Answer Set Solving in Practice

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Potassco

Rough Roadmap

1. Motivation
2. Introduction
3. Modeling
4. Language
5. Grounding
6. Foundations
7. Solving
8. Systems
9. Advanced modeling
10. Summary

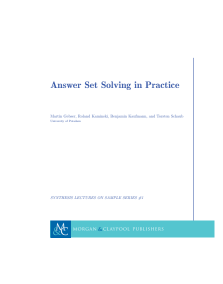
Bibliography

Resources

- Course material
  - `http://potassco.sourceforge.net/teaching.html`
  - `http://moodle.cs.uni-potsdam.de`
  - `http://www.cs.uni-potsdam.de/wv/lehre`
- Systems
  - clasp `http://potassco.sourceforge.net`
  - dlv `http://www.dlvsystem.com`
  - smodels `http://www.tcs.hut.fi/Software/smodels`
  - gringo `http://potassco.sourceforge.net`
  - lparse `http://www.tcs.hut.fi/Software/smodels`
  - clingo `http://potassco.sourceforge.net`
  - iclingo `http://potassco.sourceforge.net`
  - oclingo `http://potassco.sourceforge.net`
  - asparagus `http://asparagus.cs.uni-potsdam.de`

The Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



Resources

- http://potassco.sourceforge.net/book.html
- http://potassco.sourceforge.net/teaching.html

Literature

Books [4], [29], [53]

Surveys [50], [2], [39], [21], [11]

Articles [41], [42], [6], [61], [54], [49], [40], etc.

Motivation: Overview

1 Motivation

2 Nutshell

3 Shifting paradigms

4 Rooting ASP

5 ASP solving

6 Using ASP

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Outline

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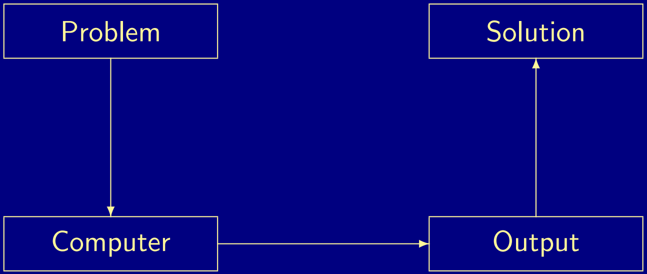
Informatics

"What is the problem?" versus *"How to solve the problem?"*



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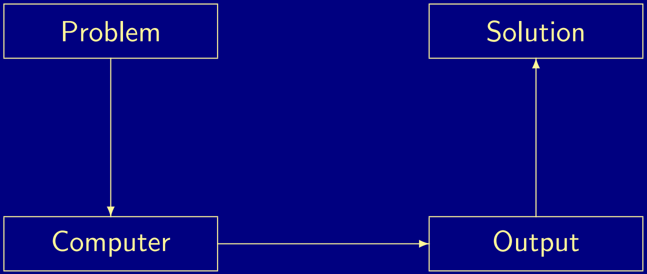

Traditional programming

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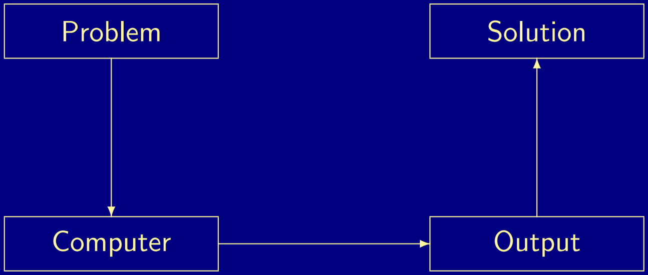

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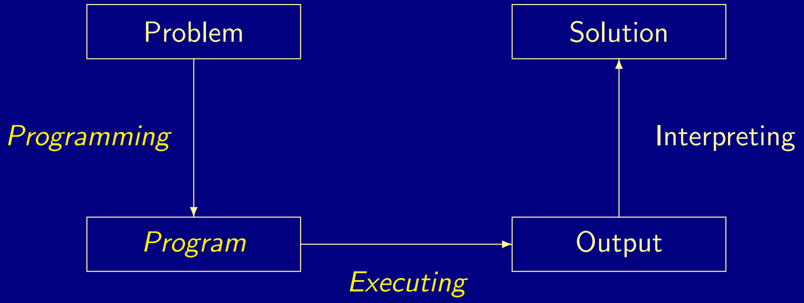

Declarative problem solving

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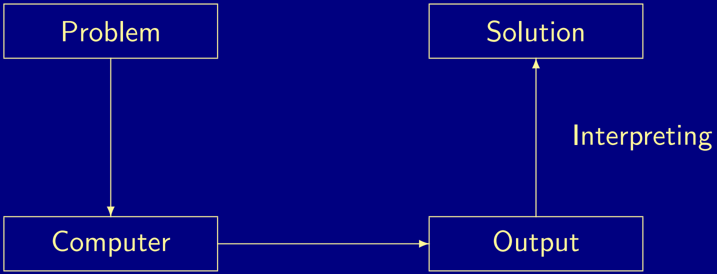

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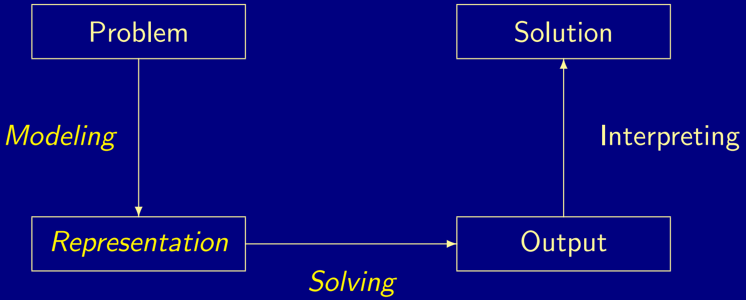

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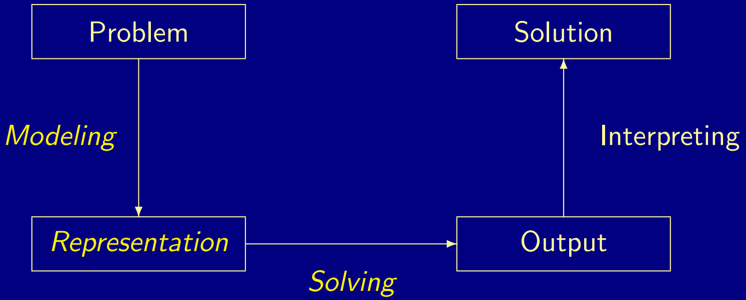

Outline

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Answer Set Programming

in a Nutshell

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
- ASP has its roots in
  - (deductive) databases
  - logic programming (with negation)
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - constraint solving (in particular, SATisfiability testing)
- ASP allows for solving all search problems in NP (and NP^{NP}) in a uniform way
- ASP is versatile as reflected by the ASP solver *clasp*, winning first places at ASP, CASC, MISC, PB, and SAT competitions
- ASP embraces many emerging application areas

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Answer Set Programming

in a Hazelnutshell

- ASP is an approach to declarative problem solving, combining
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  tailored to Knowledge Representation and Reasoning

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$$\textbf{ASP} = \textbf{DB}+\textbf{LP}+\textbf{KR}+\textbf{SAT}$$

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KR's shift of paradigm

Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem
2. A solution is given by a derivation of a query

Model Generation based approach (eg. SATisfiability testing)

1. Provide a representation of the problem
2. A solution is given by a model of the representation

Automated planning, Kautz and Selman (ECAI'92)

Represent planning problems as propositional theories so that models not proofs describe solutions

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Model Generation based Problem Solving

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first-order theories	Herbrand models
auto-epistemic theories	expansions
default theories	extensions
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LP-style playing with blocks

Prolog program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Prolog queries

```
?- above(a,c).
true.

?- above(c,a).
no.
```

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Prolog queries (testing entailment)

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true.

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```

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LP-style playing with blocks

Shuffled Prolog program

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?- above(a,c).

Fatal Error: local stack overflow.
```

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Prolog queries (answered via fixed execution)

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SAT-style playing with blocks

Formula

$$
\begin{array}{ll}
 & on(a,b) \\
\wedge & on(b,c) \\
\wedge & (on(X,Y) \rightarrow above(X,Y)) \\
\wedge & (on(X,Z) \wedge above(Z,Y) \rightarrow above(X,Y))
\end{array}
$$

Herbrand model

$$
\left\{
\begin{array}{ccccc}
on(a,b), & on(b,c), & on(a,c), & on(b,b), & \\
above(a,b), & above(b,c), & above(a,c), & above(b,b), & above(c,b)
\end{array}
\right\}
$$

Potassco

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Herbrand model (among 426!)

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➥ Answer Set Programming (ASP)

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Answer Set Programming *at large*

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Answer Set Programming *commonly*

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Answer Set Programming *in practice*

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Stable Herbrand model

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ASP versus LP

ASP	Prolog
Model generation	Query orientation
Bottom-up	Top-down
Modeling language	Programming language
Rule-based format	
Instantiation Flat terms	Unification Nested terms
(Turing +) $NP(^{NP})$	Turing

ASP versus SAT

ASP	SAT
Model generation	
Bottom-up	
Constructive Logic	Classical Logic
Closed (and open) world reasoning	Open world reasoning
Modeling language	—
Complex reasoning modes	Satisfiability testing
Satisfiability	Satisfiability
Enumeration/Projection	—
Intersection/Union	—
Optimization	—
(Turing +) $NP^{(NP)}$	NP

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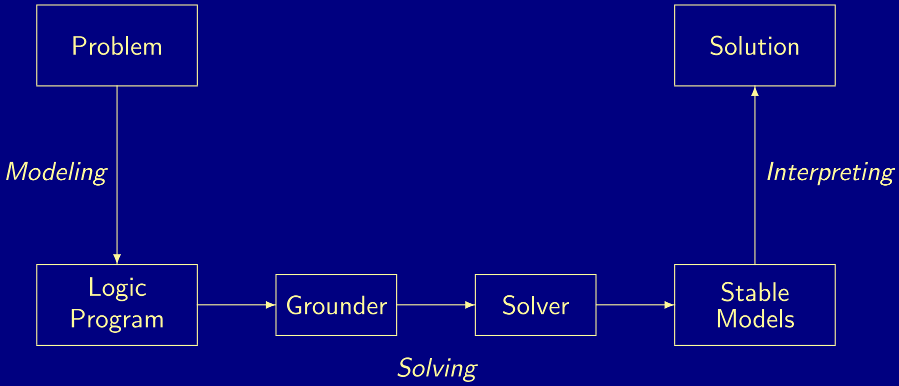
ASP solving


Problem
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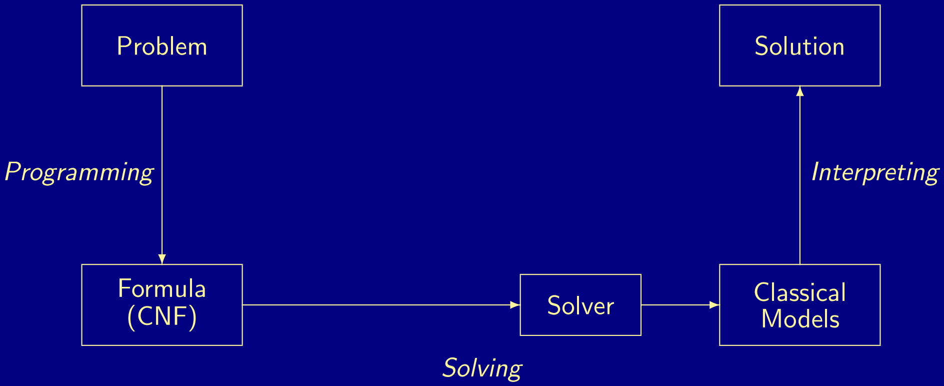
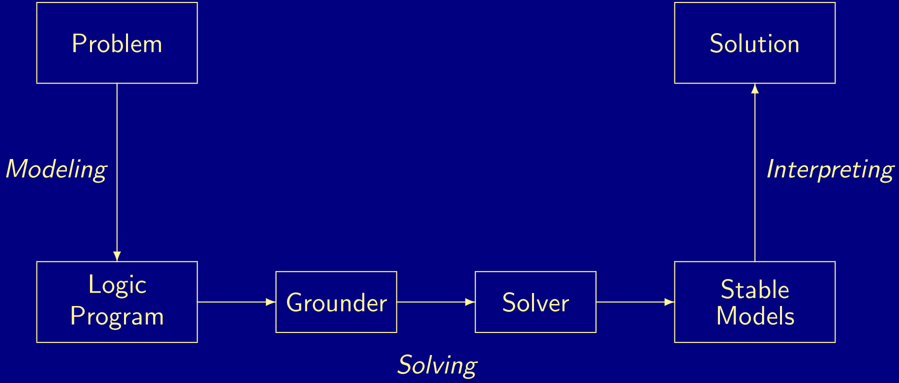
SAT solving

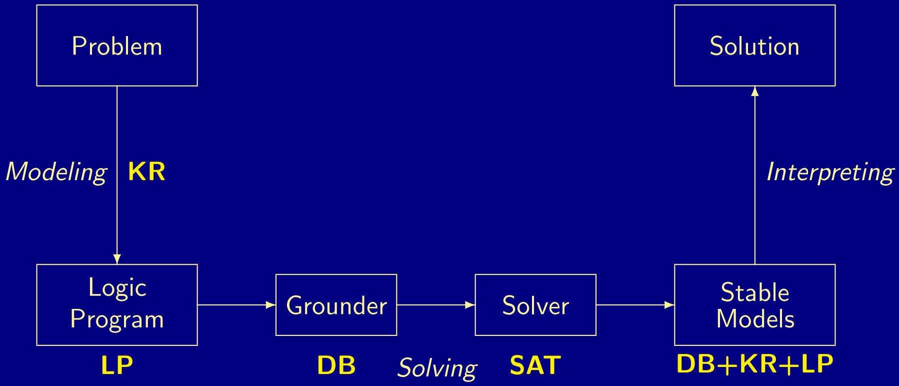


Potassco

Rooting ASP solving



Rooting ASP solving



Outline

1 Motivation

2 Nutshell

3 Shifting paradigms

4 Rooting ASP

5 ASP solving

6 Using ASP

Two sides of a coin

- ASP as High-level Language
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules
- ASP as Low-level Language
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation

What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated Planning
  - Code Optimization
  - Composition of Renaissance Music
  - Database Integration
  - Decision Support for NASA shuttle controllers
  - Model Checking
  - Product Configuration
  - Robotics
  - Systems Biology
  - System Synthesis
  - (industrial) Team-building
  - and many many more

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What does ASP offer?

- Integration of DB, KR, and SAT techniques
- Succinct, elaboration-tolerant problem representations
  - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
  - including: data, frame axioms, exceptions, defaults, closures, etc

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ASP = DB+LP+KR+SAT

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ASP = DB+LP+KR+SMT

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Introduction: Overview

7 Syntax

8 Semantics

9 Examples

10 Variables

11 Language constructs

12 Reasoning modes

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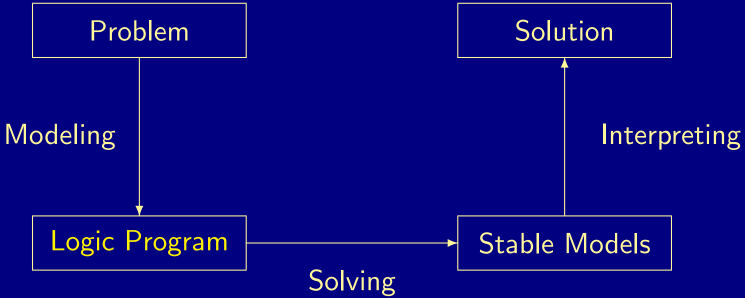
Problem solving in ASP: Syntax



Normal logic programs

- A logic program, P, over a set $\mathcal{A}$ of atoms is a finite set of rules
- A (normal) rule, r, is of the form

$$a_0 \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

$$\begin{array}{rcl} head(r) & = & a_0 \\ body(r) & = & \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \\ body(r)^+ & = & \{a_1, \dots, a_m\} \\ body(r)^- & = & \{a_{m+1}, \dots, a_n\} \\ atom(P) & = & \bigcup_{r \in P} \left(\{head(r)\} \cup body(r)^+ \cup body(r)^-\right) \\ body(P) & = & \{body(r) \mid r \in P\} \end{array}$$

- A program P is positive if $body(r)^- = \emptyset$ for all $r \in P$

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- Notation

$$\begin{aligned}
head(r) &= a_0 \\
body(r) &= \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \\
body(r)^+ &= \{a_1, \dots, a_m\} \\
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Rough notational convention

We sometimes use the following notation interchangeably
in order to stress the respective view:

	true, false	if	and	or	iff	default negation	classical negation
source code		`:-`	`,`	`|`		`not`	`-`
logic program		$\leftarrow$	$,$	$;$		$\sim$	$\neg$
formula	$\bot, \top$	$\rightarrow$	$\wedge$	$\vee$	$\leftrightarrow$	$\sim$	$\neg$

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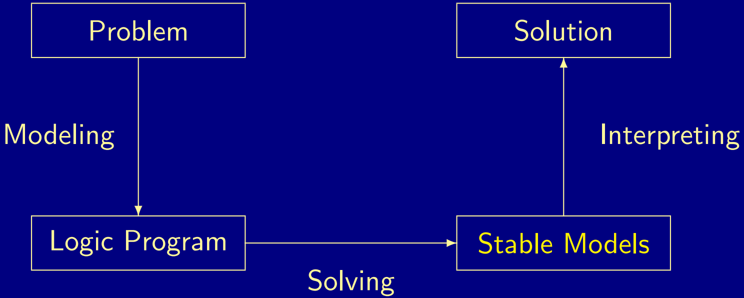
Problem solving in ASP: Semantics



Formal Definition

Stable models of positive programs

- A set of atoms X is closed under a positive program P iff for any $r \in P$, $head(r) \in X$ whenever $body(r)^+ \subseteq X$
  - X corresponds to a model of P (seen as a formula)
- The smallest set of atoms which is closed under a positive program P is denoted by $Cn(P)$
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of P (ditto)
- The set $Cn(P)$ of atoms is the stable model of a *positive* program P

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Some “logical” remarks

- Positive rules are also referred to as definite clauses
  - Definite clauses are disjunctions with exactly one positive atom:

    $$a_0 \vee \neg a_1 \vee \cdots \vee \neg a_m$$

  - A set of definite clauses has a (unique) smallest model
- Horn clauses are clauses with at most one positive atom
  - Every definite clause is a Horn clause but not vice versa
  - Non-definite Horn clauses can be regarded as integrity constraints
  - A set of Horn clauses has a smallest model or none
- This smallest model is the intended semantics of such sets of clauses
  - Given a positive program P, $Cn(P)$ corresponds to the smallest model of the set of definite clauses corresponding to P

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Basic idea

Consider the logical formula Φ and its three (classical) models:

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

Φ $\boxed{q \wedge (q \wedge \neg r \rightarrow p)}$

Formula Φ has one stable model, often called answer set:

$$\{p, q\}$$

P_Φ

$$\begin{array}{|lll|}\hline q & \leftarrow & \\ p & \leftarrow & q, \sim r \\ \hline\end{array}$$

Informally, a set X of atoms is a stable model of a logic program P

- if X is a (classical) model of P and
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(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))

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$$\Phi \quad \boxed{q \;\wedge\; (q \wedge \neg r \rightarrow p)}$$

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

$$\begin{array}{ccc} p & \mapsto & 1 \\ q & \mapsto & 1 \\ r & \mapsto & 0 \end{array}$$

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Stable model of normal programs

- The reduct, P^X, of a program P relative to a set X of atoms is defined by

$$P^X = \{head(r) \leftarrow body(r)^+ \mid r \in P \text{ and } body(r)^- \cap X = \emptyset\}$$

- A set X of atoms is a stable model of a program P, if $Cn(P^X) = X$

- Note $Cn(P^X)$ is the $\subseteq$–smallest (classical) model of P^X
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A closer look at P^X

- In other words, given a set X of atoms from P,

  P^X is obtained from P by deleting

  1. each rule having $\sim a$ in its body with $a \in X$
     and then
  2. all negative atoms of the form $\sim a$
     in the bodies of the remaining rules

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7 Syntax

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12 Reasoning modes

A first example

$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p\}$	$p \leftarrow p$	$\emptyset$
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p, q\}$	$p \leftarrow p$	$\emptyset$

A first example

$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✗
$\{p\}$	$p \leftarrow p$	$\emptyset$	
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✓
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$P = \{p \leftarrow p,\ q \leftarrow \neg p\}$

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$\{p, q\}$	$p \leftarrow p$	$\emptyset$	✘

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A second example

$P = \{p \leftarrow \sim q,\ q \leftarrow \sim p\}$

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$\{\ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$
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A second example

$P = \{p \leftarrow \neg q,\ q \leftarrow \neg p\}$

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$\{p\ \}$	$p \leftarrow$	$\{p\}$	✔
$\{\ q\}$	$q \leftarrow$	$\{q\}$	✔
$\{p, q\}$		$\emptyset$	✔

Potassco

A third example

$P = \{p \leftarrow {\sim}p\}$

X	P^X	$Cn(P^X)$
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$\{p\}$		$\emptyset$

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Some properties

- A logic program may have zero, one, or multiple stable models!
- If X is a stable model of a logic program P, then X is a model of P (seen as a formula)
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Outline

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Programs with Variables

Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \rightarrow \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground Instantiation of P: $\quad ground(P) = \bigcup_{r \in P} ground(r)$

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Programs with Variables

Let P be a logic program

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- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$ (also called alphabet or Herbrand base)
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An example

$$P = \{\ r(a,b) \leftarrow,\ \ r(b,c) \leftarrow,\ \ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a,b,c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

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Intelligent Grounding aims at reducing the ground instantiation

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Stable models of programs with Variables

Let P be a normal logic program with variables

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7 Syntax

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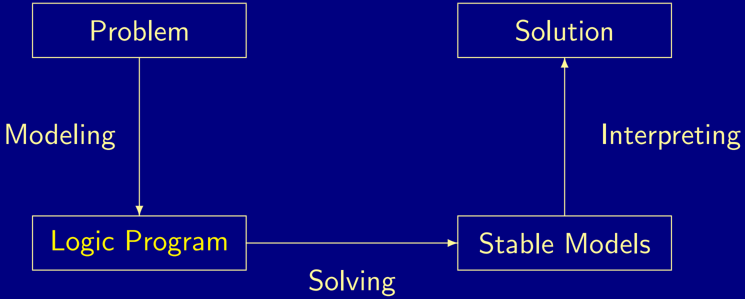
12 Reasoning modes

Problem solving in ASP: Extended Syntax



Language Constructs

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  - `p(X) :- q(X)` over constants $\{a, b, c\}$ stands for
    `p(a) :- q(a), p(b) :- q(b), p(c) :- q(c)`
- Conditional Literals
  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for
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- Disjunction
  - `p(X) | q(X) :- r(X)`
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  - `:- q(X), p(X)`
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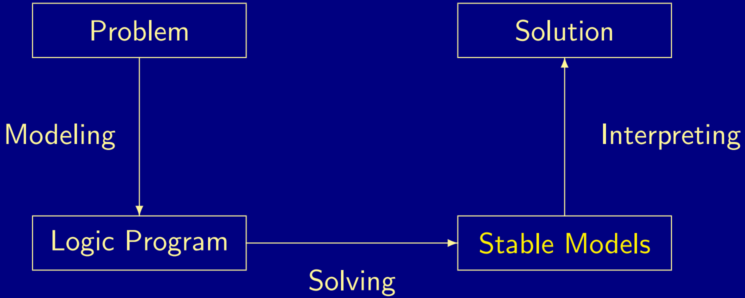
Problem solving in ASP: Reasoning Modes



Reasoning Modes

- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization
- and combinations of them

† without solution recording

‡ without solution enumeration

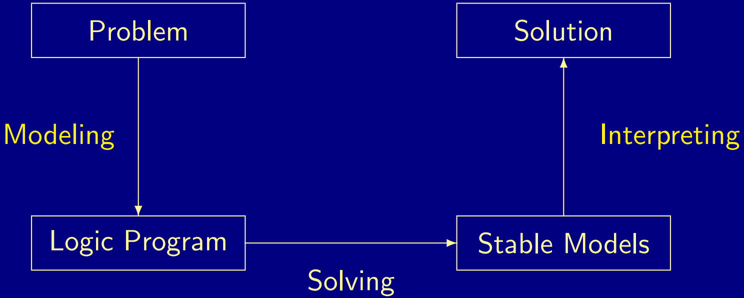
Basic Modeling: Overview

13 ASP solving process

14 Methodology

- Satisfiability
- Queens
- Traveling Salesperson
- Reviewer Assignment
- Planning

Modeling and Interpreting



- For solving a problem class **C** for a problem instance **I**, encode
  1. the problem instance **I** as a set $P_{\mathbf{I}}$ of facts and
  2. the problem class **C** as a set $P_{\mathbf{C}}$ of rules

  such that the solutions to **C** for **I** can be (polynomially) extracted from the stable models of $P_{\mathbf{I}} \cup P_{\mathbf{C}}$

- $P_{\mathbf{I}}$ is (still) called problem instance
- $P_{\mathbf{C}}$ is often called the problem encoding

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Attention!

All following examples are written in the language of gringo 3 !

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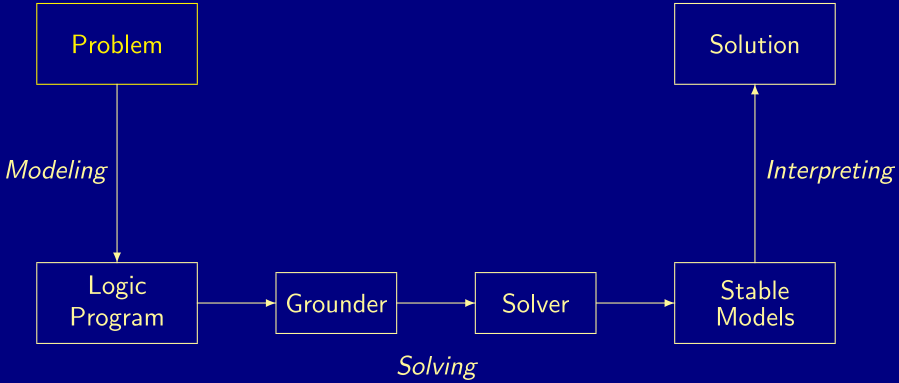
ASP solving process



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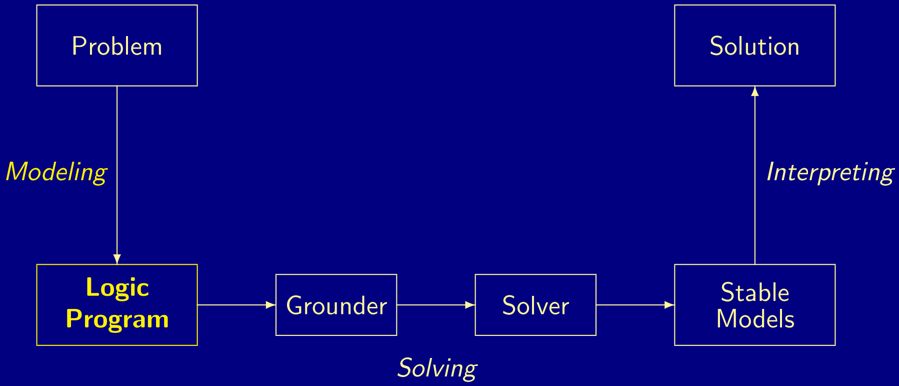

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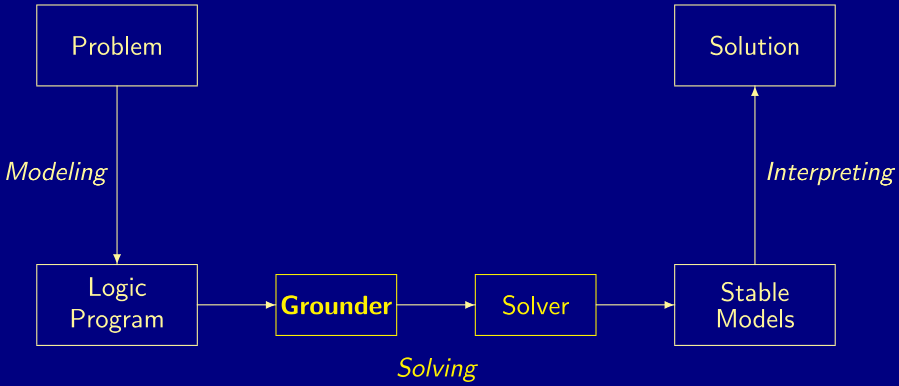

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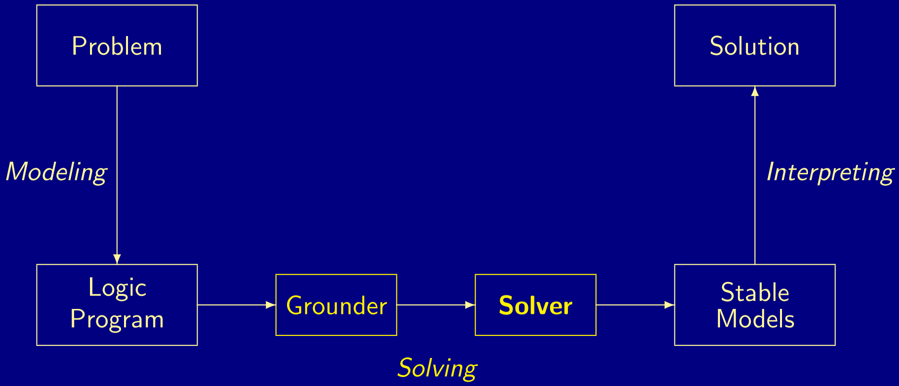

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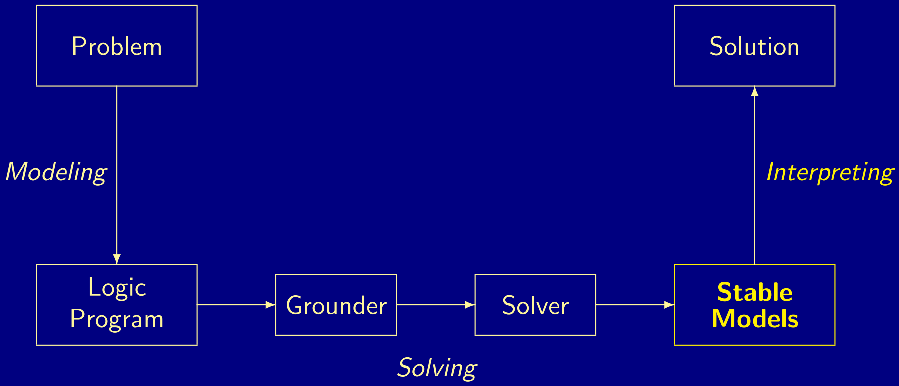

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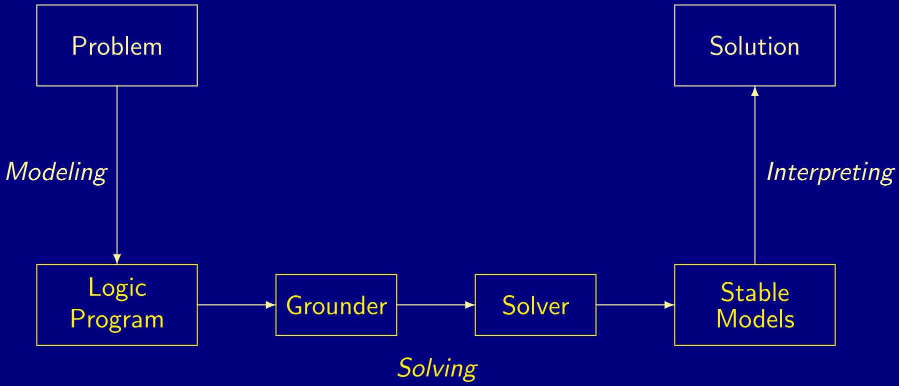

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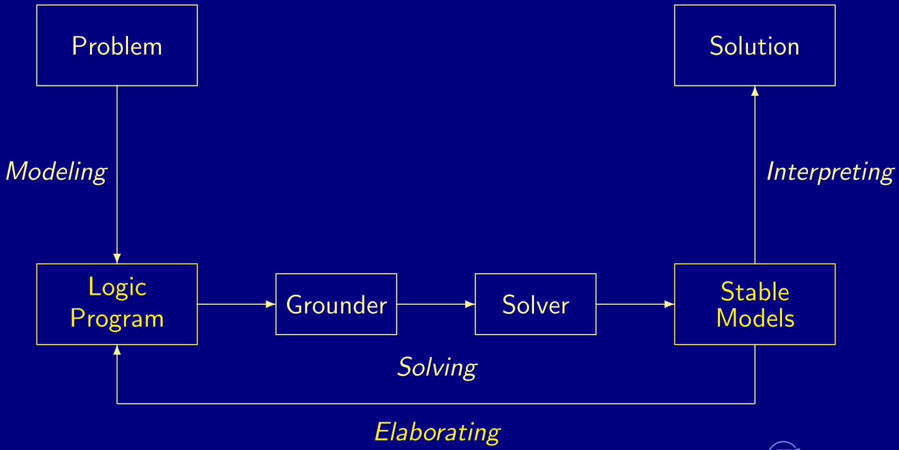
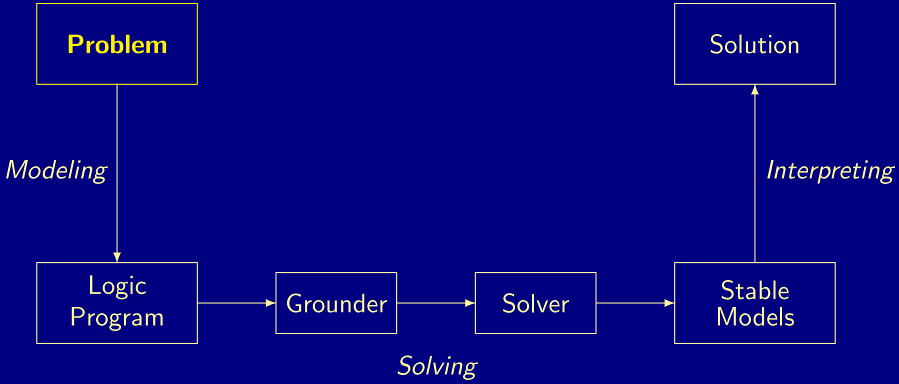
ASP solving process



A case-study: Graph coloring



Graph coloring

- Problem instance A graph consisting of nodes and edges

Graph coloring

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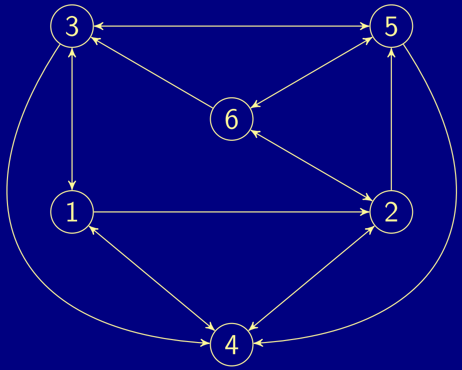

Graph coloring

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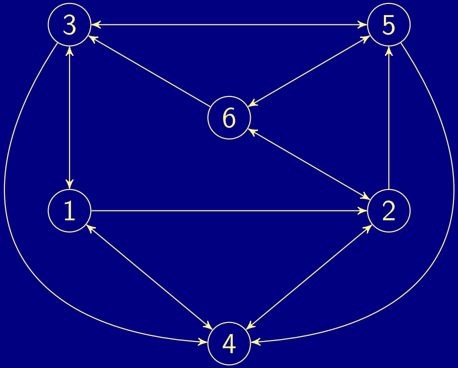
  - facts formed by predicates `node/1` and `edge/2`



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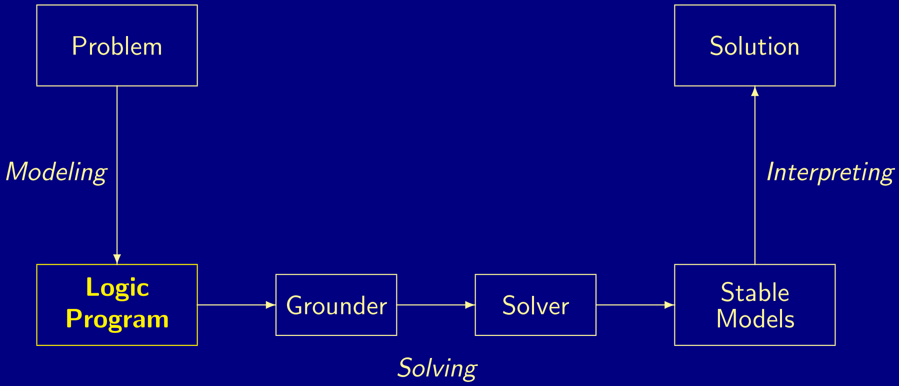
  - facts formed by predicate `col/1`
- Problem class Assign each node one color such that no two nodes connected by an edge have the same color

  In other words,

  1. Each node has a unique color
  2. Two connected nodes must not have the same color

ASP solving process



Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
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col(r).   col(b).   col(g).
```

