Towards Embedded Answer Set Solving

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Rough Roadmap

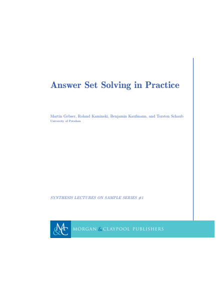
1. 09:00-10:30 Motivation, Introduction, Basic modeling
2. 10:45-11:45 Multi-shot solving and its applications

Resources

- Course material
  - http://potassco.sourceforge.net
- Systems
  - *clasp* http://potassco.sourceforge.net
  - *clingo* http://potassco.sourceforge.net
  - *dlv* http://www.dlvsystem.com
  - *smodels* http://www.tcs.hut.fi/Software/smodels
  - *wasp* https://www.mat.unical.it/ricca/wasp
  - *gringo* http://potassco.sourceforge.net
  - *lparse* http://www.tcs.hut.fi/Software/smodels
  - *asparagus* http://asparagus.cs.uni-potsdam.de

The Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



Resources

- `http://potassco.sourceforge.net/book.html`
- `http://potassco.sourceforge.net/teaching.html`

Literature

Books [4], [31], [55]

Surveys [52], [2], [41], [23], [11]

Articles [43], [44], [6], [63], [56], [51], [42], etc.

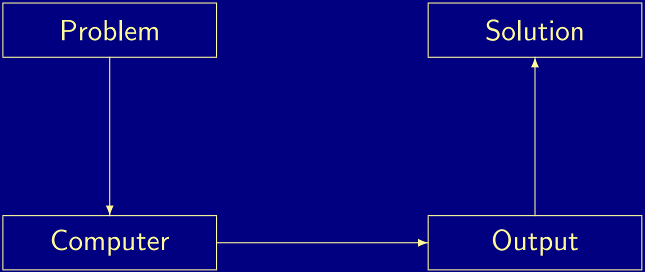
Motivation: Overview

Outline

Potassco

Informatics

"What is the problem?" versus *"How to solve the problem?"*



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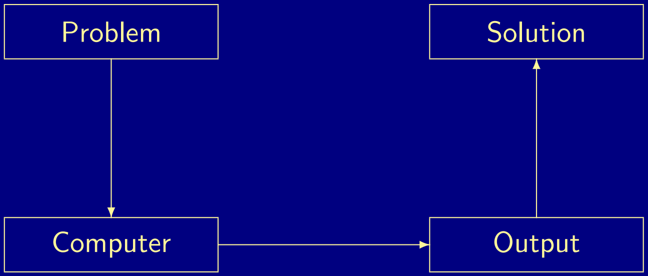

Traditional programming

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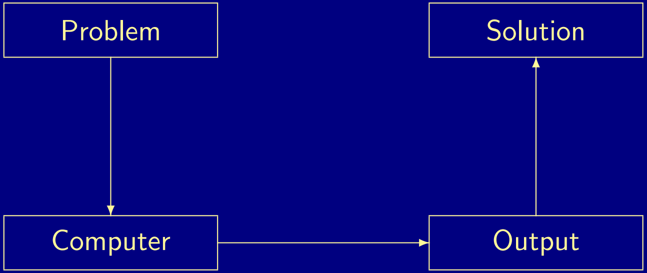

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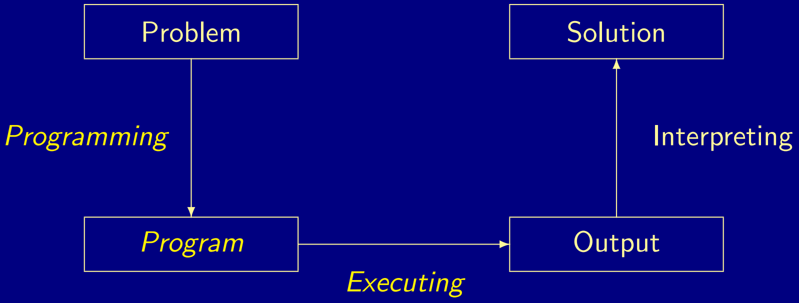
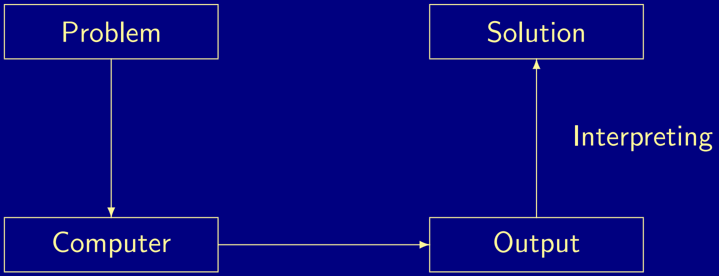


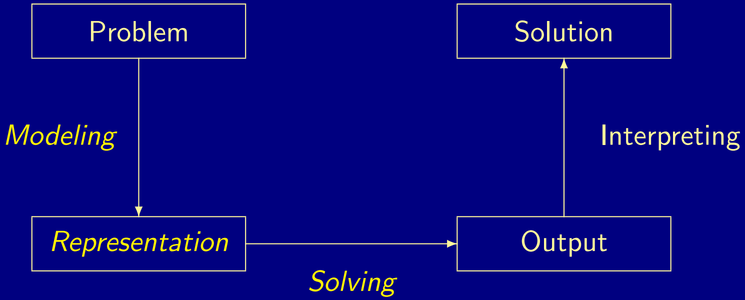
Declarative problem solving

"What is the problem?" versus *"How to solve the problem?"*



Declarative problem solving

"What is the problem?" versus *"How to solve the problem?"*



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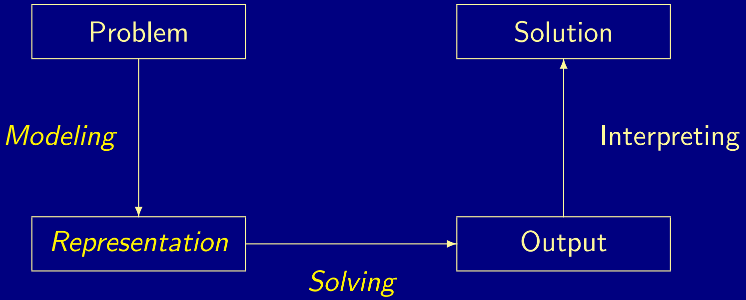

Outline

Answer Set Programming

in a Nutshell

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
- ASP has its roots in
  - (deductive) databases
  - logic programming (with negation)
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - constraint solving (in particular, SATisfiability testing)
- ASP allows for solving all search problems in NP (and NP^{NP}) in a uniform way
- ASP is versatile as reflected by the ASP solver *clasp*, winning first places at ASP, CASC, MISC, PB, and SAT competitions
- ASP embraces many emerging application areas

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Answer Set Programming

in a Hazelnutshell

- ASP is an approach to declarative problem solving, combining
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  tailored to Knowledge Representation and Reasoning

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$$\textbf{ASP} = \textbf{DB}+\textbf{LP}+\textbf{KR}+\textbf{SAT}$$

Outline

Potassco

KR's shift of paradigm

Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem
2. A solution is given by a derivation of a query

Model Generation based approach (eg. SATisfiability testing)

1. Provide a representation of the problem
2. A solution is given by a model of the representation

Automated planning, Kautz and Selman (ECAI'92)

Represent planning problems as propositional theories so that models not proofs describe solutions

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Model Generation based Problem Solving

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constraint satisfaction problem	assignment
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auto-epistemic theories	expansions
default theories	extensions
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Potassco

LP-style playing with blocks

Prolog program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Prolog queries

```
?- above(a,c).
true.

?- above(c,a).
no.
```

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Prolog queries (testing entailment)

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```

LP-style playing with blocks

Shuffled Prolog program

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above(X,Y) :- on(X,Y).
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Prolog queries

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?- above(a,c).

Fatal Error: local stack overflow.
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Potassco

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Prolog queries (answered via fixed execution)

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SAT-style playing with blocks

Formula

$$
\begin{array}{ll}
 & on(a,b) \\
\wedge & on(b,c) \\
\wedge & (on(X,Y) \rightarrow above(X,Y)) \\
\wedge & (on(X,Z) \wedge above(Z,Y) \rightarrow above(X,Y))
\end{array}
$$

Herbrand model

$$
\left\{
\begin{array}{ccccc}
on(a,b), & on(b,c), & on(a,c), & on(b,b), & \\
above(a,b), & above(b,c), & above(a,c), & above(b,b), & above(c,b)
\end{array}
\right\}
$$

SAT-style playing with blocks

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Potassco

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\end{array}
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Herbrand model (among 426!)

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➥ Answer Set Programming (ASP)

Potassco

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Potassco

Answer Set Programming *at large*

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Answer Set Programming *commonly*

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Answer Set Programming *in practice*

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ASP-style playing with blocks

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on(a,b).
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Stable Herbrand model

$$\{ \mathtt{on}(a, b),\ \mathtt{on}(b, c),\ \mathtt{above}(b, c),\ \mathtt{above}(a, b),\ \mathtt{above}(a, c) \}$$

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ASP versus LP

ASP	Prolog
Model generation	Query orientation
Bottom-up	Top-down
Modeling language	Programming language
Rule-based format	
Instantiation Flat terms	Unification Nested terms
(Turing $+$) $NP(^{NP})$	Turing

ASP versus SAT

ASP	SAT
Model generation	
Bottom-up	
Constructive Logic	Classical Logic
Closed (and open) world reasoning	Open world reasoning
Modeling language	—
Complex reasoning modes	Satisfiability testing
Satisfiability	Satisfiability
Enumeration/Projection	—
Intersection/Union	—
Optimization	—
(Turing +) $NP(^{NP})$	NP

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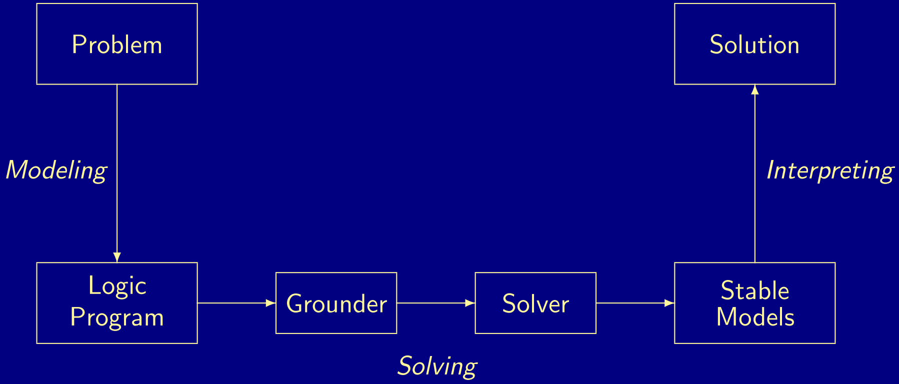
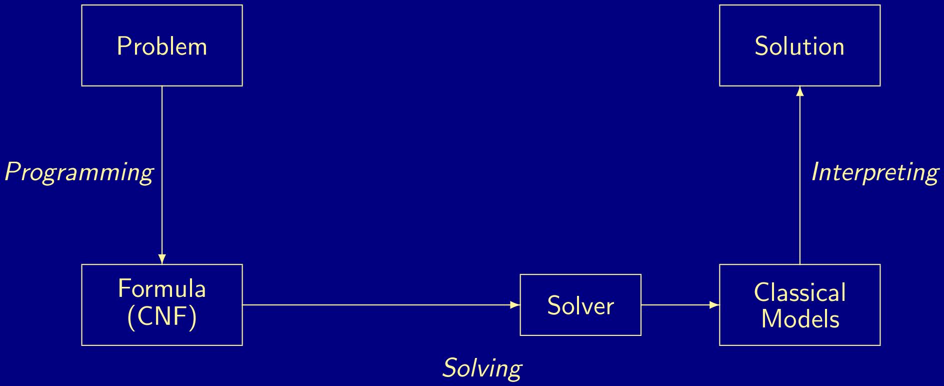
Outline

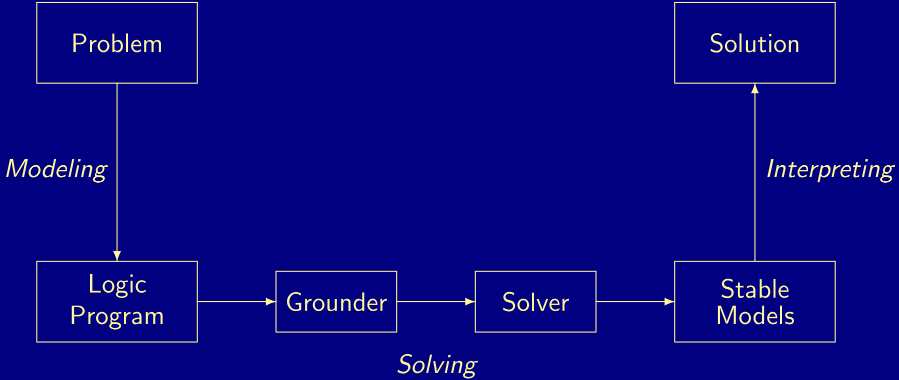
ASP grounding and solving



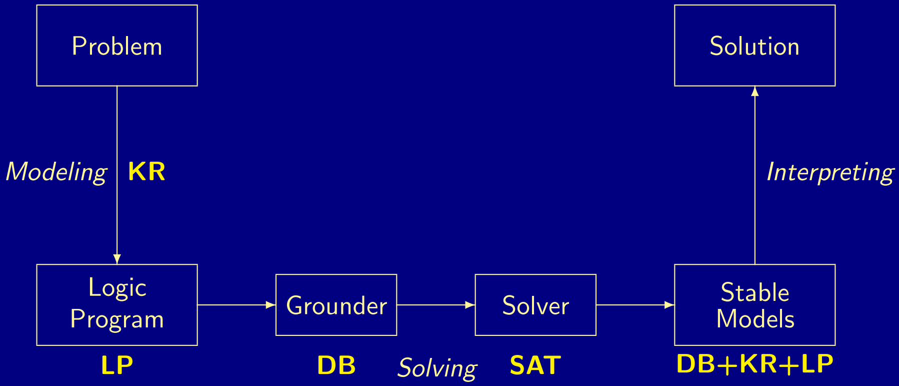
SAT solving


Problem
Solution
Programming
Interpreting
Formula (CNF)
Solver
Classical Models
Solving

Rooting ASP solving



Rooting ASP solving



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Potassco

Two sides of a coin

- ASP as High-level Language
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules
- ASP as Low-level Language
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation
- ASP and Imperative language
  - Control continuously changing logic programs

Two and a half sides of a coin

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  - Control continuously changing logic programs

What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated planning
  - Code optimization
  - Database integration
  - Decision support for NASA shuttle controllers
  - Model checking
  - Music composition
  - Product configuration
  - Robotics
  - Systems biology
  - System design
  - Team building
  - and many many more

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What does ASP offer?

