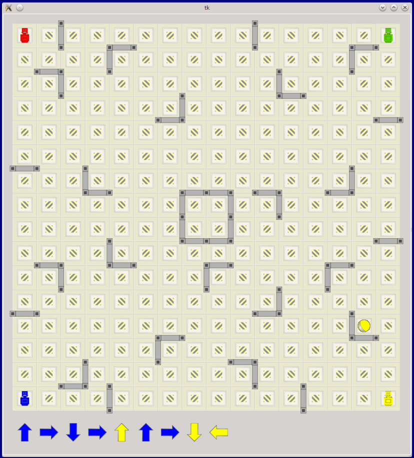
  up to blocking objects, ricocheting (optionally)

- Goal Robot on target (sharing same color)

Potassco

Alex Rudolph's Ricochet Robots

Solving goal(13) from cornered robots



- Four robots roaming
  - horizontally
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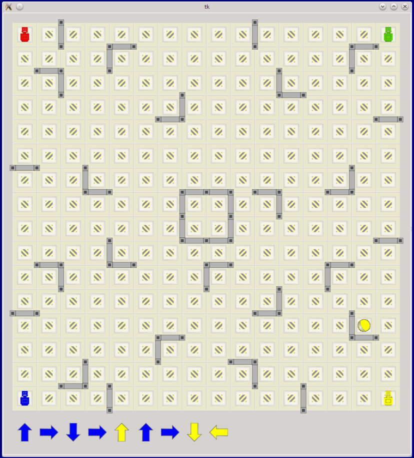
Alex Rudolph's Ricochet Robots

Solving goal(13) from cornered robots



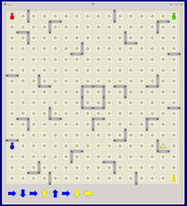
- Four robots roaming
  - horizontally
  - vertically

  up to blocking objects,

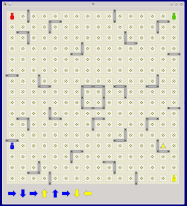
  ricocheting (optionally)

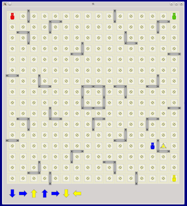
- Goal Robot on target (sharing same color)

Solving goal(13) from cornered robots (ctd)

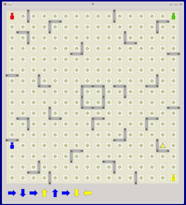


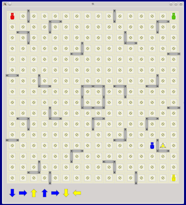
Solving `goal(13)` from cornered robots (ctd)

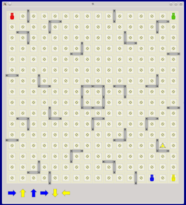




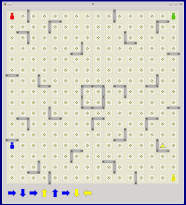
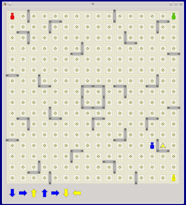
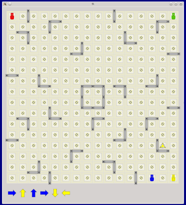
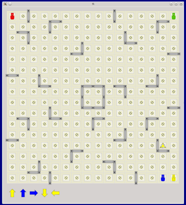
Solving `goal(13)` from cornered robots (ctd)



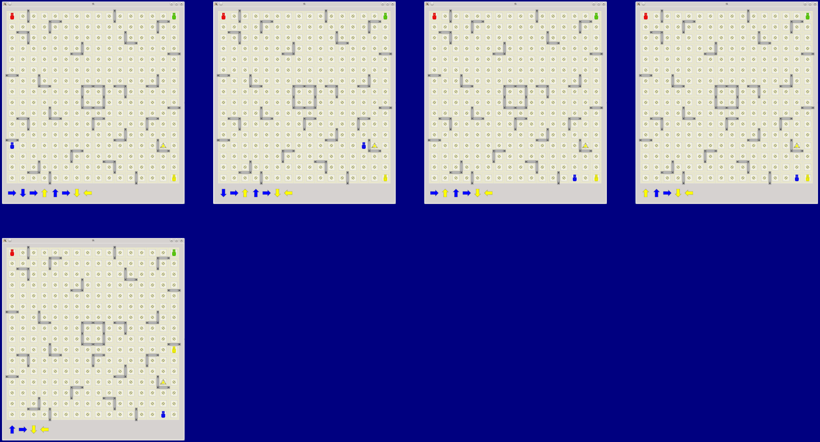




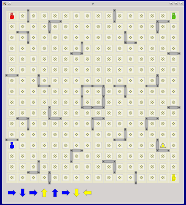
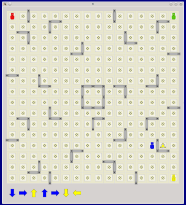
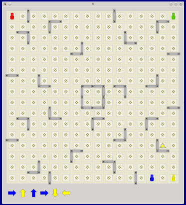
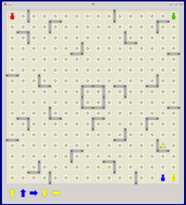
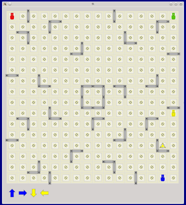
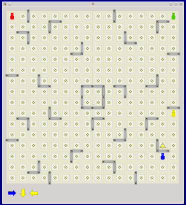
Solving `goal(13)` from cornered robots (ctd)

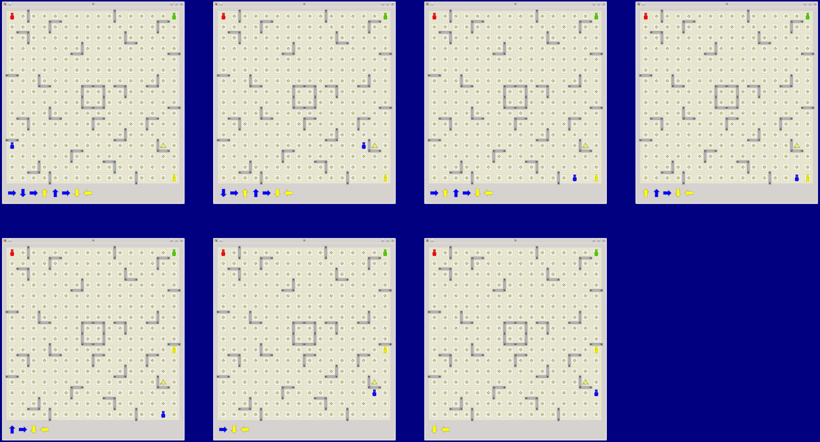
Solving `goal(13)` from cornered robots (ctd)



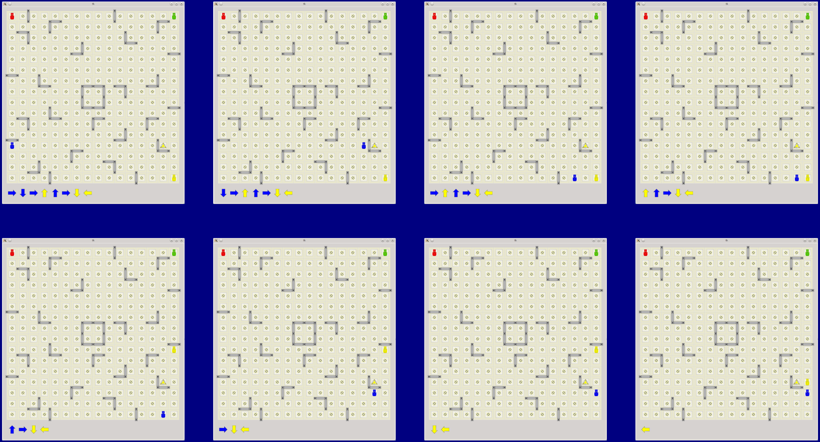
Solving goal(13) from cornered robots (ctd)

Solving goal(13) from cornered robots (ctd)



Solving `goal(13)` from cornered robots (ctd)



board.lp

```
dim(1..16).

barrier( 2, 1, 1, 0).  barrier(13,11, 1, 0).  barrier( 9, 7, 0, 1).
barrier(10, 1, 1, 0).  barrier(11,12, 1, 0).  barrier(11, 7, 0, 1).
barrier( 4, 2, 1, 0).  barrier(14,13, 1, 0).  barrier(14, 7, 0, 1).
barrier(14, 2, 1, 0).  barrier( 6,14, 1, 0).  barrier(16, 9, 0, 1).
barrier( 2, 3, 1, 0).  barrier( 3,15, 1, 0).  barrier( 2,10, 0, 1).
barrier(11, 3, 1, 0).  barrier(10,15, 1, 0).  barrier( 5,10, 0, 1).
barrier( 7, 4, 1, 0).  barrier( 4,16, 1, 0).  barrier( 8,10, 0,-1).
barrier( 3, 7, 1, 0).  barrier(12,16, 1, 0).  barrier( 9,10, 0,-1).
barrier(14, 7, 1, 0).  barrier( 5, 1, 0, 1).  barrier( 9,10, 0, 1).
barrier( 7, 8, 1, 0).  barrier(15, 1, 0, 1).  barrier(14,10, 0, 1).
barrier(10, 8,-1, 0).  barrier( 2, 2, 0, 1).  barrier( 1,12, 0, 1).
barrier(11, 8, 1, 0).  barrier(12, 3, 0, 1).  barrier(11,12, 0, 1).
barrier( 7, 9, 1, 0).  barrier( 7, 4, 0, 1).  barrier( 7,13, 0, 1).
barrier(10, 9,-1, 0).  barrier(16, 4, 0, 1).  barrier(15,13, 0, 1).
barrier( 4,10, 1, 0).  barrier( 1, 6, 0, 1).  barrier(10,14, 0, 1).
barrier( 2,11, 1, 0).  barrier( 4, 7, 0, 1).  barrier( 3,15, 0, 1).
barrier( 8,11, 1, 0).  barrier( 8, 7, 0, 1).
```

targets.lp

```
#external goal(1..16).

target(red,    5, 2) :- goal(1).
target(red,   15, 2) :- goal(2).
target(green,  2, 3) :- goal(3).
target(blue,  12, 3) :- goal(4).
target(yellow, 7, 4) :- goal(5).
target(blue,   4, 7) :- goal(6).
target(green, 14, 7) :- goal(7).
target(yellow,11, 8) :- goal(8).
target(yellow, 5,10) :- goal(9).
target(green,  2,11) :- goal(10).
target(red,   14,11) :- goal(11).
target(green, 11,12) :- goal(12).
target(yellow,15,13) :- goal(13).
target(blue,   7,14) :- goal(14).
target(red,    3,15) :- goal(15).
target(blue,  10,15) :- goal(16).

robot(red;green;blue;yellow).
#external pos((red;green;blue;yellow),1..16,1..16).
```

ricochet.lp

```
time(1..horizon).
dir(-1,0;1,0;0,-1;0,1).

stop( DX, DY,X,   Y   ) :- barrier(X,Y,DX,DY).
stop(-DX,-DY,X+DX,Y+DY) :- stop(DX,DY,X,Y).

pos(R,X,Y,0) :- pos(R,X,Y).

1 { move(R,DX,DY,T) : robot(R), dir(DX,DY) } 1 :- time(T).
move(R,T) :- move(R,_,_,T).

halt(DX,DY,X-DX,Y-DY,T) :- pos(_,X,Y,T), dir(DX,DY), dim(X-DX), dim(Y-DY),
                           not stop(-DX,-DY,X,Y), T < horizon.

goto(R,DX,DY,X,Y,T) :- pos(R,X,Y,T), dir(DX,DY), T < horizon.
goto(R,DX,DY,X+DX,Y+DY,T) :- goto(R,DX,DY,X,Y,T), dim(X+DX), dim(Y+DY),
                          not stop(DX,DY,X,Y), not halt(DX,DY,X,Y,T).

pos(R,X,Y,T) :- move(R,DX,DY,T), goto(R,DX,DY,X,Y,T-1),
                not goto(R,DX,DY,X+DX,Y+DY,T-1).
pos(R,X,Y,T) :- pos(R,X,Y,T-1), time(T), not move(R,T).

:- target(R,X,Y), not pos(R,X,Y,horizon).

#show move/4.
```