Problem instance

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1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
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Problem encoding

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:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

Potassco

Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

Problem instance

```
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Potassco

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edge(5,3).  edge(5,4).  edge(5,6).
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edge(5,3).  edge(5,4).  edge(5,6).
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Problem encoding

Graph coloring

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node(1..6).

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edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

} Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

} **Problem encoding**

Potassco

Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

} Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

} Problem encoding

Potassco

color.lp

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

Problem instance

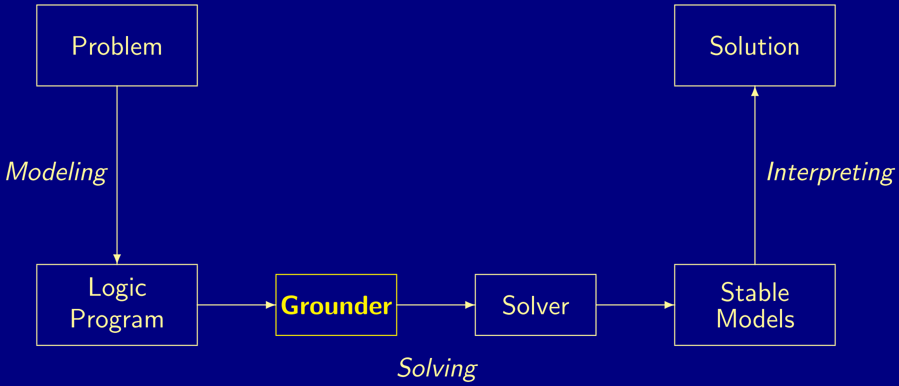
```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

Potassco

ASP solving process



Graph coloring: Grounding

$ gringo --text color.lp

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

Graph coloring: Grounding

```
$ gringo --text color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

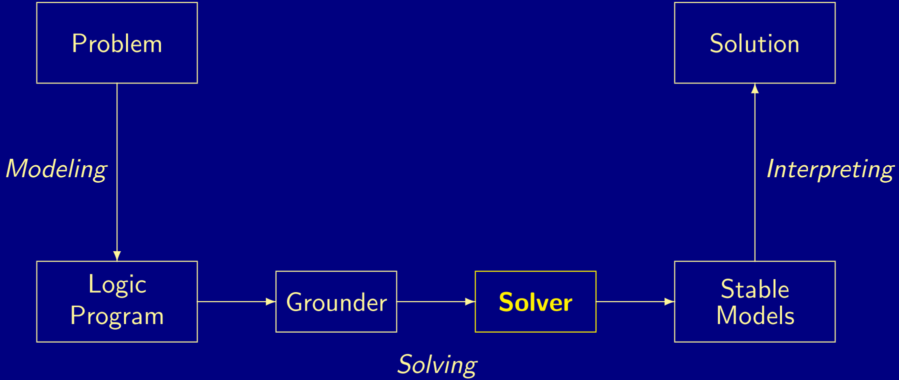
edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

ASP solving process



Graph coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

Potassco

Graph coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 2.1.0
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Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
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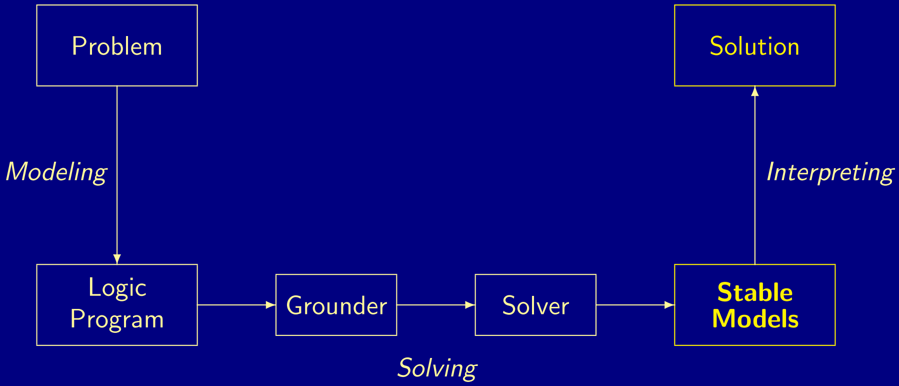
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

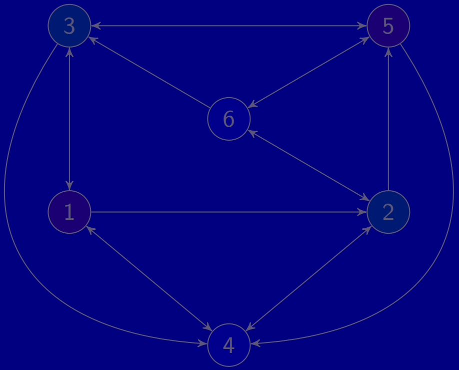
Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

ASP solving process



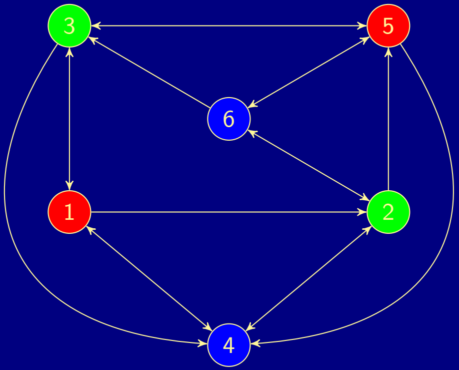
A coloring

```
Answer: 6
edge(1,2) ... col(r) ... node(1) ...                                    \
color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
```



A coloring

```
Answer: 6
edge(1,2) ... col(r) ... node(1) ...                             \
color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
```



Outline

13 ASP solving process

14 Methodology
- Satisfiability
- Queens
- Traveling Salesperson
- Reviewer Assignment
- Planning

Potassco

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

Generator: Generate potential stable model candidates (typically through non-deterministic constructs)

Tester: Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Basic methodology

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Satisfiability testing

- Problem Instance: A propositional formula ϕ in CNF
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program:

Generator	Tester	Stable models
$\{ a, b \} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
	$\leftarrow a, \sim b$	$X_2 = \{\}$

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Generator	Tester	Stable models
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	$\leftarrow \ a, \sim b$	$X_2 = \{\}$

Outline

13 ASP solving process

14 Methodology
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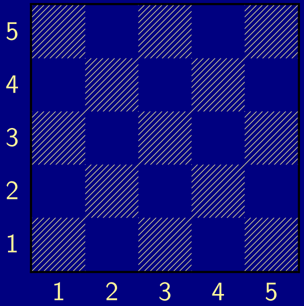
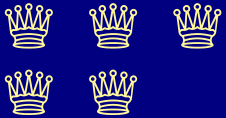
The n-Queens Problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another



Defining the Field

queens.lp

```
row(1..n).
col(1..n).
```

- Create file `queens.lp`
- Define the field
  - n rows
  - n columns

Defining the Field

Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models      : 1
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

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Placing some Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
```

- Guess a solution candidate
  by placing some queens on the board

Placing some Queens

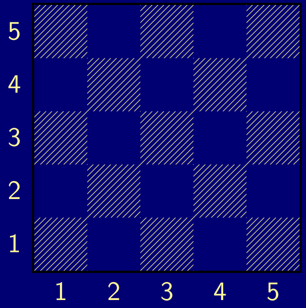
Running . . .

```
$ gringo queens.lp --const n=5 | clasp 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE

Models     : 3+
...
```

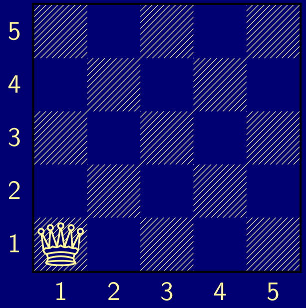
Placing some Queens: Answer 1

Answer 1



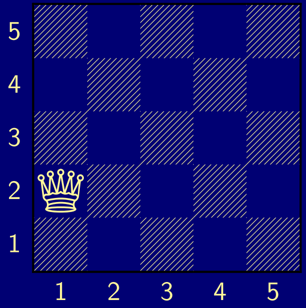
Placing some Queens: Answer 2

Answer 2



Placing some Queens: Answer 3

Answer 3



Placing n Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
```

- Place exactly n queens on the board

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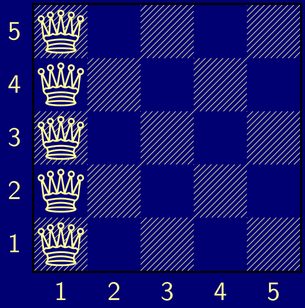
Placing n Queens

Running . . .