- Integration of DB, KR, and SAT techniques
- Succinct, elaboration-tolerant problem representations
  - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
  - including: data, frame axioms, exceptions, defaults, closures, etc

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$$\textbf{ASP} = \textbf{DB}+\textbf{LP}+\textbf{KR}+\textbf{SMT}^n$$

Introduction: Overview

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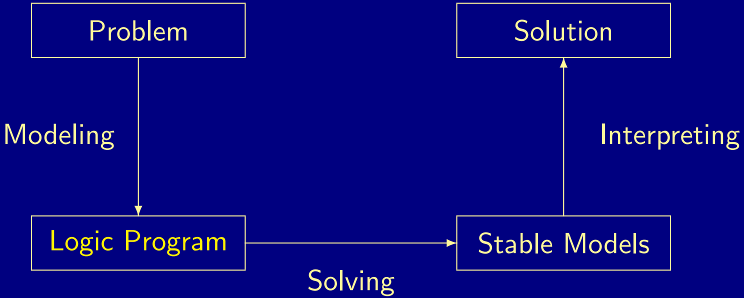
Outline

Problem solving in ASP: Syntax



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Normal logic programs

- A logic program, P, over a set $\mathcal{A}$ of atoms is a finite set of rules
- A (normal) rule, r, is of the form

$$a_0 \leftarrow a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

$$\begin{array}{rcl} head(r) & = & a_0 \\ body(r) & = & \{a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n\} \\ body(r)^+ & = & \{a_1, \ldots, a_m\} \\ body(r)^- & = & \{a_{m+1}, \ldots, a_n\} \\ atom(P) & = & \bigcup_{r \in P} \left(\{head(r)\} \cup body(r)^+ \cup body(r)^-\right) \\ body(P) & = & \{body(r) \mid r \in P\} \end{array}$$

A program P is positive if $body(r)^- = \emptyset$ for all $r \in P$

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- Notation

$$\begin{aligned}
head(r) &= a_0 \\
body(r) &= \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \\
body(r)^+ &= \{a_1, \dots, a_m\} \\
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atom(P) &= \bigcup_{r \in P} \left(\{head(r)\} \cup body(r)^+ \cup body(r)^-\right) \\
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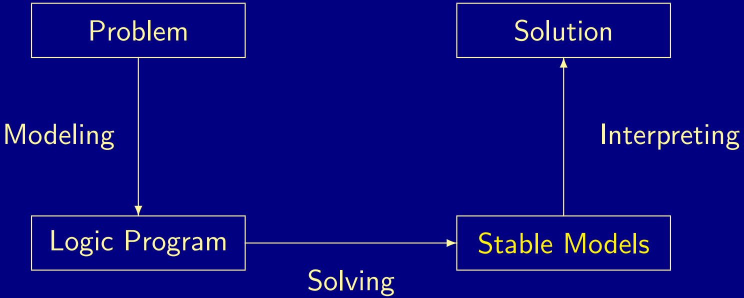
Rough notational convention

We sometimes use the following notation interchangeably in order to stress the respective view:

	true, false	if	and	or	iff	default negation	classical negation
source code		`:-`	`,`	`;`		`not`	`-`
logic program		$\leftarrow$	$,$	$;$		$\sim$	$\neg$
formula	$\bot, \top$	$\rightarrow$	$\wedge$	$\vee$	$\leftrightarrow$	$\sim$	$\neg$

Outline

Problem solving in ASP: Semantics



Formal Definition

Stable models of positive programs

- A set of atoms X is closed under a positive program P iff for any $r \in P$, $head(r) \in X$ whenever $body(r)^+ \subseteq X$
  - X corresponds to a model of P (seen as a formula)
- The smallest set of atoms which is closed under a positive program P is denoted by $Cn(P)$
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of P (ditto)
- The set $Cn(P)$ of atoms is the stable model of a *positive* program P

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Basic idea

Consider the logical formula Φ and its three (classical) models:

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

Φ $\boxed{q \;\wedge\; (q \wedge \neg r \rightarrow p)}$

Formula Φ has one stable model, often called answer set:

$$\{p, q\}$$

P_Φ $\boxed{\begin{array}{lll} q & \leftarrow & \\ p & \leftarrow & q, \; \sim r \end{array}}$

Informally, a set X of atoms is a stable model of a logic program P

- if X is a (classical) model of P and
- if all atoms in X are justified by some rule in P

(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))

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$\Phi \quad \boxed{q \;\wedge\; (q \wedge \neg r \rightarrow p)}$

$\{p, q\}, \{q, r\}$, and $\{p, q, r\}$

$$\boxed{\begin{array}{ccc} p & \mapsto & 1 \\ q & \mapsto & 1 \\ r & \mapsto & 0 \end{array}}$$

Formula Φ has one stable model, often called answer set:

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$$P_\Phi \quad \boxed{\begin{array}{lll} q & \leftarrow & \\ p & \leftarrow & q, \; \sim r \end{array}}$$

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Stable model of normal programs

- The reduct, P^X, of a program P relative to a set X of atoms is defined by

$$P^X = \{head(r) \leftarrow body(r)^+ \mid r \in P \text{ and } body(r)^- \cap X = \emptyset\}$$

- A set X of atoms is a stable model of a program P, if $Cn(P^X) = X$

- Note $Cn(P^X)$ is the $\subseteq$–smallest (classical) model of P^X
- Note Every atom in X is justified by an *"applying rule from P"*

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Outline

A first example

$P = \{p \leftarrow p,\ q \leftarrow {\sim}p\}$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p\}$	$p \leftarrow p$	$\emptyset$
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p, q\}$	$p \leftarrow p$	$\emptyset$

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A first example

$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✗
$\{p\}$	$p \leftarrow p$	$\emptyset$	
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✔
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A second example

$P = \{p \leftarrow {\sim}q,\ q \leftarrow {\sim}p\}$

X	P^X	$Cn(P^X)$
$\{\ \ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$
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A third example

$P = \{p \leftarrow \sim p\}$

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$P = \{p \leftarrow {\sim}p\}$

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Some properties

- A logic program may have zero, one, or multiple stable models!
- If X is a stable model of a logic program P,
  then X is a model of P (seen as a formula)
- If X and Y are stable models of a *normal* program P,
  then $X \not\subset Y$

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Programs with Variables

Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \to \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground Instantiation of P: $ground(P) = \bigcup_{r \in P} ground(r)$

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Programs with Variables

Let P be a logic program

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- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$ (also called alphabet or Herbrand base)
- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

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$$ground(r) = \{r\theta \mid \theta : var(r) \rightarrow \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r;
  θ is a (ground) substitution

- Ground Instantiation of P: $\quad ground(P) = \bigcup_{r \in P} ground(r)$

Programs with Variables

Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

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An example

$$P = \{\ r(a,b) \leftarrow,\ \ r(b,c) \leftarrow,\ \ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a,b,c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

$$ground(P) = \left\{ \begin{array}{lllllllll} r(a,b) & \leftarrow & , & & & & & & \\ r(b,c) & \leftarrow & , & & & & & & \\ t(a,a) & \leftarrow & r(a,a), & t(b,a) & \leftarrow & r(b,a), & t(c,a) & \leftarrow & r(c,a), \\ t(a,b) & \leftarrow & r(a,b), & t(b,b) & \leftarrow & r(b,b), & t(c,b) & \leftarrow & r(c,b), \\ t(a,c) & \leftarrow & r(a,c), & t(b,c) & \leftarrow & r(b,c), & t(c,c) & \leftarrow & r(c,c) \end{array} \right\}$$

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Potassco

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Stable models of programs with Variables

Let P be a normal logic program with variables

- A set X of (ground) atoms is a stable model of P,
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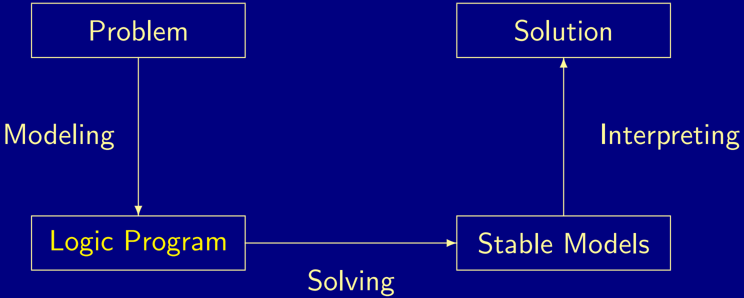
Outline

Problem solving in ASP: Extended Syntax



Language constructs

- Variables (over the Herbrand universe)
  - `p(X) :- q(X)` over constants $\{a, b, c\}$ stands for
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- Conditional Literals
  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for
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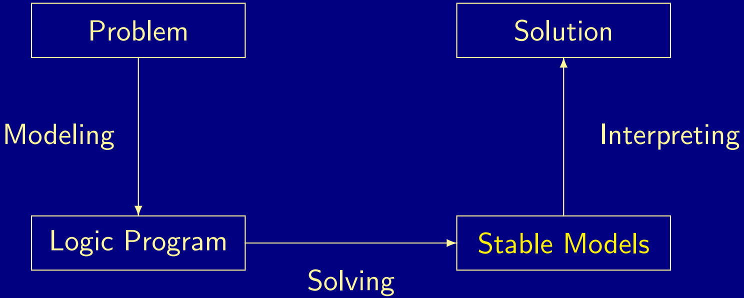
Potassco

Outline

Problem solving in ASP: Reasoning Modes



Reasoning Modes

- Satisfiability
- Enumeration[†]
- Projection[†]
- Intersection[‡]
- Union[‡]
- Optimization

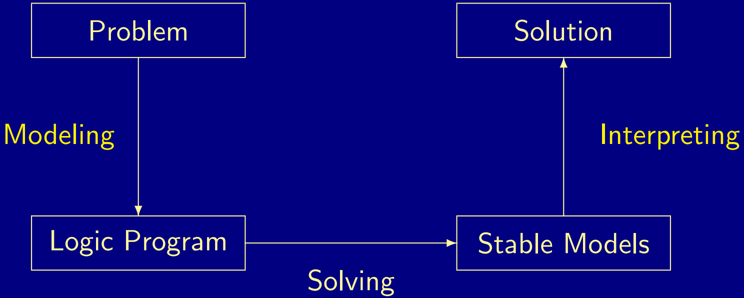
- and combinations of them

[†] without solution recording

[‡] without solution enumeration

Basic Modeling: Overview

Modeling and Interpreting



- For solving a problem class **C** for a problem instance **I**, encode
  1. the problem instance **I** as a set $P_{\mathbf{I}}$ of facts and
  2. the problem class **C** as a set $P_{\mathbf{C}}$ of rules

  such that the solutions to **C** for **I** can be (polynomially) extracted from the stable models of $P_{\mathbf{I}} \cup P_{\mathbf{C}}$

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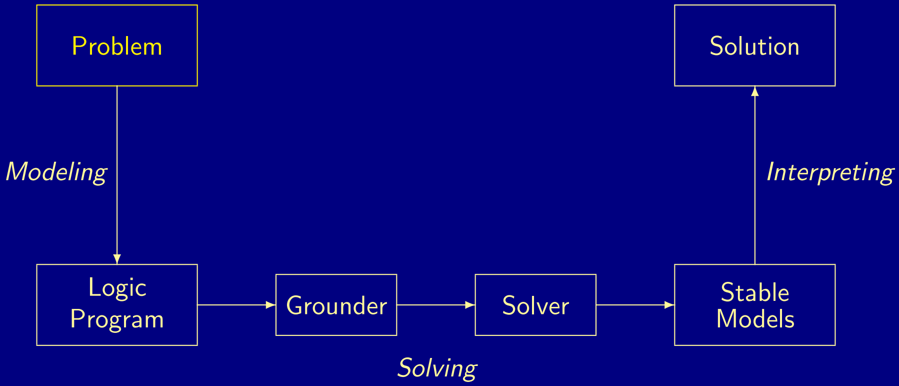
Outline

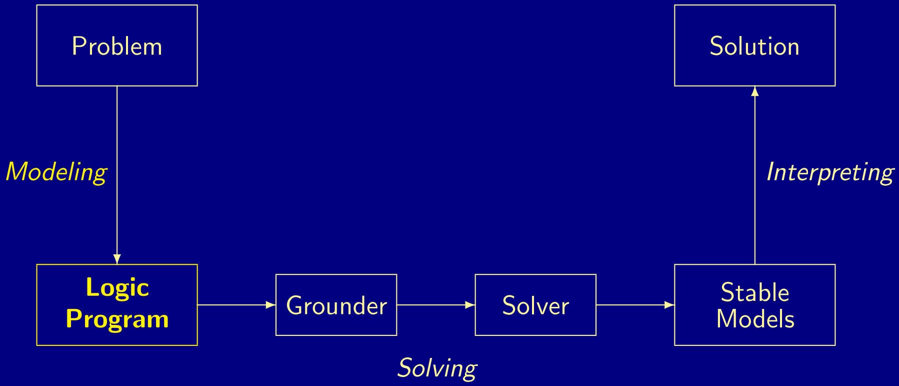
13 ASP solving process

14 Methodology

ASP solving process



ASP solving process



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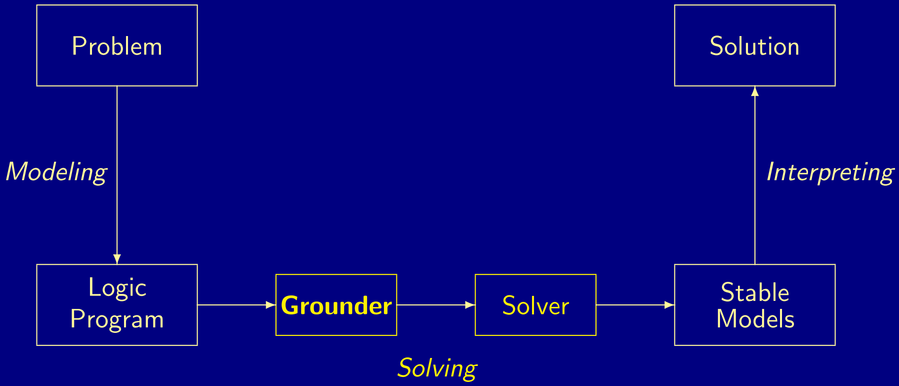

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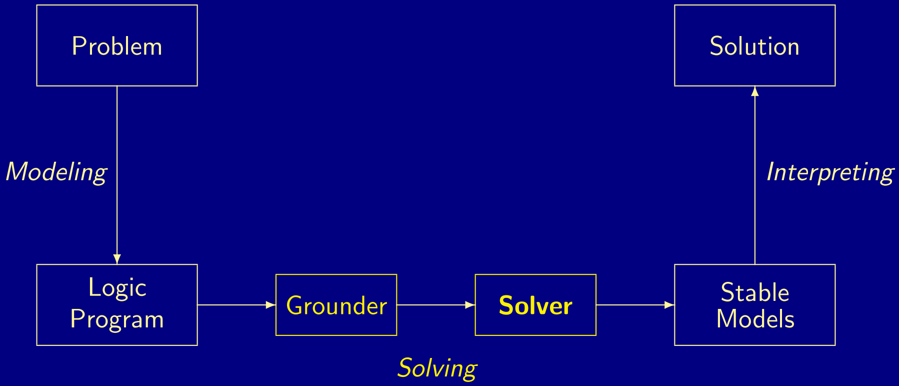

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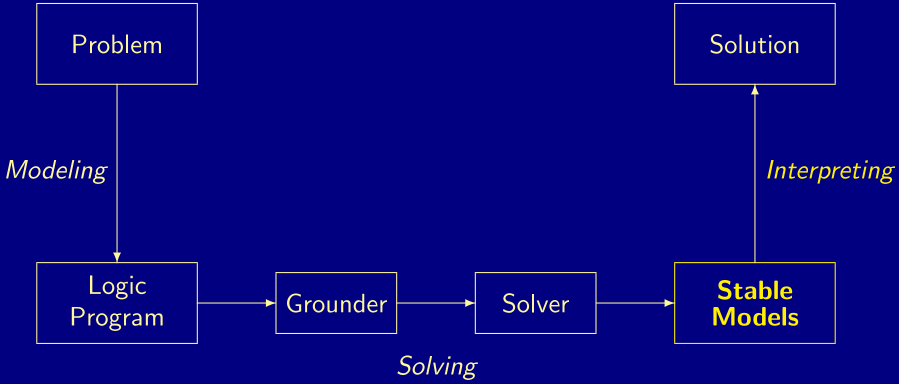