Potassco

Solving goal(13) from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=9 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
Answer: 1
move(red,0,1,1)     move(red,1,0,2) move(red,0,1,3)    move(red,-1,0,4) move(red,0,1,5) \
move(yellow,0,-1,6) move(red,1,0,7) move(yellow,0,1,8) move(yellow,-1,0,9)
SATISFIABLE

Models       : 1+
Calls        : 1
Time         : 1.895s (Solving: 1.45s 1st Model: 1.45s Unsat: 0.00s)
CPU Time     : 1.880s


$ clingo board.lp targets.lp ricochet.lp -c horizon=8 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 2.817s (Solving: 2.41s 1st Model: 0.00s Unsat: 2.41s)
CPU Time     : 2.800s
```

Potassco

Solving goal(13) from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=9 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
Answer: 1
move(red,0,1,1)     move(red,1,0,2) move(red,0,1,3)    move(red,-1,0,4) move(red,0,1,5) \
move(yellow,0,-1,6) move(red,1,0,7) move(yellow,0,1,8) move(yellow,-1,0,9)
SATISFIABLE

Models       : 1+
Calls        : 1
Time         : 1.895s (Solving: 1.45s 1st Model: 1.45s Unsat: 0.00s)
CPU Time     : 1.880s
```

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=8 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 2.817s (Solving: 2.41s 1st Model: 0.00s Unsat: 2.41s)
CPU Time     : 2.800s
```

Potassco

Solving goal(13) from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=9 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
Answer: 1
move(red,0,1,1)     move(red,1,0,2) move(red,0,1,3)    move(red,-1,0,4) move(red,0,1,5) \
move(yellow,0,-1,6) move(red,1,0,7) move(yellow,0,1,8) move(yellow,-1,0,9)
SATISFIABLE

Models       : 1+
Calls        : 1
Time         : 1.895s (Solving: 1.45s 1st Model: 1.45s Unsat: 0.00s)
CPU Time     : 1.880s
```

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=8 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 2.817s (Solving: 2.41s 1st Model: 0.00s Unsat: 2.41s)
CPU Time     : 2.800s
```

Solving goal(13) from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=9 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
Answer: 1
move(red,0,1,1)     move(red,1,0,2) move(red,0,1,3)    move(red,-1,0,4) move(red,0,1,5) \
move(yellow,0,-1,6) move(red,1,0,7) move(yellow,0,1,8) move(yellow,-1,0,9)
SATISFIABLE

Models       : 1+
Calls        : 1
Time         : 1.895s (Solving: 1.45s 1st Model: 1.45s Unsat: 0.00s)
CPU Time     : 1.880s


$ clingo board.lp targets.lp ricochet.lp -c horizon=8 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 2.817s (Solving: 2.41s 1st Model: 0.00s Unsat: 2.41s)
CPU Time     : 2.800s
```

optimization.lp

```
goon(T) :- target(R,X,Y), T = 0..horizon, not pos(R,X,Y,T).

:- move(R,DX,DY,T-1), time(T), not goon(T-1), not move(R,DX,DY,T).

#minimize{ 1,T : goon(T) }.
```

Solving goal(13) from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp optimization.lp -c horizon=20 --quiet=1,0 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")
clingo version 4.5.0
Reading from board.lp ...
Solving...
Optimization: 20
Optimization: 19
Optimization: 18
Optimization: 17
Optimization: 16
Optimization: 15
Optimization: 14
Optimization: 13
Optimization: 12
Optimization: 11
Optimization: 10
Optimization: 9
Answer: 12
move(blue,0,-1,1)   move(blue,1,0,2)    move(yellow,0,-1,3) move(blue,0,1,4)    move(yellow,-1,0,5) \
move(blue,1,0,6)    move(blue,0,-1,7)   move(yellow,1,0,8)  move(yellow,0,1,9)  move(yellow,0,1,10) \
move(yellow,0,1,11) move(yellow,0,1,12) move(yellow,0,1,13) move(yellow,0,1,14) move(yellow,0,1,15) \
move(yellow,0,1,16) move(yellow,0,1,17) move(yellow,0,1,18) move(yellow,0,1,19) move(yellow,0,1,20)
OPTIMUM FOUND

Models       : 12
  Optimum    : yes
Optimization : 9
Calls        : 1
Time         : 16.145s (Solving: 15.01s 1st Model: 3.35s Unsat: 2.02s)
CPU Time     : 16.080s
```

Potassco

Solving goal(13) from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp optimization.lp -c horizon=20 --quiet=1,0 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")
clingo version 4.5.0
Reading from board.lp ...
Solving...
Optimization: 20
Optimization: 19
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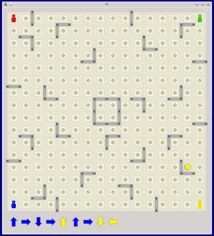
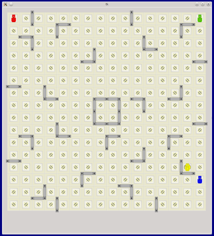
Optimization: 13
Optimization: 12
Optimization: 11
Optimization: 10
Optimization: 9
Answer: 12
move(blue,0,-1,1)   move(blue,1,0,2)    move(yellow,0,-1,3) move(blue,0,1,4)    move(yellow,-1,0,5) \
move(blue,1,0,6)    move(blue,0,-1,7)   move(yellow,1,0,8)  move(yellow,0,1,9)  move(yellow,0,1,10) \
move(yellow,0,1,11) move(yellow,0,1,12) move(yellow,0,1,13) move(yellow,0,1,14) move(yellow,0,1,15) \
move(yellow,0,1,16) move(yellow,0,1,17) move(yellow,0,1,18) move(yellow,0,1,19) move(yellow,0,1,20)
OPTIMUM FOUND

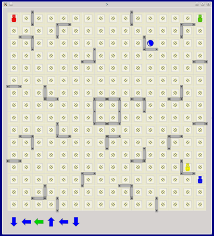
Models       : 12
  Optimum    : yes
Optimization : 9
Calls        : 1
Time         : 16.145s (Solving: 15.01s 1st Model: 3.35s Unsat: 2.02s)
CPU Time     : 16.080s
```

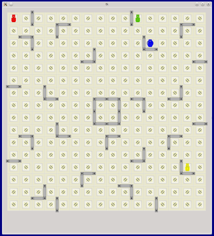
Playing in rounds





Round 1: `goal(13)`





Round 2: `goal(4)`

Control loop

1. Create an operational *clingo* object
2. Load and ground the logic programs encoding Ricochet Robot (relative to some fixed `horizon`) within the control object
3. While there is a goal, do the following
    1. Enforce the initial robot positions
    2. Enforce the current goal
    3. Solve the logic program contained in the control object

Ricochet Robot Player

ricochet.py

```
from gringo import Control, Model, Fun

class Player:
    def __init__(self, horizon, positions, files):
        self.last_positions = positions
        self.last_solution = None
        self.undo_external = []
        self.horizon = horizon
        self.ctl = Control(['-c', 'horizon={0}'.format(self.horizon)])
        for x in files:
            self.ctl.load(x)
        self.ctl.ground([("base", [])])

    def solve(self, goal):
        for x in self.undo_external:
            self.ctl.assign_external(x, False)
        self.undo_external = []
        for x in self.last_positions + [goal]:
            self.ctl.assign_external(x, True)
            self.undo_external.append(x)
        self.last_solution = None
        self.ctl.solve(on_model=self.on_model)
        return self.last_solution

    def on_model(self, model):
        self.last_solution = model.atoms()
        self.last_positions = []
        for atom in model.atoms(Model.ATOMS):
            if (atom.name() == "pos" and len(atom.args()) == 4 and atom.args()[3] == self.horizon):
                self.last_positions.append(Fun("pos", atom.args()[:-1]))

horizon   = 15
encodings = ["board.lp", "targets.lp", "ricochet.lp", "optimization.lp"]
positions = [Fun("pos", [Fun("red"),     1,  1]), Fun("pos", [Fun("blue"),    1, 16]),
             Fun("pos", [Fun("green"),  16,  1]), Fun("pos", [Fun("yellow"), 16, 16])]
sequence  = [Fun("goal", [13]), Fun("goal", [4]), Fun("goal", [7])]

player = Player(horizon, positions, encodings)
for goal in sequence:
    print player.solve(goal)
```

Variables of interest

- `last_positions` holds the starting positions of the robots for each turn
- `last_solution` holds the last solution of a search call
  (Note that callbacks cannot return values directly)
- `undo_external` holds a list containing the current goal and starting positions to be cleared upon the next step
- `horizon` holds the maximum number of moves to find a solution
- `ctl` holds the actual object providing an interface to the grounder and solver; it holds all state information necessary for multi-shot solving

Variables of interest

- last_positions holds the starting positions of the robots for each turn
- last_solution holds the last solution of a search call
  (Note that callbacks cannot return values directly)
- undo_external holds a list containing the current goal and starting positions to be cleared upon the next step
- horizon holds the maximum number of moves to find a solution
- ctl holds the actual object providing an interface to the grounder and solver; it holds all state information necessary for multi-shot solving

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Ricochet Robot Player
Setup and control loop

```
from gringo import Control, Model, Fun

class Player:
    def __init__(self, horizon, positions, files):
        self.last_positions = positions
        self.last_solution = None
        self.undo_external = []
        self.horizon = horizon
        self.ctl = Control(['-c', 'horizon={0}'.format(self.horizon)])
        for x in files:
            self.ctl.load(x)
        self.ctl.ground([("base", [])])

    def solve(self, goal):
        for x in self.undo_external:
            self.ctl.assign_external(x, False)
        self.undo_external = []
        for x in self.last_positions + [goal]:
            self.ctl.assign_external(x, True)
            self.undo_external.append(x)
        self.last_solution = None
        self.ctl.solve(on_model=self.on_model)
        return self.last_solution

    def on_model(self, model):
        self.last_solution = model.atoms()
        self.last_positions = []
        for atom in model.atoms(Model.ATOMS):
            if (atom.name() == "pos" and len(atom.args()) == 4 and atom.args()[3] == self.horizon):
                self.last_positions.append(Fun("pos", atom.args()[:-1]))

horizon   = 15
encodings = ["board.lp", "targets.lp", "ricochet.lp", "optimization.lp"]
positions = [Fun("pos", [Fun("red"),     1,  1]), Fun("pos", [Fun("blue"),    1, 16]),
             Fun("pos", [Fun("green"),  16,  1]), Fun("pos", [Fun("yellow"), 16, 16])]
sequence  = [Fun("goal", [13]), Fun("goal", [4]), Fun("goal", [7])]

player = Player(horizon, positions, encodings)
for goal in sequence:
    print player.solve(goal)
```

Setup and control loop

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horizon   = 15
encodings = ["board.lp", "targets.lp", "ricochet.lp", "optimization.lp"]
positions = [Fun("pos", [Fun("red"),     1,  1]),
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```

1. Initializing variables
2. Creating a player object (wrapping a *clingo* object)
3. Playing in rounds

Potassco

Setup and control loop

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>> horizon   = 15
>> encodings = ["board.lp", "targets.lp", "ricochet.lp", "optimization.lp"]
>> positions = [Fun("pos", [Fun("red"),     1,  1]),
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                Fun("goal",  [7])]

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>> for goal in sequence:
>>     print player.solve(goal)
```

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    print player.solve(goal)
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2. Creating a player object (wrapping a *clingo* object)
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Ricochet Robot Player
`__init__`