```
$ gringo queens.lp --const n=5 | clasp 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
...
```

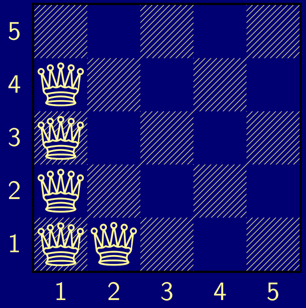
Placing *n* Queens: Answer 1

Answer 1



Placing *n* Queens: Answer 2

Answer 2



Horizontal and Vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

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Horizontal and Vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

Potassco

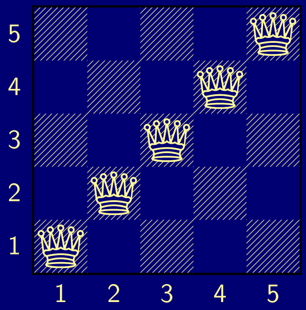
Horizontal and Vertical Attack

Running . . .

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) \
queen(2,2) queen(1,1)
...
```

Horizontal and Vertical Attack: Answer 1

Answer 1



Diagonal Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I-J == II-JJ.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I+J == II+JJ.
```

- Forbid diagonal attacks

Potassco

Diagonal Attack

Running ...

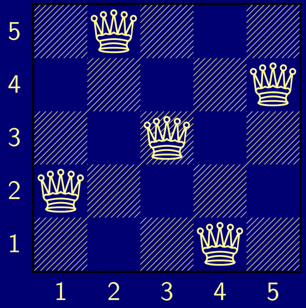
```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) queen(5,2) queen(2,1)
SATISFIABLE

Models      : 1+
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Potassco

Diagonal Attack: Answer 1

Answer 1



Optimizing

queens-opt.lp

```
1 { queen(I,1..n) } 1 :- I = 1..n.
1 { queen(1..n,J) } 1 :- J = 1..n.
 :- 2 { queen(D-J,J) }, D = 2..2*n.
 :- 2 { queen(D+J,J) }, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=3
clingo version 4.1.0
Solving...
SATISFIABLE

Models       : 1+
Time         : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time     : 3758.320s

Choices      : 288594554
Conflicts    : 3442    (Analyzed: 3442)
Restarts     : 17      (Average: 202.47 Last: 3442)
Model-Level  : 7594728.0
Problems     : 1       (Average Length: 0.00 Splits: 0)
Lemmas       : 3442    (Deleted: 0)
  Binary     : 0       (Ratio:   0.00%)
  Ternary    : 0       (Ratio:   0.00%)
  Conflict   : 3442    (Average Length: 229056.5 Ratio: 100.00%)
  Loop       : 0       (Average Length:    0.0 Ratio:   0.00%)
  Other      : 0       (Average Length:    0.0 Ratio:   0.00%)

Atoms        : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules        : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies       : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight        : Yes
Variables    : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664   (Binary: 35.6% Ternary:  0.0% Other: 64.4%)

Backjumps    : 3442    (Average: 681.19 Max: 169512 Sum: 2344658)
  Executed   : 3442    (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
```

Outline

13 ASP solving process

14 Methodology
- Satisfiability
- Queens
- Traveling Salesperson
- Reviewer Assignment
- Planning

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Potassco

Traveling Salesperson

```
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edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
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cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
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```

Traveling Salesperson

```
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize [ cycle(X,Y) = C : cost(X,Y,C) ].
```

Traveling Salesperson

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Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
...

3 { assigned(P,R) : reviewer(R) } 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { assignedB(P,R) : paper(P) : reviewer(R) }.
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Potassco

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Simplistic STRIPS Planning

```
time(1..k).     lasttime(T) :- time(T), not time(T+1).

fluent(p).      action(a).      action(b).      init(p).
fluent(q).         pre(a,p).       pre(b,q).
fluent(r).         add(a,q).       add(b,r).    query(r).
                   del(a,p).       del(b,q).

holds(P,0) :- init(P).

1 { occ(A,T) : action(A) } 1 :- time(T).
 :- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- holds(F,T-1), not nolds(F,T), time(T).
holds(F,T) :- occ(A,T), add(A,F).
nolds(F,T) :- occ(A,T), del(A,F).

 :- query(F), not holds(F,T), lasttime(T).
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Language: Overview

15 Motivation

16 Core language
- Integrity constraint
- Choice rule
- Cardinality rule
- Weight rule

17 Extended language
- Conditional literal
- Optimization statement

18 smodels format

19 ASP language standard

Potassco

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Basic language extensions

- The expressiveness of a language can be enhanced by introducing new constructs
- To this end, we must address the following issues:
  - What is the syntax of the new language construct?
  - What is the semantics of the new language construct?
  - How to implement the new language construct?
- A way of providing semantics is to furnish a translation removing the new constructs, eg. classical negation
- This translation might also be used for implementing the language extension

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Integrity constraint

- Idea Eliminate unwanted solution candidates
- Syntax An integrity constraint is of the form

$$\leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n$$

where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$

- Example `:- edge(3,7), color(3,red), color(7,red).`
- Embedding The above integrity constraint can be turned into the normal rule

$$x \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n, \sim x$$

where x is a new symbol, that is, $x \notin \mathcal{A}$.

- Another example $P = \{a \leftarrow \sim b, \ b \leftarrow \sim a\}$
versus $P' = P \cup \{\leftarrow a\}$ and $P'' = P \cup \{\leftarrow \sim a\}$

Potassco

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Choice rule

- Idea Choices over subsets
- Syntax A choice rule is of the form

$$\{a_1, \dots, a_m\} \leftarrow a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o$$

where $0 \leq m \leq n \leq o$ and each a_i is an atom for $1 \leq i \leq o$

- Informal meaning If the body is satisfied by the stable model at hand, then any subset of $\{a_1, \dots, a_m\}$ can be included in the stable model
- Example `{ buy(pizza), buy(wine), buy(corn) } :- at(grocery).`
- Another Example $P = \{\{a\} \leftarrow b, \; b \leftarrow\}$ has two stable models: $\{b\}$ and $\{a, b\}$

Potassco

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Embedding in normal rules

- A choice rule of form

$$\{a_1, \dots, a_m\} \leftarrow a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o$$

can be translated into $2m + 1$ normal rules

$$\begin{array}{lllllll} a' & \leftarrow & a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o & & & & \\ a_1 & \leftarrow & a', \sim \overline{a_1} & \dots & a_m & \leftarrow & a', \sim \overline{a_m} \\ \overline{a_1} & \leftarrow & \sim a_1 & \dots & \overline{a_m} & \leftarrow & \sim a_m \end{array}$$

by introducing new atoms $a', \overline{a_1}, \dots, \overline{a_m}$.

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Cardinality rule

- Idea Control (lower) cardinality of subsets
- Syntax A cardinality rule is the form

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n \; \}$$

where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$; l is a non-negative integer.

- Informal meaning The head atom belongs to the stable model, if at least l elements of the body are included in the stable model
- Note l acts as a lower bound on the body
- Example `pass(c42) :- 2 { pass(a1), pass(a2), pass(a3) }.`
- Another Example $P = \{a \leftarrow 1\{b, c\}, \; b \leftarrow\}$ has stable model $\{a, b\}$

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- Note l acts as a lower bound on the body
- Example `pass(c42) :- 2 { pass(a1), pass(a2), pass(a3) }.`
- Another Example $P = \{a \leftarrow 1\{b, c\},\ b \leftarrow\}$ has stable model $\{a, b\}$

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Cardinality rule

- Idea Control (lower) cardinality of subsets
- Syntax A cardinality rule is the form

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n \; \}$$

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Embedding in normal rules

- Replace each cardinality rule

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n \; \}$$

by $\quad a_0 \leftarrow ctr(1, l)$

where atom $ctr(i, j)$ represents the fact that at least j of the literals having an equal or greater index than i, are in a stable model

- The definition of $ctr/2$ is given for $0 \leq k \leq l$ by the rules

$$\begin{array}{rcll} ctr(i, k{+}1) & \leftarrow & ctr(i+1, k), a_i & \\ ctr(i, k) & \leftarrow & ctr(i+1, k) & \text{for } 1 \leq i \leq m \\ ctr(j, k{+}1) & \leftarrow & ctr(j+1, k), \sim a_j & \\ ctr(j, k) & \leftarrow & ctr(j+1, k) & \text{for } m+1 \leq j \leq n \\ ctr(n+1, 0) & \leftarrow & & \end{array}$$

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- Program $\{a \leftarrow,\ c \leftarrow 1\ \{a, b\}\}$ has the stable model $\{a, c\}$
- Translating the cardinality rule yields the rules

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. . . and vice versa

- A normal rule

$$a_0 \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n,$$

can be represented by the cardinality rule

$$a_0 \leftarrow n \; \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$$

Cardinality rules with upper bounds

- A rule of the form

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \} \; u$$

where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$;
l and u are non-negative integers

stands for

$$\begin{array}{rcl} a_0 & \leftarrow & b, {\sim}c \\ b & \leftarrow & l \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \} \\ c & \leftarrow & u{+}1 \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \} \end{array}$$

where b and c are new symbols

- The single constraint in the body of the above cardinality rule is referred to as a cardinality constraint

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- Informal meaning A cardinality constraint is satisfied by a stable model X, if the number of its contained literals satisfied by X is between l and u (inclusive)

- In other words, if

$$l \leq |\,(\{a_1, \dots, a_m\} \cap X) \cup (\{a_{m+1}, \dots, a_n\} \setminus X)\,| \leq u$$

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Cardinality constraints as heads

- A rule of the form

$$l\ \{a_1,\dots,a_m,\sim a_{m+1},\dots,\sim a_n\}\ u \leftarrow a_{n+1},\dots,a_o,\sim a_{o+1},\dots,\sim a_p$$

where $0 \le m \le n \le o \le p$ and each a_i is an atom for $1 \le i \le p$; l and u are non-negative integers

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- Example `1 { color(v42,red),color(v42,green),color(v42,blue) } 1.`

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$$l_0\ S_0\ u_0 \leftarrow l_1\ S_1\ u_1, \dots, l_n\ S_n\ u_n$$

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Outline

15 Motivation

16 Core language
- Integrity constraint
- Choice rule
- Cardinality rule
- Weight rule

17 Extended language
- Conditional literal
- Optimization statement

18 smodels format

19 ASP language standard

Weight rule

- Syntax A weight rule is the form

$$a_0 \leftarrow l \, \{ \, a_1 = w_1, \dots, a_m = w_m, \sim a_{m+1} = w_{m+1}, \dots, \sim a_n = w_n \, \}$$

  where $0 \leq m \leq n$ and each a_i is an atom;
  l and w_i are integers for $1 \leq i \leq n$
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- Note (Cardinality and) weight constraints amount to constraints on (count and) sum aggregate functions

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$$l \leq \left(\sum_{1 \leq i \leq m, a_i \in X} w_i + \sum_{m < i \leq n, a_i \notin X} w_i \right) \leq u$$

- Note (Cardinality and) weight constraints amount to constraints on (count and) sum aggregate functions

- Example `10 [course(db)=6,course(ai)=6,course(project)=8,course(xml)=3] 20`

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Outline

15 Motivation

16 Core language
- Integrity constraint
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- Cardinality rule
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17 Extended language
- Conditional literal
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Conditional literals (in *lparse* & *gringo* 3)

- Syntax A conditional literal is of the form

  $\ell : \ell_1 : \cdots : \ell_n$

  where ℓ and ℓ_i are literals for $0 \leq i \leq n$
- Informal meaning A conditional literal can be regarded as the list of elements in the set $\{\ell \mid \ell_1, \ldots, \ell_n\}$
- Note The expansion of conditional literals is context dependent
- Example Given '`p(1). p(2). p(3). q(2).`'

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r(X):p(X):not q(X) :- r(X):p(X):not q(X), 1 {r(X):p(X):not q(X)}.
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Optimization statement

- Idea Express cost functions subject to minimization and/or maximization
- Syntax A minimize statement is of the form

$$minimize\{\ \ell_1 = w_1@p_1, \dots, \ell_n = w_n@p_n\ \}.$$

where each ℓ_i is a literal; and w_i and p_i are integers for $1 \leq i \leq n$

Priority levels, p_i, allow for representing lexicographically ordered minimization objectives

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- A maximize statement of the form

$$maximize\{\ \ell_1 = w_1@p_1, \dots, \ell_n = w_n@p_n\ \}$$

  stands for $minimize\{\ \ell_1 = -w_1@p_1, \dots, \ell_n = -w_n@p_n\ \}$

- Example When configuring a computer, we may want to maximize hard disk capacity, while minimizing price

```
#maximize[ hd(1)=250@1, hd(2)=500@1, hd(3)=750@1, hd(4)=1000@1 ].
#minimize[ hd(1)=30@2, hd(2)=40@2, hd(3)=60@2, hd(4)=80@2 ].
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  The priority levels indicate that (minimizing) price is more important than (maximizing) capacity

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smodels format

- Logic programs in *smodels* format consist of
  - normal rules
  - choice rules
  - cardinality rules
  - weight rules
  - optimization statements
- Such a format is obtained by grounders *lparse* and *gringo*

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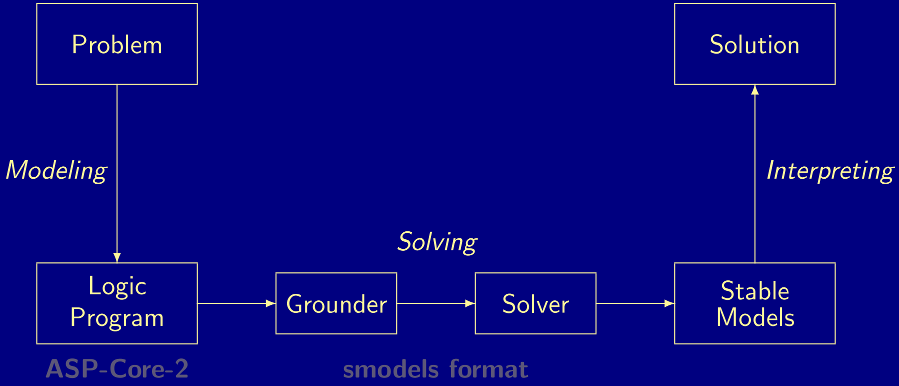
ASP-Core-2



- smodels format is a machine-oriented standard for ground programs
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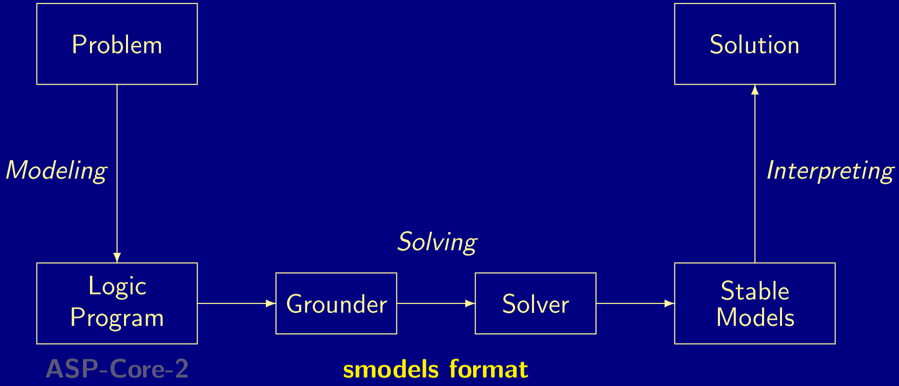
ASP-Core-2



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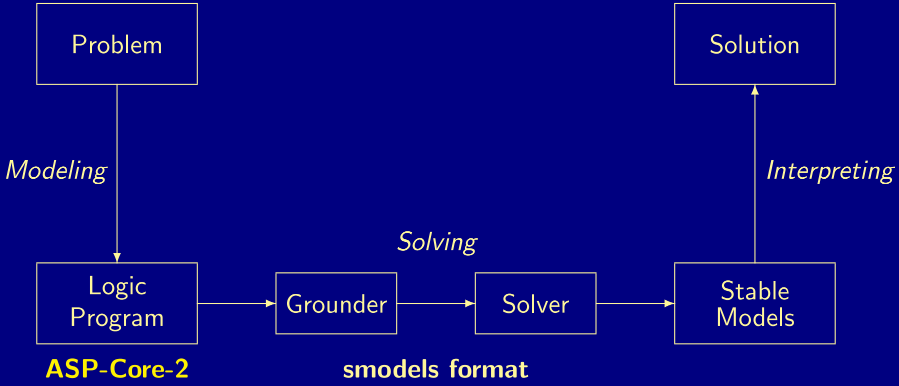

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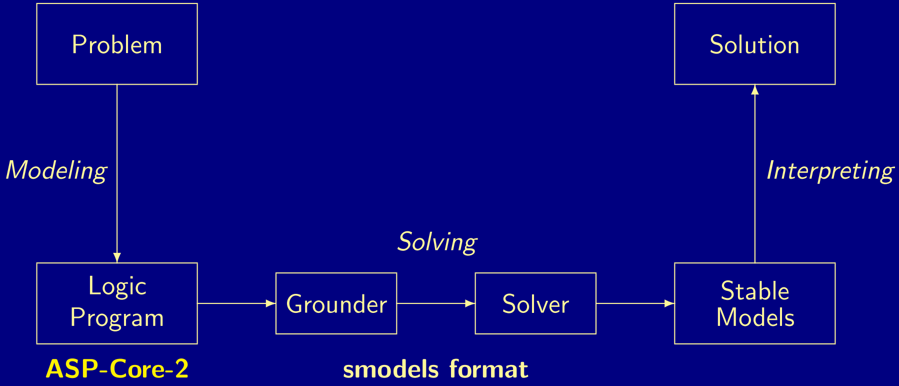

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Aggregates

- Syntax ASP-Core-2 aggregates are of the form

$$t_1 \prec_1 \#\mathtt{A}\{t_{1_1},\dots,t_{m_1} : \ell_{1_1},\dots,\ell_{n_1}\} \prec_2 t_2$$

  where
  - $\#\mathtt{A} \in \{\#\mathtt{count}, \#\mathtt{sum}, \#\mathtt{max}, \#\mathtt{min}\}$
  - $\prec_1, \prec_2 \in \{<, \leq, =, \neq, >, \geq\}$
  - $t_{1_1},\dots,t_{m_1}$ and t_1, t_2 are terms
  - $\ell_{1_1},\dots,\ell_{n_1}$ are literals

- Example Weight constraint

```
10 [course(db)=6,course(ai)=6,course(project)=8,course(xml)=3] 20
```

  is written as an ASP-Core-2 aggregate as

$$10 \leq \#\mathtt{sum}\{\mathtt{6,db:course(db);\ 6,ai:course(ai);\ 8,project:course(project);\ 3,xml:course(xml)}\} \leq 20$$

Aggregates

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$$:\sim a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n. \, [w@p, t_1, \dots, t_m]$$

  where
  - $a_1, \dots, a_n$ are atoms
  - $t_1, \dots, t_m$, w, and p are terms
- $a_1, \dots, a_n$ may contain ASP-Core-2 aggregates
- w and p stand for a weight and priority level ($p = 0$ if '@p' is omitted)
- Example Minimize statement

```
#minimize[ hd(1)=30@2, hd(2)=40@2, hd(3)=60@2, hd(4)=80@2 ].
```

  can be written in terms of weak constraints as

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:~ hd(1). [30@2,1]      :~ hd(3). [60@2,3]
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gringo 4

- The input language of *gringo* series 4 comprises
  - ASP-Core-2
  - concepts from *gringo* 3 (conditional literals, `#show` directives, ...)
- Example The *gringo* 3 rule

```
r(X):p(X):not q(X) :- r(X):p(X):not q(X), 1 {r(X):p(X):not q(X)}.
```

can be written as follows in the language of *gringo* 4:

```
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                      1 <= #count{X:r(X),p(X),not q(X)}.
```

Term-based `#show` directives as in

```
#show. #show hello. #show X : p(X).  1{p(earth);p(mars);p(venus)}1.
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The languages of *gringo* 3 and 4 are not fully compatible

Many example programs given in this tutorial are written for *gringo* 3

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Language Extensions: Overview

20 Two kinds of negation

21 Disjunctive logic programs

22 Propositional theories

Outline

20 Two kinds of negation

21 Disjunctive logic programs

22 Propositional theories

Motivation

- Classical versus default negation
  - Symbol $\neg$ and $\sim$
  - Idea
    - $\neg a \approx \neg a \in X$
    - $\sim a \approx a \notin X$
  - Example
    - $cross \leftarrow \neg train$
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Motivation

- Classical versus default negation
  - Symbol $\neg$ and $\sim$
  - Idea
    - $\neg a \;\approx\; \neg a \in X$
    - $\sim a \;\approx\; a \notin X$
  - Example
    - $cross \leftarrow \neg train$
    - $cross \leftarrow \sim train$

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Classical negation

- We consider logic programs in negation normal form
  - That is, classical negation is applied to atoms only
- Given an alphabet $\mathcal{A}$ of atoms, let $\overline{\mathcal{A}} = \{\neg a \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \overline{\mathcal{A}} = \emptyset$
- Given a program P over $\mathcal{A}$, classical negation is encoded by adding

$$P^{\neg} = \{a \leftarrow b, \neg b \mid a \in (\mathcal{A} \cup \overline{\mathcal{A}}), b \in \mathcal{A}\}$$

- A set X of atoms is a stable model of a program P over $\mathcal{A} \cup \overline{\mathcal{A}}$, if X is a stable model of $P \cup P^{\neg}$

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An example

- The program

$$P = \{a \leftarrow \sim b,\ b \leftarrow \sim a\} \cup \{c \leftarrow b,\ \neg c \leftarrow b\}$$

induces

$$P^{\neg} = \left\{ \begin{array}{rclrclrcl} a & \leftarrow & a, \neg a & a & \leftarrow & b, \neg b & a & \leftarrow & c, \neg c \\ \neg a & \leftarrow & a, \neg a & \neg a & \leftarrow & b, \neg b & \neg a & \leftarrow & c, \neg c \\ b & \leftarrow & a, \neg a & b & \leftarrow & b, \neg b & b & \leftarrow & c, \neg c \\ \neg b & \leftarrow & a, \neg a & \neg b & \leftarrow & b, \neg b & \neg b & \leftarrow & c, \neg c \\ c & \leftarrow & a, \neg a & c & \leftarrow & b, \neg b & c & \leftarrow & c, \neg c \\ \neg c & \leftarrow & a, \neg a & \neg c & \leftarrow & b, \neg b & \neg c & \leftarrow & c, \neg c \end{array} \right\}$$

- The stable models of P are given by the ones of $P \cup P^{\neg}$, viz $\{a\}$

An example

- The program

$$P \; = \; \{a \leftarrow {\sim}b,\; b \leftarrow {\sim}a\} \cup \{c \leftarrow b,\; \neg c \leftarrow b\}$$

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Properties

- The only inconsistent stable "model" is $X = \mathcal{A} \cup \overline{\mathcal{A}}$
- Note Strictly speaking, an inconsistemt set like $\mathcal{A} \cup \overline{\mathcal{A}}$ is not a model
- For a logic program P over $\mathcal{A} \cup \overline{\mathcal{A}}$, exactly one of the following two cases applies:
  1. All stable models of P are consistent or
  2. $X = \mathcal{A} \cup \overline{\mathcal{A}}$ is the only stable model of P

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Train spotting

- $P_1 = \{cross \leftarrow \sim train\}$
  - stable model: $\{cross\}$
- $P_2 = \{cross \leftarrow \neg train\}$
  - stable model: $\emptyset$
- $P_3 = \{cross \leftarrow \neg train, \ \neg train \leftarrow\}$
  - stable model: $\{cross, \neg train\}$
- $P_4 = \{cross \leftarrow \neg train, \ \neg train \leftarrow, \ \neg cross \leftarrow\}$
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- $P_5 = \{cross \leftarrow \neg train, \ \neg train \leftarrow \sim train\}$
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Default negation in rule heads

- We consider logic programs with default negation in rule heads
- Given an alphabet $\mathcal{A}$ of atoms, let $\widetilde{\mathcal{A}} = \{\widetilde{a} \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \widetilde{\mathcal{A}} = \emptyset$
- Given a program P over $\mathcal{A}$, consider the program

$$\begin{aligned}\widetilde{P} \quad = \quad & \{r \in P \mid head(r) \neq {\sim}a\} \\ & \cup \{\leftarrow body(r) \cup \{{\sim}\widetilde{a}\} \mid r \in P \text{ and } head(r) = {\sim}a\} \\ & \cup \{\widetilde{a} \leftarrow {\sim}a \mid r \in P \text{ and } head(r) = {\sim}a\}\end{aligned}$$

- A set X of atoms is a stable model of a program P (with default negation in rule heads) over $\mathcal{A}$,
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Outline

20 Two kinds of negation

21 Disjunctive logic programs

22 Propositional theories

Disjunctive logic programs

- A disjunctive rule, r, is of the form

$$a_1 ; \dots ; a_m \leftarrow a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o$$

where $0 \leq m \leq n \leq o$ and each a_i is an atom for $0 \leq i \leq o$

- A disjunctive logic program is a finite set of disjunctive rules
- Notation

$$\begin{aligned}
head(r) &= \{a_1, \dots, a_m\} \\
body(r) &= \{a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o\} \\
body(r)^+ &= \{a_{m+1}, \dots, a_n\} \\
body(r)^- &= \{a_{n+1}, \dots, a_o\} \\
atom(P) &= \bigcup_{r \in P} \left(head(r) \cup body(r)^+ \cup body(r)^-\right) \\
body(P) &= \{body(r) \mid r \in P\}
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- A program is called positive if $body(r)^- = \emptyset$ for all its rules

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Stable models

- Positive programs
  - A set X of atoms is closed under a positive program P iff for any $r \in P$, $head(r) \cap X \neq \emptyset$ whenever $body(r)^+ \subseteq X$
    - X corresponds to a model of P (seen as a formula)
  - The set of all $\subseteq$-minimal sets of atoms being closed under a positive program P is denoted by $\min_\subseteq(P)$
    - $\min_\subseteq(P)$ corresponds to the $\subseteq$-minimal models of P (ditto)
- Disjunctive programs
  - The reduct, P^X, of a disjunctive program P relative to a set X of atoms is defined by

$$P^X = \{head(r) \leftarrow body(r)^+ \mid r \in P \text{ and } body(r)^- \cap X = \emptyset\}$$

  - A set X of atoms is a stable model of a disjunctive program P, if $X \in \min_\subseteq(P^X)$

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A "positive" example

$$P = \left\{ \begin{array}{lcl} a & \leftarrow & \\ b\,;c & \leftarrow & a \end{array} \right\}$$

- The sets $\{a,b\}$, $\{a,c\}$, and $\{a,b,c\}$ are closed under P
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Graph coloring (reloaded)

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node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).


color(X,r) | color(X,b) | color(X,g) :- node(X).