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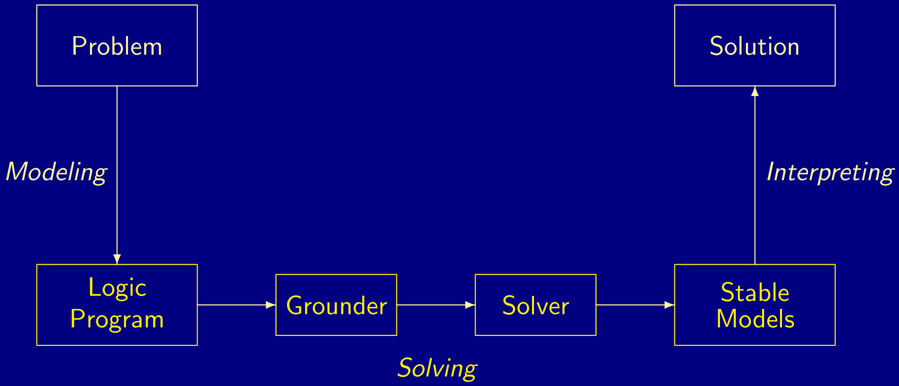

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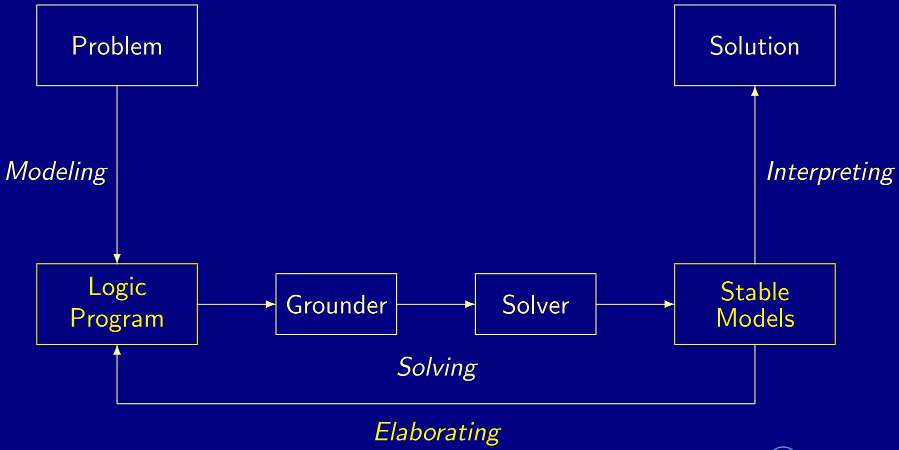
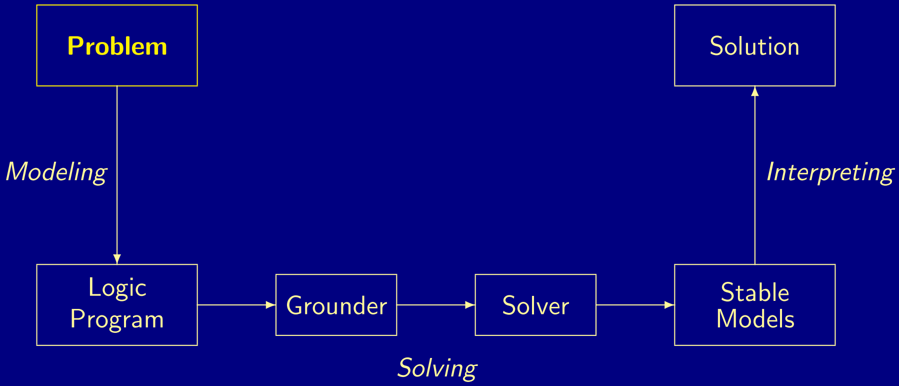


A case-study: Graph coloring



Graph coloring

- Problem instance A graph consisting of nodes and edges

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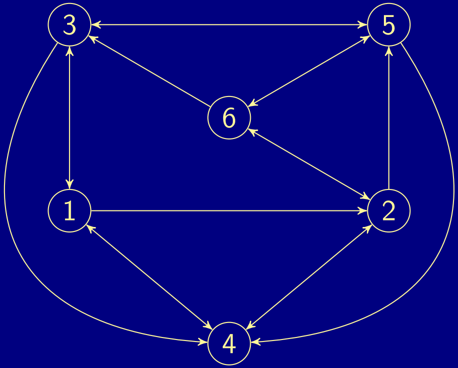

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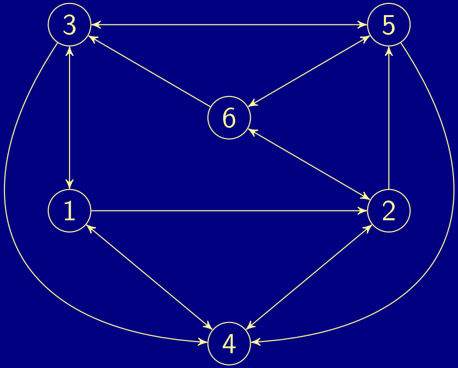

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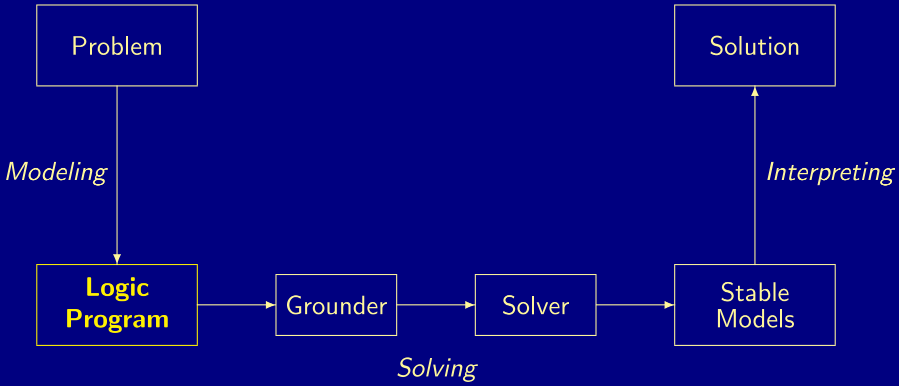
  - facts formed by predicate `col/1`
- Problem class Assign each node one color such that no two nodes connected by an edge have the same color

  In other words,

  1. Each node has one color
  2. Two connected nodes must not have the same color

ASP solving process



Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
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col(r).   col(b).   col(g).
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Problem instance

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1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
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Problem encoding

Potassco

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edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

Potassco

color.lp

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).
```

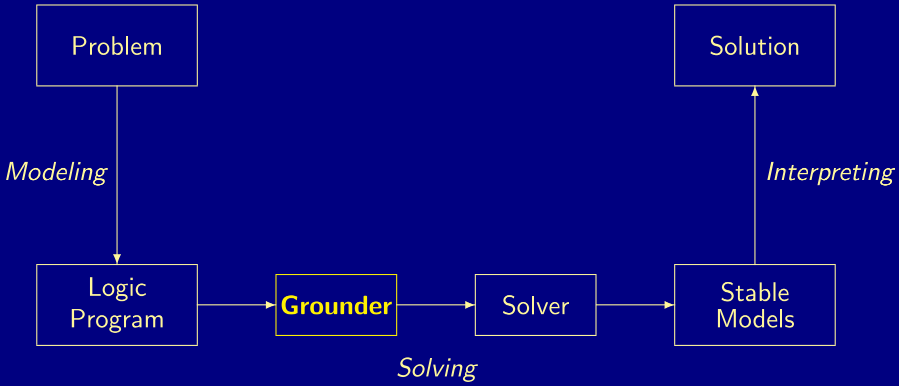
Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

ASP solving process



Graph coloring: Grounding

```
$ gringo --text color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

Graph coloring: Grounding

```
$ gringo --text color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

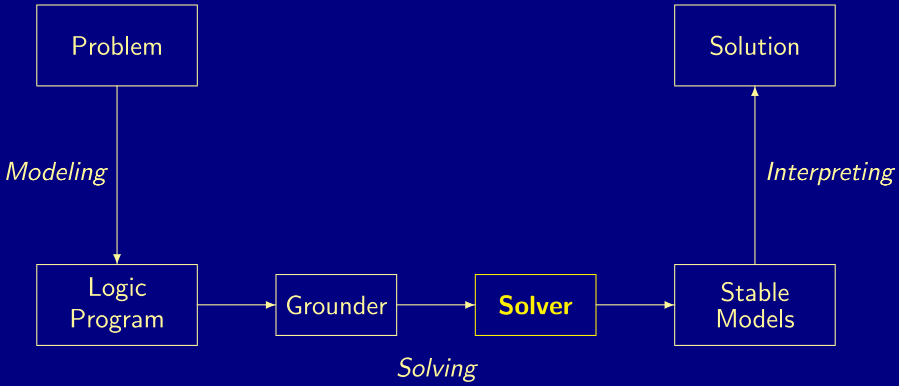
edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

ASP solving process



Graph coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

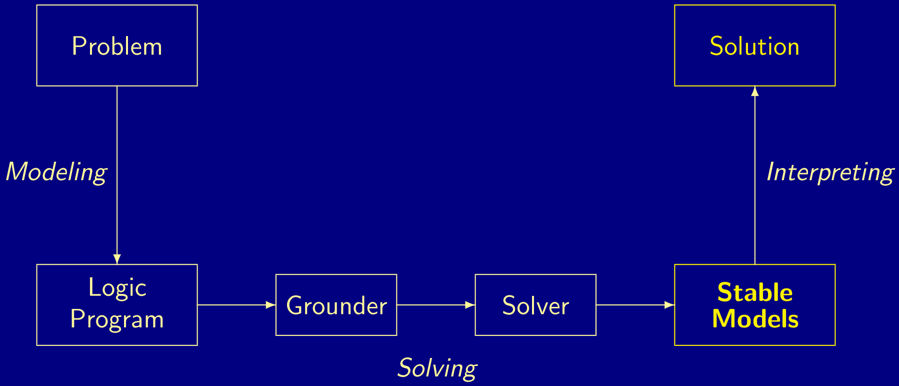
Graph coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

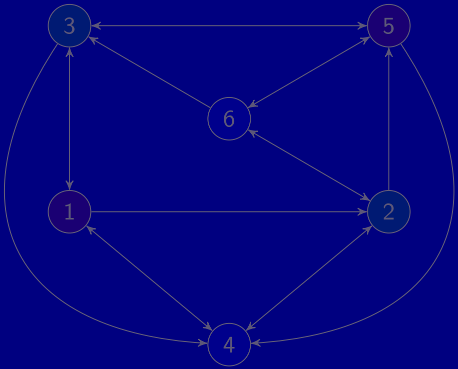
Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

ASP solving process



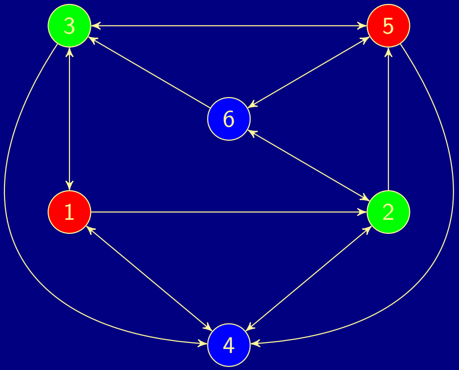
A coloring

```
Answer: 6
edge(1,2) ... col(r) ... node(1) ...                                  \
color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
```



A coloring

```
Answer: 6
edge(1,2) ... col(r) ... node(1) ...                             \
color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
```



Outline

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

- **Generator** Generate potential stable model candidates (typically through non-deterministic constructs)
- **Tester** Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program $=$ Data $+$ Generator $+$ Tester $(+$ Optimizer$)$

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

- **Generator** Generate potential stable model candidates (typically through non-deterministic constructs)
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Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Outline

Satisfiability testing

- Problem Instance: A propositional formula ϕ in CNF
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program:

Generator	Tester	Stable models
$\{ a \} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
$\{ b \} \leftarrow$	$\leftarrow a, \sim b$	$X_2 = \{\}$

Potassco

Satisfiability testing

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Generator	**Tester**	**Stable models**
$\{ a \} \leftarrow$	$\leftarrow \; {\sim}a, b$	$X_1 = \{a, b\}$
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Potassco

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- Logic Program:

Generator	**Tester**	**Stable models**
$\{ a \} \leftarrow$	$\leftarrow \; {\sim}a, b$	$X_1 = \{a, b\}$
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Satisfiability testing

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- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

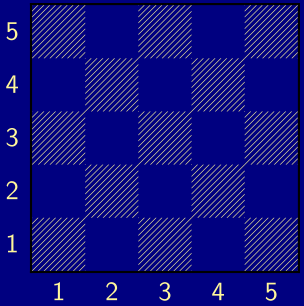
- Logic Program:

Generator	**Tester**	**Stable models**
$\{ a \} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
$\{ b \} \leftarrow$	$\leftarrow a, \sim b$	$X_2 = \{\}$

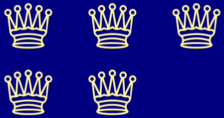
Potassco

Outline

The n-Queens Problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another



Defining the Field

queens.lp

```
row(1..n).
col(1..n).
```

- Create file `queens.lp`
- Define the field
  - n rows
  - n columns

Defining the Field

Running . . .

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models      : 1
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Placing some Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
```

- Guess a solution candidate
  by placing some queens on the board

Placing some Queens

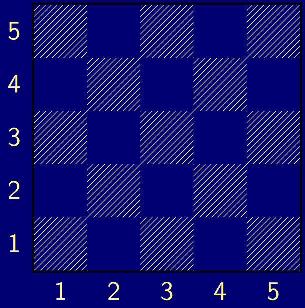
Running . . .

```
$ gringo queens.lp --const n=5 | clasp 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE

Models      : 3+
...
```

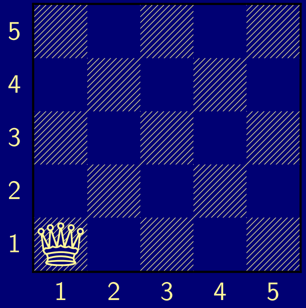
Placing some Queens: Answer 1

Answer 1



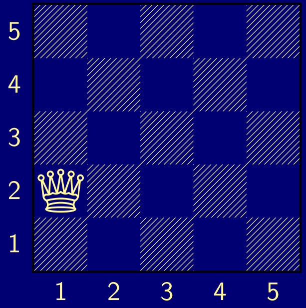
Placing some Queens: Answer 2

Answer 2



Placing some Queens: Answer 3

Answer 3



Placing n Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- not n { queen(I,J) } n.
```

- Place exactly n queens on the board

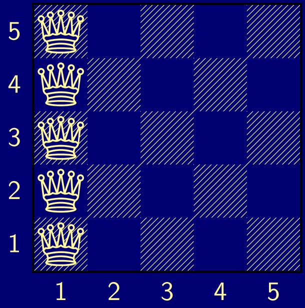
Placing n Queens

Running ...