```
from gringo import Control, Model, Fun

class Player:
    def __init__(self, horizon, positions, files):
        self.last_positions = positions
        self.last_solution = None
        self.undo_external = []
        self.horizon = horizon
        self.ctl = Control(['-c', 'horizon={0}'.format(self.horizon)])
        for x in files:
            self.ctl.load(x)
        self.ctl.ground([("base", [])])

    def solve(self, goal):
        for x in self.undo_external:
            self.ctl.assign_external(x, False)
        self.undo_external = []
        for x in self.last_positions + [goal]:
            self.ctl.assign_external(x, True)
            self.undo_external.append(x)
        self.last_solution = None
        self.ctl.solve(on_model=self.on_model)
        return self.last_solution

    def on_model(self, model):
        self.last_solution = model.atoms()
        self.last_positions = []
        for atom in model.atoms(Model.ATOMS):
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    for x in files:
        self.ctl.load(x)
    self.ctl.ground([("base", [])])
```

1. Initializing variables
2. Creating *clingo* object
3. Loading encoding and instance
4. Grounding encoding and instance

__init__

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Potassco

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```

1 Initializing variables
2 Creating *clingo* object
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Potassco

Ricochet Robot Player
solve

```
from gringo import Control, Model, Fun

class Player:
    def __init__(self, horizon, positions, files):
        self.last_positions = positions
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        self.horizon = horizon
        self.ctl = Control(['-c', 'horizon={0}'.format(self.horizon)])
        for x in files:
            self.ctl.load(x)
        self.ctl.ground([("base", [])])

    def solve(self, goal):
        for x in self.undo_external:
            self.ctl.assign_external(x, False)
        self.undo_external = []
        for x in self.last_positions + [goal]:
            self.ctl.assign_external(x, True)
            self.undo_external.append(x)
        self.last_solution = None
        self.ctl.solve(on_model=self.on_model)
        return self.last_solution

    def on_model(self, model):
        self.last_solution = model.atoms()
        self.last_positions = []
        for atom in model.atoms(Model.ATOMS):
            if (atom.name() == "pos" and len(atom.args()) == 4 and atom.args()[3] == self.horizon):
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horizon   = 15
encodings = ["board.lp", "targets.lp", "ricochet.lp", "optimization.lp"]
positions = [Fun("pos", [Fun("red"),     1,  1]), Fun("pos", [Fun("blue"),    1, 16]),
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sequence  = [Fun("goal", [13]), Fun("goal", [4]), Fun("goal", [7])]

player = Player(horizon, positions, encodings)
for goal in sequence:
    print player.solve(goal)
```

solve

```
def solve(self, goal):
      for x in self.undo_external:
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      self.ctl.solve(on_model=self.on_model)
      return self.last_solution
```

1. Unsetting previous external atoms (viz. previous goal and positions)
2. Setting next external atoms (viz. next goal and positions)
3. Computing next stable model
   by passing user-defined `on_model` method

solve

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    def solve(self, goal):
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>>          self.ctl.assign_external(x, True)
>>          self.undo_external.append(x)
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```

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            self.ctl.assign_external(x, True)
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>>      self.last_solution = None
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```

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solve

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      self.ctl.solve(on_model=self.on_model)
      return self.last_solution
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Potassco

Ricochet Robot Player

on_model

```
from gringo import Control, Model, Fun

class Player:
    def __init__(self, horizon, positions, files):
        self.last_positions = positions
        self.last_solution = None
        self.undo_external = []
        self.horizon = horizon
        self.ctl = Control(['-c', 'horizon={0}'.format(self.horizon)])
        for x in files:
            self.ctl.load(x)
        self.ctl.ground([("base", [])])

    def solve(self, goal):
        for x in self.undo_external:
            self.ctl.assign_external(x, False)
        self.undo_external = []
        for x in self.last_positions + [goal]:
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            self.undo_external.append(x)
        self.last_solution = None
        self.ctl.solve(on_model=self.on_model)
        return self.last_solution

    def on_model(self, model):
        self.last_solution = model.atoms()
        self.last_positions = []
        for atom in model.atoms(Model.ATOMS):
            if (atom.name() == "pos" and len(atom.args()) == 4 and atom.args()[3] == self.horizon):
                self.last_positions.append(Fun("pos", atom.args()[:-1]))

horizon   = 15
encodings = ["board.lp", "targets.lp", "ricochet.lp", "optimization.lp"]
positions = [Fun("pos", [Fun("red"),     1,  1]), Fun("pos", [Fun("blue"),    1, 16]),
             Fun("pos", [Fun("green"),  16,  1]), Fun("pos", [Fun("yellow"), 16, 16])]
sequence  = [Fun("goal", [13]), Fun("goal", [4]), Fun("goal", [7])]

player = Player(horizon, positions, encodings)
for goal in sequence:
    print player.solve(goal)
```

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on_model

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def on_model(self, model):
      self.last_solution = model.atoms()
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```

1. Storing stable model
2. Extracting atoms (viz. last robot positions) by adding `pos(R,X,Y)` for each `pos(R,X,Y,horizon)`

Potassco

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Potassco

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1. Storing stable model
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ricochet.py

```
from gringo import Control, Model, Fun

class Player:
    def __init__(self, horizon, positions, files):
        self.last_positions = positions
        self.last_solution = None
        self.undo_external = []
        self.horizon = horizon
        self.ctl = Control(['-c', 'horizon={0}'.format(self.horizon)])
        for x in files:
            self.ctl.load(x)
        self.ctl.ground([("base", [])])

    def solve(self, goal):
        for x in self.undo_external:
            self.ctl.assign_external(x, False)
        self.undo_external = []
        for x in self.last_positions + [goal]:
            self.ctl.assign_external(x, True)
            self.undo_external.append(x)
        self.last_solution = None
        self.ctl.solve(on_model=self.on_model)
        return self.last_solution

    def on_model(self, model):
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        for atom in model.atoms(Model.ATOMS):
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horizon   = 15
encodings = ["board.lp", "targets.lp", "ricochet.lp", "optimization.lp"]
positions = [Fun("pos", [Fun("red"),     1,  1]), Fun("pos", [Fun("blue"),    1, 16]),
             Fun("pos", [Fun("green"),  16,  1]), Fun("pos", [Fun("yellow"), 16, 16])]
sequence  = [Fun("goal", [13]), Fun("goal", [4]), Fun("goal", [7])]

player = Player(horizon, positions, encodings)
for goal in sequence:
    print player.solve(goal)
```

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Let's play!

```
$ python ricochet.py
[move(red,0,1,1), move(yellow,-1,0,14), move(yellow,-1,0,12), move(yellow,-1,0,11),
 move(yellow,-1,0,9), move(red,1,0,7), move(red,1,0,2), move(yellow,-1,0,10),
 move(yellow,-1,0,13), move(yellow,-1,0,15), move(red,-1,0,4), move(yellow,0,-1,6),
 move(red,0,1,3), move(red,0,1,5), move(yellow,0,1,8)]
[move(blue,0,1,15), move(blue,0,1,11), move(blue,0,1,8), move(blue,0,1,3),
 move(blue,1,0,2), move(blue,0,1,9), move(blue,-1,0,7), move(blue,0,1,10),
 move(blue,0,1,13), move(blue,-1,0,4), move(blue,0,-1,1), move(blue,0,-1,6),
 move(green,-1,0,5), move(blue,0,1,12), move(blue,0,1,14)]
[move(green,1,0,15), move(green,1,0,8), move(green,1,0,5), move(green,1,0,4),
 move(green,1,0,3), move(green,1,0,10), move(green,1,0,7), move(green,1,0,12),
 move(green,1,0,9), move(green,1,0,2), move(green,1,0,11), move(green,1,0,13),
 move(green,1,0,6), move(green,1,0,14), move(green,0,1,1)]

$ python robotviz
```

Let's play!

```
$ python ricochet.py
[move(red,0,1,1), move(yellow,-1,0,14), move(yellow,-1,0,12), move(yellow,-1,0,11),
 move(yellow,-1,0,9), move(red,1,0,7), move(red,1,0,2), move(yellow,-1,0,10),
 move(yellow,-1,0,13), move(yellow,-1,0,15), move(red,-1,0,4), move(yellow,0,-1,6),
 move(red,0,1,3), move(red,0,1,5), move(yellow,0,1,8)]
[move(blue,0,1,15), move(blue,0,1,11), move(blue,0,1,8), move(blue,0,1,3),
 move(blue,1,0,2), move(blue,0,1,9), move(blue,-1,0,7), move(blue,0,1,10),
 move(blue,0,1,13), move(blue,-1,0,4), move(blue,0,-1,1), move(blue,0,-1,6),
 move(green,-1,0,5), move(blue,0,1,12), move(blue,0,1,14)]
[move(green,1,0,15), move(green,1,0,8), move(green,1,0,5), move(green,1,0,4),
 move(green,1,0,3), move(green,1,0,10), move(green,1,0,7), move(green,1,0,12),
 move(green,1,0,9), move(green,1,0,2), move(green,1,0,11), move(green,1,0,13),
 move(green,1,0,6), move(green,1,0,14), move(green,0,1,1)]

$ python robotviz
```

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Let's play!

```
$ python ricochet.py
[move(red,0,1,1), move(yellow,-1,0,14), move(yellow,-1,0,12), move(yellow,-1,0,11),
 move(yellow,-1,0,9), move(red,1,0,7), move(red,1,0,2), move(yellow,-1,0,10),
 move(yellow,-1,0,13), move(yellow,-1,0,15), move(red,-1,0,4), move(yellow,0,-1,6),
 move(red,0,1,3), move(red,0,1,5), move(yellow,0,1,8)]
[move(blue,0,1,15), move(blue,0,1,11), move(blue,0,1,8), move(blue,0,1,3),
 move(blue,1,0,2), move(blue,0,1,9), move(blue,-1,0,7), move(blue,0,1,10),
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[move(green,1,0,15), move(green,1,0,8), move(green,1,0,5), move(green,1,0,4),
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 move(green,1,0,6), move(green,1,0,14), move(green,0,1,1)]