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color(X,C) : col(C) :- node(X).

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More Examples

- $P_1 = \{a \,;b \,;c \leftarrow\}$
  - stable models $\{a\}$, $\{b\}$, and $\{c\}$
- $P_2 = \{a \,;b \,;c \leftarrow \;,\; \leftarrow a\}$
  - stable models $\{b\}$ and $\{c\}$
- $P_3 = \{a \,;b \,;c \leftarrow \;,\; \leftarrow a \;,\; b \leftarrow c \;,\; c \leftarrow b\}$
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Some properties

- A disjunctive logic program may have zero, one, or multiple stable models
- If X is a stable model of a disjunctive logic program P, then X is a model of P (seen as a formula)
- If X and Y are stable models of a disjunctive logic program P, then $X \not\subset Y$
- If $A \in X$ for some stable model X of a disjunctive logic program P, then there is a rule $r \in P$ such that $body(r)^+ \subseteq X$, $body(r)^- \cap X = \emptyset$, and $head(r) \cap X = \{A\}$

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$$P = \left\{ \begin{array}{lcl} a(1,2) & \leftarrow & \\ b(X) \,;\, c(Y) & \leftarrow & a(X,Y), {\sim}c(Y) \end{array} \right\}$$

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Default negation in rule heads

- Consider disjunctive rules of the form

$$a_1 \,;\ldots;\, a_m \,;\, \sim a_{m+1} \,;\ldots;\, \sim a_n \leftarrow a_{n+1}, \ldots, a_o, \sim a_{o+1}, \ldots, \sim a_p$$

where $0 \leq m \leq n \leq o \leq p$ and each a_i is an atom for $0 \leq i \leq p$

- Given a program P over $\mathcal{A}$, consider the program

$$\begin{aligned} \widetilde{P} \quad = \quad & \{head(r)^+ \leftarrow body(r) \cup \{\sim\widetilde{a} \mid a \in head(r)^-\} \mid r \in P\} \\ & \cup \{\widetilde{a} \leftarrow \sim a \mid r \in P \text{ and } a \in head(r)^-\} \end{aligned}$$

- A set X of atoms is a stable model of a disjunctive program P (with default negation in rule heads) over $\mathcal{A}$, if $X = Y \cap \mathcal{A}$ for some stable model Y of $\widetilde{P}$ over $\mathcal{A} \cup \widetilde{\mathcal{A}}$

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Outline

20 Two kinds of negation

21 Disjunctive logic programs

22 Propositional theories

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Propositional theories

- Formulas are formed from
  - atoms in $\mathcal{A}$
  - $\bot$

  using
  - conjunction ($\wedge$)
  - disjunction ($\vee$)
  - implication ($\rightarrow$)
- Notation

$$\begin{array}{rcl} \top & = & (\bot \rightarrow \bot) \\ \sim\phi & = & (\phi \rightarrow \bot) \end{array}$$

A propositional theory is a finite set of formulas

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Reduct

- The satisfaction relation $X \models \phi$ between a set X of atoms and a (set of) formula(s) ϕ is defined as in propositional logic
- The reduct, ϕ^X, of a formula ϕ relative to a set X of atoms is defined recursively as follows:

$$\phi^X = \bot \quad \text{if } X \not\models \phi$$
$$\phi^X = \phi \quad \text{if } \phi \in X$$
$$\phi^X = (\psi^X \circ H^X) \quad \text{if } X \models \phi \text{ and } \phi = (\psi \circ H) \text{ for } \circ \in \{\wedge, \vee, \rightarrow\}$$

If $\phi = \sim\psi = (\psi \rightarrow \bot)$,
then $\phi^X = (\bot \rightarrow \bot) = \top$, if $X \not\models \psi$, and $\phi^X = \bot$, otherwise

The reduct, Φ^X, of a propositional theory Φ relative to a set X of atoms is defined as $\Phi^X = \{\phi^X \mid \phi \in \Phi\}$

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  - If $\phi = {\sim}\psi = (\psi \rightarrow \bot)$, then $\phi^X = (\bot \rightarrow \bot) = \top$, if $X \not\models \psi$, and $\phi^X = \bot$, otherwise
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Stable models

- A set X of atoms satisfies a propositional theory Φ, written $X \models \Phi$, if $X \models \phi$ for each $\phi \in \Phi$
- The set of all $\subseteq$-minimal sets of atoms satisfying a propositional theory Φ is denoted by $\min_{\subseteq}(\Phi)$
- A set X of atoms is a stable model of a propositional theory Φ, if $X \in \min_{\subseteq}(\Phi^X)$
- If X is a stable model of Φ, then
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Two examples

- $\Phi_1 = \{p \vee (p \rightarrow (q \wedge r))\}$
  - For $X = \{p, q, r\}$, we get
    $\Phi_1^{\{p,q,r\}} = \{p \vee (p \rightarrow (q \wedge r))\}$ and $\min_\subseteq(\Phi_1^{\{p,q,r\}}) = \{\emptyset\}$
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- The translation, $\tau[(\phi \leftarrow \psi)]$, of a rule $(\phi \leftarrow \psi)$ is defined as follows:
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The translation of a logic program P is $\tau[P] = \{\tau[r] \mid r \in P\}$

Given a logic program P and a set X of atoms,
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Logic programs as propositional theories

- The normal logic program $P = \{p \leftarrow \sim q,\ q \leftarrow \sim p\}$ corresponds to $\tau[P] = \{\sim q \rightarrow p,\ \sim p \rightarrow q\}$
  - stable models: $\{p\}$ and $\{q\}$
- The disjunctive logic program $P = \{p\,;q \leftarrow\}$ corresponds to $\tau[P] = \{\top \rightarrow p \vee q\}$
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- The nested logic program $P = \{p \leftarrow {\sim}{\sim}p\}$ corresponds to $\tau[P] = \{{\sim}{\sim}p \rightarrow p\}$
  - stable models: $\emptyset$ and $\{p\}$

Logic programs as propositional theories

- The normal logic program $P = \{p \leftarrow {\sim}q,\ q \leftarrow {\sim}p\}$ corresponds to $\tau[P] = \{{\sim}q \rightarrow p,\ {\sim}p \rightarrow q\}$
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Grounding: Overview

Grounding by example

$d(a)$
$d(c)$
$d(d)$

$p(a, b)$
$p(b, c)$
$p(c, d)$
$p(X, Z) \leftarrow p(X, Y), p(Y, Z)$

$q(a)$
$q(b)$
$q(X) \leftarrow \sim r(X), d(X)$

$r(X) \leftarrow \sim q(X), d(X)$
$s(X) \leftarrow \sim r(X), p(X, Y), q(Y)$

Grounding by example

Safe ?

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Grounding by example

	Safe ?
$d(a)$	✔
$d(c)$	✔
$d(d)$	✔
$p(a,b)$	✔
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$p(c,d)$	✔
$p(X,Z) \leftarrow p(X,Y), p(Y,Z)$	
$q(a)$	✔
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Grounding by example

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Match

- A substitution is a mapping from variables to terms
- Given sets B and D of atoms, a substitution θ is a match of B in D, if $B\theta \subseteq D$
- Given a set B of atoms and a set D of ground atoms, define

$$\Theta(B, D) = \{\, \theta \mid \theta \text{ is a } \subseteq\text{-minimal match of } B \text{ in } D \,\}$$

- Example $\{X \mapsto 1\}$ and $\{X \mapsto 2\}$ are $\subseteq$-minimal matches of $\{p(X)\}$ in $\{p(1), p(2), p(3)\}$, while match $\{X \mapsto 1, Y \mapsto 2\}$ is not

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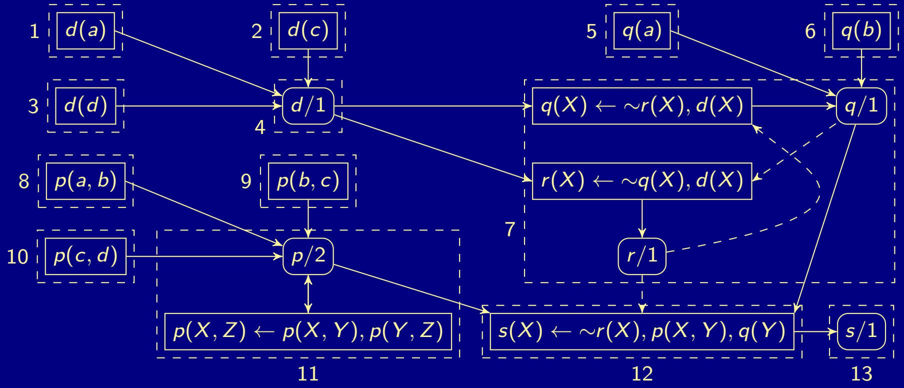
Naive instantiation

Algorithm 1: NaiveInstantiation

Input : A safe (first-order) logic program P
Output : A ground logic program P'

```
D := ∅
P' := ∅
repeat
    D' := D
    foreach r ∈ P do
        B := body(r)^+
        foreach θ ∈ Θ(B, D) do
            D := D ∪ {head(r)θ}
            P' := P' ∪ {rθ}
until D = D'
```

Predicate-rule dependency graph



Instantiation

SCC	$\Theta(B, D)$	D	P'
1	$\{\emptyset\}$	$d(a)$	$d(a) \leftarrow$
2	$\{\emptyset\}$	$d(c)$	$d(c) \leftarrow$
3	$\{\emptyset\}$	$d(d)$	$d(d) \leftarrow$
5	$\{\emptyset\}$	$q(a)$	$q(a) \leftarrow$
6	$\{\emptyset\}$	$q(b)$	$q(b) \leftarrow$
7	$\{\{X \mapsto a\},$		$q(a) \leftarrow \sim r(a), d(a)$
	$\{X \mapsto c\},$	$q(c)$	$q(c) \leftarrow \sim r(c), d(c)$
	$\{X \mapsto d\},$	$q(d)$	$q(d) \leftarrow \sim r(d), d(d)$
	$\{X \mapsto a\},$		$r(a) \leftarrow \sim q(a), d(a)$
	$\{X \mapsto c\},$	$r(c)$	$r(c) \leftarrow \sim q(c), d(c)$
	$\{X \mapsto d\}\}$	$r(d)$	$r(d) \leftarrow \sim q(d), d(d)$

Instantiation

SCC	$\Theta(B, D)$	D	P'
8	$\{\emptyset\}$	$p(a,b)$	$p(a,b) \leftarrow$
9	$\{\emptyset\}$	$p(b,c)$	$p(b,c) \leftarrow$
10	$\{\emptyset\}$	$p(c,d)$	$p(c,d) \leftarrow$
11	$\{\{X \mapsto a, Y \mapsto b, Z \mapsto c\},$	$p(a,c)$	$p(a,c) \leftarrow p(a,b), p(b,c)$
	$\{X \mapsto b, Y \mapsto c, Z \mapsto d\}\}$	$p(b,d)$	$p(b,d) \leftarrow p(b,c), p(c,d)$
	$\{\{X \mapsto a, Y \mapsto c, Z \mapsto d\},$	$p(a,d)$	$p(a,d) \leftarrow p(a,c), p(c,d)$
	$\{X \mapsto a, Y \mapsto b, Z \mapsto d\}\}$		$p(a,d) \leftarrow p(a,b), p(b,d)$
12	$\{\{X \mapsto a, Y \mapsto b\},$	$s(a)$	$s(a) \leftarrow \sim r(a), p(a,b), q(b)$
	$\{X \mapsto a, Y \mapsto c\},$		$s(a) \leftarrow \sim r(a), p(a,c), q(c)$
	$\{X \mapsto a, Y \mapsto d\},$		$s(a) \leftarrow \sim r(a), p(a,d), q(d)$
	$\{X \mapsto b, Y \mapsto c\},$	$s(b)$	$s(b) \leftarrow \sim r(b), p(b,c), q(c)$
	$\{X \mapsto b, Y \mapsto d\},$		$s(b) \leftarrow \sim r(b), p(b,d), q(d)$
	$\{X \mapsto c, Y \mapsto d\}\}$	$s(c)$	$s(c) \leftarrow \sim r(c), p(c,d), q(d)$

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Computational Aspects: Overview

23 Consequence operator

24 Computation from first principles

25 Complexity

Outline

23 Consequence operator

24 Computation from first principles

25 Complexity

Consequence operator

- Let P be a positive program and X a set of atoms
  - The consequence operator T_P is defined as follows:

$$T_P X = \{head(r) \mid r \in P \text{ and } body(r) \subseteq X\}$$

  - Iterated applications of T_P are written as T_P^j for $j \geq 0$, where
    - $T_P^0 X = X$ and
    - $T_P^i X = T_P T_P^{i-1} X$ for $i \geq 1$
- For any positive program P, we have
  - $Cn(P) = \bigcup_{i \geq 0} T_P^i \emptyset$
  - $X \subseteq Y$ implies $T_P X \subseteq T_P Y$
  - $Cn(P)$ is the smallest fixpoint of T_P

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An example

- Consider the program

$$P = \{p \leftarrow,\ q \leftarrow,\ r \leftarrow p,\ s \leftarrow q, t,\ t \leftarrow r,\ u \leftarrow v\}$$

- We get

$$\begin{array}{lclclcl}
T_P^0\emptyset & = & \emptyset & & & & \\
T_P^1\emptyset & = & \{p,q\} & = & T_P T_P^0\emptyset & = & T_P\emptyset \\
T_P^2\emptyset & = & \{p,q,r\} & = & T_P T_P^1\emptyset & = & T_P\{p,q\} \\
T_P^3\emptyset & = & \{p,q,r,t\} & = & T_P T_P^2\emptyset & = & T_P\{p,q,r\} \\
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- $Cn(P) = \{p,q,r,t,s\}$ is the smallest fixpoint of T_P because
  - $T_P\{p,q,r,t,s\} = \{p,q,r,t,s\}$ and
  - $T_P X \neq X$ for each $X \subset \{p,q,r,t,s\}$

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Approximating stable models

- First Idea Approximate a stable model X by two sets of atoms L and U such that $L \subseteq X \subseteq U$
  - L and U constitute lower and upper bounds on X
  - L and $(\mathcal{A} \setminus U)$ describe a three-valued model of the program
- Observation

$$X \subseteq Y \text{ implies } P^Y \subseteq P^X \text{ implies } Cn(P^Y) \subseteq Cn(P^X)$$

- Properties Let X be a stable model of normal logic program P
  - If $L \subseteq X$, then $X \subseteq Cn(P^L)$
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Approximating stable models

- Second Idea

  repeat

  replace L **by** $L \cup Cn(P^U)$
  replace U **by** $U \cap Cn(P^L)$

  until *L and U do not change anymore*

- Observations
  - At each iteration step
    - L becomes larger (or equal)
    - U becomes smaller (or equal)
  - $L \subseteq X \subseteq U$ is invariant for every stable model X of P

  If $L \not\subseteq U$, then P has no stable model
  If $L = U$, then L is a stable model of P

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The simplistic expand algorithm

$\textbf{expand}_P(L, U)$

repeat

$L' \leftarrow L$
$U' \leftarrow U$

$L \leftarrow L' \cup Cn(P^{U'})$
$U \leftarrow U' \cap Cn(P^{L'})$

if $L \not\subseteq U$ **then return**

until $L = L'$ and $U = U'$

An example

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, \sim c \\ d \leftarrow b, \sim e \\ e \leftarrow \ \sim d \end{array} \right\}$$

	L'	$Cn(P^{U'})$	L	U'	$Cn(P^{L'})$	U
1	$\emptyset$	$\{a\}$	$\{a\}$	$\{a,b,c,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$
2	$\{a\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$
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- Note We have $\{a,b\} \subseteq X$ and $(\mathcal{A} \setminus \{a,b,d,e\}) \cap X = (\{c\} \cap X) = \emptyset$ for every stable model X of P

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An example

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, \sim c \\ d \leftarrow b, \sim e \\ e \leftarrow \ \sim d \end{array} \right\}$$

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The simplistic expand algorithm

- **expand_P**
  - tightens the approximation on stable models
  - is stable model preserving

Let's expand with d !

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, \sim c \\ d \leftarrow b, \sim e \\ e \leftarrow \ \sim d \end{array} \right\}$$

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2	$\{a, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$
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A simplistic solving algorithm

$solve_P(L, U)$

$(L, U) \leftarrow expand_P(L, U)$ // *propagation*

if $L \not\subseteq U$ **then failure** // *failure*

if $L = U$ **then output** L // *success*

else choose $a \in U \setminus L$ // *choice*

$solve_P(L \cup \{a\}, U)$

$solve_P(L, U \setminus \{a\})$

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A simplistic solving algorithm

- Close to the approach taken by the ASP solver `smodels`, inspired by the Davis-Putman-Logemann-Loveland (DPLL) procedure
  - Backtracking search building a binary search tree
  - A node in the search tree corresponds to a three-valued interpretation
  - The search space is pruned by
    - deriving deterministic consequences and detecting conflicts (**expand**)
    - making one choice at a time by appeal to a heuristic (**choose**)
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Outline

23 Consequence operator

24 Computation from first principles

25 Complexity

Complexity

Let a be an atom and X be a set of atoms

- For a positive normal logic program P:
  - Deciding whether X is the stable model of P is P-complete
  - Deciding whether a is in the stable model of P is P-complete
- For a normal logic program P:
  - Deciding whether X is a stable model of P is P-complete
  - Deciding whether a is in a stable model of P is NP-complete
- For a normal logic program P with optimization statements:
  - Deciding whether X is an optimal stable model of P is *co-NP*-complete
  - Deciding whether a is in an optimal stable model of P is Δ_2^p-complete

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Complexity

Let a be an atom and X be a set of atoms

- For a positive disjunctive logic program P:
  - Deciding whether X is a stable model of P is *co-NP*-complete
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- For a propositional theory Φ:
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Axiomatic Characterization: Overview

26 Completion

27 Tightness

28 Loops and Loop Formulas

Outline

26 Completion

27 Tightness

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Motivation

- Question Is there a propositional formula $F(P)$ such that the models of $F(P)$ correspond to the stable models of P ?