```
$ gringo queens.lp --const n=5 | clasp 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
...
```

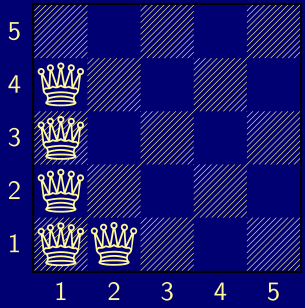
Placing n Queens: Answer 1

Answer 1



Placing n Queens: Answer 2

Answer 2



Horizontal and Vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
```

- Forbid horizontal attacks
- Forbid vertical attacks

Horizontal and Vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
```

- Forbid horizontal attacks
- Forbid vertical attacks

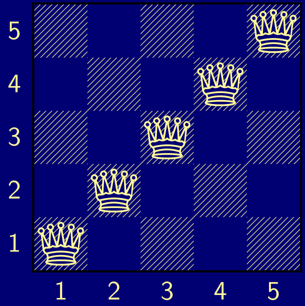
Horizontal and Vertical Attack

Running . . .

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) \
queen(2,2) queen(1,1)
...
```

Horizontal and Vertical Attack: Answer 1

Answer 1



Diagonal Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I-J == I'-J'.
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I+J == I'+J'.
```

- Forbid diagonal attacks

Diagonal Attack

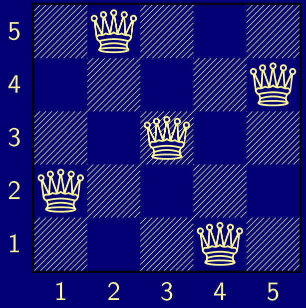
Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) queen(5,2) queen(2,1)
SATISFIABLE

Models      : 1+
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Diagonal Attack: Answer 1

Answer 1



Optimizing

queens-opt.lp

```
1 { queen(I,1..n) } 1 :- I = 1..n.
1 { queen(1..n,J) } 1 :- J = 1..n.
 :- 2 { queen(D-J,J) }, D = 2..2*n.
 :- 2 { queen(D+J,J) }, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=2
clingo version 4.1.0
Solving...
SATISFIABLE

Models       : 1+
Time         : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time     : 3758.320s

Choices      : 288594554
Conflicts    : 3442    (Analyzed: 3442)
Restarts     : 17      (Average: 202.47 Last: 3442)
Model-Level  : 7594728.0
Problems     : 1       (Average Length: 0.00 Splits: 0)
Lemmas       : 3442    (Deleted: 0)
  Binary     : 0       (Ratio:   0.00%)
  Ternary    : 0       (Ratio:   0.00%)
  Conflict   : 3442    (Average Length: 229056.5 Ratio: 100.00%)
  Loop       : 0       (Average Length:    0.0 Ratio:   0.00%)
  Other      : 0       (Average Length:    0.0 Ratio:   0.00%)

Atoms        : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules        : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies       : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight        : Yes
Variables    : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664   (Binary: 35.6% Ternary:  0.0% Other: 64.4%)

Backjumps    : 3442    (Average: 681.19 Max: 169512 Sum: 2344658)
  Executed   : 3442    (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
  Bounded    : 0       (Average:  0.00 Max:    0 Sum:      0 Ratio:   0.00%)
```

Outline

Traveling Salesperson

```
node(1..6).

edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Traveling Salesperson

```
node(1..6).

edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Traveling Salesperson

```
node(1..6).

edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Traveling Salesperson

```
node(1..6).

edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).

edge(X,Y) :- cost(X,Y,_).
```

Traveling Salesperson

```
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

Potassco

Traveling Salesperson

```
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

Traveling Salesperson

```
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
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Potassco

Traveling Salesperson

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1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
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#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

Outline

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
...

3 { assigned(P,R) : reviewer(R) } 3 :- paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :- classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Potassco

Reviewer Assignment

by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
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...

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 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
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...

3 { assigned(P,R) : reviewer(R) } 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
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assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Potassco

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
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 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment

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```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
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 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Potassco

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
...

3 <= #count { P,R : assigned(P,R) : reviewer(R) } <= 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 <= #count { P,R : assigned(P,R), paper(P) } <= 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 <= #count { P,R : assignedB(P,R), paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Reviewer Assignment
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```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
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3 <= #count { P,R : assigned(P,R) : reviewer(R) } <= 3 :-  paper(P).

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 :- not 6 <= #count { P,R : assigned(P,R), paper(P) } <= 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 <= #count { P,R : assignedB(P,R), paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Outline

Simplistic STRIPS Planning

```
time(1..k).

fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).    query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

1 { occ(A,T) : action(A) } 1 :- time(T).
 :- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- holds(F,T-1), not nolds(F,T), time(T).
holds(F,T) :- occ(A,T), add(A,F).
nolds(F,T) :- occ(A,T), del(A,F).

 :- query(F), not holds(F,k).
```

Simplistic STRIPS Planning

```
time(1..k).

fluent(p).     action(a).     action(b).     init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).   query(r).
                  del(a,p).      del(b,q).

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1 { occ(A,T) : action(A) } 1 :- time(T).
 :- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- holds(F,T-1), not nolds(F,T), time(T).
holds(F,T) :- occ(A,T), add(A,F).
nolds(F,T) :- occ(A,T), del(A,F).

 :- query(F), not holds(F,k).
```

Simplistic STRIPS Planning

```
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fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).    query(r).
                  del(a,p).      del(b,q).

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 :- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- holds(F,T-1), not nolds(F,T), time(T).
holds(F,T) :- occ(A,T), add(A,F).
nolds(F,T) :- occ(A,T), del(A,F).

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Simplistic STRIPS Planning

```
time(1..k).

fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).     query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

1 { occc(A,T) : action(A) } 1 :- time(T).
 :- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- holds(F,T-1), not nolds(F,T), time(T).
holds(F,T) :- occ(A,T), add(A,F).
nolds(F,T) :- occ(A,T), del(A,F).

 :- query(F), not holds(F,k).
```

Multi-shot ASP Solving: Overview

Potassco

Outline

Motivation

- Claim ASP is an under-the-hood technology

  That is, in practice, it mainly serves as a solving engine within an encompassing software environment

- Single-shot solving: *ground* | *solve*

  Multi-shot solving: *ground* | *solve*

  ➥ *continuously changing logic programs*

Agents, Assisted Living, Robotics, Planning, Query-answering, etc

clingo 4

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- Implementation *clingo* 4

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  That is, in practice, it mainly serves as a solving engine within an encompassing software environment

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- Multi-shot solving: (*input* | *ground** | *solve**)*

  ➥ *continuously changing logic programs*

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- Claim ASP is an under-the-hood technology

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- Single-shot solving: *ground* | *solve*
- Multi-shot solving: (*input* | *ground** | *solve**| *theory*)*
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  ➥ *continuously changing logic programs*

- Application areas

  Agents, Assisted Living, Robotics, Planning, Query-answering, etc

- Implementation *clingo* 4

Clingo = ASP + Control

- ASP
  - `#program <name> [ (<parameters>) ]`

    `#program play(t).`
  - `#external <atom> [ : <body> ]`

    `#external mark(X,Y,P,t) : field(X,Y), player(P).`

- Control
  - Lua (www.lua.org)

    `prg:solve(), prg:ground(parts), ...`
  - Python (www.python.org)

    `prg.solve(), prg.ground(parts), ...`

- Integration
  - in ASP: embedded scripting language (#script)
  - in Lua/Python: library import (import gringo)

Clingo = ASP + Control

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  - `#program <name> [ (<parameters>) ]`
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Clingo = ASP + Control

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Potassco

Vanilla *clingo*

- Emulating *clingo* in *clingo* 4

```
#script (python)
def main(prg):
    parts = []
    parts.append(("base", []))
    prg.ground(parts)
    prg.solve()
#end.
```

Vanilla *clingo*

- Emulating *clingo* in *clingo* 4

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Vanilla *clingo*

- Emulating *clingo* in *clingo* 4

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#script (python)
def main(prg):
    parts = []
    parts.append(("base", []))
    prg.ground(parts)
    prg.solve()
#end.
```

Hello world!

```
#script (python)
def main(prg):
    print("Hello world!")
#end.
```

```
$ clingo hello.lp
clingo version 4.5.0
Reading from hello.lp
Hello world!
UNKNOWN

Models       : 0+
Calls        : 1
Time         : 0.009s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
```

Potassco

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CPU Time     : 0.000s
```

Outline

#program declaration

- A program declaration is of form

  `#program` $n\,(p_1,\ldots,p_k)$

  where $n, p_1, \ldots, p_k$ are non-integer constants
- We call n the name of the declaration and $p_1, \ldots, p_k$ its parameters
- Convention Different occurrences of program declarations with the same name share the same parameters
- Example

```
#program acid(k).
b(k).
c(X,k) :- a(X).
#program base.
a(2).
```

Potassco

#program declaration

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- We call n the name of the declaration and $p_1, \dots, p_k$ its parameters
- Convention Different occurrences of program declarations with the same name share the same parameters
- Example

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#program acid(k).
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c(X,k) :- a(X).
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#program declaration

- A program declaration is of form

  #program $n\,(p_1, \dots, p_k)$

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Scope of #program declarations

- The scope of an occurrence of a program declaration in a list of rules and declarations consists of the set of all rules and non-program declarations appearing between the occurrence and the next occurrence of a program declaration or the end of the list
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- Given a list R of (non-ground) rules and declarations and a name n, we define $R(n)$ as the set of all rules and non-program declarations in the scope of all occurrences of program declarations with name n
- We often refer to $R(n)$ as a subprogram of R
- Example
  - $R(\texttt{base}) = \{a(1), a(2)\}$
  - $R(\texttt{acid}) = \{b(k), c(X, k) \leftarrow a(X)\}$
- Given a name n with associated parameters $(p_1, \dots, p_k)$, the instantiation of $R(n)$ with a term tuple $(t_1, \dots, t_k)$ results in the set

  $$R(n)[p_1/t_1, \dots, p_k/t_k]$$

  obtained by replacing in $R(n)$ each occurrence of p_i by t_i

Potassco

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Contextual grounding

- Rules are grounded relative to a set of atoms, called atom base
- Given a set R of (non-ground) rules and two sets C, D of ground atoms, we define an instantiation of R relative to C as a ground program $ground_C(R)$ over D subject to the following conditions:

$$C \subseteq D \subseteq C \cup head(ground_C(R))$$

$$ground_C(R) \subseteq \{head(r) \leftarrow body(r)^+ \cup \{\sim a \mid a \in body(r)^- \cap D\} \mid r \in ground(R), body(r)^+ \subseteq D\}$$

- Example Given $R = \{\ a(X) \leftarrow f(X), e(X);\ \ b(X) \leftarrow f(X), \sim e(X)\ \}$ and $C = \{f(1), f(2), e(1)\}$, we obtain

$$ground_C(R) = \left\{ \begin{array}{ll} a(1) \leftarrow f(1), e(1); & b(1) \leftarrow f(1), \sim e(1) \\ & b(2) \leftarrow f(2) \end{array} \right\}$$

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#external declaration

- An external declaration is of form

  #external $a : B$

  where a is an atom and B a rule body

- A logic program with external declarations is said to be extensible

- Example

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#external e(X) : f(X), X < 2.
f(1..2).
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Grounding extensible logic programs

- Given an extensible program R, we define

$$Q = \{a \leftarrow B, \varepsilon \mid (\texttt{\#external}\ a : B) \in R\}$$
$$R' = \{a \leftarrow B \in R\}$$

- Note An external declaration is treated as a rule $a \leftarrow B, \varepsilon$ where ε is a ground marking atom
- Given an atom base C, the ground instantiation of an extensible logic program R is defined as a (ground) logic program P with externals E where

$$P = \{r \in ground_{C \cup \{\varepsilon\}}(R' \cup Q) \mid \varepsilon \notin body(r)\}$$
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Outline

Module

- The assembly of subprograms can be characterized by means of modules:

- A module $\mathbb{P}$ is a triple (P, I, O) consisting of
  - a (ground) program P over $ground(\mathcal{A})$ and
  - sets $I, O \subseteq ground(\mathcal{A})$ such that
    - $I \cap O = \emptyset$,
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Composing modules

- Two modules $\mathbb{P}$ and $\mathbb{Q}$ are compositional, if
  - $O(\mathbb{P}) \cap O(\mathbb{Q}) = \emptyset$ and
  - $O(\mathbb{P}) \cap S = \emptyset$ or $O(\mathbb{Q}) \cap S = \emptyset$
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  - Recursion between two modules to be joined is disallowed
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- The join, $\mathbb{P} \sqcup \mathbb{Q}$, of two modules $\mathbb{P}$ and $\mathbb{Q}$ is defined as the module

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  provided that $\mathbb{P}$ and $\mathbb{Q}$ are compositional

Composing modules

- Two modules $\mathbb{P}$ and $\mathbb{Q}$ are compositional, if
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Composing logic programs with externals

- Idea Each `ground` instruction induces a module to be joined with the module representing the current program state

- Given an atom base C, a (non-ground) extensible program R induces the module

$$\mathbb{R}(C) = (P, (C \cup E) \setminus head(P), head(P))$$

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Example

- Atom base $C = \{\texttt{g(1)}\}$
- Extensible program R

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#external e(X) : f(X), g(X)
f(1). f(2).
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Capturing program states by modules

- Each program state is captured by a module
  - The input and output atoms of each module provide the atom base
- The initial program state is given by the empty module

$$\mathbb{P}_0 = (\emptyset, \emptyset, \emptyset)$$

- The program state succeeding $\mathbb{P}_i$ is captured by the module

$$\mathbb{P}_{i+1} = \mathbb{P}_i \sqcup \mathbb{R}_{i+1}(I(\mathbb{P}_i) \cup O(\mathbb{P}_i))$$

  where $\mathbb{R}_{i+1}(I(\mathbb{P}_i) \cup O(\mathbb{P}_i))$ captures the result of grounding an extensible program R relative to atom base $I(\mathbb{P}_i) \cup O(\mathbb{P}_i)$

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If $\bigsqcup_{i\geq 0} \mathbb{P}_i$ is compositional, then

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Outline

Clingo state

- A *clingo* state is a triple

$$(\mathbf{R}, \mathbb{P}, V)$$

where

  - $\mathbf{R}$ is a collection of extensible (non-ground) logic programs
  - $\mathbb{P}$ is a module
  - V is a three-valued assignment over $I(\mathbb{P})$

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- Note Input atoms in $I(\mathbb{P})$ are taken to be false by default

create

- $create(R) : \mapsto (\mathbf{R}, \mathbb{P}, V)$

  for a list R of (non-ground) rules and declarations where

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- $ground((n, \mathbf{p}_n)_{n \in N}) : (\mathbf{R}, \mathbb{P}_1, V_1) \mapsto (\mathbf{R}, \mathbb{P}_2, V_2)$

  for a collection $(n, \mathbf{p}_n)_{n \in N}$ such that $N \subseteq \mathcal{C}$ and $\mathbf{p}_n \in \mathcal{T}^k$ for some k where

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- Notes
  - The external status of an atom is eliminated once it becomes defined by a rule in some added program
    This is accomplished by module composition, namely, the elimination of output atoms from input atoms
  - Jointly grounded subprograms are treated as a single subprogram
  - $ground((n, \mathbf{p}), (n, \mathbf{p}))(s) = ground((n, \mathbf{p}))(s)$ while $ground((n, \mathbf{p}))(ground((n, \mathbf{p}))(s))$ leads to two non-compositional modules whenever $head(R_n) \neq \emptyset$
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assignExternal

- $\mathit{assignExternal}(a, v) : (\mathbf{R}, \mathbb{P}, V_1) \mapsto (\mathbf{R}, \mathbb{P}, V_2)$

  for a ground atom a and $v \in \{t, u, f\}$ where

  - if $v = t$
    - $V_2^t = V_1^t \cup \{a\}$ if $a \in I(\mathbb{P})$, and $V_2^t = V_1^t$ otherwise
    - $V_2^u = V_1^u \setminus \{a\}$
  - if $v = u$
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- Notes
  - *releaseExternal* only affects input atoms; defined atoms remain unaffected
  - A released atom can never be re-defined, neither by a rule nor an external declaration
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releaseExternal

- *releaseExternal*$(a) : (\mathbf{R}, \mathbb{P}_1, V_1) \mapsto (\mathbf{R}, \mathbb{P}_2, V_2)$

  for a ground atom a where

  - $\mathbb{P}_2 = (P(\mathbb{P}_1), I(\mathbb{P}_1) \setminus \{a\}, O(\mathbb{P}_1) \cup \{a\})$ if $a \in I(\mathbb{P}_1)$, and
    $\mathbb{P}_2 = \mathbb{P}_1$ otherwise
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solve

- $solve((A^t, A^f)) : (\mathbf{R}, \mathbb{P}, V) \mapsto (\mathbf{R}, \mathbb{P}, V)$ prints the set

$$\{X \mid X \text{ is a stable model of } \mathbb{P} \text{ wrt } V \text{ st } A^t \subseteq X \text{ and } A^f \cap X = \emptyset\}$$

where the stable models of a module $\mathbb{P}$ wrt an assignment V are given by the stable models of the program

$$P(\mathbb{P}) \cup \{a \leftarrow \mid a \in V^t\} \cup \{\{a\} \leftarrow \mid a \in V^u\}$$