$ python robotviz
```

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ASP modulo theories: Overview

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Motivation

- Input ASP = DB+KRR+LP+SAT
- Output ASPmT = DB+KRR+LP+S
- ASP solving *ground* | *solve*
  - ➥ **logic programs with elusive theory atoms**
- Application areas

  Agents, Assisted Living, Robotics, Planning, Scheduling, Bio- and Cheminformatics, etc

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Motivation

- Input ASP = DB+KRR+LP+SAT
- Output ASPmT = DB+KRR+LP+SMT — **NO!**
- ASP solving *ground* | *solve*

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Potassco

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- Output $\quad \text{ASPmT} = (\text{DB+KRR+LP+SAT})\text{mT}$
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- Input ASP = DB+KRR+LP+SAT
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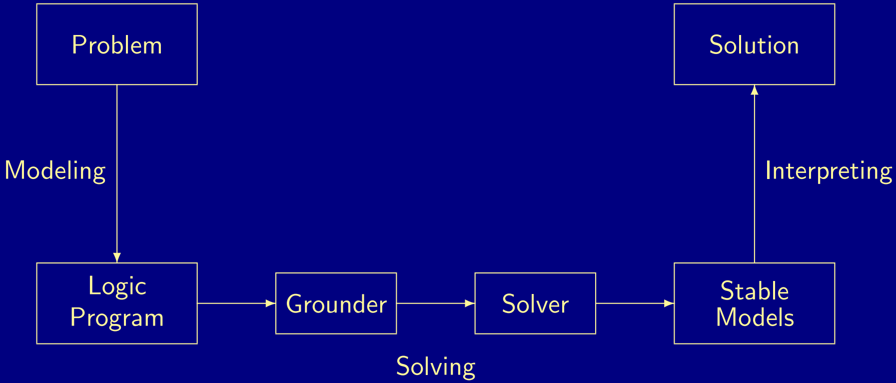
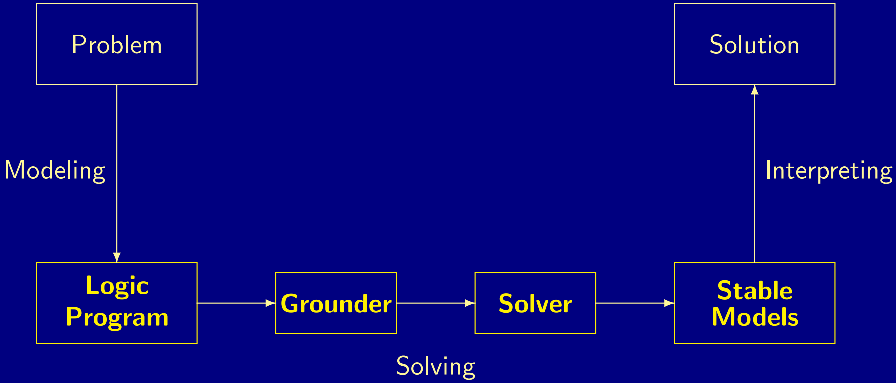
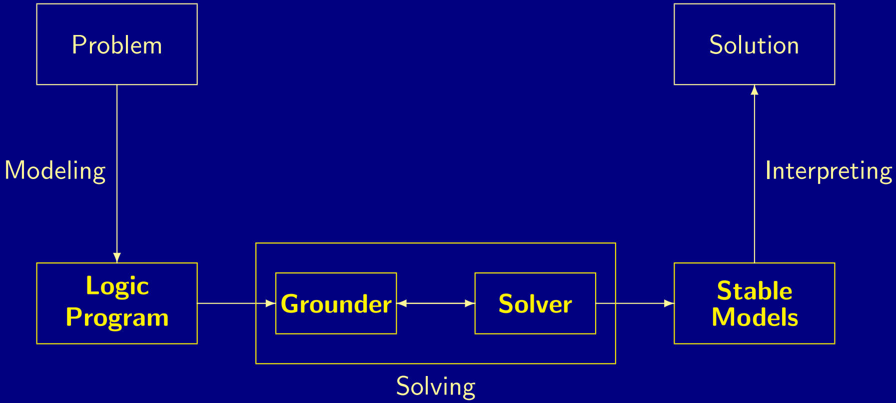
ASP solving process



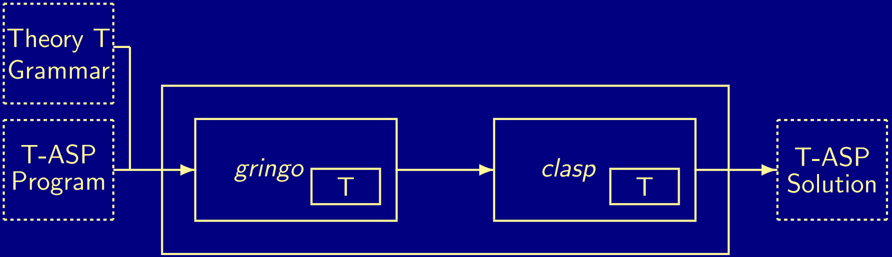
ASP solving process modulo theories



ASP solving process modulo theories

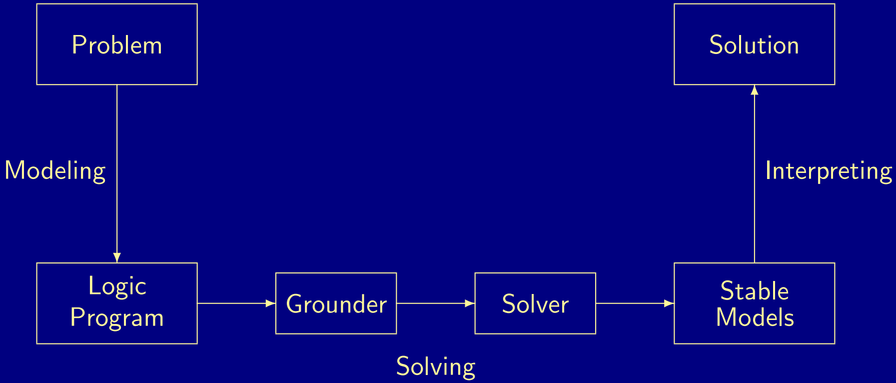


clingo's approach

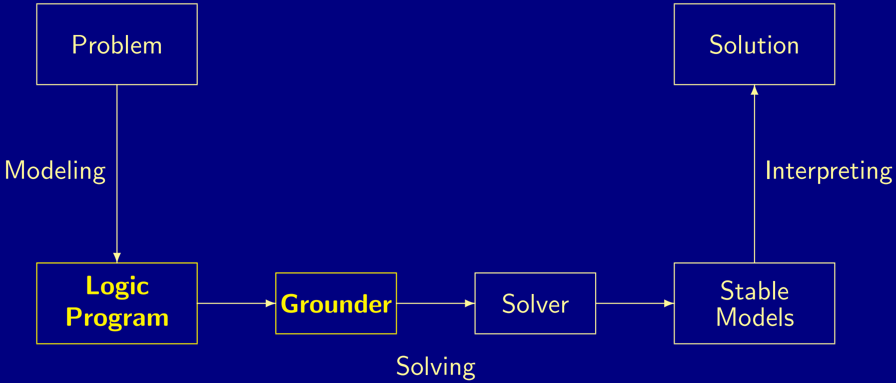


Outline

ASP solving process modulo theories



ASP solving process modulo theories



Linear constraints

```
#theory csp {
    linear_term {
      + : 5, unary;
      - : 5, unary;
      * : 4, binary, left;
      + : 3, binary, left;
      - : 3, binary, left
    };

    dom_term {
      + : 5, unary;
      - : 5, unary;
      .. : 1, binary, left
    };

    show_term {
      / : 1, binary, left
    };

    minimize_term {
      + : 5, unary;
      - : 5, unary;
      * : 4, binary, left;
      + : 3, binary, left;
      - : 3, binary, left;
      @ : 0, binary, left
    };

    &dom/0 : dom_term, {=}, linear_term, any;
    &sum/0 : linear_term, {<=,=,>=,<,>,!=}, linear_term, any;
    &show/0 : show_term, directive;
    &distinct/0 : linear_term, any;
    &minimize/0 : minimize_term, directive
}.
```

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	s	e	n	d
+	m	o	r	e
m	o	n	e	y

Each letter corresponds exactly to one digit and all variables have to be pairwise distinct

	9	5	6	7
+	1	0	8	5
1	0	6	5	2

The example has exactly one solution

$$\{\ s \mapsto 9, e \mapsto 5, n \mapsto 6, d \mapsto 7, m \mapsto 1, o \mapsto 0, r \mapsto 8, y \mapsto 2\ \}$$

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	s	e	n	d
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Each letter corresponds exactly to one digit and all variables have to be pairwise distinct

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The example has exactly one solution

$$\{\ s \mapsto 9, e \mapsto 5, n \mapsto 6, d \mapsto 7, m \mapsto 1, o \mapsto 0, r \mapsto 8, y \mapsto 2\ \}$$

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```
#include "csp.lp".

digit(1,3,s).   digit(2,3,m).   digit(sum,4,m).
digit(1,2,e).   digit(2,2,o).   digit(sum,3,o).
digit(1,1,n).   digit(2,1,r).   digit(sum,2,n).
digit(1,0,d).   digit(2,0,e).   digit(sum,1,e).
                                digit(sum,0,y).
base(10).
exp(E) :- digit(_,E,_).

power(1,0).
power(B*P,E) :- base(B), power(P,E-1), exp(E), E>0.

number(N) :- digit(N,_,_), N!= sum.
high(D) :- digit(N,E,D), not digit(N,E+1,_).

&dom {0..9} = X :- digit(_,_,X).

&sum { M*D : digit(N,E,D),   power(M,E), number(N);
      -M*D : digit(sum,E,D), power(M,E)           } = 0.


&sum { D } > 0 :- high(D).

&distinct { D : digit(_,_,D) }.

&show { D : digit(_,_,D) }.
```

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```
#include "csp.lp".

digit(1,3,s).   digit(2,3,m).   digit(sum,4,m).
digit(1,2,e).   digit(2,2,o).   digit(sum,3,o).
digit(1,1,n).   digit(2,1,r).   digit(sum,2,n).
digit(1,0,d).   digit(2,0,e).   digit(sum,1,e).
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```

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&sum { D } > 0 :- high(D).

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send+more=money

```
digit(1,3,s).   digit(2,3,m).   digit(sum,4,m).
digit(1,2,e).   digit(2,2,o).   digit(sum,3,o).
digit(1,1,n).   digit(2,1,r).   digit(sum,2,n).
digit(1,0,d).   digit(2,0,e).   digit(sum,1,e).
                                digit(sum,0,y).
base(10).
exp(0).  exp(1).  exp(2).  exp(3).  exp(4).

power(1,0).
power(10,1).  power(100,2).  power(1000,3).  power(10000,4).

number(1).  number(2).
high(s).  high(m).

&dom{0..9}=s. &dom{0..9}=m. &dom{0..9}=e. &dom{0..9}=o. &dom{0..9}=n. &dom{0..9}=r. &dom{0..9

&sum{  1000*s;   100*e;   10*n;   1*d;
       1000*m;   100*o;   10*r;   1*e;
     -10000*m; -1000*o; -100*n; -10*e; -1*y } = 0.

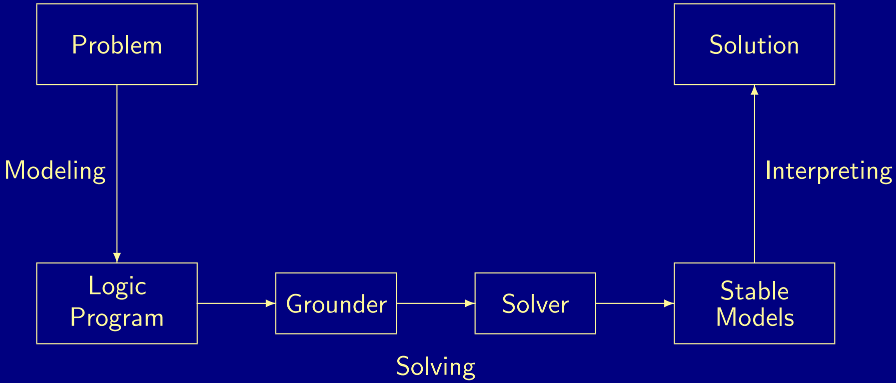
&sum{s} > 0.  &sum{m} > 0.

&distinct{s; m; e; o; n; r; d; y}.

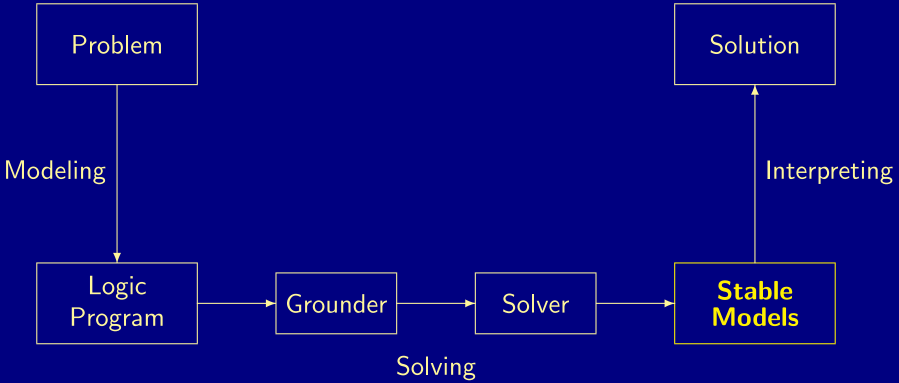
&show{s; m; e; o; n; r; d; y}.
```

Outline

ASP solving process modulo theories



ASP solving process modulo theories



ASP modulo theories

- We distinguish theory atoms depending upon whether they are
  - defined via rules in the logic program, or
  - external otherwise, or
  - strict being equivalent to the associated constraint, or
  - non-strict only implying the associated constraint.
- Informally, a set $X \subseteq \mathcal{A} \cup \mathcal{T}$ of atoms is a $\mathbb{T}$-stable model of a program P if there is some $\mathbb{T}$-solution $\mathcal{S}$ such that X is a (regular) stable model of the program

$$
\begin{aligned}
P \cup \{a \leftarrow \mid a \in (\mathcal{T}_e \setminus \mathit{head}(P)) \cap \mathcal{S}\} \\
\cup \{\leftarrow {\sim}a \mid a \in (\mathcal{T}_e \cap \mathit{head}(P)) \cap \mathcal{S}\} \\
\cup \{\{a\} \leftarrow \mid a \in (\mathcal{T}_i \setminus \mathit{head}(P)) \cap \mathcal{S}\} \\
\cup \{\leftarrow a \mid a \in (\mathcal{T} \cap \mathit{head}(P)) \setminus \mathcal{S}\}
\end{aligned}
$$