- Observation Although each atom is defined through a set of rules, each such rule provides only a sufficient condition for its head atom

- Idea The idea of program completion is to turn such implications into a definition by adding the corresponding necessary counterpart

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Program completion

Let P be a normal logic program

- The completion $CF(P)$ of P is defined as follows

$$CF(P) = \left\{ a \leftrightarrow \bigvee_{r \in P, head(r)=a} BF(body(r)) \mid a \in atom(P) \right\}$$

where

$$BF(body(r)) = \bigwedge_{a \in body(r)^+} a \wedge \bigwedge_{a \in body(r)^-} \neg a$$

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An example

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow \sim a \\ c \leftarrow a, \sim d \\ d \leftarrow \sim c, \sim e \\ e \leftarrow b, \sim f \\ e \leftarrow e \end{array} \right\} \qquad CF(P) = \left\{ \begin{array}{l} a \leftrightarrow \top \\ b \leftrightarrow \neg a \\ c \leftrightarrow a \wedge \neg d \\ d \leftrightarrow \neg c \wedge \neg e \\ e \leftrightarrow (b \wedge \neg f) \vee e \\ f \leftrightarrow \bot \end{array} \right\}$$

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A closer look

- $CF(P)$ is logically equivalent to $\overleftarrow{CF}(P) \cup \overrightarrow{CF}(P)$, where

$$\begin{aligned} \overleftarrow{CF}(P) &= \left\{a \leftarrow \bigvee_{B \in body_P(a)} BF(B) \mid a \in atom(P)\right\} \\ \overrightarrow{CF}(P) &= \left\{a \rightarrow \bigvee_{B \in body_P(a)} BF(B) \mid a \in atom(P)\right\} \end{aligned}$$

$$body_P(a) = \{body(r) \mid r \in P \text{ and } head(r) = a\}$$

- $\overleftarrow{CF}(P)$ characterizes the classical models of P
- $\overrightarrow{CF}(P)$ completes P by adding necessary conditions for all atoms

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Supported models

- Every stable model of P is a model of $CF(P)$, but not vice versa

  Models of $CF(P)$ are called the supported models of P

  In other words, every stable model of P is a supported model of P

  By definition, every supported model of P is also a model of P

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- P has 21 models, including $\{a, c\}$, $\{a, d\}$, but also $\{a, b, c, d, e, f\}$
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Outline

26 Completion

27 Tightness

28 Loops and Loop Formulas

The mismatch

- Question What causes the mismatch between supported models and stable models?
- Hint Consider the unstable yet supported model $\{a, c, e\}$ of P !
- Answer Cyclic derivations are causing the mismatch between supported and stable models

  Atoms in a stable model can be "derived" from a program in a finite number of steps

  Atoms in a cycle (not being "supported from outside the cycle") cannot be "derived" from a program in a finite number of steps

  But such atoms do not contradict the completion of a program and do thus not eliminate an unstable supported model

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The mismatch

- Question What causes the mismatch between supported models and stable models?
- Hint Consider the unstable yet supported model $\{a, c, e\}$ of P !
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Non-cyclic derivations

Let X be a stable model of normal logic program P

- For every atom $A \in X$, there is a finite sequence of positive rules

  $$\langle r_1, \dots, r_n \rangle$$

  such that
  1. $head(r_1) = A$
  2. $body(r_i)^+ \subseteq \{head(r_j) \mid i < j \leq n\}$ for $1 \leq i \leq n$
  3. $r_i \in P^X$ for $1 \leq i \leq n$

- That is, each atom of X has a non-cyclic derivation from P^X

- Example There is no finite sequence of rules providing a derivation for e from $P^{\{a,c,e\}}$

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Positive atom dependency graph

- The origin of (potential) circular derivations can be read off the positive atom dependency graph $G(P)$ of a logic program P given by

$$(atom(P), \{(a,b) \mid r \in P, a \in body(r)^+, head(r) = b\})$$

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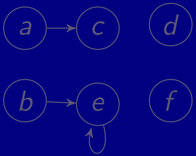

- P has supported models: $\{a, c\}$, $\{a, d\}$, and $\{a, c, e\}$
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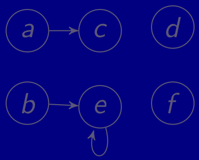

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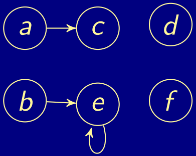

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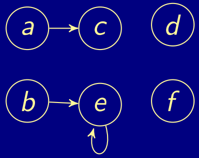

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Tight programs

- A logic program P is called tight, if $G(P)$ is acyclic
- For tight programs, stable and supported models coincide:

Fages' Theorem

Let P be a tight normal logic program and $X \subseteq atom(P)$
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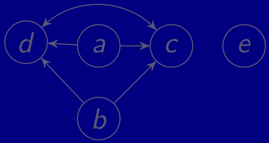

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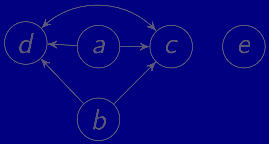

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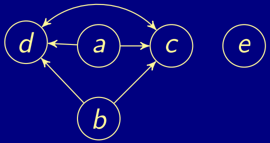

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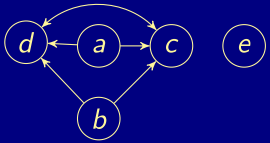

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Outline

26 Completion

27 Tightness

28 Loops and Loop Formulas

Motivation

- Question Is there a propositional formula $F(P)$ such that the models of $F(P)$ correspond to the stable models of P ?

- Observation Starting from the completion of a program, the problem boils down to eliminating the circular support of atoms holding in the supported models of the program

- Idea Add formulas prohibiting circular support of sets of atoms
- Note Circular support between atoms a and b is possible, if a has a path to b and b has a path to a in the program's positive atom dependency graph

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Loops

Let P be a normal logic program, and
let $G(P) = (atom(P), E)$ be the positive atom dependency graph of P

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  That is, each pair of atoms in L is connected by a path of non-zero length in $(L, E \cap (L \times L))$

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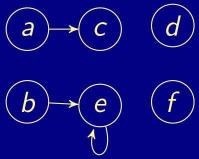

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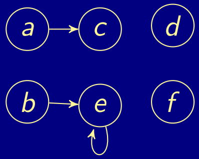

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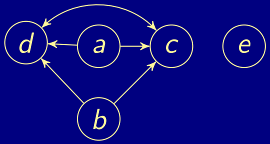

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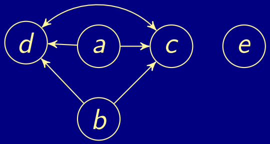

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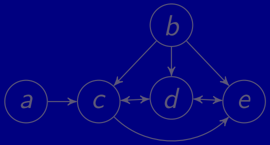

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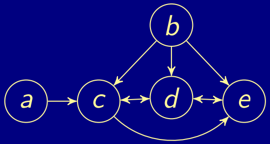

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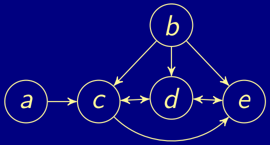

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Loop formulas

Let P be a normal logic program

- For $L \subseteq atom(P)$, define the external supports of L for P as

$$ES_P(L) = \{r \in P \mid head(r) \in L \text{ and } body(r)^+ \cap L = \emptyset\}$$

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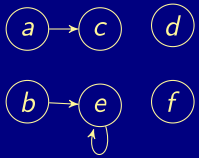

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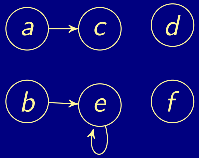

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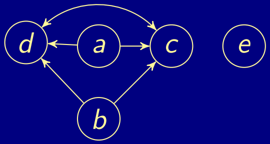

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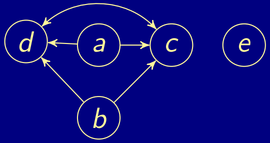

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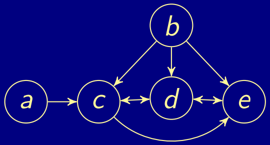

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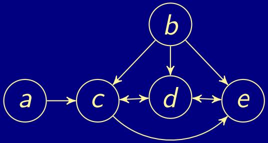

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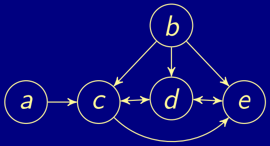

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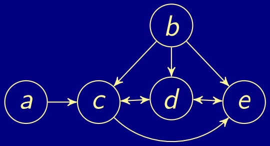

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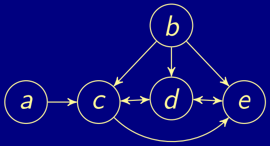

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Lin-Zhao Theorem

Theorem

Let P be a normal logic program and $X \subseteq atom(P)$
Then, X is a stable model of P iff $X \models CF(P) \cup LF(P)$

Loops and loop formulas: Properties

Let X be a supported model of normal logic program P

- Then, X is a stable model of P iff
  - $X \models \{LF_P(U) \mid U \subseteq atom(P)\}$;
  - $X \models \{LF_P(U) \mid U \subseteq X\}$;
  - $X \models \{LF_P(L) \mid L \in loop(P)\}$, that is, $X \models LF(P)$;
  - $X \models \{LF_P(L) \mid L \in loop(P) \text{ and } L \subseteq X\}$

- Note If X is not a stable model of P, then there is a loop $L \subseteq X \setminus Cn(P^X)$ such that $X \not\models LF_P(L)$

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Loops and loop formulas: Properties (ctd)

If $\mathcal{P} \not\subseteq \mathcal{NC}^1/poly$,[1] then there is no translation $\mathcal{T}$ from logic programs to propositional formulas such that, for each normal logic program P, both of the following conditions hold:

1. The propositional variables in $\mathcal{T}[P]$ are a subset of $atom(P)$
2. The size of $\mathcal{T}[P]$ is polynomial in the size of P
   - Note Every vocabulary-preserving translation from normal logic programs to propositional formulas must be exponential (in the worst case).

Observations

- Translation $CF(P) \cup LF(P)$ preserves the vocabulary of P
- The number of loops in $loop(P)$ may be exponential in $|atom(P)|$

[1] A conjecture from the theory of complexity that is widely believed to be true.

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Operational Characterization: Overview

29 Partial Interpretations

30 Fitting Operator

31 Unfounded Sets

32 Well-Founded Operator

Outline

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Interlude: Partial interpretations

or: 3-valued interpretations

A partial interpretation maps atoms onto truth values *true*, *false*, and *unknown*

- Representation $\langle T, F \rangle$, where
  - T is the set of all *true* atoms and
  - F is the set of all *false* atoms
  - Truth of atoms in $\mathcal{A} \setminus (T \cup F)$ is *unknown*
- Properties
  - $\langle T, F \rangle$ is conflicting if $T \cap F \neq \emptyset$
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Outline

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31 Unfounded Sets

32 Well-Founded Operator

Basic idea

- Idea Extend T_P to normal logic programs
- Logical background The idea is to turn a program's completion into an operator such that
  - the head atom of a rule must be *true*
    if the rule's body is *true*
  - an atom must be *false*
    if the body of each rule having it as head is *false*

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Definition

- Let P be a normal logic program
- Define

$$\Phi_P\langle T,F\rangle = \langle \mathbf{T}_P\langle T,F\rangle, \mathbf{F}_P\langle T,F\rangle\rangle$$

where

$$\mathbf{T}_P\langle T,F\rangle = \{head(r) \mid r \in P, body(r)^+ \subseteq T, body(r)^- \subseteq F\}$$

$$\mathbf{F}_P\langle T,F\rangle = \{a \in atom(P) \mid body(r)^+ \cap F \neq \emptyset \text{ or } body(r)^- \cap T \neq \emptyset \text{ for each } r \in P \text{ such that } head(r) = a\}$$

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\end{aligned}$$

Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- Let's iterate $\mathbf{\Phi}_P$ on $\langle \{a\}, \{d\} \rangle$:

$$\begin{array}{rcl} \mathbf{\Phi}_P\langle \{a\}, \{d\} \rangle & = & \langle \{a, c\}, \{b, f\} \rangle \\ \mathbf{\Phi}_P\langle \{a, c\}, \{b, f\} \rangle & = & \langle \{a\}, \{b, d, f\} \rangle \\ \mathbf{\Phi}_P\langle \{a\}, \{b, d, f\} \rangle & = & \langle \{a, c\}, \{b, f\} \rangle \\ & \vdots & \end{array}$$

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Fitting semantics

- Define the iterative variant of $\mathbf{\Phi}_P$ analogously to T_P:

$$\mathbf{\Phi}_P^0\langle T,F\rangle = \langle T,F\rangle \qquad \mathbf{\Phi}_P^{i+1}\langle T,F\rangle = \mathbf{\Phi}_P\mathbf{\Phi}_P^i\langle T,F\rangle$$

- Define the Fitting semantics of a normal logic program P as the partial interpretation:

$$\textstyle\bigsqcup_{i\geq 0}\mathbf{\Phi}_P^i\langle\emptyset,\emptyset\rangle$$

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Properties

Let P be a normal logic program

- $\mathbf{\Phi}_P\langle\emptyset,\emptyset\rangle$ is monotonic
  That is, $\mathbf{\Phi}^i_P\langle\emptyset,\emptyset\rangle \sqsubseteq \mathbf{\Phi}^{i+1}_P\langle\emptyset,\emptyset\rangle$
- The Fitting semantics of P is
  - not conflicting,
  - and generally not total

Fitting fixpoints

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Define $\langle T, F\rangle$ as a Fitting fixpoint of P if $\mathbf{\Phi}_P\langle T, F\rangle = \langle T, F\rangle$
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- P has three total Fitting fixpoints:
  - $\langle\{a, c\}, \{b, d, e\}\rangle$
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P has three supported models, two of them are stable models

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- Let $\mathbf{\Phi}_P\langle T, F\rangle = \langle T', F'\rangle$
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  That is, $\mathbf{\Phi}_P$ is stable model preserving

  Hence, $\mathbf{\Phi}_P$ can be used for approximating stable models and so for propagation in ASP-solvers

- However, $\mathbf{\Phi}_P$ is still insufficient, because total fixpoints correspond to supported models, not necessarily stable models

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  Hence, $\mathbf{\Phi}_P$ can be used for approximating stable models and so for propagation in ASP-solvers

- However, $\mathbf{\Phi}_P$ is still insufficient, because total fixpoints correspond to supported models, not necessarily stable models

  Note The problem is the same as with program completion

  The missing piece is non-circularity of derivations !

Properties

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Let $\mathbf{\Phi}_P\langle T, F\rangle = \langle T', F'\rangle$
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Example

$$P = \left\{ \begin{array}{lcl} a & \leftarrow & b \\ b & \leftarrow & a \end{array} \right\}$$

$$\begin{array}{lcl} \Phi^0_P\langle\emptyset,\emptyset\rangle & = & \langle\emptyset,\emptyset\rangle \\ \Phi^1_P\langle\emptyset,\emptyset\rangle & = & \langle\emptyset,\emptyset\rangle \end{array}$$

- That is, Fitting semantics cannot assign *false* to a and b, although they can never become *true* !

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Outline

29 Partial Interpretations

30 Fitting Operator

31 Unfounded Sets

32 Well-Founded Operator

Unfounded sets

Let P be a normal logic program,
and let $\langle T, F \rangle$ be a partial interpretation

- A set $U \subseteq atom(P)$ is an unfounded set of P wrt $\langle T, F \rangle$,
  if we have for each rule $r \in P$ such that $head(r) \in U$ either

  1. $body(r)^+ \cap F \neq \emptyset$ or $body(r)^- \cap T \neq \emptyset$ or
  2. $body(r)^+ \cap U \neq \emptyset$

- Intuitively, $\langle T, F \rangle$ is what we already know about P
- Rules satisfying Condition 1 are not usable for further derivations
- Condition 2 is the unfounded set condition treating cyclic derivations:
  All rules still being usable to derive an atom in U require an(other) atom in U to be true

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Example

$$P = \left\{ \begin{array}{lcl} a & \leftarrow & b \\ b & \leftarrow & a \end{array} \right\}$$

- $\emptyset$ is an unfounded set (by definition)
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- $\{a\}$ is an unfounded set of P wrt $\langle \emptyset, \{b\} \rangle$
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- Analogously for $\{b\}$
- $\{a, b\}$ is an unfounded set of P wrt $\langle \emptyset, \emptyset \rangle$
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Greatest unfounded sets

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Observation The union of two unfounded sets is an unfounded set
- The greatest unfounded set of P wrt $\langle T, F\rangle$ is the union of all unfounded sets of P wrt $\langle T, F\rangle$

  It is denoted by $\mathbf{U}_P\langle T, F\rangle$
- Alternatively, we may define

$$\mathbf{U}_P\langle T, F\rangle = atom(P) \setminus Cn(\{r \in P \mid body(r)^+ \cap F = \emptyset\}^T)$$

- Note $Cn(\{r \in P \mid body(r)^+ \cap F = \emptyset\}^T)$ contains all non-circularly derivable atoms from P wrt $\langle T, F\rangle$

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Outline

29 Partial Interpretations

30 Fitting Operator

31 Unfounded Sets

32 Well-Founded Operator

Well-founded operator

Let P be a normal logic program,
and let $\langle T, F \rangle$ be a partial interpretation

- Observation Condition 1 (in the definition of an unfounded set) corresponds to $\mathbf{F}_P\langle T, F\rangle$ of Fitting's $\mathbf{\Phi}_P\langle T, F\rangle$
- Idea Extend (negative part of) Fitting's operator $\mathbf{\Phi}_P$
  That is,
  - keep definition of $\mathbf{T}_P\langle T, F\rangle$ from $\mathbf{\Phi}_P\langle T, F\rangle$ and
  - replace $\mathbf{F}_P\langle T, F\rangle$ from $\mathbf{\Phi}_P\langle T, F\rangle$ by $\mathbf{U}_P\langle T, F\rangle$
- In words, an atom must be *false*
  if it belongs to the greatest unfounded set
- Definition $\mathbf{\Omega}_P\langle T, F\rangle = \langle \mathbf{T}_P\langle T, F\rangle, \mathbf{U}_P\langle T, F\rangle\rangle$

$$\mathbf{\Phi}_P\langle T, F\rangle \sqsubseteq \mathbf{\Omega}_P\langle T, F\rangle$$

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Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- Let's iterate $\mathbf{\Omega}_{P_1}$ on $\langle \{c\}, \emptyset \rangle$:

$$\begin{array}{rcl} \mathbf{\Omega}_P \langle \{c\}, \emptyset \rangle & = & \langle \{a\}, \{d, f\} \rangle \\ \mathbf{\Omega}_P \langle \{a\}, \{d, f\} \rangle & = & \langle \{a, c\}, \{b, e, f\} \rangle \\ \mathbf{\Omega}_P \langle \{a, c\}, \{b, e, f\} \rangle & = & \langle \{a\}, \{b, d, e, f\} \rangle \\ \mathbf{\Omega}_P \langle \{a\}, \{b, d, e, f\} \rangle & = & \langle \{a, c\}, \{b, e, f\} \rangle \\ & \vdots & \end{array}$$

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Example

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Well-founded semantics

- Define the iterative variant of $\mathbf{\Omega}_P$ analogously to $\mathbf{\Phi}_P$:

$$\mathbf{\Omega}^0_P\langle T,F\rangle=\langle T,F\rangle \qquad \mathbf{\Omega}^{i+1}_P\langle T,F\rangle=\mathbf{\Omega}_P\mathbf{\Omega}^i_P\langle T,F\rangle$$

- Define the well-founded semantics of a normal logic program P as the partial interpretation:

$$\bigsqcup_{i\geq 0}\mathbf{\Omega}^i_P\langle\emptyset,\emptyset\rangle$$

Potassco

Well-founded semantics

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- Define the well-founded semantics of a normal logic program P as the partial interpretation:

$$\bigsqcup_{i\geq 0}\mathbf{\Omega}_P^i\langle \emptyset, \emptyset\rangle$$

Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

$$\begin{array}{lllll} \mathbf{\Omega}^0\langle\emptyset,\emptyset\rangle & = & & & \langle\emptyset,\emptyset\rangle \\ \mathbf{\Omega}^1\langle\emptyset,\emptyset\rangle & = & \mathbf{\Omega}\langle\emptyset,\emptyset\rangle & = & \langle\{a\},\{f\}\rangle \\ \mathbf{\Omega}^2\langle\emptyset,\emptyset\rangle & = & \mathbf{\Omega}\langle\{a\},\{f\}\rangle & = & \langle\{a\},\{b,e,f\}\rangle \\ \mathbf{\Omega}^3\langle\emptyset,\emptyset\rangle & = & \mathbf{\Omega}\langle\{a\},\{b,e,f\}\rangle & = & \langle\{a\},\{b,e,f\}\rangle \end{array}$$

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Properties

Let P be a normal logic program

- $\mathbf{\Omega}_P\langle\emptyset,\emptyset\rangle$ is monotonic
  That is, $\mathbf{\Omega}_P^i\langle\emptyset,\emptyset\rangle \sqsubseteq \mathbf{\Omega}_P^{i+1}\langle\emptyset,\emptyset\rangle$
- The well-founded semantics of P is
  - not conflicting,
  - and generally not total
- We have $\bigsqcup_{i\geq 0}\mathbf{\Phi}_P^i\langle\emptyset,\emptyset\rangle \sqsubseteq \bigsqcup_{i\geq 0}\mathbf{\Omega}_P^i\langle\emptyset,\emptyset\rangle$

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Well-founded fixpoints

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Define $\langle T, F\rangle$ as a well-founded fixpoint of P if $\mathbf{\Omega}_P\langle T, F\rangle = \langle T, F\rangle$
  - The well-founded semantics is the $\sqsubseteq$-least well-founded fixpoint of P
  - Any other well-founded fixpoint extends the well-founded semantics
  - Total well-founded fixpoints correspond to stable models

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Well-founded fixpoints: Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, {\sim}d & e \leftarrow b, {\sim}f \\ b \leftarrow {\sim}a & d \leftarrow {\sim}c, {\sim}e & e \leftarrow e \end{array} \right\}$$

- P has two total well-founded fixpoints:
  1. $\langle \{a, c\}, \{b, d, e\} \rangle$
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- Let $\mathbf{\Omega}_P\langle T, F\rangle = \langle T', F'\rangle$
- If X is a stable model of P such that $T \subseteq X$ and $X \cap F = \emptyset$, then $T' \subseteq X$ and $X \cap F' = \emptyset$

  That is, $\mathbf{\Omega}_P$ is stable model preserving

  Hence, $\mathbf{\Omega}_P$ can be used for approximating stable models and so for propagation in ASP-solvers
- In contrast to $\mathbf{\Phi}_P$, operator $\mathbf{\Omega}_P$ is sufficient for propagation because total fixpoints correspond to stable models
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Proof-theoretic Characterization: Overview

Motivation

- Goal Analyze computations in ASP solvers
- Wanted A declarative and fine-grained instrument for characterizing operations as well as strategies of ASP solvers
- Idea View stable model computations as derivations in an inference system

  Consider Tableau-based proof systems for analyzing ASP solving

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Tableau calculi

- Traditionally, tableau calculi are used for
  - automated theorem proving and
  - proof theoretical analysis

  in classical as well as non-classical logics
- General idea Given an input, prove some property by decomposition
  Decomposition is done by applying deduction rules
- For details, see *Handbook of Tableau Methods*, Kluwer, 1999

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General definitions

- A tableau is a (mostly binary) tree
- A branch in a tableau is a path from the root to a leaf
- A branch containing $\gamma_1, \dots, \gamma_m$ can be extended by applying tableau rules of form

$$\frac{\gamma_1, \dots, \gamma_m}{\begin{array}{c}\alpha_1\\ \vdots\\ \alpha_n\end{array}} \qquad\qquad \frac{\gamma_1, \dots, \gamma_m}{\beta_1 \mid \dots \mid \beta_n}$$

Rules of the former format append entries $\alpha_1, \dots, \alpha_n$ to the branch

Rules of the latter format create multiple sub-branches for $\beta_1, \dots, \beta_n$

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Example

- A simple tableau calculus for proving unsatisfiability of propositional formulas, composed from $\neg$, $\wedge$, and $\vee$, consists of rules

$$\frac{\neg\neg\alpha}{\alpha} \qquad \frac{\alpha_1 \wedge \alpha_2}{\begin{array}{c}\alpha_1\\ \alpha_2\end{array}} \qquad \frac{\beta_1 \vee \beta_2}{\beta_1 \mid \beta_2}$$

- All rules are semantically valid, when interpreting entries in a branch conjunctively and distinct (sub-)branches as connected disjunctively
- A propositional formula φ is unsatisfiable iff there is a tableau with φ as the root node such that
  1. all other entries can be produced by tableau rules and
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(2)	a	[1]
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All three branches of the tableau are contradictory (cf 2, 5, 7, 8, 10)

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Tableaux and ASP

- A tableau rule captures an elementary inference scheme in an ASP solver
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ASP-specific definitions

- A (signed) tableau for a logic program P is a binary tree such that
  - the root node of the tree consists of the rules in P;
  - the other nodes in the tree are entries of the form $\mathbf{T}v$ or $\mathbf{F}v$, called signed literals, where v is a variable,
  - generated by extending a tableau using deduction rules (given below)
- An entry $\mathbf{T}v$ ($\mathbf{F}v$) reflects that variable v is *true* (*false*) in a corresponding variable assignment