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#script declaration

- A script declaration is of form

  `#script(python)` P `#end`

  where P is a Python program
- Analogously for Lua
- `main` routine exercises control (from within *clingo*, not from Python)

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#script(python)
def main(prg):
    prg.ground([("base",[])])
    prg.solve()
#end.
```

```
#script(python)
def main(prg):
    prg.ground([("acid",[42])])
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Potassco

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Extensible programs

- Initial *clingo* state

$$(\mathbf{R}_0, \mathbb{P}_0, V_0) = ((R(\texttt{base}), R(\texttt{succ})), (\emptyset, \emptyset, \emptyset), (\emptyset, \emptyset))$$

  where

$$R(\texttt{base}) = \left\{ \begin{array}{ll} \texttt{\#external}\ p(1) & p(0) \leftarrow p(3) \\ \texttt{\#external}\ p(2) & p(0) \leftarrow {\sim}p(0) \\ \texttt{\#external}\ p(3) & \end{array} \right\}$$

$$R(\texttt{succ}) = \left\{ \begin{array}{l} \texttt{\#external}\ p(n+3) \\ p(n) \leftarrow p(n+3) \\ p(n) \leftarrow {\sim}p(n+1), {\sim}p(n+2) \end{array} \right\}$$

- Initial atom base $I(\mathbb{P}_0) \cup O(\mathbb{P}_0) = \emptyset$

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Extensible programs

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Extensible programs

- Initial *clingo* state, or more precisely, state of *clingo* object '`prg`'

$$create(R) = ((R(\mathtt{base}), R(\mathtt{succ})), (\emptyset, \emptyset, \emptyset), (\emptyset, \emptyset))$$

where R is the list of rules and declarations in Line 1-8 and

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- Note $create(R)$ is invoked implicitly to create *clingo* object ‘prg’

Example

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#end.
```

prg.ground([("base", [])])

- Global *clingo* state $(\mathbf{R}_0, \mathbb{P}_0, V_0)$, including atom base $\emptyset$
- Input Extensible program $R(\texttt{base})$
- Output Module

$$\mathbb{R}_1(\emptyset) = (P_1, E_1, \{p(0)\}) \qquad \text{where}$$
$$P_1 = \{p(0) \leftarrow p(3);\ p(0) \leftarrow {\sim}p(0)\}$$
$$E_1 = \{p(1), p(2), p(3)\}$$

- Result *clingo* state

$$(\mathbf{R}_1, \mathbb{P}_1, V_1) = (\mathbf{R}_0, \mathbb{P}_0 \sqcup \mathbb{R}_1(\emptyset), V_0)$$

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```

Potassco

prg.assign_external(Fun("p",[3]),True)

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- Input assignment $p(3) \mapsto t$
- Result *clingo* state

$$(\mathbf{R}_2, \mathbb{P}_2, V_2) = (\mathbf{R}_0, \mathbb{P}_1, (\{p(3)\}, \emptyset))$$

`prg.assign_external(Fun("p",[3]),True)`

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```

Potassco

prg.solve()

- Global *clingo* state $(\mathbf{R}_2, \mathbb{P}_2, V_2)$
- Input empty assignment
- Result *clingo* state

$$(\mathbf{R}_2, \mathbb{P}_2, V_2) = (\mathbf{R}_0, \mathbb{P}_1, (\{p(3)\}, \emptyset))$$

stable model $\{p(0), p(3)\}$ of $\mathbb{P}_2$ wrt V_2

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- Print stable model $\{p(0), p(3)\}$ of $\mathbb{P}_2$ wrt V_2

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#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from gringo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
>>  prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
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    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Potassco

prg.assign_external(Fun("p",[3]),False)

- Global *clingo* state $(\mathbf{R}_2, \mathbb{P}_2, V_2)$
- Input assignment $p(3) \mapsto f$
- Result *clingo* state

$$(\mathbf{R}_3, \mathbb{P}_3, V_3) = (\mathbf{R}_0, \mathbb{P}_1, (\emptyset, \emptyset))$$

`prg.assign_external(Fun("p",[3]),False)`

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    prg.solve()
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    prg.solve()
#end.
```

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#end.
```

`prg.solve()`

- Global *clingo* state $(\mathbf{R}_3, \mathbb{P}_3, V_3)$
- Input empty assignment
- Result *clingo* state

$$(\mathbf{R}_3, \mathbb{P}_3, V_3) = (\mathbf{R}_0, \mathbb{P}_1, (\emptyset, \emptyset))$$

- Print no stable model of $\mathbb{P}_3$ wrt V_3

`prg.solve()`

- Global *clingo* state $(\mathbf{R}_3, \mathbb{P}_3, V_3)$
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Potassco

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Example

```
#external p(1;2;3).
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```

Example

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   p(n) :- p(n+3).
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>>     prg.ground([("succ", [1]),("succ", [2])])
       prg.solve()
       prg.ground([("succ", [3])])
       prg.solve()
   #end.
```

`prg.ground([("succ",[1]),("succ",[2])])`

- Global *clingo* state $(\mathbf{R}_3, \mathbb{P}_3, V_3)$, including atom base
  $I(\mathbb{P}_3) \cup O(\mathbb{P}_3) = \{p(0), p(1), p(2), p(3)\}$
- Input Extensible program $R(\mathtt{succ})[\mathtt{n}/1] \cup R(\mathtt{succ})[\mathtt{n}/2]$
- Output Module

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(P_4, \left\{\begin{matrix} p(0), p(4), \\ p(3), p(5) \end{matrix}\right\}, \left\{\begin{matrix} p(1), \\ p(2) \end{matrix}\right\}\right) \quad \text{where}$$

$$P_4 = \left\{\begin{matrix} p(1) \leftarrow p(4);\ \ p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(2) \leftarrow p(5);\ \ p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{matrix}\right\}$$

$$E_4 = \{p(4), p(5)\}$$

- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

```
prg.ground([("succ",[1]),("succ",[2])])
```

- Global *clingo* state $(\mathbf{R}_3, \mathbb{P}_3, V_3)$, including atom base
  $$I(\mathbb{P}_3) \cup O(\mathbb{P}_3) = \{p(0), p(1), p(2), p(3)\}$$
- Input Extensible program $R(\mathtt{succ})[\mathtt{n}/1] \cup R(\mathtt{succ})[\mathtt{n}/2]$
- Output Module

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(P_4, \left\{\begin{array}{l} p(0), p(4), \\ p(3), p(5) \end{array}\right\}, \left\{\begin{array}{l} p(1), \\ p(2) \end{array}\right\}\right) \quad \text{where}$$

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$$E_4 = \{p(4), p(5)\}$$

- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

`prg.ground([("succ",[1]),("succ",[2])])`

- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\begin{Bmatrix} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(0) \leftarrow {\sim}p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{Bmatrix}, \begin{Bmatrix} p(4), \\ p(3), p(5) \end{Bmatrix}, \begin{Bmatrix} p(0), p(1), \\ p(2) \end{Bmatrix}\right)$$

$$\mathbb{P}_3 = \left(\begin{Bmatrix} p(0) \leftarrow p(3); \\ p(0) \leftarrow {\sim}p(0) \end{Bmatrix}, \{p(1), p(2), p(3)\}, \{p(0)\}\right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\begin{Bmatrix} p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{Bmatrix}, \begin{Bmatrix} p(0), p(4), \\ p(3), p(5) \end{Bmatrix}, \begin{Bmatrix} p(1), \\ p(2) \end{Bmatrix}\right)$$

`prg.ground([("succ",[1]),("succ",[2])])`

- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\left\{ \begin{array}{llll} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(0) \leftarrow \sim p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{array} \right\}, \left\{ \begin{array}{r} p(4), \\ p(3), p(5) \end{array} \right\}, \left\{ \begin{array}{r} p(0), p(1), \\ p(2) \end{array} \right\} \right)$$

$$\mathbb{P}_3 = \left(\left\{ \begin{array}{l} p(0) \leftarrow p(3); \\ p(0) \leftarrow \sim p(0) \end{array} \right\}, \{p(1), p(2), p(3)\}, \{p(0)\} \right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\left\{ \begin{array}{ll} p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{array} \right\}, \left\{ \begin{array}{l} p(0), p(4), \\ p(3), p(5) \end{array} \right\}, \left\{ \begin{array}{l} p(1), \\ p(2) \end{array} \right\} \right)$$

Potassco

`prg.ground([("succ",[1]),("succ",[2])])`

- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\begin{Bmatrix} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(0) \leftarrow \sim p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{Bmatrix}, \begin{Bmatrix} p(4), \\ p(3), p(5) \end{Bmatrix}, \begin{Bmatrix} p(0), p(1), \\ p(2) \end{Bmatrix} \right)$$

$$\mathbb{P}_3 = \left(\begin{Bmatrix} p(0) \leftarrow p(3); \\ p(0) \leftarrow \sim p(0) \end{Bmatrix}, \{p(1), p(2), p(3)\}, \{p(0)\} \right)$$

$$\mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)) = \left(\begin{Bmatrix} p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4) \end{Bmatrix}, \begin{Bmatrix} p(0), p(4), \\ p(3), p(5) \end{Bmatrix}, \begin{Bmatrix} p(1), \\ p(2) \end{Bmatrix} \right)$$

prg.ground([("succ",[1]),("succ",[2])])

- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\begin{Bmatrix} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(0) \leftarrow {\sim}p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{Bmatrix}, \begin{Bmatrix} p(4), \\ p(3), p(5) \end{Bmatrix}, \begin{Bmatrix} p(0), p(1), \\ p(2) \end{Bmatrix}\right)$$

$$\mathbb{P}_3 = \left(\begin{Bmatrix} p(0) \leftarrow p(3); \\ p(0) \leftarrow {\sim}p(0) \end{Bmatrix}, \{p(1), p(2), p(3)\}, \{p(0)\}\right)$$

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```
prg.ground([("succ",[1]),("succ",[2])])
```

- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

where

$$\mathbb{P}_4 = \left(\left\{\begin{matrix} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow {\sim}p(2), {\sim}p(3); \\ p(0) \leftarrow {\sim}p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow {\sim}p(3), {\sim}p(4) \end{matrix}\right\}, \left\{\begin{matrix} p(4), \\ p(3), p(5) \end{matrix}\right\}, \left\{\begin{matrix} p(0), p(1), \\ p(2) \end{matrix}\right\}\right)$$

$$\mathbb{P}_3 = \left(\left\{\begin{matrix} p(0) \leftarrow p(3); \\ p(0) \leftarrow {\sim}p(0) \end{matrix}\right\}, \{p(1), p(2), p(3)\}, \{p(0)\}\right)$$

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prg.ground([("succ",[1]),("succ",[2])])

- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_3 \sqcup \mathbb{R}_4(I(\mathbb{P}_3) \cup O(\mathbb{P}_3)), V_3)$$

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prg.ground([("succ",[1]),("succ",[2])])

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`prg.ground([("succ",[1]),("succ",[2])])`

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`prg.ground([("succ",[1]),("succ",[2])])`

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$$\mathbb{P}_3 = \left(\left\{\begin{matrix} p(0) \leftarrow p(3); \\ p(0) \leftarrow {\sim}p(0) \end{matrix}\right\}, \{p(1), p(2), p(3)\}, \{p(0)\}\right)$$

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Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from gringo import Fun
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    prg.ground([("base", [])])
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    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
>>  prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Example

```
#external p(1;2;3).
p(0) :- p(3).
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>>  prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

`prg.solve()`

- Global *clingo* state $(\mathbf{R}_4, \mathbb{P}_4, V_4)$
- Input empty assignment
- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_4, V_3)$$

- Print no stable model of $\mathbb{P}_4$ wrt V_4

Potassco

`prg.solve()`

- Global *clingo* state $(\mathbf{R}_4, \mathbb{P}_4, V_4)$
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`prg.solve()`

- Global *clingo* state $(\mathbf{R}_4, \mathbb{P}_4, V_4)$
- Input empty assignment
- Result *clingo* state

$$(\mathbf{R}_4, \mathbb{P}_4, V_4) = (\mathbf{R}_0, \mathbb{P}_4, V_3)$$

- Print no stable model of $\mathbb{P}_4$ wrt V_4

Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from gringo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
>>  prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from gringo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
>>  prg.ground([("succ", [3])])
    prg.solve()
#end.
```

`prg.ground([("succ", [3])])`

- Global *clingo* state $(\mathbf{R}_4, \mathbb{P}_4, V_4)$, including atom base
$$I(\mathbb{P}_4) \cup O(\mathbb{P}_4) = \{p(0), p(1), p(2), p(3), p(4), p(5)\}$$
- Input Extensible program $R(\mathtt{succ})[\mathtt{n}/3]$
- Output Module

$$\mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)) = \left(P_5, \left\{\begin{array}{l} p(0), p(1), p(2), \\ p(4), p(5), p(6) \end{array}\right\}, \{p(3)\}\right)$$

$$\text{where } P_5 = \{p(3) \leftarrow p(6);\ p(3) \leftarrow {\sim}p(4), {\sim}p(5)\}$$
$$E_5 = \{p(6)\}$$

- Result *clingo* state

$$(\mathbf{R}_5, \mathbb{P}_5, V_5) = (\mathbf{R}_0, \mathbb{P}_4 \sqcup \mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)), V_3)$$

`prg.ground([("succ", [3])])`

- Global *clingo* state $(\mathbf{R}_4, \mathbb{P}_4, V_4)$, including atom base
  $I(\mathbb{P}_4) \cup O(\mathbb{P}_4) = \{p(0), p(1), p(2), p(3), p(4), p(5)\}$
- Input Extensible program $R(\mathtt{succ})[\mathtt{n}/3]$
- Output Module

$$\mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)) = \left(P_5, \left\{\begin{array}{l} p(0), p(1), p(2), \\ p(4), p(5), p(6) \end{array}\right\}, \{p(3)\}\right)$$

$$\text{where } P_5 = \{p(3) \leftarrow p(6);\ p(3) \leftarrow {\sim}p(4), {\sim}p(5)\}$$

$$E_5 = \{p(6)\}$$

- Result *clingo* state

$$(\mathbf{R}_5, \mathbb{P}_5, V_5) = (\mathbf{R}_0, \mathbb{P}_4 \sqcup \mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)), V_3)$$

`prg.ground([("succ", [3])])`

- Global *clingo* state $(\mathbf{R}_4, \mathbb{P}_4, V_4)$, including atom base
  $$I(\mathbb{P}_4) \cup O(\mathbb{P}_4) = \{p(0), p(1), p(2), p(3), p(4), p(5)\}$$
- Input Extensible program $R(\mathtt{succ})[\mathtt{n}/3]$
- Output Module

$$\mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)) = \left(P_5, \left\{\begin{array}{l} p(0), p(1), p(2), \\ p(4), p(5), p(6) \end{array}\right\}, \{p(3)\}\right)$$

$$\text{where } P_5 = \{p(3) \leftarrow p(6);\ p(3) \leftarrow {\sim}p(4), {\sim}p(5)\}$$

$$E_5 = \{p(6)\}$$

- Result *clingo* state

$$(\mathbf{R}_5, \mathbb{P}_5, V_5) = (\mathbf{R}_0, \mathbb{P}_4 \sqcup \mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)), V_3)$$