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ASP modulo theories

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\cup \{\leftarrow {\sim}a & \mid a \in (\mathcal{T}_e \cap head(P)) \cap \mathcal{S}\} \\
\cup \{\{a\} \leftarrow & \mid a \in (\mathcal{T}_i \setminus head(P)) \cap \mathcal{S}\} \\
\cup \{\leftarrow a & \mid a \in (\mathcal{T} \cap head(P)) \setminus \mathcal{S}\}
\end{aligned}
$$

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ASP modulo theories

- We distinguish theory atoms depending upon whether they are
  - defined via rules in the logic program, or
  - external otherwise, or
  - strict being equivalent to the associated constraint, or
  - non-strict only implying the associated constraint.
- Informally, a set $X \subseteq \mathcal{A} \cup \mathcal{T}$ of atoms is a $\mathbb{T}$-stable model of a program P if there is some $\mathbb{T}$-solution $\mathcal{S}$ such that X is a (regular) stable model of the program

$$\begin{aligned} P \cup \{a \leftarrow \mid a \in (\mathcal{T}_e \setminus head(P)) \cap \mathcal{S}\} \\ \cup \{\leftarrow \sim a \mid a \in (\mathcal{T}_e \cap head(P)) \cap \mathcal{S}\} \\ \cup \{\{a\} \leftarrow \mid a \in (\mathcal{T}_i \setminus head(P)) \cap \mathcal{S}\} \\ \cup \{\leftarrow a \mid a \in (\mathcal{T} \cap head(P)) \setminus \mathcal{S}\} \end{aligned}$$

ASP modulo theories

- We distinguish theory atoms depending upon whether they are
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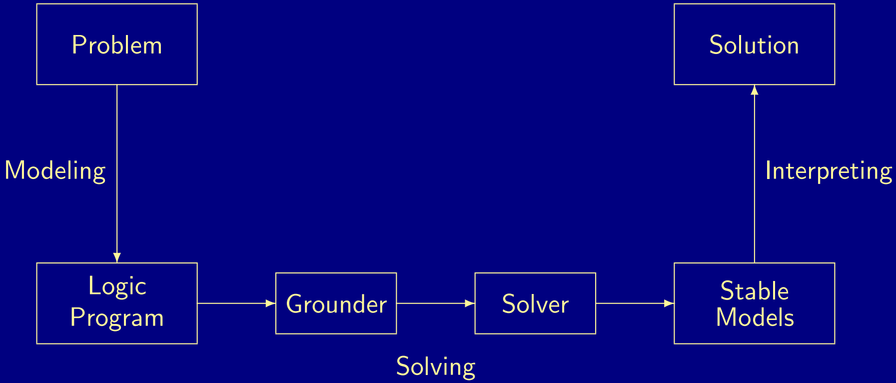
Outline

ASP solving process modulo theories



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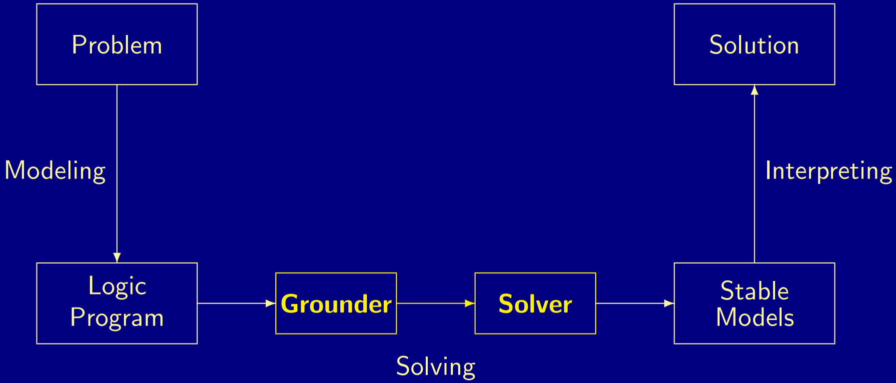

aspif example

```
{a}.
b :- a.
c :- not a.
```

```
asp 1 0 0
1 1 1 1 0 0
1 0 1 2 0 1 1
1 0 1 3 0 1 -1
4 1 a 1 1
4 1 b 1 2
4 1 c 1 3
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aspif overview

- Rule statements
- Minimize statements
- Projection statements
- Output statements
- External statements
- Assumption statements
- Heuristic statements
- Edge statements
- Theory terms and atoms
- Comments

aspif theory example

```
task(1).
task(2).

duration(1,200).
duration(2,400).

&dom {1..1000} = beg(1).
&dom {1..1000} = end(1).
&dom {1..1000} = beg(2).
&dom {1..1000} = end(2).

&diff{end(1)-beg(1)}<=200.
&diff{end(2)-beg(2)}<=400.

&show{ beg/1; end/1 }.
```

```
asp 1 0 0
1 0 1 1 0 0
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1 0 1 3 0 0
1 0 1 4 0 0
1 0 1 5 0 0
1 0 1 6 0 0
4 7 task(1) 0
4 7 task(2) 0
4 15 duration(1,200) 0
4 15 duration(2,400) 0
9 0 1 200
9 0 3 400
9 0 6 1
9 0 11 2
9 1 0 4 diff
9 1 2 2 <=
9 1 4 1 -
9 1 5 3 end
9 1 8 3 beg
9 2 7 5 1 6
9 2 9 8 1 6
9 2 10 4 2 7 9
9 2 12 5 1 11
9 2 13 8 1 11
9 2 14 4 2 12 13
9 4 0 1 10 0
9 4 1 1 14 0
9 6 5 0 1 0 2 1
9 6 6 0 1 1 2 3
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```

Only 6 (theory) atoms!

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1 0 1 4 0 0
1 0 1 5 0 0
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9 1 8 3 beg
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9 4 1 1 14 0
9 6 5 0 1 0 2 1
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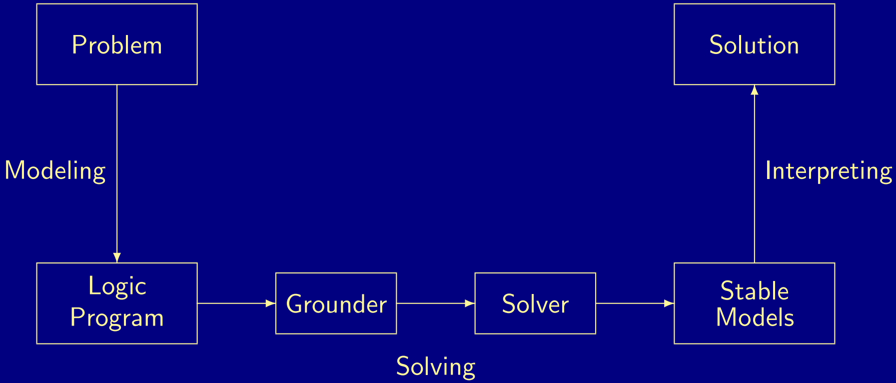
0
```

Outline

ASP solving process modulo theories



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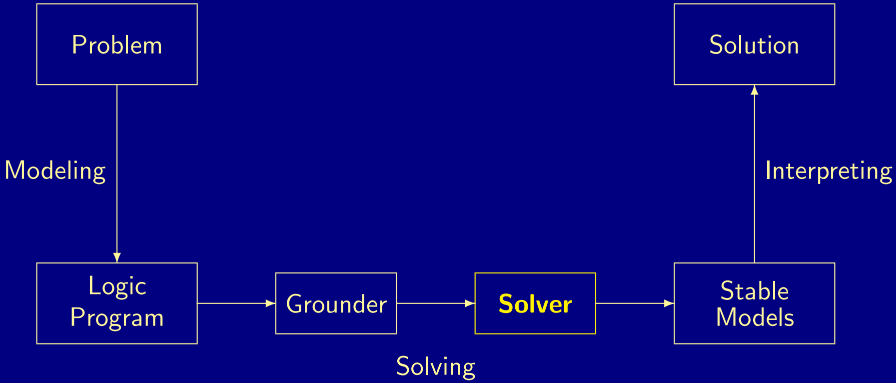
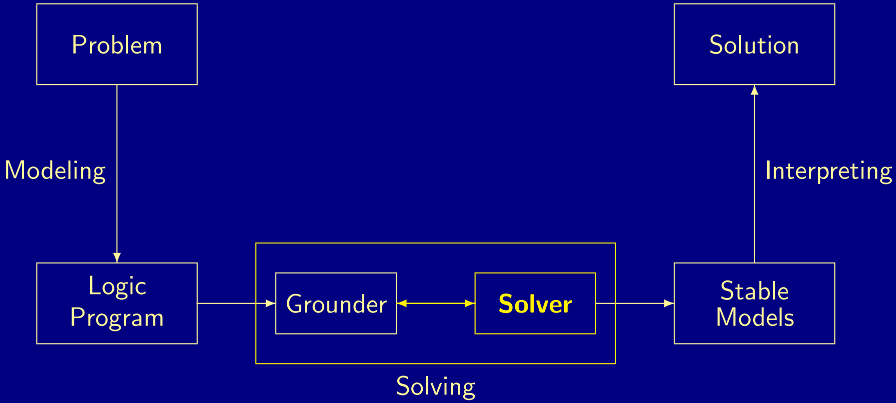
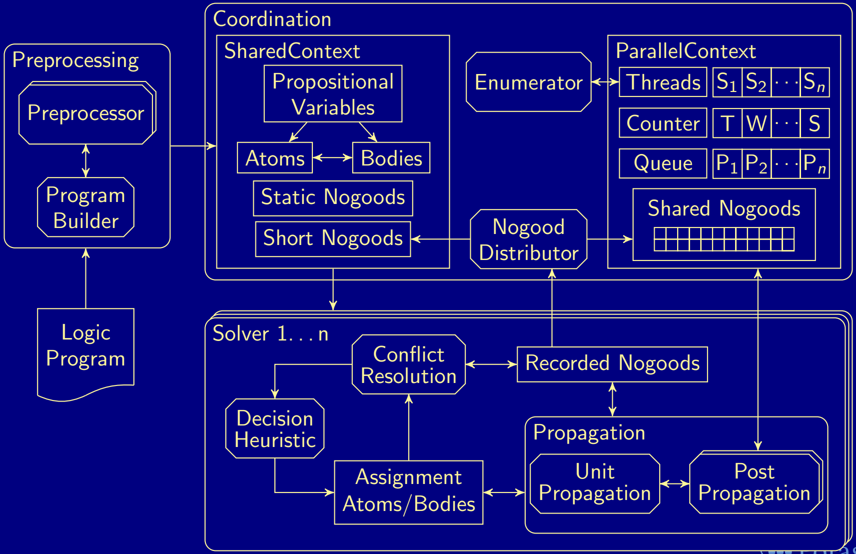


ASP solving process modulo theories



Architecture of *clasp*



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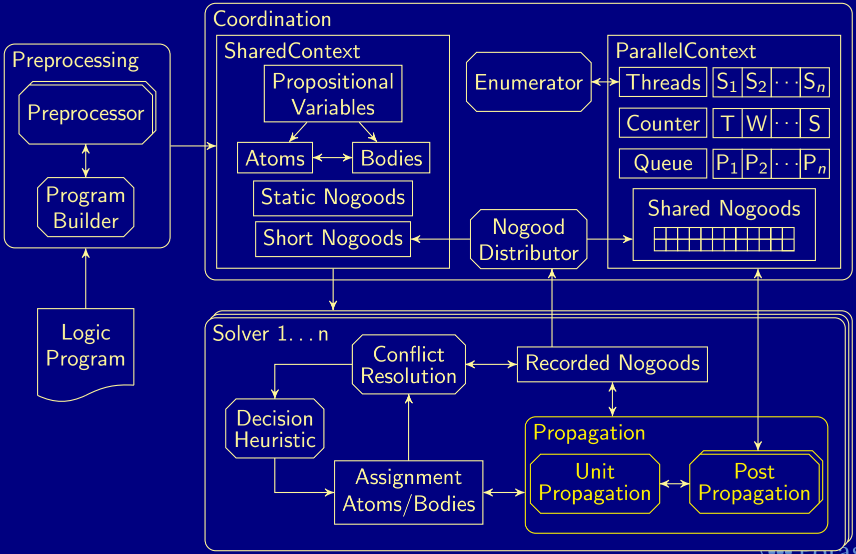
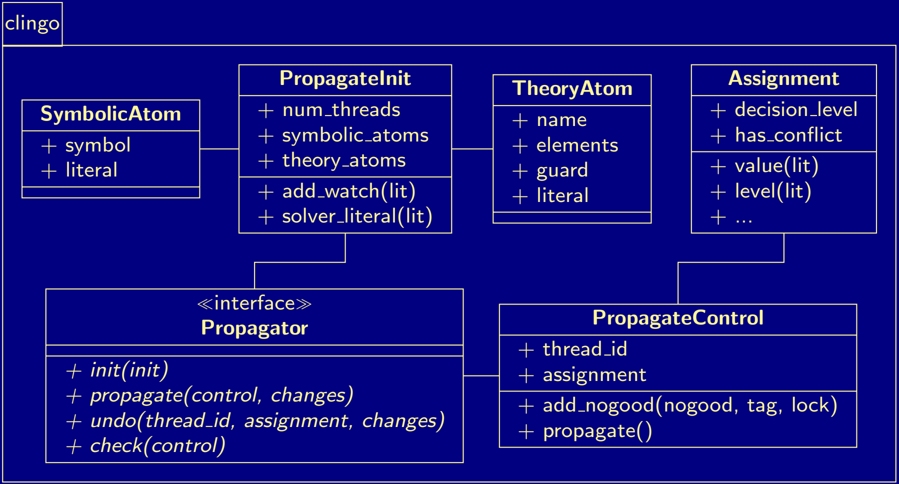


Conflict-driven constraint learning modulo theories