  A set of signed literals constitutes a partial assignment
- For a normal logic program P,
  - atoms of P in $atom(P)$ and
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Tableau rules for ASP at a glance

$$(FTB)\quad \frac{p \leftarrow l_1,\dots,l_n \quad \mathbf{t}l_1,\dots,\mathbf{t}l_n}{\mathbf{T}\{l_1,\dots,l_n\}} \qquad (BFB)\quad \frac{\mathbf{F}\{l_1,\dots,l_i,\dots,l_n\} \quad \mathbf{t}l_1,\dots,\mathbf{t}l_{i-1},\mathbf{t}l_{i+1},\dots,\mathbf{t}l_n}{\mathbf{f}l_i}$$

$$(FTA)\quad \frac{p \leftarrow l_1,\dots,l_n \quad \mathbf{T}\{l_1,\dots,l_n\}}{\mathbf{T}p} \qquad (BFA)\quad \frac{p \leftarrow l_1,\dots,l_n \quad \mathbf{F}p}{\mathbf{F}\{l_1,\dots,l_n\}}$$

$$(FFB)\quad \frac{p \leftarrow l_1,\dots,l_i,\dots,l_n \quad \mathbf{f}l_i}{\mathbf{F}\{l_1,\dots,l_i,\dots,l_n\}} \qquad (BTB)\quad \frac{\mathbf{T}\{l_1,\dots,l_i,\dots,l_n\}}{\mathbf{t}l_i}$$

$$(FFA)\quad \frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\ (\S) \qquad (BTA)\quad \frac{\mathbf{T}p \quad \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m}{\mathbf{T}B_i}\ (\S)$$

$$(WFN)\quad \frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\ (\dagger) \qquad (WFJ)\quad \frac{\mathbf{T}p \quad \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m}{\mathbf{T}B_i}\ (\dagger)$$

$$(FL)\quad \frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\ (\ddagger) \qquad (BL)\quad \frac{\mathbf{T}p \quad \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m}{\mathbf{T}B_i}\ (\ddagger)$$

$$(Cut[X])\quad \frac{}{\mathbf{T}v \mid \mathbf{F}v}\ (\sharp[X])$$

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More concepts

- A tableau calculus is a set of tableau rules
- A branch in a tableau is conflicting,
  if it contains both $\mathbf{T}v$ and $\mathbf{F}v$ for some variable v
- A branch in a tableau is total for a program P,
  if it contains either $\mathbf{T}v$ or $\mathbf{F}v$ for each $v \in atom(P) \cup body(P)$
- A branch in a tableau of some calculus $\mathcal{T}$ is closed,
  if no rule in $\mathcal{T}$ other than *Cut* can produce any new entries
- A branch in a tableau is complete,
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Example

- Consider the program

$$P = \left\{ \begin{array}{l} a \leftarrow \\ c \leftarrow \sim b, \sim d \\ d \leftarrow a, \sim c \end{array} \right\}$$

having stable models $\{a, c\}$ and $\{a, d\}$

(Previewed) Example

$$
\begin{array}{c}
a \leftarrow \\
c \leftarrow \sim b, \sim d \\
d \leftarrow a, \sim c
\end{array}
$$

(FTB)		$\mathbf{T}\emptyset$		
(FTA)		$\mathbf{T}a$		
(FFA)		$\mathbf{F}b$		
(Cut[$atom(P)$])	$\mathbf{T}c$			$\mathbf{F}c$
(BTA)	$\mathbf{T}\{\sim b, \sim d\}$		(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
(BTB)	$\mathbf{F}d$		(BFB)	$\mathbf{T}d$
(FFB)	$\mathbf{F}\{a, \sim c\}$		(FTB)	$\mathbf{T}\{a, \sim c\}$

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(Previewed) Example

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$$c \leftarrow \sim b, \sim d$$
$$d \leftarrow a, \sim c$$

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Auxiliary definitions

- For a literal l, define conjugation functions $\mathbf{t}$ and $\mathbf{f}$ as follows

$$\mathbf{t}l = \begin{cases} \mathbf{T}l & \text{if } l \text{ is an atom} \\ \mathbf{F}a & \text{if } l = {\sim}a \text{ for an atom } a \end{cases}$$

$$\mathbf{f}l = \begin{cases} \mathbf{F}l & \text{if } l \text{ is an atom} \\ \mathbf{T}a & \text{if } l = {\sim}a \text{ for an atom } a \end{cases}$$

- Examples $\mathbf{t}a = \mathbf{T}a$, $\mathbf{f}a = \mathbf{F}a$, $\mathbf{t}{\sim}a = \mathbf{F}a$, and $\mathbf{f}{\sim}a = \mathbf{T}a$

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Auxiliary definitions

- Some tableau rules require conditions for their application
- Such conditions are specified as provisos

$$\frac{\mathit{prerequisites}}{\mathit{consequence}}\ (\mathit{proviso})$$

proviso: some condition(s)

- Note All tableau rules given in the sequel are stable model preserving

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Forward True Body (FTB)

- Prerequisites All of a body's literals are *true*
- Consequence The body is *true*
- Tableau Rule FTB

$$\frac{\begin{array}{c} p \leftarrow l_1, \dots, l_n \\ \mathbf{t}l_1, \dots, \mathbf{t}l_n \end{array}}{\mathbf{T}\{l_1, \dots, l_n\}}$$

- Example

$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{T}b \\ \mathbf{F}c \end{array}}{\mathbf{T}\{b, \sim c\}}$$

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- Example

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Backward False Body (BFB)

- Prerequisites A body is *false*, and all its literals except for one are *true*
- Consequence The residual body literal is *false*
- Tableau Rule BFB

$$\frac{\mathbf{F}\{l_1,\dots,l_i,\dots,l_n\} \quad \mathbf{t}l_1,\dots,\mathbf{t}l_{i-1},\mathbf{t}l_{i+1},\dots,\mathbf{t}l_n}{\mathbf{f}l_i}$$

- Examples

$$\frac{\mathbf{F}\{b,\sim c\} \quad \mathbf{T}b}{\mathbf{T}c} \qquad \frac{\mathbf{F}\{b,\sim c\} \quad \mathbf{F}c}{\mathbf{F}b}$$

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- Examples

$$\frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{T}b\end{array}}{\mathbf{T}c} \qquad \frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{F}c\end{array}}{\mathbf{F}b}$$

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Forward False Body (FFB)

- Prerequisites Some literal of a body is *false*
- Consequence The body is *false*
- Tableau Rule FFB

$$\frac{p \leftarrow l_1, \dots, l_i, \dots, l_n \qquad \mathbf{f}\, l_i}{\mathbf{F}\{l_1, \dots, l_i, \dots, l_n\}}$$

- Examples

$$\frac{a \leftarrow b, \sim c \qquad \mathbf{F}\, b}{\mathbf{F}\{b, \sim c\}} \qquad \frac{a \leftarrow b, \sim c \qquad \mathbf{T}\, c}{\mathbf{F}\{b, \sim c\}}$$

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- Examples

$$\frac{a \leftarrow b, \sim c \quad \mathbf{F}b}{\mathbf{F}\{b, \sim c\}} \qquad \frac{a \leftarrow b, \sim c \quad \mathbf{T}c}{\mathbf{F}\{b, \sim c\}}$$

Backward True Body (BTB)

- Prerequisites A body is *true*
- Consequence The body's literals are *true*
- Tableau Rule BTB

$$\frac{\mathbf{T}\{l_1, \dots, l_i, \dots, l_n\}}{\mathbf{t}l_i}$$

- Examples

$$\frac{\mathbf{T}\{b, \sim c\}}{\mathbf{T}b} \qquad \frac{\mathbf{T}\{b, \sim c\}}{\mathbf{F}c}$$

Backward True Body (BTB)

- Prerequisites A body is *true*
- Consequence The body's literals are *true*
- Tableau Rule BTB

$$\frac{\mathbf{T}\{l_1, \dots, l_i, \dots, l_n\}}{\mathbf{t}l_i}$$

- Examples

$$\frac{\mathbf{T}\{b, {\sim}c\}}{\mathbf{T}b} \qquad \frac{\mathbf{T}\{b, {\sim}c\}}{\mathbf{F}c}$$

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Tableau rules for bodies

Consider rule body $B = \{l_1, \dots, l_n\}$

- Rules FTB and BFB amount to implication

$$l_1 \wedge \cdots \wedge l_n \rightarrow B$$

- Rules FFB and BTB amount to implication

$$B \rightarrow l_1 \wedge \cdots \wedge l_n$$

- Together they yield

$$B \equiv l_1 \wedge \cdots \wedge l_n$$

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Potassco

Forward True Atom (FTA)

- Prerequisites Some of an atom's bodies is *true*
- Consequence The atom is *true*
- Tableau Rule FTA

$$\frac{p \leftarrow l_1, \dots, l_n \quad \mathbf{T}\{l_1, \dots, l_n\}}{\mathbf{T}p}$$

- Examples

$$\frac{a \leftarrow b, \sim c \quad \mathbf{T}\{b, \sim c\}}{\mathbf{T}a} \qquad \frac{a \leftarrow d, \sim e \quad \mathbf{T}\{d, \sim e\}}{\mathbf{T}a}$$

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Backward False Atom (BFA)

- Prerequisites An atom is *false*
- Consequence The bodies of all rules with the atom as head are *false*
- Tableau Rule BFA

$$\frac{p \leftarrow l_1, \ldots, l_n \quad \mathbf{F}p}{\mathbf{F}\{l_1, \ldots, l_n\}}$$

- Examples

$$\frac{a \leftarrow b, \sim c \quad \mathbf{F}a}{\mathbf{F}\{b, \sim c\}} \qquad \frac{a \leftarrow d, \sim e \quad \mathbf{F}a}{\mathbf{F}\{d, \sim e\}}$$

Backward False Atom (BFA)

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- Tableau Rule BFA

$$\frac{\begin{array}{c} p \leftarrow l_1, \dots, l_n \\ \mathbf{F}p \end{array}}{\mathbf{F}\{l_1, \dots, l_n\}}$$

- Examples

$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{F}a \end{array}}{\mathbf{F}\{b, \sim c\}} \qquad \frac{\begin{array}{c} a \leftarrow d, \sim e \\ \mathbf{F}a \end{array}}{\mathbf{F}\{d, \sim e\}}$$

Potassco

Forward False Atom (FFA)

- Prerequisites For some atom, the bodies of all rules with the atom as head are *false*
- Consequence The atom is *false*
- Tableau Rule FFA

$$\frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\ (body_P(p)=\{B_1,\dots,B_m\})$$

- Example

$$\frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{F}\{d,\sim e\}\end{array}}{\mathbf{F}a}\ (body_P(a)=\{\{b,\sim c\},\{d,\sim e\}\})$$

Potassco

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- Consequence The atom is *false*
- Tableau Rule FFA

$$\frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\ (body_P(p)=\{B_1,\dots,B_m\})$$

- Example

$$\frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{F}\{d,\sim e\}\end{array}}{\mathbf{F}a}\ (body_P(a)=\{\{b,\sim c\},\{d,\sim e\}\})$$

Potassco

Backward True Atom (BTA)

- Prerequisites An atom is *true*, and the bodies of all rules with the atom as head except for one are *false*
- Consequence The residual body is *true*
- Tableau Rule BTA

$$\frac{\mathbf{T}p \qquad \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m}{\mathbf{T}B_i}\ (body_P(p)=\{B_1,\dots,B_m\})$$

- Examples

$$\frac{\mathbf{T}a \qquad \mathbf{F}\{b,\sim c\}}{\mathbf{T}\{d,\sim e\}}\ (*) \qquad \frac{\mathbf{T}a \qquad \mathbf{F}\{d,\sim e\}}{\mathbf{T}\{b,\sim c\}}\ (*)$$

$$(*) \quad body_P(a)=\{\{b,\sim c\},\{d,\sim e\}\}$$

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- Examples

$$\frac{\mathbf{T}a \qquad \mathbf{F}\{b,\sim c\}}{\mathbf{T}\{d,\sim e\}}\ (*) \qquad \frac{\mathbf{T}a \qquad \mathbf{F}\{d,\sim e\}}{\mathbf{T}\{b,\sim c\}}\ (*)$$

$$(*) \quad body_P(a)=\{\{b,\sim c\},\{d,\sim e\}\}$$

Potassco

Tableau rules for atoms

Consider an atom p such that $body_P(p) = \{B_1, \dots, B_m\}$

- Rules FTA and BFA amount to implication

  $$B_1 \vee \cdots \vee B_m \rightarrow p$$

- Rules FFA and BTA amount to implication

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- Together they yield

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Potassco

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Relationship with program completion

Let P be a normal logic program

- The eight tableau rules introduced so far essentially provide (straightforward) inferences from $CF(P)$

Preliminaries for unfounded sets

Let P be a normal logic program

- For $P' \subseteq P$, define the greatest unfounded set of P wrt P' as

$$\mathbf{U}_P(P') = atom(P) \setminus Cn((P')^{\emptyset})$$

- For a loop $L \in loop(P)$, define the external bodies of L as

$$EB_P(L) = \{body(r) \mid r \in P, head(r) \in L, body(r)^+ \cap L = \emptyset\}$$

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- For a loop $L \in loop(P)$, define the external bodies of L as

$$EB_P(L) = \{body(r) \mid r \in P, head(r) \in L, body(r)^+ \cap L = \emptyset\}$$

Well-Founded Negation (WFN)

- Prerequisites An atom is in the greatest unfounded set wrt rules whose bodies are *false*
- Consequence The atom is *false*
- Tableau Rule WFN

$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \quad (p \in \mathbf{U}_P(\{r \in P \mid body(r) \notin \{B_1, \dots, B_m\}\}))$$

- Examples

$$\frac{\begin{array}{c} a \leftarrow \sim b \\ \mathbf{F}\{\sim b\} \end{array}}{\mathbf{F}a} \ (*) \qquad \frac{\begin{array}{c} a \leftarrow a \\ a \leftarrow \sim b \\ \mathbf{F}\{\sim b\} \end{array}}{\mathbf{F}a} \ (*)$$

$$(*) \quad a \in \mathbf{U}_P(P \setminus \{a \leftarrow \sim b\})$$

Well-Founded Negation (WFN)

- Prerequisites An atom is in the greatest unfounded set wrt rules whose bodies are *false*
- Consequence The atom is *false*
- Tableau Rule WFN

$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \quad (p \in \mathbf{U}_P(\{r \in P \mid body(r) \notin \{B_1, \dots, B_m\}\}))$$

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Well-Founded Justification (WFJ)

- Prerequisites A *true* atom is in the greatest unfounded set wrt rules whose bodies are *false*, if a particular body is made *false*
- Consequence The respective body is *true*
- Tableau Rule WFJ

$$\frac{\begin{array}{c}\mathbf{T}p\\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i}\quad (p \in \mathbf{U}_P(\{r \in P \mid body(r) \notin \{B_1,\dots,B_m\}\}))$$

- Examples

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Well-founded tableau rules

- Tableau rules WFN and WFJ ensure non-circular support for *true* atoms
- Note
  1. WFN subsumes falsifying atoms via FFA,
  2. WFJ can be viewed as "backward propagation" for unfounded sets,
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Relationship with well-founded operator

Let P be a normal logic program, $\langle T, F\rangle$ a partial interpretation, and $P' = \{r \in P \mid body(r)^+ \cap F = \emptyset \text{ and } body(r)^- \cap T = \emptyset\}$.

- The following conditions are equivalent
  1. $p \in \mathbf{U}_P\langle T, F\rangle$
  2. $p \in \mathbf{U}_P(P')$
- Hence, the well-founded operator $\mathbf{\Omega}$ and WFN coincide
- Note In contrast to $\mathbf{\Omega}$, WFN does not necessarily require a rule body to contain a *false* literal for the rule being inapplicable

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Forward Loop (FL)

- Prerequisites The external bodies of a loop are *false*
- Consequence The atoms in the loop are *false*
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$$\frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\ (p \in L, L \in loop(P), EB_P(L) = \{B_1,\dots,B_m\})$$

- Example

$$\begin{array}{c} a \leftarrow a \\ a \leftarrow {\sim}b \end{array}$$

$$\frac{\mathbf{F}\{{\sim}b\}}{\mathbf{F}a}\ (EB_P(\{a\}) = \{\{{\sim}b\}\})$$

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- Tableau rules FL and BL ensure non-circular support for *true* atoms
- For a loop L such that $EB_P(L) = \{B_1, \dots, B_m\}$,
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- Comparison to well-founded tableau rules yields
  - FL (plus FFA and FFB) is equivalent to WFN (plus FFB),
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Relationship with loop formulas

- Tableau rules FL and BL essentially provide (straightforward) inferences from loop formulas
  - Impractical to precompute exponentially many loop formulas
- In practice, ASP solvers such as *smodels* and *clasp*
  - exploit strongly connected components of positive atom dependency graphs
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Case analysis by *Cut*

- Up to now, all tableau rules are deterministic
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- Prerequisites None
- Consequence Two alternative (complementary) entries
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$$\frac{}{\mathbf{T}v \mid \mathbf{F}v}\ (v \in \mathcal{C})$$

- Examples

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Well-known tableau calculi

- Fitting's operator $\mathbf{\Phi}$ applies forward propagation without sophisticated unfounded set checks

$$\mathcal{T}_{\mathbf{\Phi}} = \{FTB, FTA, FFB, FFA\}$$

- Well-founded operator $\mathbf{\Omega}$ replaces negation of single atoms with negation of unfounded sets

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- "Local" propagation via a program's completion can be determined by elementary inferences on atoms and rule bodies

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Tableau calculi characterizing ASP solvers

- ASP solvers combine propagation with case analysis
- We obtain the following tableau calculi characterizing

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\mathcal{T}_{\mathit{cmodels\text{-}1}} &= \mathcal{T}_{\mathit{completion}} \cup \{\mathit{Cut}[\mathit{atom}(P) \cup \mathit{body}(P)]\} \\
\mathcal{T}_{\mathit{assat}} &= \mathcal{T}_{\mathit{completion}} \cup \{\mathit{FL}\} \cup \{\mathit{Cut}[\mathit{atom}(P) \cup \mathit{body}(P)]\} \\
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- SAT-based ASP solvers, *assat* and *cmodels*, incrementally add loop formulas to a program's completion
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Proof complexity

- Proof complexity is used for describing the relative efficiency of different proof systems
  - It compares proof systems based on minimal refutations
  - It is independent of heuristics
- A proof system $\mathcal{T}$ polynomially simulates a proof system $\mathcal{T}'$, if every refutation of $\mathcal{T}'$ can be polynomially mapped to a refutation of $\mathcal{T}$
  - Otherwise, $\mathcal{T}$ does not polynomially simulate $\mathcal{T}'$
- For showing that proof system $\mathcal{T}$ does not polynomially simulate $\mathcal{T}'$, we have to provide an infinite witnessing family of programs such that minimal refutations of $\mathcal{T}$ asymptotically are exponentially larger than minimal refutations of $\mathcal{T}'$
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$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

- $\mathcal{T}_{smodels}$ restricts Cut to $atom(P)$ and $\mathcal{T}_{noMoRe}$ to $body(P)$
  Are both approaches similar or is one of them superior to the other?
- Let $\{P^n_a\}$, $\{P^n_b\}$, and $\{P^n_c\}$ be infinite families of programs where

$$P^n_a = \left\{ \begin{array}{c} x \leftarrow \sim x \\ x \leftarrow a_1, b_1 \\ \vdots \\ x \leftarrow a_n, b_n \end{array} \right\} \quad P^n_b = \left\{ \begin{array}{cc} \multicolumn{2}{c}{x \leftarrow c_1, \dots, c_n, \sim x} \\ c_1 \leftarrow a_1 & c_1 \leftarrow b_1 \\ \vdots & \vdots \\ c_n \leftarrow a_n & c_n \leftarrow b_n \end{array} \right\} \quad P^n_c = \left\{ \begin{array}{c} a_1 \leftarrow \sim b_1 \\ b_1 \leftarrow \sim a_1 \\ \vdots \\ a_n \leftarrow \sim b_n \\ b_n \leftarrow \sim a_n \end{array} \right\}$$

- In minimal refutations for $P^n_a \cup P^n_c$, the number of applications of $Cut[body(P^n_a \cup P^n_c)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $Cut[atom(P^n_a \cup P^n_c)]$
  Vice versa, minimal refutations for $P^n_b \cup P^n_c$ require linearly many applications of $Cut[atom(P^n_b \cup P^n_c)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $Cut[body(P^n_b \cup P^n_c)]$ with $\mathcal{T}_{noMoRe}$

Potassco

$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

- $\mathcal{T}_{smodels}$ restricts Cut to $atom(P)$ and $\mathcal{T}_{noMoRe}$ to $body(P)$
  Are both approaches similar or is one of them superior to the other?
- Let $\{P_a^n\}$, $\{P_b^n\}$, and $\{P_c^n\}$ be infinite families of programs where

$$P_a^n = \left\{ \begin{array}{c} x \leftarrow \sim x \\ x \leftarrow a_1, b_1 \\ \vdots \\ x \leftarrow a_n, b_n \end{array} \right\} \quad P_b^n = \left\{ \begin{array}{cc} \multicolumn{2}{c}{x \leftarrow c_1, \dots, c_n, \sim x} \\ c_1 \leftarrow a_1 & c_1 \leftarrow b_1 \\ \vdots & \vdots \\ c_n \leftarrow a_n & c_n \leftarrow b_n \end{array} \right\} \quad P_c^n = \left\{ \begin{array}{c} a_1 \leftarrow \sim b_1 \\ b_1 \leftarrow \sim a_1 \\ \vdots \\ a_n \leftarrow \sim b_n \\ b_n \leftarrow \sim a_n \end{array} \right\}$$

- In minimal refutations for $P_a^n \cup P_c^n$, the number of applications of $Cut[body(P_a^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $Cut[atom(P_a^n \cup P_c^n)]$
  Vice versa, minimal refutations for $P_b^n \cup P_c^n$ require linearly many applications of $Cut[atom(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $Cut[body(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$

Potassco

$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

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  Are both approaches similar or is one of them superior to the other?