`prg.ground([("succ", [3])])`

- Result *clingo* state

$$(\mathbf{R}_5, \mathbb{P}_5, V_5) = (\mathbf{R}_0, \mathbb{P}_4 \sqcup \mathbb{R}_5(I(\mathbb{P}_4) \cup O(\mathbb{P}_4)), V_3)$$

where

$$\mathbf{R}_5 = (R(\mathtt{base}), R(\mathtt{succ}))$$

$$P(\mathbb{P}_5) = \left\{\begin{array}{lll} p(0) \leftarrow p(3); & p(1) \leftarrow p(4); & p(1) \leftarrow \sim p(2), \sim p(3); \\ p(0) \leftarrow \sim p(0); & p(2) \leftarrow p(5); & p(2) \leftarrow \sim p(3), \sim p(4); \\ & p(3) \leftarrow p(6); & p(3) \leftarrow \sim p(4), \sim p(5) \end{array}\right\}$$

$$I(\mathbb{P}_5) = \{p(4), p(5), p(6)\}$$

$$O(\mathbb{P}_5) = \{p(0), p(1), p(2), p(3)\}$$

$$V_5 = (\emptyset, \emptyset)$$

Potassco

Example

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from gringo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
>>  prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Example

```
    #external p(1;2;3).
    p(0) :- p(3).
    p(0) :- not p(0).

    #program succ(n).
    #external p(n+3).
    p(n) :- p(n+3).
    p(n) :- not p(n+1), not p(n+2).

    #script(python)
    from gringo import Fun
    def main(prg):
        prg.ground([("base", [])])
        prg.assign_external(Fun("p", [3]), True)
        prg.solve()
        prg.assign_external(Fun("p", [3]), False)
        prg.solve()
        prg.ground([("succ", [1]),("succ", [2])])
        prg.solve()
        prg.ground([("succ", [3])])
>>      prg.solve()
    #end.
```

Potassco

`prg.solve()`

- Global *clingo* state $(\mathbf{R}_5, \mathbb{P}_5, V_5)$
- Input empty assignment
- Result *clingo* state

$$(\mathbf{R}_5, \mathbb{P}_5, V_5) = (\mathbf{R}_0, \mathbb{P}_5, V_3)$$

- Print stable model $\{p(0), p(3)\}$ of $\mathbb{P}_5$ wrt V_5

`prg.solve()`

- Global *clingo* state $(\mathbf{R}_5, \mathbb{P}_5, V_5)$
- Input empty assignment
- Result *clingo* state

$$(\mathbf{R}_5, \mathbb{P}_5, V_5) = (\mathbf{R}_0, \mathbb{P}_5, V_3)$$

- Print stable model $\{p(0), p(3)\}$ of $\mathbb{P}_5$ wrt V_5

Potassco

prg.solve()

- Global *clingo* state $(\mathbf{R}_5, \mathbb{P}_5, V_5)$
- Input empty assignment
- Result *clingo* state

$$(\mathbf{R}_5, \mathbb{P}_5, V_5) = (\mathbf{R}_0, \mathbb{P}_5, V_3)$$

- Print stable model $\{p(0), p(3)\}$ of $\mathbb{P}_5$ wrt V_5

Potassco

simple.lp

```
#external p(1;2;3).
p(0) :- p(3).
p(0) :- not p(0).

#program succ(n).
#external p(n+3).
p(n) :- p(n+3).
p(n) :- not p(n+1), not p(n+2).

#script(python)
from gringo import Fun
def main(prg):
    prg.ground([("base", [])])
    prg.assign_external(Fun("p", [3]), True)
    prg.solve()
    prg.assign_external(Fun("p", [3]), False)
    prg.solve()
    prg.ground([("succ", [1]),("succ", [2])])
    prg.solve()
    prg.ground([("succ", [3])])
    prg.solve()
#end.
```

Clingo on the run

```
$ clingo simple.lp
clingo version 4.5.0
Reading from simple.lp
Solving...
Answer: 1
p(3) p(0)
Solving...
Solving...
Solving...
Answer: 1
p(3) p(0)
SATISFIABLE

Models       : 2+
Calls        : 4
Time         : 0.019s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.010s
```

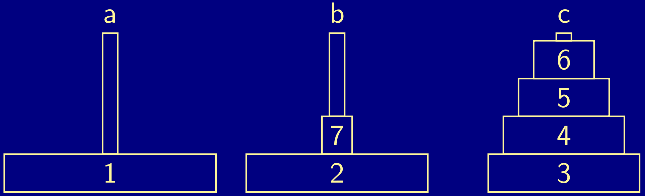
Clingo on the run

```
$ clingo simple.lp
clingo version 4.5.0
Reading from simple.lp
Solving...
Answer: 1
p(3) p(0)
Solving...
Solving...
Solving...
Answer: 1
p(3) p(0)
SATISFIABLE

Models       : 2+
Calls        : 4
Time         : 0.019s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.010s
```

Outline

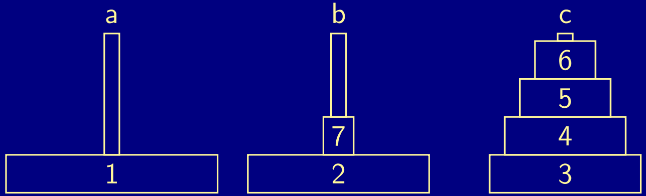
Towers of Hanoi Instance




```
peg(a;b;c).   disk(1..7).

    init_on(1,a).  init_on((2;7),b).  init_on((3;4;5;6),c).
goal_on((3;4),a).                   goal_on((1;2;5;6;7),c).
```

Towers of Hanoi Instance




```
peg(a;b;c).   disk(1..7).

    init_on(1,a).  init_on((2;7),b).  init_on((3;4;5;6),c).
goal_on((3;4),a).                   goal_on((1;2;5;6;7),c).
```

Towers of Hanoi Encoding

```
#program base.

on(D,P,0) :- init_on(D,P).
```

Towers of Hanoi Encoding

```
#program step(t).

1 { move(D,P,t) : disk(D), peg(P) } 1.

moved(D,t) :- move(D,_,t).
blocked(D,P,t) :- on(D+1,P,t-1), disk(D+1).
blocked(D,P,t) :- blocked(D+1,P,t), disk(D+1).
:- move(D,P,t), blocked(D-1,P,t).
:- moved(D,t), on(D,P,t-1), blocked(D,P,t).

on(D,P,t) :- on(D,P,t-1), not moved(D,t).
on(D,P,t) :- move(D,P,t).
:- not 1 { on(D,P,t) : peg(P) } 1, disk(D).
```

Towers of Hanoi Encoding

```
#program check(t).
#external query(t).

:- goal_on(D,P), not on(D,P,t), query(t).
```

Incremental Solving (ASP)

```
#script (python)

from gringo import SolveResult, Fun

def main(prg):
    ret, parts, step = SolveResult.UNSAT, [], 1
    parts.append(("base", []))
    while ret == SolveResult.UNSAT:
        parts.append(("step", [step]))
        parts.append(("check", [step]))
        prg.ground(parts)
        prg.release_external(Fun("query", [step-1]))
        prg.assign_external(Fun("query", [step]), True)
        ret, parts, step = prg.solve(), [], step+1

#end.
```

Incremental Solving (tohCtrl.lp)

```
#script (python)

from gringo import SolveResult, Fun

def main(prg):
    ret, parts, step = SolveResult.UNSAT, [], 1
    parts.append(("base", []))
    while ret == SolveResult.UNSAT:
        parts.append(("step", [step]))
        parts.append(("check", [step]))
        prg.ground(parts)
        prg.release_external(Fun("query", [step-1]))
        prg.assign_external(Fun("query", [step]), True)
        ret, parts, step = prg.solve(), [], step+1

#end.
```

Incremental Solving

```
$ clingo toh.lp tohCtrl.lp
clingo version 4.5.0
Reading from toh.lp ...
Solving...
Solving...
[...]
Solving...
Answer: 1
move(7,a,1)  move(6,b,2)  move(7,b,3)  move(5,a,4)  move(7,c,5)  move(6,a,6)  \
move(7,a,7)  move(4,b,8)  move(7,b,9)  move(6,c,10) move(7,c,11) move(5,b,12) \
move(1,c,13) move(7,a,14) move(6,b,15) move(7,b,16) move(3,a,17) move(7,c,18) \
move(6,a,19) move(7,a,20) move(5,c,21) move(7,b,22) move(6,c,23) move(7,c,24) \
move(4,a,25) move(7,a,26) move(6,b,27) move(7,b,28) move(5,a,29) move(7,c,30) \
move(6,a,31) move(7,a,32) move(2,c,33) move(7,c,34) move(6,b,35) move(7,b,36) \
move(5,c,37) move(7,a,38) move(6,c,39) move(7,c,40)
SATISFIABLE

Models       : 1+
Calls        : 40
Time         : 0.312s (Solving: 0.22s 1st Model: 0.01s Unsat: 0.21s)
CPU Time     : 0.300s
```

Incremental Solving

```
$ clingo toh.lp tohCtrl.lp
clingo version 4.5.0
Reading from toh.lp ...
Solving...
Solving...
[...]
Solving...
Answer: 1
move(7,a,1)  move(6,b,2)  move(7,b,3)  move(5,a,4)  move(7,c,5)  move(6,a,6)  \
move(7,a,7)  move(4,b,8)  move(7,b,9)  move(6,c,10) move(7,c,11) move(5,b,12) \
move(1,c,13) move(7,a,14) move(6,b,15) move(7,b,16) move(3,a,17) move(7,c,18) \
move(6,a,19) move(7,a,20) move(5,c,21) move(7,b,22) move(6,c,23) move(7,c,24) \
move(4,a,25) move(7,a,26) move(6,b,27) move(7,b,28) move(5,a,29) move(7,c,30) \
move(6,a,31) move(7,a,32) move(2,c,33) move(7,c,34) move(6,b,35) move(7,b,36) \
move(5,c,37) move(7,a,38) move(6,c,39) move(7,c,40)
SATISFIABLE

Models       : 1+
Calls        : 40
Time         : 0.312s (Solving: 0.22s 1st Model: 0.01s Unsat: 0.21s)
CPU Time     : 0.300s
```

Incremental Solving (Python)

```
from sys import stdout
from gringo import SolveResult, Fun, Control

prg = Control()
prg.load("toh.lp")

ret, parts, step = SolveResult.UNSAT, [], 1
parts.append(("base", []))
while ret == SolveResult.UNSAT:
    parts.append(("step", [step]))
    parts.append(("check", [step]))
    prg.ground(parts)
    prg.release_external(Fun("query", [step-1]))
    prg.assign_external(Fun("query", [step]), True)
    f = lambda m: stdout.write(str(m))
    ret, parts, step = prg.solve(on_model=f), [], step+1
```

Incremental Solving (`tohCtrl.py`)

```
from sys import stdout
from gringo import SolveResult, Fun, Control

prg = Control()
prg.load("toh.lp")

ret, parts, step = SolveResult.UNSAT, [], 1
parts.append(("base", []))
while ret == SolveResult.UNSAT:
    parts.append(("step", [step]))
    parts.append(("check", [step]))
    prg.ground(parts)
    prg.release_external(Fun("query", [step-1]))
    prg.assign_external(Fun("query", [step]), True)
    f = lambda m: stdout.write(str(m))
    ret, parts, step = prg.solve(on_model=f), [], step+1
```

Incremental Solving (tohCtrl.py)

```
from sys import stdout
from gringo import SolveResult, Fun, Control

prg = Control()
prg.load("toh.lp")

ret, parts, step = SolveResult.UNSAT, [], 1
parts.append(("base", []))
while ret == SolveResult.UNSAT:
    parts.append(("step", [step]))
    parts.append(("check", [step]))
    prg.ground(parts)
    prg.release_external(Fun("query", [step-1]))
    prg.assign_external(Fun("query", [step]), True)
    f = lambda m: stdout.write(str(m))
    ret, parts, step = prg.solve(on_model=f), [], step+1
```

Incremental Solving (tohCtrl.py)

```
from sys import stdout
from gringo import SolveResult, Fun, Control

prg = Control()
prg.load("toh.lp")

ret, parts, step = SolveResult.UNSAT, [], 1
parts.append(("base", []))
while ret == SolveResult.UNSAT:
    parts.append(("step", [step]))
    parts.append(("check", [step]))
    prg.ground(parts)
    prg.release_external(Fun("query", [step-1]))
    prg.assign_external(Fun("query", [step]), True)
    f = lambda m: stdout.write(str(m))
    ret, parts, step = prg.solve(on_model=f), [], step+1
```

Incremental Solving (tohCtrl.py)

```
from sys import stdout
from gringo import SolveResult, Fun, Control

prg = Control()
prg.load("toh.lp")

ret, parts, step = SolveResult.UNSAT, [], 1
parts.append(("base", []))
while ret == SolveResult.UNSAT:
    parts.append(("step", [step]))
    parts.append(("check", [step]))
    prg.ground(parts)
    prg.release_external(Fun("query", [step-1]))
    prg.assign_external(Fun("query", [step]), True)
    f = lambda m: stdout.write(str(m))
    ret, parts, step = prg.solve(on_model=f), [], step+1
```

Incremental Solving (Python)

```
$ python tohCtrl.py
move(7,c,40) move(7,a,20) move(7,c,18) move(6,a,31) move(6,b,15) move(7,b,36) \
move(7,c,24) move(7,c,11) move(3,a,17) move(6,a,19) move(7,b,3)  move(7,c,5)  \
move(7,a,1)  move(6,b,35) move(6,c,10) move(6,a,6)  move(6,b,2)  move(7,b,9)  \
move(7,a,7)  move(4,b,8)  move(7,a,38) move(7,b,16) move(5,a,29) move(7,b,22) \
move(6,c,39) move(6,c,23) move(5,b,12) move(4,a,25) move(1,c,13) move(5,a,4)  \
move(7,a,14) move(7,a,26) move(6,b,27) move(7,a,32) move(7,b,28) move(7,c,30) \
move(2,c,33) move(5,c,21) move(7,c,34) move(5,c,37)
```

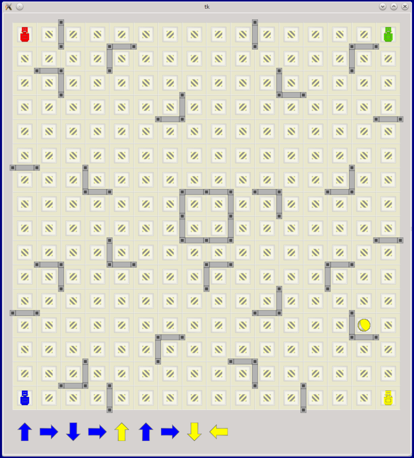
Incremental Solving (Python)

```
$ python tohCtrl.py
move(7,c,40) move(7,a,20) move(7,c,18) move(6,a,31) move(6,b,15) move(7,b,36) \
move(7,c,24) move(7,c,11) move(3,a,17) move(6,a,19) move(7,b,3)  move(7,c,5)  \
move(7,a,1)  move(6,b,35) move(6,c,10) move(6,a,6)  move(6,b,2)  move(7,b,9)  \
move(7,a,7)  move(4,b,8)  move(7,a,38) move(7,b,16) move(5,a,29) move(7,b,22) \
move(6,c,39) move(6,c,23) move(5,b,12) move(4,a,25) move(1,c,13) move(5,a,4)  \
move(7,a,14) move(7,a,26) move(6,b,27) move(7,a,32) move(7,b,28) move(7,c,30) \
move(2,c,33) move(5,c,21) move(7,c,34) move(5,c,37)
```

Outline

Alex Rudolph's Ricochet Robots

Solving goal(13) from cornered robots

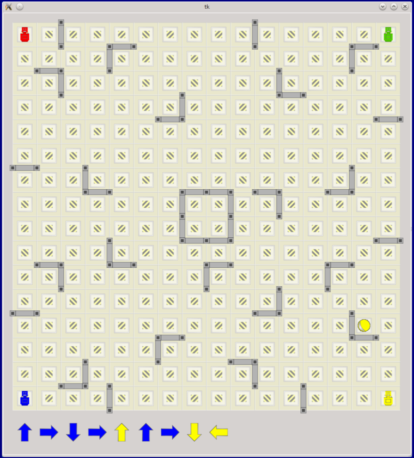


- Four robots roaming
  - horizontally
  - vertically

  up to blocking objects, ricocheting (optionally)
- Goal Robot on target (sharing same color)