```
(I)  initialize                                     // register theory propagators and initialize watches
     loop
       propagate completion, loop, and recorded nogoods      // deterministically assign literals
       if no conflict then
         if all variables assigned then
(C)        if some δ ∈ Δ_T is violated for T ∈ 𝕋 then record δ      // theory propagator's check
           else return variable assignment                        // 𝕋-stable model found
         else
(P)        propagate theories T ∈ 𝕋              // theory propagators may record theory nogoods
           if no nogood recorded then decide     // non-deterministically assign some literal
       else
         if top-level conflict then return unsatisfiable
         else
           analyze                               // resolve conflict and record a conflict constraint
(U)        backjump                              // undo assignments until conflict constraint is unit
```

Propagator interface



The *dot* propagator

```
#script (python)

import sys
import time

class Propagator:
    def init(self, init):
        self.sleep = .1
        for atom in init.symbolic_atoms:
            init.add_watch(init.solver_literal(atom.literal))

    def propagate(self, ctl, changes):
        for l in changes:
            sys.stdout.write(".")
            sys.stdout.flush()
            time.sleep(self.sleep)
        return True

    def undo(self, solver_id, assign, undo):
        for l in undo:
            sys.stdout.write("\b \b")
            sys.stdout.flush()
            time.sleep(self.sleep)

def main(prg):
    prg.register_propagator(Propagator())
    prg.ground([("base", [])])
    prg.solve()
    sys.stdout.write("\n")