Alex Rudolph's Ricochet Robots

Solving goal(13) from cornered robots



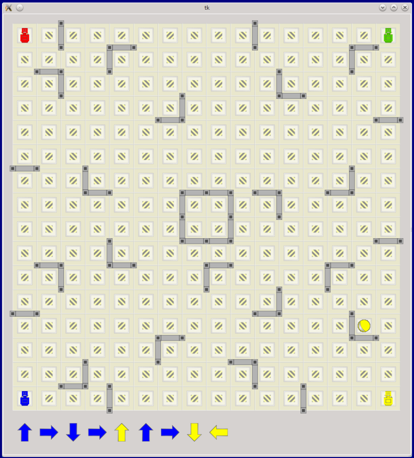
- Four robots roaming
  - horizontally
  - vertically

  up to blocking objects, ricocheting (optionally)

- Goal Robot on target (sharing same color)

Alex Rudolph's Ricochet Robots

Solving goal(13) from cornered robots



- Four robots
  roaming
  - horizontally
  - vertically

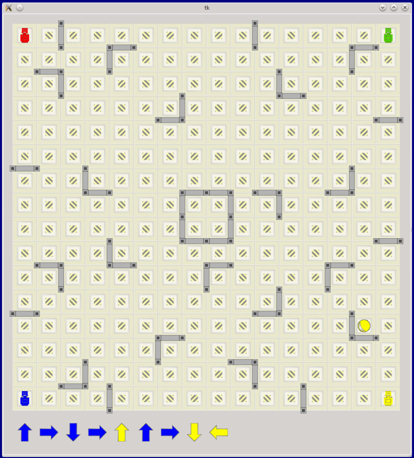
  up to blocking objects,
  ricocheting (optionally)

- Goal Robot on target
  (sharing same color)

Potassco

Alex Rudolph's Ricochet Robots

Solving goal(13) from cornered robots

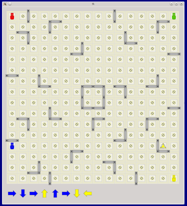


- Four robots
  roaming
  - horizontally
  - vertically

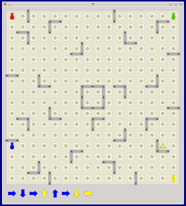
  up to blocking objects,
  ricocheting (optionally)
- Goal Robot on target
  (sharing same color)

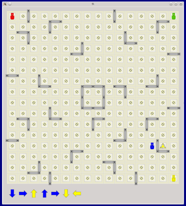
Potassco

Solving `goal(13)` from cornered robots (ctd)

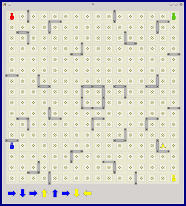


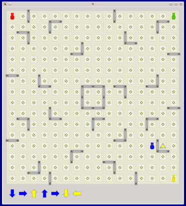
Solving `goal(13)` from cornered robots (ctd)

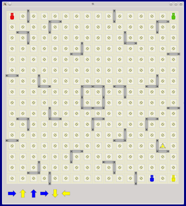




Solving `goal(13)` from cornered robots (ctd)







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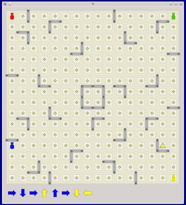
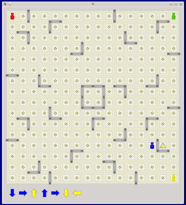
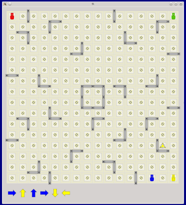
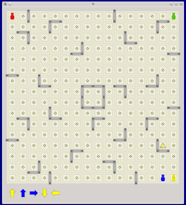
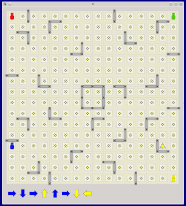
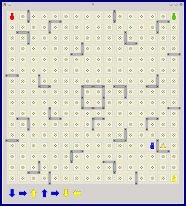
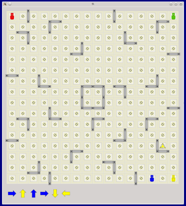
Solving `goal(13)` from cornered robots (ctd)

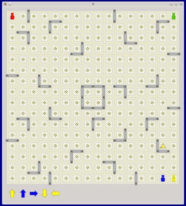
   

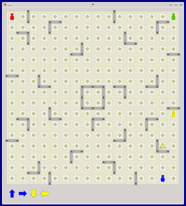
Solving `goal(13)` from cornered robots (ctd)



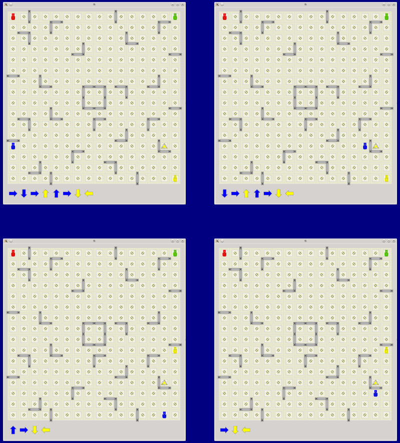




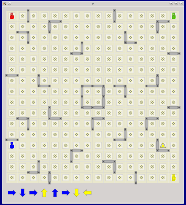
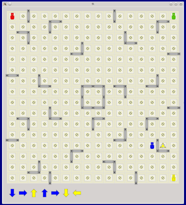
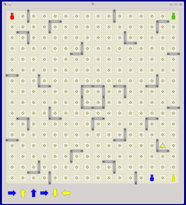
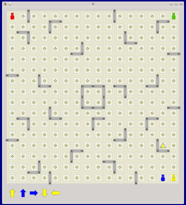
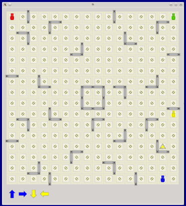
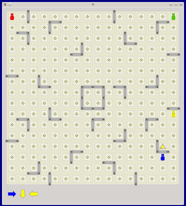
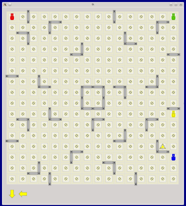




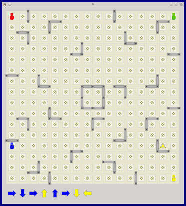
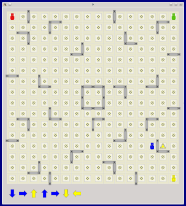
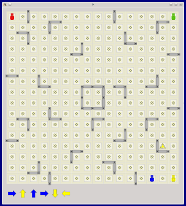
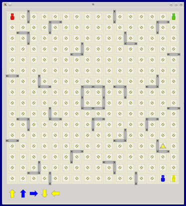
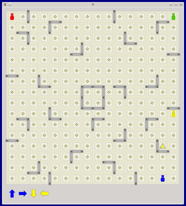
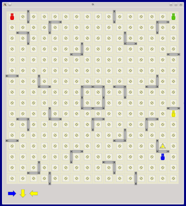
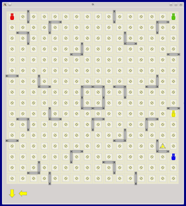
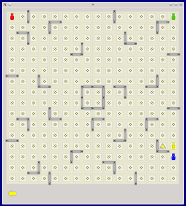
Solving `goal(13)` from cornered robots (ctd)



Solving `goal(13)` from cornered robots (ctd)

Solving `goal(13)` from cornered robots (ctd)

board.lp

```
dim(1..16).

barrier( 2, 1, 1, 0). barrier(13,11, 1, 0). barrier( 9, 7, 0, 1).
barrier(10, 1, 1, 0). barrier(11,12, 1, 0). barrier(11, 7, 0, 1).
barrier( 4, 2, 1, 0). barrier(14,13, 1, 0). barrier(14, 7, 0, 1).
barrier(14, 2, 1, 0). barrier( 6,14, 1, 0). barrier(16, 9, 0, 1).
barrier( 2, 3, 1, 0). barrier( 3,15, 1, 0). barrier( 2,10, 0, 1).
barrier(11, 3, 1, 0). barrier(10,15, 1, 0). barrier( 5,10, 0, 1).
barrier( 7, 4, 1, 0). barrier( 4,16, 1, 0). barrier( 8,10, 0,-1).
barrier( 3, 7, 1, 0). barrier(12,16, 1, 0). barrier( 9,10, 0,-1).
barrier(14, 7, 1, 0). barrier( 5, 1, 0, 1). barrier( 9,10, 0, 1).
barrier( 7, 8, 1, 0). barrier(15, 1, 0, 1). barrier(14,10, 0, 1).
barrier(10, 8,-1, 0). barrier( 2, 2, 0, 1). barrier( 1,12, 0, 1).
barrier(11, 8, 1, 0). barrier(12, 3, 0, 1). barrier(11,12, 0, 1).
barrier( 7, 9, 1, 0). barrier( 7, 4, 0, 1). barrier( 7,13, 0, 1).
barrier(10, 9,-1, 0). barrier(16, 4, 0, 1). barrier(15,13, 0, 1).
barrier( 4,10, 1, 0). barrier( 1, 6, 0, 1). barrier(10,14, 0, 1).
barrier( 2,11, 1, 0). barrier( 4, 7, 0, 1). barrier( 3,15, 0, 1).
barrier( 8,11, 1, 0). barrier( 8, 7, 0, 1).
```

targets.lp

```
#external goal(1..16).

target(red,   5, 2) :- goal(1).
target(red,  15, 2) :- goal(2).
target(green, 2, 3) :- goal(3).
target(blue, 12, 3) :- goal(4).
target(yellow, 7, 4) :- goal(5).
target(blue,  4, 7) :- goal(6).
target(green, 14, 7) :- goal(7).
target(yellow,11, 8) :- goal(8).
target(yellow, 5,10) :- goal(9).
target(green, 2,11) :- goal(10).
target(red,  14,11) :- goal(11).
target(green, 11,12) :- goal(12).
target(yellow,15,13) :- goal(13).
target(blue,  7,14) :- goal(14).
target(red,   3,15) :- goal(15).
target(blue, 10,15) :- goal(16).

robot(red;green;blue;yellow).
#external pos((red;green;blue;yellow),1..16,1..16).
```

Potassco

ricochet.lp

```
time(1..horizon).
dir(-1,0;1,0;0,-1;0,1).

stop( DX, DY,X, Y   ) :- barrier(X,Y,DX,DY).
stop(-DX,-DY,X+DX,Y+DY) :- stop(DX,DY,X,Y).

pos(R,X,Y,0) :- pos(R,X,Y).

1 { move(R,DX,DY,T) : robot(R), dir(DX,DY) } 1 :- time(T).
move(R,T) :- move(R,_,_,T).

halt(DX,DY,X-DX,Y-DY,T) :- pos(_,X,Y,T), dir(DX,DY), dim(X-DX), dim(Y-DY),
                           not stop(-DX,-DY,X,Y), T < horizon.

goto(R,DX,DY,X,Y,T) :- pos(R,X,Y,T), dir(DX,DY), T < horizon.
goto(R,DX,DY,X+DX,Y+DY,T) :- goto(R,DX,DY,X,Y,T), dim(X+DX), dim(Y+DY),
                          not stop(DX,DY,X,Y), not halt(DX,DY,X,Y,T).

pos(R,X,Y,T) :- move(R,DX,DY,T), goto(R,DX,DY,X,Y,T-1),
                not goto(R,DX,DY,X+DX,Y+DY,T-1).
pos(R,X,Y,T) :- pos(R,X,Y,T-1), time(T), not move(R,T).

:- target(R,X,Y), not pos(R,X,Y,horizon).

#show move/4.
```

Potassco

Solving `goal(13)` from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=9 \
       <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
Answer: 1
move(red,0,1,1)     move(red,1,0,2) move(red,0,1,3)    move(red,-1,0,4) move(red,0,1,5) \
move(yellow,0,-1,6) move(red,1,0,7) move(yellow,0,1,8) move(yellow,-1,0,9)
SATISFIABLE

Models       : 1+
Calls        : 1
Time         : 1.895s (Solving: 1.45s 1st Model: 1.45s Unsat: 0.00s)
CPU Time     : 1.880s


$ clingo board.lp targets.lp ricochet.lp -c horizon=8 \
       <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 2.817s (Solving: 2.41s 1st Model: 0.00s Unsat: 2.41s)
CPU Time     : 2.800s
```

Solving `goal(13)` from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=9 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
Answer: 1
move(red,0,1,1)     move(red,1,0,2) move(red,0,1,3)    move(red,-1,0,4) move(red,0,1,5) \
move(yellow,0,-1,6) move(red,1,0,7) move(yellow,0,1,8) move(yellow,-1,0,9)
SATISFIABLE

Models       : 1+
Calls        : 1
Time         : 1.895s (Solving: 1.45s 1st Model: 1.45s Unsat: 0.00s)
CPU Time     : 1.880s
```

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=8 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 2.817s (Solving: 2.41s 1st Model: 0.00s Unsat: 2.41s)
CPU Time     : 2.800s
```

Solving `goal(13)` from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=9 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
Answer: 1
move(red,0,1,1)     move(red,1,0,2) move(red,0,1,3)    move(red,-1,0,4) move(red,0,1,5) \
move(yellow,0,-1,6) move(red,1,0,7) move(yellow,0,1,8) move(yellow,-1,0,9)
SATISFIABLE

Models       : 1+
Calls        : 1
Time         : 1.895s (Solving: 1.45s 1st Model: 1.45s Unsat: 0.00s)
CPU Time     : 1.880s


$ clingo board.lp targets.lp ricochet.lp -c horizon=8 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 2.817s (Solving: 2.41s 1st Model: 0.00s Unsat: 2.41s)
CPU Time     : 2.800s
```

Solving `goal(13)` from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp -c horizon=9 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
Answer: 1
move(red,0,1,1)     move(red,1,0,2) move(red,0,1,3)    move(red,-1,0,4) move(red,0,1,5) \
move(yellow,0,-1,6) move(red,1,0,7) move(yellow,0,1,8) move(yellow,-1,0,9)
SATISFIABLE

Models       : 1+
Calls        : 1
Time         : 1.895s (Solving: 1.45s 1st Model: 1.45s Unsat: 0.00s)
CPU Time     : 1.880s


$ clingo board.lp targets.lp ricochet.lp -c horizon=8 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")

clingo version 4.5.0
Reading from board.lp ...
Solving...
UNSATISFIABLE

Models       : 0
Calls        : 1
Time         : 2.817s (Solving: 2.41s 1st Model: 0.00s Unsat: 2.41s)
CPU Time     : 2.800s
```

optimization.lp

```
goon(T) :- target(R,X,Y), T = 0..horizon, not pos(R,X,Y,T).

:- move(R,DX,DY,T-1), time(T), not goon(T-1), not move(R,DX,DY,T).

#minimize{ 1,T : goon(T) }.
```

Solving `goal(13)` from cornered robots

```
$ clingo board.lp targets.lp ricochet.lp optimization.lp -c horizon=20 --quiet=1,0 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")
clingo version 4.5.0
Reading from board.lp ...
Solving...
Optimization: 20
Optimization: 19
Optimization: 18
Optimization: 17
Optimization: 16
Optimization: 15
Optimization: 14
Optimization: 13
Optimization: 12
Optimization: 11
Optimization: 10
Optimization: 9
Answer: 12
move(blue,0,-1,1)   move(blue,1,0,2)    move(yellow,0,-1,3) move(blue,0,1,4)    move(yellow,-1,0,5) \
move(blue,1,0,6)    move(blue,0,-1,7)   move(yellow,1,0,8)  move(yellow,0,1,9)  move(yellow,0,1,10) \
move(yellow,0,1,11) move(yellow,0,1,12) move(yellow,0,1,13) move(yellow,0,1,14) move(yellow,0,1,15) \
move(yellow,0,1,16) move(yellow,0,1,17) move(yellow,0,1,18) move(yellow,0,1,19) move(yellow,0,1,20)
OPTIMUM FOUND

Models       : 12
  Optimum    : yes
Optimization : 9
Calls        : 1
Time         : 16.145s (Solving: 15.01s 1st Model: 3.35s Unsat: 2.02s)
CPU Time     : 16.080s
```

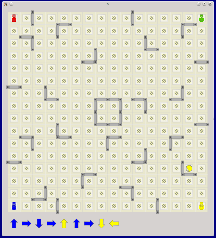
Solving goal(13) from cornered robots

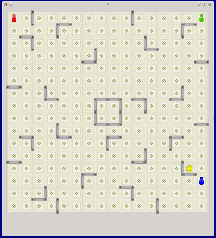
```
$ clingo board.lp targets.lp ricochet.lp optimization.lp -c horizon=20 --quiet=1,0 \
        <(echo "pos(red,1,1). pos(green,16,1). pos(blue,1,16). pos(yellow,16,16).   goal(13).")
clingo version 4.5.0
Reading from board.lp ...
Solving...
Optimization: 20
Optimization: 19
Optimization: 18
Optimization: 17
Optimization: 16
Optimization: 15
Optimization: 14
Optimization: 13
Optimization: 12
Optimization: 11
Optimization: 10
Optimization: 9
Answer: 12
move(blue,0,-1,1)   move(blue,1,0,2)    move(yellow,0,-1,3) move(blue,0,1,4)    move(yellow,-1,0,5) \
move(blue,1,0,6)    move(blue,0,-1,7)   move(yellow,1,0,8)  move(yellow,0,1,9)  move(yellow,0,1,10) \
move(yellow,0,1,11) move(yellow,0,1,12) move(yellow,0,1,13) move(yellow,0,1,14) move(yellow,0,1,15) \
move(yellow,0,1,16) move(yellow,0,1,17) move(yellow,0,1,18) move(yellow,0,1,19) move(yellow,0,1,20)
OPTIMUM FOUND

Models       : 12
  Optimum    : yes
Optimization : 9
Calls        : 1
Time         : 16.145s (Solving: 15.01s 1st Model: 3.35s Unsat: 2.02s)
CPU Time     : 16.080s
```

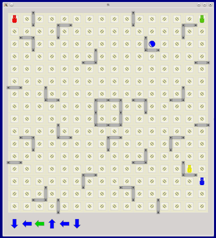
Playing in rounds

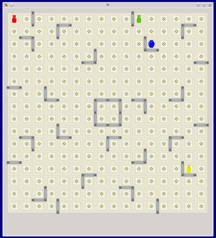
Round 1: `goal(13)`





Round 2: `goal(4)`





Control loop

1. Create an operational *clingo* object
2. Load and ground the logic programs encoding Ricochet Robot (relative to some fixed `horizon`) within the control object
3. While there is a goal, do the following
   1. Enforce the initial robot positions
   2. Enforce the current goal
   3. Solve the logic program contained in the control object

Ricochet Robot Player

ricochet.py

```
from gringo import Control, Model, Fun

class Player:
    def __init__(self, horizon, positions, files):
        self.last_positions = positions
        self.last_solution = None
        self.undo_external = []
        self.horizon = horizon
        self.ctl = Control(['-c', 'horizon={0}'.format(self.horizon)])
        for x in files:
            self.ctl.load(x)
        self.ctl.ground([("base", [])])

    def solve(self, goal):
        for x in self.undo_external:
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Variables of interest

- `last_positions` holds the starting positions of the robots for each turn
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Ricochet Robot Player
Setup and control loop

```
from gringo import Control, Model, Fun

class Player:
    def __init__(self, horizon, positions, files):
        self.last_positions = positions
        self.last_solution = None
        self.undo_external = []
        self.horizon = horizon
        self.ctl = Control(['-c', 'horizon={0}'.format(self.horizon)])
        for x in files:
            self.ctl.load(x)
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    def solve(self, goal):
        for x in self.undo_external:
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    def on_model(self, model):
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1. Initializing variables
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3. Playing in rounds

Potassco

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Ricochet Robot Player

__init__

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from gringo import Control, Model, Fun

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    def __init__(self, horizon, positions, files):
        self.last_positions = positions
        self.last_solution = None
        self.undo_external = []
        self.horizon = horizon
        self.ctl = Control(['-c', 'horizon={0}'.format(self.horizon)])
        for x in files:
            self.ctl.load(x)
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        for x in self.undo_external:
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Ricochet Robot Player

solve

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1. Unsetting previous external atoms (viz. previous goal and positions)
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Ricochet Robot Player

on_model

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class Player:
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1. Storing stable model
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Potassco

ricochet.py

```
from gringo import Control, Model, Fun

class Player:
    def __init__(self, horizon, positions, files):
        self.last_positions = positions
        self.last_solution = None
        self.undo_external = []
        self.horizon = horizon
        self.ctl = Control(['-c', 'horizon={0}'.format(self.horizon)])
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horizon   = 15
encodings = ["board.lp", "targets.lp", "ricochet.lp", "optimization.lp"]
positions = [Fun("pos", [Fun("red"),     1,  1]), Fun("pos", [Fun("blue"),    1, 16]),
             Fun("pos", [Fun("green"),  16,  1]), Fun("pos", [Fun("yellow"), 16, 16])]
sequence  = [Fun("goal", [13]), Fun("goal", [4]), Fun("goal", [7])]

player = Player(horizon, positions, encodings)
for goal in sequence:
    print player.solve(goal)
```

Let's play!

```
$ python ricochet.py
[move(red,0,1,1), move(yellow,-1,0,14), move(yellow,-1,0,12), move(yellow,-1,0,11),
 move(yellow,-1,0,9), move(red,1,0,7), move(red,1,0,2), move(yellow,-1,0,10),
 move(yellow,-1,0,13), move(yellow,-1,0,15), move(red,-1,0,4), move(yellow,0,-1,6),
 move(red,0,1,3), move(red,0,1,5), move(yellow,0,1,8)]
[move(blue,0,1,15), move(blue,0,1,11), move(blue,0,1,8), move(blue,0,1,3),
 move(blue,1,0,2), move(blue,0,1,9), move(blue,-1,0,7), move(blue,0,1,10),
 move(blue,0,1,13), move(blue,-1,0,4), move(blue,0,-1,1), move(blue,0,-1,6),
 move(green,-1,0,5), move(blue,0,1,12), move(blue,0,1,14)]
[move(green,1,0,15), move(green,1,0,8), move(green,1,0,5), move(green,1,0,4),
 move(green,1,0,3), move(green,1,0,10), move(green,1,0,7), move(green,1,0,12),
 move(green,1,0,9), move(green,1,0,2), move(green,1,0,11), move(green,1,0,13),
 move(green,1,0,6), move(green,1,0,14), move(green,0,1,1)]

$ python robotviz
```

Let's play!

```
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[move(red,0,1,1), move(yellow,-1,0,14), move(yellow,-1,0,12), move(yellow,-1,0,11),
 move(yellow,-1,0,9), move(red,1,0,7), move(red,1,0,2), move(yellow,-1,0,10),
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[move(blue,0,1,15), move(blue,0,1,11), move(blue,0,1,8), move(blue,0,1,3),
 move(blue,1,0,2), move(blue,0,1,9), move(blue,-1,0,7), move(blue,0,1,10),
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 move(green,1,0,3), move(green,1,0,10), move(green,1,0,7), move(green,1,0,12),
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$ python robotviz
```

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Preferences and optimization: Overview

Outline

Motivation

- Preferences are pervasive
- The identification of preferred, or optimal, solutions is often indispensable in real-world applications

  In many cases, this also involves the combination of various qualitative and quantitative preferences

- Only optimization statements representing objective functions using sum or count aggregates are established components of ASP systems
- Example $\#minimize\{40 : sauna, 70 : dive\}$

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Approach

- asprin is a framework for handling preferences among the stable models of logic programs
  - general because it captures numerous existing approaches to preference from the literature
  - flexible because it allows for an easy implementation of new or extended existing approaches
- asprin builds upon advanced control capacities for incremental and meta solving, allowing for

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Example

$$\#\mathit{preference}(\mathit{costs}, \mathit{less}(\mathit{weight}))\{40 : \mathit{sauna}, 70 : \mathit{dive}\}$$
$$\#\mathit{preference}(\mathit{fun}, \mathit{superset})\{\mathit{sauna}, \mathit{dive}, \mathit{hike}, {\sim}\mathit{bunji}\}$$
$$\#\mathit{preference}(\mathit{temps}, \mathit{aso})\{\mathit{dive} > \mathit{sauna} \parallel \mathit{hot}, \mathit{sauna} > \mathit{dive} \parallel \neg\mathit{hot}\}$$
$$\#\mathit{preference}(\mathit{all}, \mathit{pareto})\{\mathit{name}(\mathit{costs}), \mathit{name}(\mathit{fun}), \mathit{name}(\mathit{temps})\}$$

$$\#\mathit{optimize}(\mathit{all})$$

Outline

Preference

- A strict partial order $\succ$ on the stable models of a logic program

  That is, $X \succ Y$ means that X is preferred to Y
- A stable model X is $\succ$-preferred, if there is no other stable model Y such that $Y \succ X$

- A preference type is a (parametric) class of preference relations

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- weighted formula $w_1, \dots, w_l : \phi$
  where each w_i is a term and ϕ is a Boolean formula
- naming atom $name(s)$
  where s is the name of a preference
- preference element $\Phi_1 > \cdots > \Phi_m \parallel \Phi$
  where each Φ_r is a set of weighted formulas and Φ is a non-weighted formula
- preference statement $\#preference(s, t)\{e_1, \dots, e_n\}$
  where s and t represent the preference statement and its type
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- optimization directive $\#optimize(s)$
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Preference type

- A preference type t is a function mapping a set of preference elements, E, to a (strict) preference relation, $t(E)$, on sets of atoms
- The domain of t, $dom(t)$, fixes its admissible preference elements
- Example $less(cardinality)$

$$(X, Y) \in less(cardinality)(E)$$
$$\text{if } |\{\ell \in E \mid X \models \ell\}| < |\{\ell \in E \mid Y \models \ell\}|$$
$$dom(less(cardinality)) = \mathcal{P}(\{a, \neg a \mid a \in \mathcal{A}\})$$

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More examples

- *more*(*weight*) is defined as
  - $(X, Y) \in more(weight)(E)$ if $\sum_{(w:\ell)\in E, X\models\ell} w > \sum_{(w:\ell)\in E, Y\models\ell} w$
  - $dom(more(weight)) = \mathcal{P}(\{w : a, w : \neg a \mid w \in \mathbb{Z}, a \in \mathcal{A}\})$; and
- *subset* is defined as
  - $(X, Y) \in subset(E)$ if $\{\ell \in E \mid X \models \ell\} \subset \{\ell \in E \mid Y \models \ell\}$
  - $dom(less(cardinality)) = \mathcal{P}(\{a, \neg a \mid a \in \mathcal{A}\})$.
- *pareto* is defined as
  - $(X, Y) \in pareto(E)$ if $\bigwedge_{name(s)\in E}(X \succeq_s Y) \wedge \bigvee_{name(s)\in E}(X \succ_s Y)$
  - $dom(pareto) = \mathcal{P}(\{n \mid n \in N\})$;
- *lexico* is defined as
  - $(X, Y) \in lexico(E)$ if $\bigvee_{w:name(s)\in E}((X \succ_s Y) \wedge \bigwedge_{v:name(s')\in E, v<w}(X =_{s'} Y))$
  - $dom(lexico) = \mathcal{P}(\{w : n \mid w \in \mathbb{Z}, n \in N\})$.

Preference relation

- A preference relation is obtained by applying a preference type to an admissible set of preference elements
- $\#preference(s,t)\,E$ declares preference relation $t(E)$ denoted by $\succ_s$

$$\#preference(1, less(cardinality))\{a, \neg b, c\}) \quad \text{declares}$$

$$X \succ_1 Y \text{ as } |\{\ell \in \{a, \neg b, c\} \mid X \models \ell\}| < |\{\ell \in \{a, \neg b, c\} \mid Y \models \ell\}|$$

where $\succ_1$ stands for $less(cardinality)(\{a, \neg b, c\})$

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Outline

Preference program

- Reification $H_X = \{holds(a) \mid a \in X\}$ and $H'_X = \{holds'(a) \mid a \in X\}$
- Preference program Let s be a preference statement declaring $\succ_s$ and let P_s be a logic program

  We define P_s as a preference program for s, if for all sets $X, Y \subseteq \mathcal{A}$, we have

  $$X \succ_s Y \quad \text{iff} \quad P_s \cup H_X \cup H'_Y \text{ is satisfiable}$$

- Note P_s usually consists of an encoding E_{t_s} of t_s, facts F_s representing the preference statement, and auxiliary rules A
- Note Dynamic versions of H_X and H_Y must be used for optimization

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$$\#preference(3, subset)\{a, \neg b, c\}$$

$E_{subset} =$

```
better(P) :- preference(P,subset),
             holds'(X) : preference(P,_,_,for(X),_), holds(X);
             1 #sum { 1,X : not holds(X), holds'(X),
                            preference(P,_,_,for(X),_) }.
```

$F_3 =$

```
preference(3,subset). preference(3,1,1,for(a),()).
                      preference(3,2,1,for(neg(b)),()).
                      preference(3,3,1,for(c),()).
```

$A =$

```
holds(neg(A)) :- not holds(A), preference(_,_,_,for(neg(A)),_).
holds'(neg(A)) :- not holds'(A),preference(_,_,_,for(neg(A)),_).
```

$H_{\{a,b\}} =$

```
holds(a). holds(b).
```

$H'_{\{a\}} =$

```
holds'(a).
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We get a stable model containing `better(3)` indicating that $\{a, b\} \succ_3 \{a\}$, or $\{a\} \subset \{a, \neg b\}$

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Basic algorithm $solveOpt(P, s)$

Input : A program P over $\mathcal{A}$ and preference statement s
Output : A $\succ_s$-preferred stable model of P, if P is satisfiable, and $\bot$ otherwise

$Y \leftarrow solve(P)$
if $Y = \bot$ **then return** $\bot$

repeat
  $X \leftarrow Y$
  $Y \leftarrow solve(P \cup E_{t_s} \cup F_s \cup R_{\mathcal{A}} \cup H'_X) \cap \mathcal{A}$
until $Y = \bot$
return X

where $R_X = \{holds(a) \leftarrow a \mid a \in X\}$

Sketched Python Implementation

```
#script (python)

from gringo import *
holds = []

def getHolds():
    global holds
    return holds

def onModel(model):
    global holds
    holds = []
    for a in model.atoms():
        if (a.name() == "_holds"): holds.append(a.args()[0])

def main(prg):
    step = 1
    prg.ground([("base", [])])
    while True:
        if step > 1: prg.ground([("doholds",[step-1]),("preference",[0,step-1])]
        ret = prg.solve(on_model=onModel)
        if ret == SolveResult.UNSAT: break
        step = step+1
#end.

#program base.                     #program doholds(m).
#show _holds(X,0) : _holds(X,0).   _holds(X,m) :- X = @getHolds().
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Vanilla `minimize` statements

- Emulating the `minimize` statement

```
#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

  in *asprin* amounts to

```
#preference(myminimize,less(weight))
          { C,(X,Y) :: cycle(X,Y) : cost(X,Y,C) }.
#optimize(myminimize).
```

- Note *asprin* separates the declaration of preferences from the actual optimization directive

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Example

in *asprin*'s input language

```
#preference(costs,less(weight)){
  C :: sauna : cost(sauna,C);
  C ::  dive : cost(dive,C)
}.
#preference(fun,superset){ sauna; dive; hike; not bunji }.
#preference(temps,aso){
  dive > sauna ||     hot;
 sauna > dive  || not hot
}.
#preference(all,pareto){name(costs); name(fun); name(temps)}.

#optimize(all).
```

asprin's library

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  - `subset` and `superset`
  - `less(cardinality)` and `more(cardinality)`
  - `less(weight)` and `more(weight)`
  - `aso` (Answer Set Optimization)
  - `poset` (Qualitative Preferences)
- Composite preference types
  - `neg`
  - `and`
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`http://potassco.sourceforge.net`

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