#end.
```

Outline

Difference logic propagation

Problem	#	ASP		ASP modulo *DL* (stateless)				ASP modulo *DL* (stateful)			
				defined		external		defined		external	
		T	TO	T	TO	T	TO	T	TO	T	TO
Flow shop	120	569	110	283	40	382	70	**177**	**30**	281	50
Job shop	80	600	80	600	80	600	80	**37**	**0**	43	**0**
Open shop	60	405	40	214	20	213	20	**2**	**0**	**2**	**0**
Total	260	525	230	366	140	398	170	**72**	**30**	109	50

- only non-strict interpretation of theory atoms
- defined versus external amounts to the difference between
  - `&diff { end(T)-beg(T) } <= D :- duration(T,D).`
  - `:- duration(T,D), not &diff { end(T)-beg(T) } <= D.`
- propagation
  - stateless Bellman-Ford algorithm
  - stateful Cotton-Maler algorithm

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Difference logic propagation

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Outline

Builtin acyclicity checking

- Edge statement

$$\#\text{edge}\,(u,v)\ :\ l_1,\ldots,l_n. \qquad (1)$$

- A set X of atoms is an acyclic stable of a logic program P, if
  1. X is a stable model of P and
  2. the graph

  $$(\{u,v \mid X \models l_1,\ldots,l_n,(1)\in P\},\{(u,v)\mid X\models l_1,\ldots,l_n,(1)\in P\})$$

  is acyclic

Potassco

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  is acyclic

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Outline

Constraint Satisfaction Problem

- A constraint satisfaction problem (CSP) consists of
  - a set V of variables,
  - a set D of domains, and
  - a set C of constraints

  such that
  - each variable $v \in V$ has an associated domain $dom(v) \in D$;
  - a constraint c is a pair (S, R) consisting of a k-ary relation R on a vector $S \subseteq V^k$ of variables, called the scope of R
- Note For $S = (v_1, \ldots, v_k)$, we have $R \subseteq dom(v_1) \times \cdots \times dom(v_k)$

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Example

$$\begin{array}{ccccc} & s & e & n & d \\ + & m & o & r & e \\ \hline m & o & n & e & y \end{array}$$

Each letter corresponds exactly to one digit and all variables have to be pairwisely distinct

$$V = \{s, e, n, d, m, o, r, y\}$$

$$D = \{dom(v) = \{0, \dots, 9\} \mid v \in V\}$$

$$\begin{aligned} C = \{ & (\vec{V}, \quad allDistinct(V)), \\ & (\vec{V}, \quad s \times 1000 + e \times 100 + n \times 10 + d + \\ & \qquad m \times 1000 + o \times 100 + r \times 10 + e == \\ & \qquad m \times 10000 + o \times 1000 + n \times 100 + e \times 10 + y), \\ & ((m), m == 1) \} \end{aligned}$$

Example

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+	m	o	r	e
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Example

	s	e	n	d
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m	o	n	e	y

Each letter corresponds exactly to one digit and all variables have to be pairwisely distinct

	9	5	6	7
+	1	0	8	5
1	0	6	5	2

The example has exactly one solution

$$\{ s \mapsto 9, e \mapsto 5, n \mapsto 6, d \mapsto 7, m \mapsto 1, o \mapsto 0, r \mapsto 8, y \mapsto 2 \}$$

Constraint satisfaction problem

- Notation We use $S(c) = S$ and $R(c) = R$ to access the scope and the relation of a constraint $c = (S, R)$

- For an assignment $A : V \rightarrow \bigcup_{v \in V} dom(v)$ and a constraint (S, R) with scope $S = (v_1, \dots, v_k)$, define

$$sat_C(A) = \{c \in C \mid A(S(c)) \in R(c)\}$$

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Constraint Answer Set Programming

- A constraint logic program P is a logic program over an extended alphabet $\mathcal{A} \cup \mathcal{C}$ where
  - $\mathcal{A}$ is a set of regular atoms and
  - $\mathcal{C}$ is a set of constraint atoms,

  such that $head(r) \in \mathcal{A}$ for each $r \in P$

- Given a set of literals B and some set $\mathcal{B}$ of atoms, we define
  $B|_{\mathcal{B}} = (B^{+} \cap \mathcal{B}) \cup \{\sim a \mid a \in B^{-} \cap \mathcal{B}\}$

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- We identify constraint atoms with constraints via a function

  $$\gamma : \mathcal{C} \rightarrow C$$

- Furthermore, $\gamma(Y) = \{\gamma(c) \mid c \in Y\}$ for any $Y \subseteq \mathcal{C}$
- Note Unlike regular atoms $\mathcal{A}$, constraint atoms $\mathcal{C}$ are not subject to the unique names assumption, eg.

  $$\gamma(x < y) \;=\; \gamma(((-y - 1) \leq -(x + 1)) \wedge (x \neq y))$$

- A constraint logic program P is associated with a CSP as follows
  - $C[P] = \gamma(atom(P) \cap \mathcal{C})$,
  - $V[P]$ is obtained from the constraint scopes in $C[P]$,
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Constraint Answer Set Programming

- Let P be a constraint logic program over $\mathcal{A} \cup \mathcal{C}$ and let $A : V[P] \rightarrow D[P]$ be an assignment,

  define the constraint reduct of as P wrt A as follows

$$
\begin{aligned}
P^A \quad = \quad \{ \, head(r) \leftarrow body(r)|_{\mathcal{A}} \mid r \in P, \\
\gamma(body(r)|_{\mathcal{C}}{}^{+}) \subseteq sat_{C[P]}(A), \\
\gamma(body(r)|_{\mathcal{C}}{}^{-}) \cap sat_{C[P]}(A) = \emptyset \, \}
\end{aligned}
$$

- A set $X \subseteq \mathcal{A}$ of (regular) atoms is a constraint answer set of P wrt A, if X is an stable model of P^A.

  That is, if X is the $\subseteq$-smallest model of $(P^A)^X$

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- Note That is, if X is the $\subseteq$-smallest model of $(P^A)^X$

Constraint Answer Set Programming

- Let P be a constraint logic program over $\mathcal{A} \cup \mathcal{C}$ and let $A : V[P] \rightarrow D[P]$ be an assignment,

  define the constraint reduct of as P wrt A as follows

$$
\begin{aligned}
P^A \quad = \quad \{\, head(r) \leftarrow body(r)|_{\mathcal{A}} \mid r \in P,\\
\gamma(body(r)|_{\mathcal{C}}{}^{+}) \subseteq sat_{C[P]}(A),\\
\gamma(body(r)|_{\mathcal{C}}{}^{-}) \cap sat_{C[P]}(A) = \emptyset \,\}
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Some Constraint Answer Set Programming (CASP) systems

- *adsolver*
  - extension of ASP solver *smodels*
- *clingcon*
  - extension of ASP system *clingo* (viz. *gringo* and *clasp*)
  - lazy approach
- *aspartame*
  - translational approach (independent of ASP system)
  - eager approach
- *aspmt*, *dlvhex*, *ezcsp*, *gasp*, *inca*, ...

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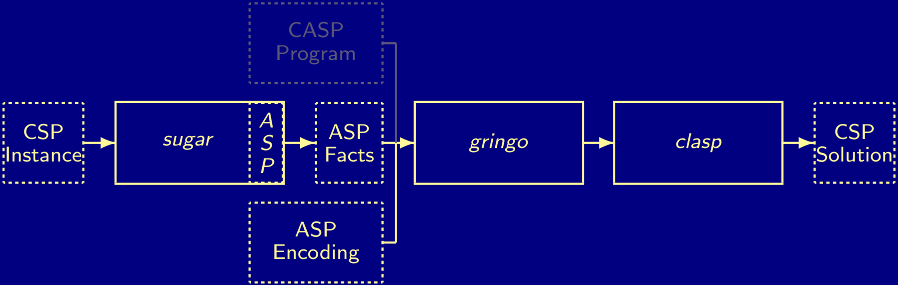
aspartame's eager approach



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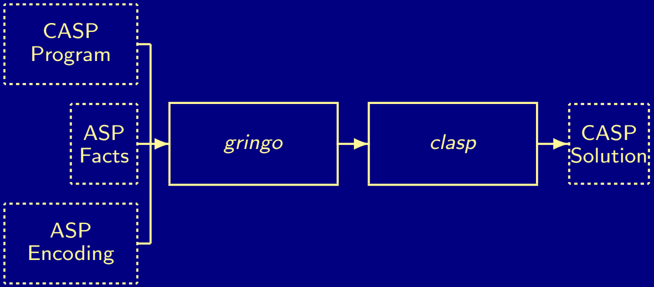

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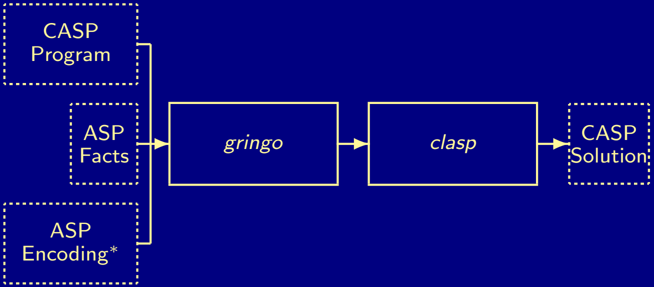

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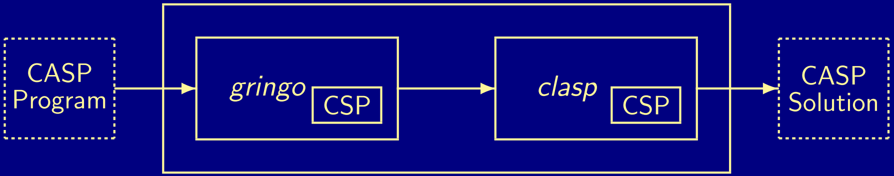

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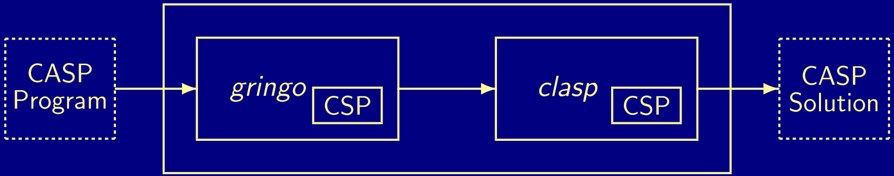

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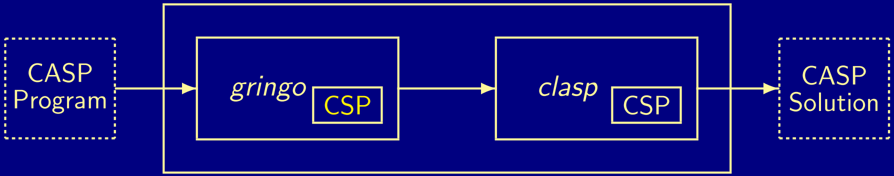

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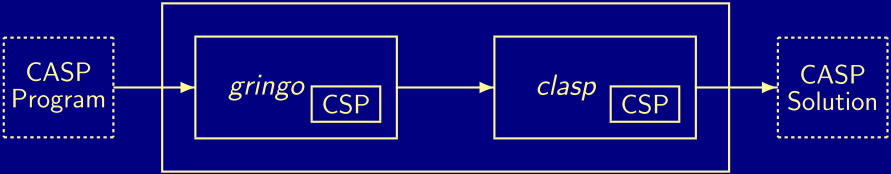

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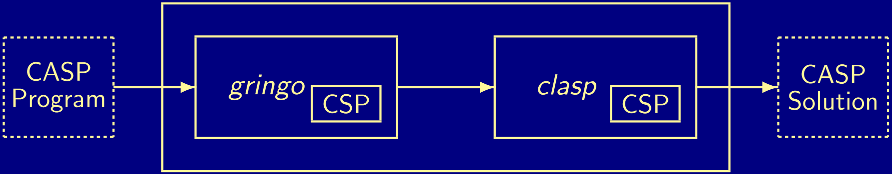

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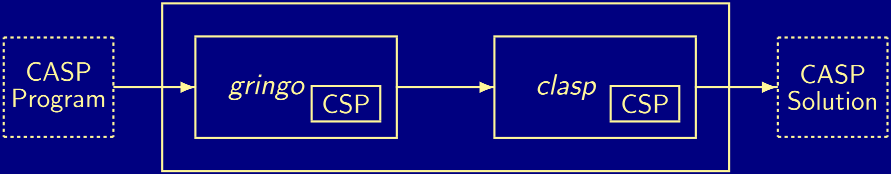

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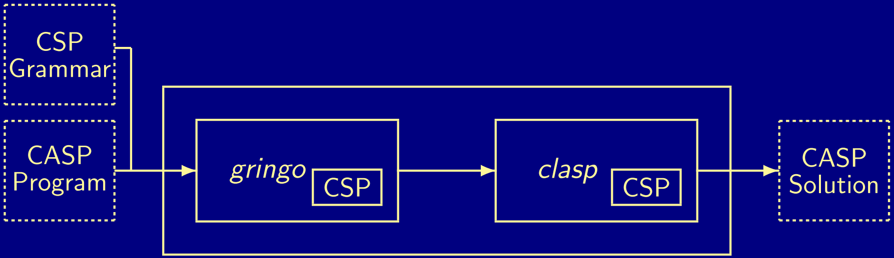

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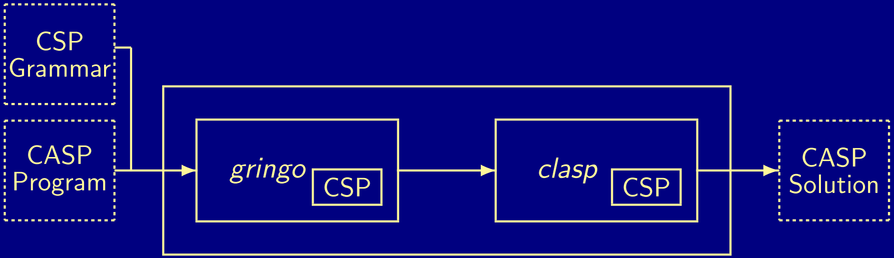

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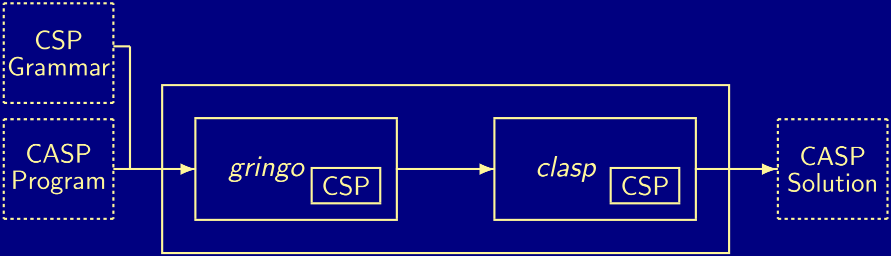
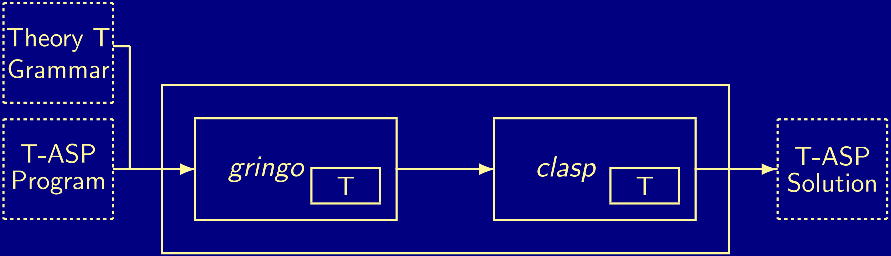


clingcon instantiates *clingo*



Heuristic programming: Overview

Outline

Motivation

- Observation Sometimes it is advantageous to take a more application-oriented approach by including domain-specific information
  - domain-specific knowledge can be added for improving propagation
  - domain-specific heuristics can be used for making better choices
- Idea Incorporation of domain-specific heuristics by extending
  - input language and/or solver options for expressing domain-specific heuristics
  - solving capacities for integrating domain-specific heuristics

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Basic CDCL decision algorithm

loop

  propagate // compute deterministic consequences

  if no conflict **then**

    if all variables assigned **then return** variable assignment

    else decide // non-deterministically assign some literal

  else

    if top-level conflict **then return** unsatisfiable

    else

      analyze // analyze conflict and add a conflict constraint

      backjump // undo assignments until conflict constraint is unit

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Inside *decide*

- Basic concepts
  - Atoms, $\mathcal{A}$
  - Assignments, $A : \mathcal{A} \rightarrow \{\mathbf{T}, \mathbf{F}\}$

$$A^{\mathbf{T}} = \{a \in \mathcal{A} \mid \mathbf{T}a \in A\} \text{ and } A^{\mathbf{F}} = \{a \in \mathcal{A} \mid \mathbf{F}a \in A\}$$

- Heuristic functions

$$h : \mathcal{A} \rightarrow [0, +\infty) \quad \text{and} \quad s : \mathcal{A} \rightarrow \{\mathbf{T}, \mathbf{F}\}$$

- Algorithmic scheme

1. $h(a) := \alpha \times h(a) + \beta(a)$ for each $a \in \mathcal{A}$
2. $U := \mathcal{A} \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}})$
3. $C := argmax_{a \in U} h(a)$
4. $a := \tau(C)$
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Outline

Heuristic language

- Heuristic directive

  `#heuristic` a : $l_1, \dots, l_n$. $[k@p, m]$

  where

  - a is an atom, and $l_1, \dots, l_n$ are literals
  - k and p are integers
  - m is a heuristic modifier

- Heuristic modifiers

  `init` for initializing the heuristic value of a with k

  `factor` for amplifying the heuristic value of a by factor k

  `level` for ranking all atoms; the rank of a is k

  `sign` for attributing the sign of k as truth value to a

- Example

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`true/false`	combine `level` and `sign`

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#heuristic occurs(mv,5) : action(mv), time(5). [5, factor]
```

Simple STRIPS planning

```
time(1..k).

holds(P,0) :- init(P).

{ occ(A,T) : action(A) } = 1 :- time(T).
:- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- occ(A,T), add(A,F).
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#heuristic holds(F,T-1) :     holds(F,T). [t-T+1, true]
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```

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Heuristic options

- Alternative for specifying structure-oriented heuristics in *clasp*

```
--dom-mod=<arg> : Default modification for
                  domain heuristic
   <arg>: <mod>[,<pick>]
     <mod>  : Modifier
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               5=false|6=init|7=factor}
     <pick> : Apply <mod> to
              {0=all|1=scc|2=hcc|4=disj|
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Engage heuristic modifications (in both settings!)

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Inclusion-minimal stable models

- Consider a logic program containing a mimimize statement of form
  - `#minimize`$\{a_1, \ldots, a_n\}$
- Computing one inclusion-minimal stable model can be done either via
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Heuristic modifications to functions h and s

- $\nu_{a,m}(A)$ — *"value for modifier m on atom a wrt assignment A"*

- `init` and `factor`

$$d_0(a) = \quad \nu_{a,\texttt{init}}(A_0) + h_0(a)$$

$$d_i(a) = \begin{cases} \nu_{a,\texttt{factor}}(A_i) \times h_i(a) & \text{if } V_{a,\texttt{factor}}(A_i) \neq \emptyset \\ h_i(a) & \text{otherwise} \end{cases}$$

- `sign`

$$t_i(a) = \begin{cases} \mathbf{T} & \text{if } \nu_{a,\texttt{sign}}(A_i) > 0 \\ \mathbf{F} & \text{if } \nu_{a,\texttt{sign}}(A_i) < 0 \\ s_i(a) & \text{otherwise} \end{cases}$$

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- `level` $\ell_{A_i}(\mathcal{A}') = argmax_{a \in \mathcal{A}'} \nu_{a,\texttt{level}}(A_i) \qquad \mathcal{A}' \subseteq \mathcal{A}$

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Inside *decide*, heuristically modified

0 $h(a) := d(a)$ for each $a \in \mathcal{A}$

1 $h(a) := \alpha \times h(a) + \beta(a)$ for each $a \in \mathcal{A}$

2 $U := \ell_A(\mathcal{A} \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}}))$

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Abductive problems with optimization

Setting		Diagnosis	Expansion	Repair (H)	Repair (S)
base configuration		111.1s (115)	161.5s (100)	101.3s (113)	33.3s (27)
`sign,-1`		324.5s (407)	7.6s (3)	8.4s (5)	3.1s (0)
`sign,-1`	`factor,2`	310.1s (387)	7.4s (2)	3.5s (0)	3.2s (1)
`sign,-1`	`factor,8`	305.9s (376)	7.7s (2)	3.1s (0)	2.9s (0)
`sign,-1`	`level,1`	76.1s (83)	6.6s (2)	0.8s (0)	2.2s (1)
	`level,1`	77.3s (86)	12.9s (5)	3.4s (0)	2.1s (0)

(abducibles subject to optimization via `#minimize` statements)

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Planning benchmarks

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#heuristic holds(F,T-1) :     holds(F,T). [t-T+1, true]
#heuristic holds(F,T-1) : not holds(F,T), fluent(F),time(T).
                                          [t-T+1, false]
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Problem	*base configuration*		#heuristic		*base config.* (SAT)		#heu. (SAT)	
Blocks'00	134.4*s*	(180/61)	9.2*s*	(239/3)	163.2*s*	(59)	2.6*s*	(0)
Elevator'00	3.1*s*	(279/0)	0.0*s*	(279/0)	3.4*s*	(0)	0.0*s*	(0)
Freecell'00	288.7*s*	(147/115)	184.2*s*	(194/74)	226.4*s*	(47)	52.0*s*	(0)
Logistics'00	145.8*s*	(148/61)	115.3*s*	(168/52)	113.9*s*	(23)	15.5*s*	(3)
Depots'02	400.3*s*	(51/184)	297.4*s*	(115/135)	389.0*s*	(64)	61.6*s*	(0)
Driverlog'02	308.3*s*	(108/143)	189.6*s*	(169/92)	245.8*s*	(61)	6.1*s*	(0)
Rovers'02	245.8*s*	(138/112)	165.7*s*	(179/79)	162.9*s*	(41)	5.7*s*	(0)
Satellite'02	398.4*s*	(73/186)	229.9*s*	(155/106)	364.6*s*	(82)	30.8*s*	(0)
Zenotravel'02	350.7*s*	(101/169)	239.0*s*	(154/116)	224.5*s*	(53)	6.3*s*	(0)
Total	252.8*s*	(1225/1031)	158.9*s*	(1652/657)	187.2*s*	(430)	17.1*s*	(3)

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Preferences and optimization: Overview

Outline

Motivation

- Preferences are pervasive
- The identification of preferred, or optimal, solutions is often indispensable in real-world applications

  In many cases, this also involves the combination of various qualitative and quantitative preferences

- Only optimization statements representing objective functions using sum or count aggregates are established components of ASP systems
- Example $\#minimize\{40 : sauna, 70 : dive\}$

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Approach

- asprin is a framework for handling preferences among the stable models of logic programs
  - general because it captures numerous existing approaches to preference from the literature
  - flexible because it allows for an easy implementation of new or extended existing approaches
- asprin builds upon advanced control capacities for incremental and meta solving, allowing for
  - without any modifications to the ASP solver
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Example

$$\#preference(costs, less(weight))\{40 : sauna, 70 : dive\}$$
$$\#preference(fun, superset)\{sauna, dive, hike, {\sim}bunji\}$$
$$\#preference(temps, aso)\{dive > sauna \parallel hot, sauna > dive \parallel \neg hot\}$$
$$\#preference(all, pareto)\{name(costs), name(fun), name(temps)\}$$
$$\#optimize(all)$$

Outline

Preference

- A strict partial order $\succ$ on the stable models of a logic program
  That is, $X \succ Y$ means that X is preferred to Y
- A stable model X is $\succ$-preferred, if there is no other stable model Y such that $Y \succ X$
- A preference type is a (parametric) class of preference relations

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- weighted formula $w_1, \dots, w_l : \phi$
  where each w_i is a term and ϕ is a Boolean formula
- naming atom $name(s)$
  where s is the name of a preference
- preference element $\Phi_1 > \cdots > \Phi_m \parallel \Phi$
  where each Φ_r is a set of weighted formulas and Φ is a non-weighted formula
- preference statement $\#preference(s, t)\{e_1, \dots, e_n\}$
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Preference type

- A preference type t is a function mapping a set of preference elements, E, to a (strict) preference relation, $t(E)$, on sets of atoms
- The domain of t, $dom(t)$, fixes its admissible preference elements
- Example $less(cardinality)$

$$(X, Y) \in less(cardinality)(E)$$
$$\text{if } |\{l \in E \mid X \models l\}| < |\{l \in E \mid Y \models l\}|$$
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More examples

- *more*(*weight*) is defined as
  - $(X,Y) \in \mathit{more}(\mathit{weight})(E)$ if $\sum_{(w:l)\in E, X\models l} w > \sum_{(w:l)\in E, Y\models l} w$
  - $\mathit{dom}(\mathit{more}(\mathit{weight})) = \mathcal{P}(\{w : a, w : \neg a \mid w \in \mathbb{Z}, a \in \mathcal{A}\})$; and
- *subset* is defined as
  - $(X,Y) \in \mathit{subset}(E)$ if $\{l \in E \mid X \models l\} \subset \{l \in E \mid Y \models l\}$
  - $\mathit{dom}(\mathit{less}(\mathit{cardinality})) = \mathcal{P}(\{a, \neg a \mid a \in \mathcal{A}\})$.
- *pareto* is defined as
  - $(X,Y) \in \mathit{pareto}(E)$ if $\bigwedge_{\mathit{name}(s)\in E}(X \succeq_s Y) \wedge \bigvee_{\mathit{name}(s)\in E}(X \succ_s Y)$
  - $\mathit{dom}(\mathit{pareto}) = \mathcal{P}(\{n \mid n \in N\})$;
- *lexico* is defined as
  - $(X,Y) \in \mathit{lexico}(E)$ if $\bigvee_{w:\mathit{name}(s)\in E}((X \succ_s Y) \wedge \bigwedge_{v:\mathit{name}(s')\in E, v<w}(X =_{s'} Y))$
  - $\mathit{dom}(\mathit{lexico}) = \mathcal{P}(\{w : n \mid w \in \mathbb{Z}, n \in N\})$.

Preference relation

- A preference relation is obtained by applying a preference type to an admissible set of preference elements
- $\#preference(s,t)\,E$ declares preference relation $t(E)$ denoted by $\succ_s$

$$\#preference(1, less(cardinality))\{a, \neg b, c\}) \text{ declares}$$

$$X \succ_1 Y \text{ as } |\{l \in \{a, \neg b, c\} \mid X \models l\}| < |\{l \in \{a, \neg b, c\} \mid Y \models l\}|$$

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Outline

Preference program

- Reification $H_X = \{holds(a) \mid a \in X\}$ and $H'_X = \{holds'(a) \mid a \in X\}$
- Preference program Let s be a preference statement declaring $\succ_s$ and let P_s be a logic program

  We define P_s as a preference program for s, if for all sets $X, Y \subseteq \mathcal{A}$, we have

  $$X \succ_s Y \quad \text{iff} \quad P_s \cup H_X \cup H'_Y \text{ is satisfiable}$$

- Note P_s usually consists of an encoding E_{t_s} of t_s, facts F_s representing the preference statement, and auxiliary rules A
- Note Dynamic versions of H_X and H_Y must be used for optimization

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$\#preference(3, subset)\{a, \neg b, c\}$

$E_{subset} =$
```
better(P) :- preference(P,subset),
             holds'(X) : preference(P,_,_,for(X),_), holds(X);
             1 #sum { 1,X : not holds(X), holds'(X),
                            preference(P,_,_,for(X),_) }.
```

$F_3 =$
```
preference(3,subset).   preference(3,1,1,for(a),()).
                        preference(3,2,1,for(neg(b)),()).
                        preference(3,3,1,for(c),()).
```

$A =$
```
holds(neg(A))  :- not holds(A), preference(_,_,_,for(neg(A)),_).
holds'(neg(A)) :- not holds'(A),preference(_,_,_,for(neg(A)),_).
```

$H_{\{a,b\}} =$
```
holds(a).  holds(b).
```

$H'_{\{a\}} =$
```
holds'(a).
```

We get a stable model containing `better(3)` indicating that $\{a, b\} \succ_3 \{a\}$, or $\{a\} \subset \{a, \neg b\}$

$\#preference(3, subset)\{a, \neg b, c\}$

$E_{subset} =$
```
better(P) :- preference(P,subset),
             holds'(X) : preference(P,_,_,for(X),_), holds(X);
             1 #sum { 1,X : not holds(X), holds'(X),
                            preference(P,_,_,for(X),_) }.
```

$F_3 =$
```
preference(3,subset).   preference(3,1,1,for(a),()).
                        preference(3,2,1,for(neg(b)),()).
                        preference(3,3,1,for(c),()).
```

$A =$
```
holds(neg(A))  :- not holds(A), preference(_,_,_,for(neg(A)),_).
holds'(neg(A)) :- not holds'(A),preference(_,_,_,for(neg(A)),_).
```

$H_{\{a,b\}} =$
```
holds(a).  holds(b).
```

$H'_{\{a\}} =$
```
holds'(a).
```

We get a stable model containing `better(3)` indicating that $\{a, b\} \succ_3 \{a\}$, or $\{a\} \subset \{a, \neg b\}$

Basic algorithm $solveOpt(P, s)$

Input : A program P over $\mathcal{A}$ and preference statement s
Output : A $\succ_s$-preferred stable model of P, if P is satisfiable, and $\bot$ otherwise

$Y \leftarrow solve(P)$
if $Y = \bot$ **then return** $\bot$

repeat
  $X \leftarrow Y$
  $Y \leftarrow solve(P \cup E_{t_s} \cup F_s \cup R_{\mathcal{A}} \cup H'_X) \cap \mathcal{A}$
until $Y = \bot$
return X

where $R_X = \{holds(a) \leftarrow a \mid a \in X\}$

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Sketched Python Implementation

```
#script (python)

from gringo import *
holds = []

def getHolds():
    global holds
    return holds

def onModel(model):
    global holds
    holds = []
    for a in model.atoms():
        if (a.name() == "_holds"): holds.append(a.args()[0])

def main(prg):
    step = 1
    prg.ground([("base", [])])
    while True:
        if step > 1: prg.ground([("doholds",[step-1]),("preference",[0,step-1])])
        ret = prg.solve(on_model=onModel)
        if ret == SolveResult.UNSAT: break
        step = step+1
#end.

#program base.                      #program doholds(m).
#show _holds(X,0) : _holds(X,0).    _holds(X,m) :- X = @getHolds().

#end.
```

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#end.
```

Vanilla `minimize` statements

- Emulating the `minimize` statement

```
#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

  in *asprin* amounts to

```
#preference(myminimize,less(weight))
          { C,(X,Y) :: cycle(X,Y) : cost(X,Y,C) }.
#optimize(myminimize).
```

- Note *asprin* separates the declaration of preferences from the actual optimization directive

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- Note *asprin* separates the declaration of preferences from the actual optimization directive

Example

in *asprin*'s input language

```
#preference(costs,less(weight)){
  C :: sauna : cost(sauna,C);
  C ::  dive : cost(dive,C)
}.
#preference(fun,superset){ sauna; dive; hike; not bunji }.
#preference(temps,aso){
  dive > sauna ||     hot;
 sauna > dive  || not hot
}.
#preference(all,pareto){name(costs); name(fun); name(temps)}.

#optimize(all).
```

asprin's library

- Basic preference types
  - `subset` and `superset`
  - `less(cardinality)` and `more(cardinality)`
  - `less(weight)` and `more(weight)`
  - `aso` (Answer Set Optimization)
  - `poset` (Qualitative Preferences)
- Composite preference types
  - `neg`
  - `and`
  - `pareto`
  - `lexico`
- See *Potassco Guide* on how to define further types

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Outline

Summary

- asprin stands for “ASP for Preference handling”
- asprin is a general, flexible, and extendable framework for preference handling in ASP
- asprin caters to
  - off-the-shelf users using the preference relations in *asprin*’s library
  - preference engineers customizing their own preference relations

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Outline

Take home message

Take home message

ASP = DB+LP+KR+SAT

Take home message

$$\textbf{ASP} = \textbf{DB}+\textbf{LP}+\textbf{KR}+\textbf{SMT}^{n}$$

Take home message

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