- Let $\{P_a^n\}$, $\{P_b^n\}$, and $\{P_c^n\}$ be infinite families of programs where

$$P_a^n = \left\{ \begin{array}{c} x \leftarrow \sim x \\ x \leftarrow a_1, b_1 \\ \vdots \\ x \leftarrow a_n, b_n \end{array} \right\} \quad P_b^n = \left\{ \begin{array}{cc} \multicolumn{2}{c}{x \leftarrow c_1, \dots, c_n, \sim x} \\ c_1 \leftarrow a_1 & c_1 \leftarrow b_1 \\ \vdots & \vdots \\ c_n \leftarrow a_n & c_n \leftarrow b_n \end{array} \right\} \quad P_c^n = \left\{ \begin{array}{c} a_1 \leftarrow \sim b_1 \\ b_1 \leftarrow \sim a_1 \\ \vdots \\ a_n \leftarrow \sim b_n \\ b_n \leftarrow \sim a_n \end{array} \right\}$$

- In minimal refutations for $P_a^n \cup P_c^n$, the number of applications of $Cut[body(P_a^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $Cut[atom(P_a^n \cup P_c^n)]$
  Vice versa, minimal refutations for $P_b^n \cup P_c^n$ require linearly many applications of $Cut[atom(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $Cut[body(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$

Potassco

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- In minimal refutations for $P_a^n \cup P_c^n$, the number of applications of $Cut[body(P_a^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $Cut[atom(P_a^n \cup P_c^n)]$
- Vice versa, minimal refutations for $P_b^n \cup P_c^n$ require linearly many applications of $Cut[atom(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $Cut[body(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$

Potassco

$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

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- In minimal refutations for $P_a^n \cup P_c^n$, the number of applications of $Cut[body(P_a^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $Cut[atom(P_a^n \cup P_c^n)]$

- Vice versa, minimal refutations for $P_b^n \cup P_c^n$ require linearly many applications of $Cut[atom(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $Cut[body(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$

Potassco

$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

- $\mathcal{T}_{smodels}$ restricts *Cut* to *atom*(P) and $\mathcal{T}_{noMoRe}$ to *body*(P)
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- Let $\{P_a^n\}$, $\{P_b^n\}$, and $\{P_c^n\}$ be infinite families of programs where

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- Vice versa, minimal refutations for $P_b^n \cup P_c^n$ require linearly many applications of $Cut[atom(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $Cut[body(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$

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$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

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  Are both approaches similar or is one of them superior to the other?
- Let $\{P^n_a\}$, $\{P^n_b\}$, and $\{P^n_c\}$ be infinite families of programs where

$$P^n_a = \left\{ \begin{array}{c} x \leftarrow \sim x \\ x \leftarrow a_1, b_1 \\ \vdots \\ x \leftarrow a_n, b_n \end{array} \right\} \quad P^n_b = \left\{ \begin{array}{cc} \multicolumn{2}{c}{x \leftarrow c_1, \dots, c_n, \sim x} \\ c_1 \leftarrow a_1 & c_1 \leftarrow b_1 \\ \vdots & \vdots \\ c_n \leftarrow a_n & c_n \leftarrow b_n \end{array} \right\} \quad P^n_c = \left\{ \begin{array}{c} a_1 \leftarrow \sim b_1 \\ b_1 \leftarrow \sim a_1 \\ \vdots \\ a_n \leftarrow \sim b_n \\ b_n \leftarrow \sim a_n \end{array} \right\}$$

- In minimal refutations for $P^n_a \cup P^n_c$, the number of applications of $Cut[body(P^n_a \cup P^n_c)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $Cut[atom(P^n_a \cup P^n_c)]$
- Vice versa, minimal refutations for $P^n_b \cup P^n_c$ require linearly many applications of $Cut[atom(P^n_b \cup P^n_c)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $Cut[body(P^n_b \cup P^n_c)]$ with $\mathcal{T}_{noMoRe}$

Potassco

$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

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Relative efficiency

- As witnessed by $\{P_a^n \cup P_c^n\}$ and $\{P_b^n \cup P_c^n\}$, respectively, $\mathcal{T}_{smodels}$ and $\mathcal{T}_{noMoRe}$ do not polynomially simulate one another
- Any refutation of $\mathcal{T}_{smodels}$ or $\mathcal{T}_{noMoRe}$ is a refutation of $\mathcal{T}_{nomore^{++}}$ (but not vice versa)
- Hence
  - both $\mathcal{T}_{smodels}$ and $\mathcal{T}_{noMoRe}$ are polynomially simulated by $\mathcal{T}_{nomore^{++}}$ and
  - $\mathcal{T}_{nomore^{++}}$ is polynomially simulated by neither $\mathcal{T}_{smodels}$ nor $\mathcal{T}_{noMoRe}$
- More generally, the proof system obtained with $Cut[atom(P) \cup body(P)]$ is exponentially stronger than the ones with either $Cut[atom(P)]$ or $Cut[body(P)]$
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$\mathcal{T}_{smodels}$: Example tableau

(r_1)	$a \leftarrow {\sim}b$	(r_2)	$b \leftarrow d, {\sim}a$	(r_3)	$c \leftarrow b, d$
(r_4)	$c \leftarrow g$	(r_5)	$d \leftarrow c$	(r_6)	$d \leftarrow g$
(r_7)	$e \leftarrow f, {\sim}c$	(r_8)	$f \leftarrow {\sim}g$	(r_9)	$g \leftarrow {\sim}a, {\sim}f$

(1)	$\mathbf{T}a$	[*Cut*]
(2)	$\mathbf{T}\{{\sim}b\}$	[*BTA*: r_1, 1]
(3)	$\mathbf{F}b$	[*BTB*: 2]
(4)	$\mathbf{F}\{d, {\sim}a\}$	[*BFA*: r_2, 3]
(5)	$\mathbf{F}\{{\sim}a, {\sim}f\}$	[*FFB*: r_9, 1]
(6)	$\mathbf{F}g$	[*FFA*: r_9, 5]
(7)	$\mathbf{T}\{{\sim}g\}$	[*FTB*: r_8, 6]
(8)	$\mathbf{T}f$	[*FTA*: r_8, 7]
(9)	$\mathbf{F}\{b, d\}$	[*FFB*: r_3, 3]
(10)	$\mathbf{F}\{g\}$	[*FFB*: r_4, r_6, 6]
(11)	$\mathbf{F}c$	[*FFA*: r_3, r_4, 9, 10]
(12)	$\mathbf{F}\{c\}$	[*FFB*: r_5, 11]
(13)	$\mathbf{F}d$	[*FFA*: r_5, r_6, 10, 12]
(14)	$\mathbf{T}\{f, {\sim}c\}$	[*FTB*: r_7, 8, 11]
(15)	$\mathbf{T}e$	[*FTA*: r_7, 14]

(16)	$\mathbf{F}a$	[*Cut*]
(17)	$\mathbf{F}\{{\sim}b\}$	[*BFA*: r_1, 16]
(18)	$\mathbf{T}b$	[*BFB*: 17]
(19)	$\mathbf{T}\{d, {\sim}a\}$	[*BTA*: r_2, 18]
(20)	$\mathbf{T}d$	[*BTB*: 19]
(21)	$\mathbf{T}\{b, d\}$	[*FTB*: r_3, 18, 20]
(22)	$\mathbf{T}c$	[*FTA*: r_3, 21]
(23)	$\mathbf{F}\{f, {\sim}c\}$	[*FFB*: r_7, 22]
(24)	$\mathbf{F}e$	[*FFA*: r_7, 23]
(25)	$\mathbf{T}\{c\}$	[*FTB*: r_5, 22]

(26)	$\mathbf{T}f$	[*Cut*]
(27)	$\mathbf{F}\{{\sim}a, {\sim}f\}$	[*FFB*: r_9, 26]
(28)	$\mathbf{F}c$	[*WFN*: 27]

(29)	$\mathbf{F}f$	[*Cut*]
(30)	$\mathbf{T}\{{\sim}a, {\sim}f\}$	[*FTB*: r_9, 16, 29]
(31)	$\mathbf{T}g$	[*FTA*: r_9, 30]
(32)	$\mathbf{T}\{g\}$	[*FTB*: r_4, r_6, 31]
(33)	$\mathbf{F}\{{\sim}g\}$	[*FFB*: r_8, 31]

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$\mathcal{T}_{noMoRe}$: Example tableau

(r_1)	$a \leftarrow \sim b$	(r_2)	$b \leftarrow d, \sim a$	(r_3)	$c \leftarrow b, d$
(r_4)	$c \leftarrow g$	(r_5)	$d \leftarrow c$	(r_6)	$d \leftarrow g$
(r_7)	$e \leftarrow f, \sim c$	(r_8)	$f \leftarrow \sim g$	(r_9)	$g \leftarrow \sim a, \sim f$

(1)	$\mathbf{T}\{\sim b\}$	[*Cut*]
(2)	$\mathbf{T}a$	[*FTA*: r_1, 1]
(3)	$\mathbf{F}b$	[*BTB*: 1]
(4)	$\mathbf{F}\{d, \sim a\}$	[*BFA*: r_2, 3]
(5)	$\mathbf{F}\{\sim a, \sim f\}$	[*FFB*: r_9, 2]
(6)	$\mathbf{F}g$	[*FFA*: r_9, 5]
(7)	$\mathbf{T}\{\sim g\}$	[*FTB*: r_8, 6]
(8)	$\mathbf{T}f$	[*FTA*: r_8, 7]
(9)	$\mathbf{F}\{b, d\}$	[*FFB*: r_3, 3]
(10)	$\mathbf{F}\{g\}$	[*FFB*: r_4, r_6, 6]
(11)	$\mathbf{F}c$	[*FFA*: r_3, r_4, 9, 10]
(12)	$\mathbf{F}\{c\}$	[*FFB*: r_5, 11]
(13)	$\mathbf{F}d$	[*FFA*: r_5, r_6, 10, 12]
(14)	$\mathbf{T}\{f, \sim c\}$	[*FTB*: r_7, 8, 11]
(15)	$\mathbf{T}e$	[*FTA*: r_7, 14]

(16)	$\mathbf{F}\{\sim b\}$	[*Cut*]
(17)	$\mathbf{F}a$	[*FFA*: r_1, 16]
(18)	$\mathbf{T}b$	[*BFB*: 16]
(19)	$\mathbf{T}\{d, \sim a\}$	[*BTA*: r_2, 18]
(20)	$\mathbf{T}d$	[*BTB*: 19]
(21)	$\mathbf{T}\{b, d\}$	[*FTB*: r_3, 18, 20]
(22)	$\mathbf{T}c$	[*FTA*: r_3, 21]
(23)	$\mathbf{F}\{f, \sim c\}$	[*FFB*: r_7, 22]
(24)	$\mathbf{F}e$	[*FFA*: r_7, 23]
(25)	$\mathbf{T}\{c\}$	[*FTB*: r_5, 22]

(26)	$\mathbf{T}\{\sim g\}$	[*Cut*]
(27)	$\mathbf{F}g$	[*BTB*: 26]
(28)	$\mathbf{F}\{g\}$	[*FFB*: r_4, r_6, 27]
(29)	$\mathbf{F}c$	[*WFN*: 28]

(30)	$\mathbf{F}\{\sim g\}$	[*Cut*]
(31)	$\mathbf{T}g$	[*BFB*: 30]
(32)	$\mathbf{T}\{g\}$	[*FTB*: r_4, r_6, 31]
(33)	$\mathbf{F}f$	[*FFA*: r_8, 30]
(34)	$\mathbf{T}\{\sim a, \sim f\}$	[*FTB*: r_9, 17, 33]

$\mathcal{T}_{nomore^{++}}$: Example tableau

(r_1)	$a \leftarrow \sim b$	(r_2)	$b \leftarrow d, \sim a$	(r_3)	$c \leftarrow b, d$
(r_4)	$c \leftarrow g$	(r_5)	$d \leftarrow c$	(r_6)	$d \leftarrow g$
(r_7)	$e \leftarrow f, \sim c$	(r_8)	$f \leftarrow \sim g$	(r_9)	$g \leftarrow \sim a, \sim f$

(1)	$\mathbf{T}a$	$[Cut]$
(2)	$\mathbf{T}\{\sim b\}$	$[BTA\colon r_1, 1]$
(3)	$\mathbf{F}b$	$[BTB\colon 2]$
(4)	$\mathbf{F}\{d, \sim a\}$	$[BFA\colon r_2, 3]$
(5)	$\mathbf{F}\{\sim a, \sim f\}$	$[FFB\colon r_9, 1]$
(6)	$\mathbf{F}g$	$[FFA\colon r_9, 5]$
(7)	$\mathbf{T}\{\sim g\}$	$[FTB\colon r_8, 6]$
(8)	$\mathbf{T}f$	$[FTA\colon r_8, 7]$
(9)	$\mathbf{F}\{b, d\}$	$[FFB\colon r_3, 3]$
(10)	$\mathbf{F}\{g\}$	$[FFB\colon r_4, r_6, 6]$
(11)	$\mathbf{F}c$	$[FFA\colon r_3, r_4, 9, 10]$
(12)	$\mathbf{F}\{c\}$	$[FFB\colon r_5, 11]$
(13)	$\mathbf{F}d$	$[FFA\colon r_5, r_6, 10, 12]$
(14)	$\mathbf{T}\{f, \sim c\}$	$[FTB\colon r_7, 8, 11]$
(15)	$\mathbf{T}e$	$[FTA\colon r_7, 14]$

(16)	$\mathbf{F}a$	$[Cut]$
(17)	$\mathbf{F}\{\sim b\}$	$[BFA\colon r_1, 16]$
(18)	$\mathbf{T}b$	$[BFB\colon 17]$
(19)	$\mathbf{T}\{d, \sim a\}$	$[BTA\colon r_2, 18]$
(20)	$\mathbf{T}d$	$[BTB\colon 19]$
(21)	$\mathbf{T}\{b, d\}$	$[FTB\colon r_3, 18, 20]$
(22)	$\mathbf{T}c$	$[FTA\colon r_3, 21]$
(23)	$\mathbf{F}\{f, \sim c\}$	$[FFB\colon r_7, 22]$
(24)	$\mathbf{F}e$	$[FFA\colon r_7, 23]$
(25)	$\mathbf{T}\{c\}$	$[FTB\colon r_5, 22]$

(26)	$\mathbf{T}\{\sim g\}$	$[Cut]$
(27)	$\mathbf{F}g$	$[BTB\colon 26]$
(28)	$\mathbf{F}\{g\}$	$[FFB\colon r_4, r_6, 27]$
(29)	$\mathbf{F}c$	$[WFN\colon 28]$

(30)	$\mathbf{F}\{\sim g\}$	$[Cut]$
(31)	$\mathbf{T}g$	$[BFB\colon 30]$
(32)	$\mathbf{T}\{g\}$	$[FTB\colon r_4, r_6, 31]$
(33)	$\mathbf{F}f$	$[FFA\colon r_8, 30]$
(34)	$\mathbf{T}\{\sim a, \sim f\}$	$[FTB\colon r_9, 16, 33]$

Conflict-driven ASP Solving: Overview

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Motivation

- Goal Approach to computing stable models of logic programs, based on concepts from
  - Constraint Processing (CP) and
  - Satisfiability Testing (SAT)
- Idea View inferences in ASP as unit propagation on nogoods
- Benefits
  - A uniform constraint-based framework for different kinds of inferences in ASP
  - Advanced techniques from the areas of CP and SAT
  - Highly competitive implementation

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Assignments

- An assignment A over $dom(A) = atom(P) \cup body(P)$ is a sequence

$$(\sigma_1, \dots, \sigma_n)$$

  of signed literals σ_i of form $\mathbf{T}v$ or $\mathbf{F}v$ for $v \in dom(A)$ and $1 \leq i \leq n$
- $\mathbf{T}v$ expresses that v is *true* and $\mathbf{F}v$ that it is *false*
- The complement, $\overline{\sigma}$, of a literal σ is defined as $\overline{\mathbf{T}v} = \mathbf{F}v$ and $\overline{\mathbf{F}v} = \mathbf{T}v$
- $A \circ \sigma$ stands for the result of appending σ to A
- Given $A = (\sigma_1, \dots, \sigma_{k-1}, \sigma_k, \dots, \sigma_n)$, we let $A[\sigma_k] = (\sigma_1, \dots, \sigma_{k-1})$
- We sometimes identify an assignment with the set of its literals
- Given this, we access *true* and *false* propositions in A via

$$A^{\mathbf{T}} = \{v \in dom(A) \mid \mathbf{T}v \in A\} \text{ and } A^{\mathbf{F}} = \{v \in dom(A) \mid \mathbf{F}v \in A\}$$

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Nogoods, solutions, and unit propagation

- A nogood is a set $\{\sigma_1, \dots, \sigma_n\}$ of signed literals, expressing a constraint violated by any assignment containing $\sigma_1, \dots, \sigma_n$
- An assignment A such that $A^{\mathbf{T}} \cup A^{\mathbf{F}} = dom(A)$ and $A^{\mathbf{T}} \cap A^{\mathbf{F}} = \emptyset$ is a solution for a set Δ of nogoods, if $\delta \not\subseteq A$ for all $\delta \in \Delta$
- For a nogood δ, a literal $\sigma \in \delta$, and an assignment A, we say that $\overline{\sigma}$ is unit-resulting for δ wrt A, if
  1. $\delta \setminus A = \{\sigma\}$ and
  2. $\overline{\sigma} \notin A$
- For a set Δ of nogoods and an assignment A, unit propagation is the iterated process of extending A with unit-resulting literals until no further literal is unit-resulting for any nogood in Δ

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Nogoods from logic programs
via program completion

The completion of a logic program P can be defined as follows:

$$
\begin{aligned}
& \{v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n \mid \\
& \qquad B \in \mathit{body}(P) \text{ and } B = \{a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n\}\} \\
\cup\; & \{a \leftrightarrow v_{B_1} \vee \cdots \vee v_{B_k} \mid \\
& \qquad a \in \mathit{atom}(P) \text{ and } \mathit{body}_P(a) = \{B_1, \dots, B_k\}\}\ ,
\end{aligned}
$$

where $\mathit{body}_P(a) = \{\mathit{body}(r) \mid r \in P \text{ and } \mathit{head}(r) = a\}$

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Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \land \cdots \land a_m \land \neg a_{m+1} \land \cdots \land \neg a_n$$

can be decomposed into two implications:

Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$$

can be decomposed into two implications:

1. $v_B \rightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$

   is equivalent to the conjunction of

   $$\neg v_B \vee a_1, \ \ldots, \ \neg v_B \vee a_m, \ \neg v_B \vee \neg a_{m+1}, \ \ldots, \ \neg v_B \vee \neg a_n$$

   and induces the set of nogoods

   $$\Delta(B) = \{ \{\mathbf{T}B, \mathbf{F}a_1\}, \ldots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \ldots, \{\mathbf{T}B, \mathbf{T}a_n\} \}$$

Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \land \cdots \land a_m \land \neg a_{m+1} \land \cdots \land \neg a_n$$

can be decomposed into two implications:

2. $a_1 \land \cdots \land a_m \land \neg a_{m+1} \land \cdots \land \neg a_n \rightarrow v_B$

   gives rise to the nogood

$$\delta(B) = \{\mathbf{F}B, \mathbf{T}a_1, \ldots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \ldots, \mathbf{F}a_n\}$$

Nogoods from logic programs
via program completion

- Analogously, the (atom-oriented) equivalence

  $$a \leftrightarrow v_{B_1} \vee \cdots \vee v_{B_k}$$

  yields the nogoods

  1. $\Delta(a) = \{\, \{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\, \}$ and
  2. $\delta(a) = \{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\}$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

$\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
$\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\}$$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

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Nogoods from logic programs
atom-oriented nogoods

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Nogoods from logic programs
atom-oriented nogoods

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Nogoods from logic programs
atom-oriented nogoods

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$$\begin{array}{|lcl|}\hline x & \leftarrow & y \\ x & \leftarrow & \sim z \\ \hline\end{array}$$

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Nogoods from logic programs
atom-oriented nogoods

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Nogoods from logic programs
atom-oriented nogoods

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Nogoods from logic programs
atom-oriented nogoods

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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$$\{\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

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atom-oriented nogoods

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atom-oriented nogoods

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x	$\leftarrow$	y
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atom-oriented nogoods

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Nogoods from logic programs
body-oriented nogoods

- For a body $B = \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$, we get

$$\{\mathbf{F}B, \mathbf{T}a_1, \dots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \dots, \mathbf{F}a_n\}$$
$$\{\,\{\mathbf{T}B, \mathbf{F}a_1\}, \dots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \dots, \{\mathbf{T}B, \mathbf{T}a_n\}\,\}$$

- Example Given Body $\{x, \sim y\}$, we obtain

$$\dots \leftarrow x, \sim y$$
$$\vdots$$
$$\dots \leftarrow x, \sim y$$

$$\{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$$
$$\{\,\{\mathbf{T}\{x, \sim y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, \sim y\}, \mathbf{T}y\}\,\}$$

For nogood $\delta(\{x, \sim y\}) = \{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$, the signed literal

- $\mathbf{T}\{x, \sim y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
- $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, \sim y\}, \mathbf{T}x)$

Nogoods from logic programs
body-oriented nogoods

- For a body $B = \{a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n\}$, we get

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Nogoods from logic programs
body-oriented nogoods

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Characterization of stable models

for tight logic programs

Let P be a logic program and

$$\begin{array}{rcl} \Delta_P & = & \{\delta(a) \mid a \in atom(P)\} \cup \{\delta \in \Delta(a) \mid a \in atom(P)\} \\ & \cup & \{\delta(B) \mid B \in body(P)\} \cup \{\delta \in \Delta(B) \mid B \in body(P)\} \end{array}$$

Theorem

Let P be a tight logic program. Then,
$X \subseteq atom(P)$ is a stable model of P iff
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Characterization of stable models

for tight logic programs, ie. free of positive recursion

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Nogoods from logic programs
via loop formulas

Let P be a normal logic program and recall that:

- For $L \subseteq atom(P)$, the external supports of L for P are

$$ES_P(L) = \{r \in P \mid head(r) \in L \text{ and } body(r)^+ \cap L = \emptyset\}$$

- The (disjunctive) loop formula of L for P is

$$\begin{aligned} LF_P(L) &= (\bigvee_{A \in L} A) \rightarrow (\bigvee_{r \in ES_P(L)} body(r)) \\ &\equiv (\bigwedge_{r \in ES_P(L)} \neg body(r)) \rightarrow (\bigwedge_{A \in L} \neg A) \end{aligned}$$

  - Note The loop formula of L enforces all atoms in L to be *false* whenever L is not externally supported

- The external bodies of L for P are

$$EB_P(L) = \{body(r) \mid r \in ES_P(L)\}$$

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via loop formulas

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Nogoods from logic programs
loop nogoods

- For a logic program P and some $\emptyset \subset U \subseteq atom(P)$, define the loop nogood of an atom $a \in U$ as

$$\lambda(a, U) \quad = \quad \{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\}$$

where $EB_P(U) = \{B_1, \dots, B_k\}$

- We get the following set of loop nogoods for P:

$$\Lambda_P \quad = \quad \bigcup_{\emptyset \subset U \subseteq atom(P)} \{\lambda(a, U) \mid a \in U\}$$

- The set Λ_P of loop nogoods denies cyclic support among *true* atoms

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Example

- Consider the program

$$\left\{\begin{array}{ll} & u \leftarrow x \\ x \leftarrow {\sim}y & u \leftarrow v \\ y \leftarrow {\sim}x & v \leftarrow u, y \end{array}\right\}$$

- For u in the set $\{u, v\}$, we obtain the loop nogood:

$$\lambda(u, \{u, v\}) = \{\mathbf{T}u, \mathbf{F}\{x\}\}$$

Similarly for v in $\{u, v\}$, we get:

$$\lambda(v, \{u, v\}) = \{\mathbf{T}v, \mathbf{F}\{x\}\}$$

Example

- Consider the program

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Characterization of stable models

Theorem

Let P be a logic program. Then,

$X \subseteq atom(P)$ *is a stable model of* P *iff*

$X = A^{\mathbf{T}} \cap atom(P)$ *for a (unique) solution* A *for* $\Delta_P \cup \Lambda_P$

Some remarks

- Nogoods in Λ_P augment Δ_P with conditions checking for unfounded sets, in particular, those being loops
- While $|\Delta_P|$ is linear in the size of P, Λ_P may contain exponentially many (non-redundant) loop nogoods

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Towards conflict-driven search

Boolean constraint solving algorithms pioneered for SAT led to:

- Traditional DPLL-style approach
  (DPLL stands for 'Davis-Putnam-Logemann-Loveland')
  - (Unit) propagation
  - (Chronological) backtracking
  - in ASP, eg *smodels*
- Modern CDCL-style approach
  (CDCL stands for 'Conflict-Driven Constraint Learning')
  - (Unit) propagation
  - Conflict analysis (via resolution)
  - Learning + Backjumping + Assertion
  - in ASP, eg *clasp*

DPLL-style solving

loop

 propagate // deterministically assign literals

 if no conflict **then**

  if all variables assigned **then return** solution

  else *decide* // non-deterministically assign some literal

 else

  if top-level conflict **then return** unsatisfiable

  else

   backtrack // unassign literals propagated after last decision

   flip // assign complement of last decision literal

CDCL-style solving

loop

propagate // deterministically assign literals

if no conflict **then**

if all variables assigned **then return** solution

else *decide* // non-deterministically assign some literal

else

if top-level conflict **then return** unsatisfiable

else

analyze // analyze conflict and add conflict constraint

backjump // unassign literals until conflict constraint is unit

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Outline of CDNL-ASP algorithm

- Keep track of deterministic consequences by unit propagation on:
  - Program completion [Δ_P]
  - Loop nogoods, determined and recorded on demand [Λ_P]
  - Dynamic nogoods, derived from conflicts and unfounded sets [∇]
- When a nogood in $\Delta_P \cup \nabla$ becomes violated:
  - Analyze the conflict by resolution (until reaching a Unique Implication Point, short: UIP)
  - Learn the derived conflict nogood δ
  - Backjump to the earliest (heuristic) choice such that the complement of the UIP is unit-resulting for δ
  - Assert the complement of the UIP and proceed (by unit propagation)
- Terminate when either:
  - Finding a stable model (a solution for $\Delta_P \cup \Lambda_P$)
  - Deriving a conflict independently of (heuristic) choices

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Algorithm 2: CDNL-ASP

Input : A normal program P
Output : A stable model of P or "no stable model"

$A := \emptyset$ // *assignment over* $atom(P) \cup body(P)$
$\nabla := \emptyset$ // *set of recorded nogoods*
$dl := 0$ // *decision level*

loop
  $(A, \nabla) := \textsc{NogoodPropagation}(P, \nabla, A)$
  if $\varepsilon \subseteq A$ **for some** $\varepsilon \in \Delta_P \cup \nabla$ **then** // *conflict*
    if $\max(\{dlevel(\sigma) \mid \sigma \in \varepsilon\} \cup \{0\}) = 0$ **then return** no stable model
    $(\delta, dl) := \textsc{ConflictAnalysis}(\varepsilon, P, \nabla, A)$
    $\nabla := \nabla \cup \{\delta\}$ // *(temporarily) record conflict nogood*
    $A := A \setminus \{\sigma \in A \mid dl < dlevel(\sigma)\}$ // *backjumping*
  else if $A^{\mathbf{T}} \cup A^{\mathbf{F}} = atom(P) \cup body(P)$ **then** // *stable model*
    return $A^{\mathbf{T}} \cap atom(P)$
  else
    $\sigma_d := \textsc{Select}(P, \nabla, A)$ // *decision*
    $dl := dl + 1$
    $dlevel(\sigma_d) := dl$
    $A := A \circ \sigma_d$

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Observations

- Decision level dl, initially set to 0, is used to count the number of heuristically chosen literals in assignment A
- For a heuristically chosen literal $\sigma_d = \mathbf{T}a$ or $\sigma_d = \mathbf{F}a$, respectively, we require $a \in (atom(P) \cup body(P)) \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}})$
- For any literal $\sigma \in A$, $dl(\sigma)$ denotes the decision level of σ, viz. the value dl had when σ was assigned
- A conflict is detected from violation of a nogood $\varepsilon \subseteq \Delta_P \cup \nabla$
- A conflict at decision level 0 (where A contains no heuristically chosen literals) indicates non-existence of stable models
- A nogood δ derived by conflict analysis is asserting, that is, some literal is unit-resulting for δ at a decision level $k < dl$
  - After learning δ and backjumping to decision level k, at least one literal is newly derivable by unit propagation
  - No explicit flipping of heuristically chosen literals !

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Example: CDNL-ASP

Consider

$$P = \left\{ \begin{array}{llll} x \leftarrow \sim y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \sim x, \sim y \\ y \leftarrow \sim x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ
1	$\mathbf{T}u$		
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Outline

33 Motivation

34 Boolean constraints

35 Nogoods from logic programs
- Nogoods from program completion
- Nogoods from loop formulas

36 Conflict-driven nogood learning
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- Conflict Analysis

Outline of NogoodPropagation

- Derive deterministic consequences via:
  - Unit propagation on Δ_P and ∇;
  - Unfounded sets $U \subseteq atom(P)$
- Note that U is unfounded if $EB_P(U) \subseteq A^{\mathbf{F}}$
  - Note For any $a \in U$, we have $(\lambda(a, U) \setminus \{\mathbf{T}a\}) \subseteq A$
- An "interesting" unfounded set U satisfies:

$$\emptyset \subset U \subseteq (atom(P) \setminus A^{\mathbf{F}})$$

- Wrt a fixpoint of unit propagation, such an unfounded set contains some loop of P
  - Note Tight programs do not yield "interesting" unfounded sets !
- Given an unfounded set U and some $a \in U$, adding $\lambda(a, U)$ to ∇ triggers a conflict or further derivations by unit propagation
  - Note Add loop nogoods atom by atom to eventually falsify all $a \in U$

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Algorithm 3: NogoodPropagation

Input : A normal program P, a set ∇ of nogoods, and an assignment A.
Output : An extended assignment and set of nogoods.

```
U := ∅                                                        // unfounded set
loop
    repeat
        if δ ⊆ A for some δ ∈ Δ_P ∪ ∇ then return (A, ∇)                // conflict
        Σ := {δ ∈ Δ_P ∪ ∇ | δ \ A = {σ̄}, σ ∉ A}              // unit-resulting nogoods
        if Σ ≠ ∅ then let σ̄ ∈ δ \ A for some δ ∈ Σ in
            dlevel(σ) := max({dlevel(ρ) | ρ ∈ δ \ {σ̄}} ∪ {0})
            A := A ∘ σ
    until Σ = ∅
    if loop(P) = ∅ then return (A, ∇)
    U := U \ A^F
    if U = ∅ then U := UnfoundedSet(P, A)
    if U = ∅ then return (A, ∇)        // no unfounded set ∅ ⊂ U ⊆ atom(P) \ A^F
    let a ∈ U in
        ∇ := ∇ ∪ {{Ta} ∪ {FB | B ∈ EB_P(U)}}                   // record loop nogood
```

Requirements for UnfoundedSet

- Implementations of UnfoundedSet must guarantee the following for a result U
  1. $U \subseteq (atom(P) \setminus A^{\mathbf{F}})$
  2. $EB_P(U) \subseteq A^{\mathbf{F}}$
  3. $U = \emptyset$ iff there is no nonempty unfounded subset of $(atom(P) \setminus A^{\mathbf{F}})$
- Beyond that, there are various alternatives, such as:
  - Calculating the greatest unfounded set
  - Calculating unfounded sets within strongly connected components of the positive atom dependency graph of P
  - Usually, the latter option is implemented in ASP solvers

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Example: NogoodPropagation

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Outline of ConflictAnalysis

- Conflict analysis is triggered whenever some nogood $\delta \in \Delta_P \cup \nabla$ becomes violated, viz. $\delta \subseteq A$, at a decision level $dl > 0$
  - Note that all but the first literal assigned at dl have been unit-resulting for nogoods $\varepsilon \in \Delta_P \cup \nabla$
  - If $\sigma \in \delta$ has been unit-resulting for ε, we obtain a new violated nogood by resolving δ and ε as follows:

$$(\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$$

- Resolution is directed by resolving first over the literal $\sigma \in \delta$ derived last, viz. $(\delta \setminus A[\sigma]) = \{\sigma\}$
  - Iterated resolution progresses in inverse order of assignment
- Iterated resolution stops as soon as it generates a nogood δ containing exactly one literal σ assigned at decision level dl
  - This literal σ is called First Unique Implication Point (First-UIP)
  - All literals in $(\delta \setminus \{\sigma\})$ are assigned at decision levels smaller than dl

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Algorithm 4: CONFLICTANALYSIS

Input : A non-empty violated nogood δ, a normal program P, a set ∇ of nogoods, and an assignment A.

Output : A derived nogood and a decision level.

```
loop
    let σ ∈ δ such that δ \ A[σ] = {σ} in
        k := max({dlevel(ρ) | ρ ∈ δ \ {σ}} ∪ {0})
        if k = dlevel(σ) then
            let ε ∈ Δ_P ∪ ∇ such that ε \ A[σ] = {σ̄} in
                δ := (δ \ {σ}) ∪ (ε \ {σ̄})        // resolution
        else return (δ, k)
```

Example: ConflictAnalysis

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Potassco

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Remarks

- There always is a First-UIP at which conflict analysis terminates
  - In the worst, resolution stops at the heuristically chosen literal assigned at decision level dl
- The nogood δ containing First-UIP σ is violated by A, viz. $\delta \subseteq A$
- We have $k = max(\{dl(\rho) \mid \rho \in \delta \setminus \{\sigma\}\} \cup \{0\}) < dl$
  - After recording δ in ∇ and backjumping to decision level k, $\overline{\sigma}$ is unit-resulting for δ !
  - Such a nogood δ is called asserting
- Asserting nogoods direct conflict-driven search into a different region of the search space than traversed before, without explicitly flipping any heuristically chosen literal !

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Potassco

Systems: Overview

37 Potassco

38 gringo

39 clasp

40 Siblings

- hclasp
- claspfolio
- claspD
- clingcon
- iclingo
- oclingo
- clavis

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Potassco

potassco.sourceforge.net

Potassco, the Potsdam Answer Set Solving Collection,
bundles tools for ASP developed at the University of Potsdam,
for instance:

- Grounder gringo, lingo, pyngo
- Solver clasp, {a,h,un}clasp, claspD, claspfolio, claspar, aspeed
- Grounder+Solver Clingo, iClingo, {ros}oClingo, Clingcon
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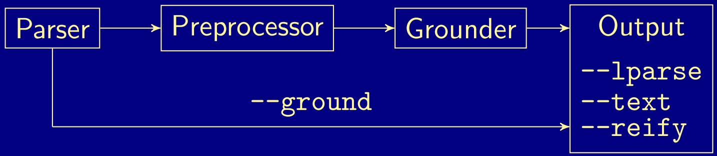
gringo

- Accepts safe programs with aggregates
- Tolerates unrestricted use of function symbols
  (as long as it yields a finite ground instantiation :)
- Expressive power of a Turing machine

- Basic architecture of *gringo*:



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- *clasp* is a native ASP solver combining conflict-driven search with sophisticated reasoning techniques:
  - advanced preprocessing, including equivalence reasoning
  - lookback-based decision heuristics
  - restart policies
  - nogood deletion
  - progress saving
  - dedicated data structures for binary and ternary nogoods
  - lazy data structures (watched literals) for long nogoods
  - dedicated data structures for cardinality and weight constraints
  - lazy unfounded set checking based on "source pointers"
  - tight integration of unit propagation and unfounded set checking
  - various reasoning modes
  - parallel search
  - ...

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Reasoning modes of *clasp*

- Beyond deciding (stable) model existence, *clasp* allows for:
  - Optimization
  - Enumeration (without solution recording)
  - Projective enumeration (without solution recording)
  - Intersection and Union (linear solution computation)
  - and combinations thereof
- *clasp* allows for
  - ASP solving (*smodels* format)
  - MaxSAT and SAT solving (extended *dimacs* format)
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Parallel search in *clasp*

- *clasp*
  - pursues a coarse-grained, task-parallel approach to parallel search via shared memory multi-threading
    - up to 64 configurable (non-hierarchic) threads
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    - both supported by solver portfolios
  - features different nogood exchange policies

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Sequential CDLC-style solving

loop
    propagate // deterministically assign literals
    if no conflict **then**
        if all variables assigned **then return** solution
        else *decide* // non-deterministically assign some literal
    else
        if top-level conflict **then return** unsatisfiable
        else
            analyze // analyze conflict and add conflict constraint
            backjump // unassign literals until conflict constraint is unit

Parallel CDCL-style solving in *clasp*

```
while work available
  while no (result) message to send
    communicate                    // exchange information with other solver
    propagate                      // deterministically assign literals
    if no conflict then
      if all variables assigned then send solution
      else decide                  // non-deterministically assign some literal
    else
      if root-level conflict then send unsatisfiable
      else if external conflict then send unsatisfiable
      else
        analyze                    // analyze conflict and add conflict constraint
        backjump                   // unassign literals until conflict constraint is unit
  communicate                      // exchange results (and receive work)
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         if all variables assigned then send solution
         else decide                  // non-deterministically assign some literal
      else
         if root-level conflict then send unsatisfiable
         else if external conflict then send unsatisfiable
         else
            analyze                   // analyze conflict and add conflict constraint
            backjump                  // unassign literals until conflict constraint is unit
   communicate                        // exchange results (and receive work)
```

Parallel CDCL-style solving in *clasp*

while work available
  while no (result) message to send
    communicate // exchange information with other solver
    propagate // deterministically assign literals
    if no conflict **then**
      if all variables assigned **then send** solution
      else *decide* // non-deterministically assign some literal
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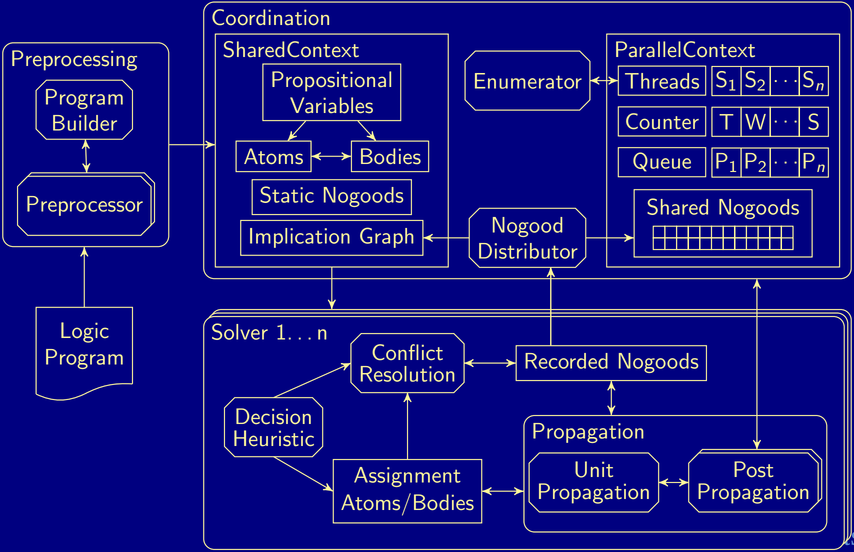
Multi-threaded architecture of *clasp*



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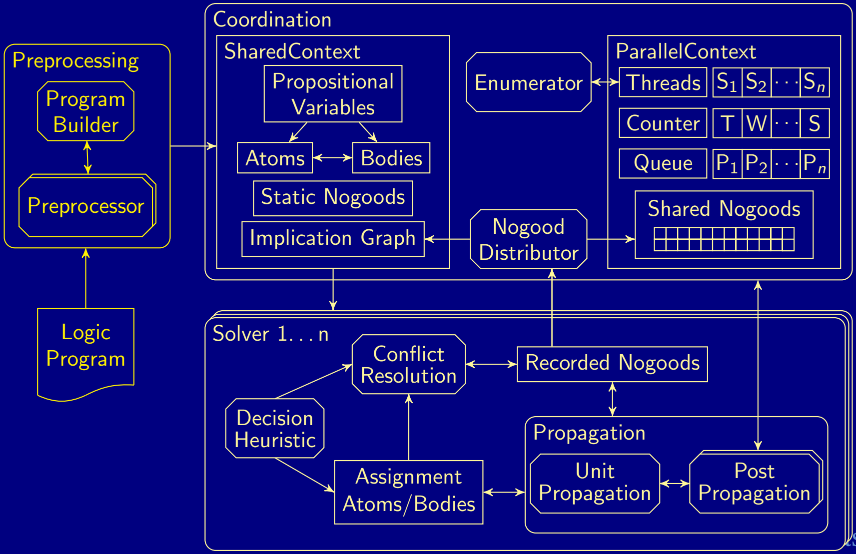

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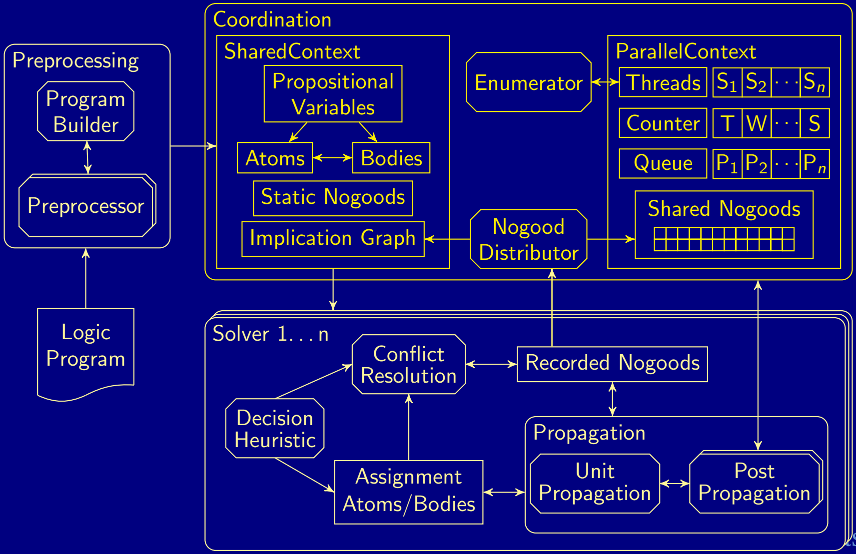

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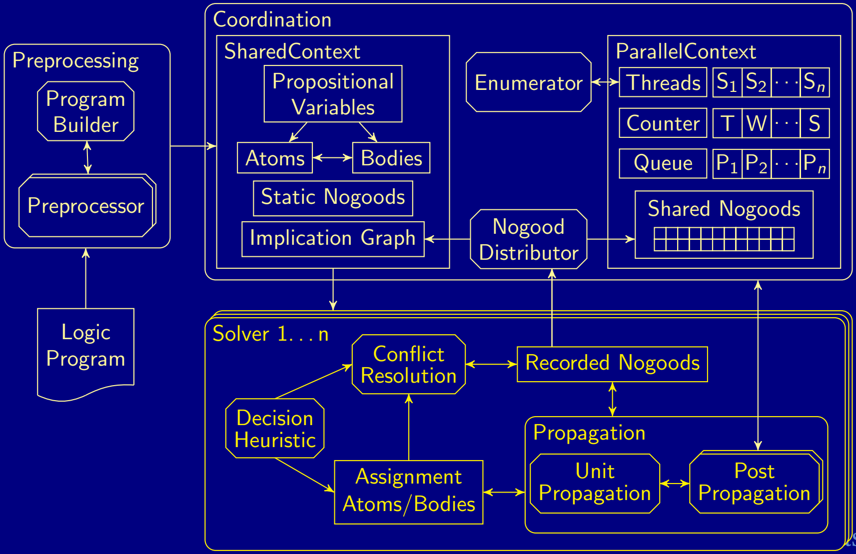

clasp in context

- Compare *clasp* (2.0.5) to the multi-threaded SAT solvers
  - *cryptominisat* (2.9.2)
  - *manysat* (1.1)
  - *miraxt* (2009)
  - *plingeling* (587f)

  all run with four and eight threads in their default settings

- 160/300 benchmarks from crafted category at SAT'11
  - all solvable by *ppfolio* in 1000 seconds
  - crafted SAT benchmarks are closest to ASP benchmarks

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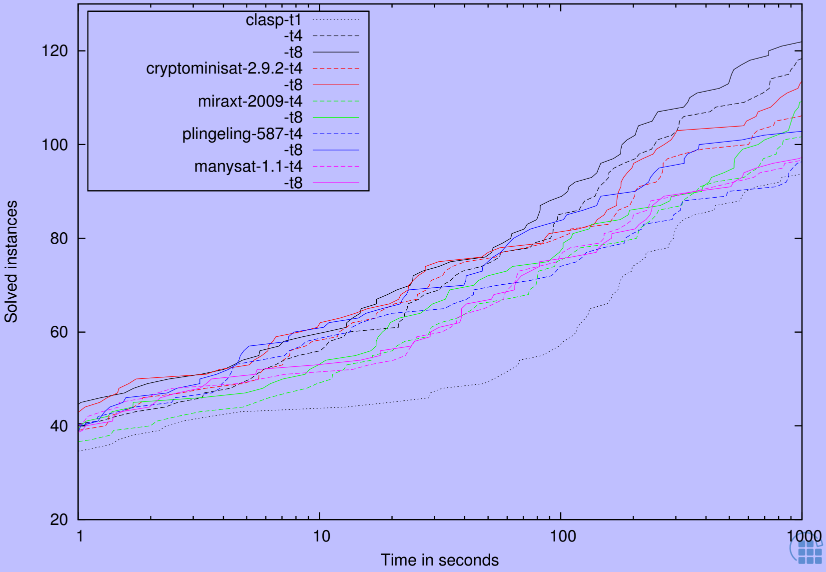
clasp in context



Using *clasp*

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--help[=<n>],-h          : Print {1=basic|2=more|3=full} help and exit

--parallel-mode,-t <arg>: Run parallel search with given number of threads
    <arg>: <n {1..64}>[,<mode {compete|split}>]
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--configuration=<arg>    : Configure default configuration [frumpy]
    <arg>: {frumpy|jumpy|handy|crafty|trendy|chatty}
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      jumpy : Use aggressive defaults
      handy : Use defaults geared towards large problems
      crafty: Use defaults geared towards crafted problems
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Potassco

Outline

37 Potasco

38 gringo

39 clasp

40 Siblings
- hclasp
- claspfolio
- claspD
- clingcon
- iclingo
- oclingo
- clavis

Potassco

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hclasp

- *hclasp* allows for incorporating domain-specific heuristics into ASP solving
  - input language for expressing domain-specific heuristics
  - solving capacities for integrating domain-specific heuristics
- Example

```
_heuristics(occ(A,T),factor,T) :- action(A), time(T).
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hclasp

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Basic CDCL decision algorithm

loop

    propagate // compute deterministic consequences
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Inside *decide*

- Heuristic functions

$$h : \mathcal{A} \rightarrow [0, +\infty) \quad \text{and} \quad s : \mathcal{A} \rightarrow \{\mathbf{T}, \mathbf{F}\}$$

- Algorithmic scheme

1. $h(a) := \alpha \times h(a) + \beta(a)$ for each $a \in \mathcal{A}$
2. $U := \mathcal{A} \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}})$
3. $C := argmax_{a \in U} h(a)$
4. $a := \tau(C)$
5. $A := A \cup \{a \mapsto s(a)\}$

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Heuristic language elements

- Heuristic predicate `_heuristic`
- Heuristic modifiers (atom, *a*, and integer, *v*)
  - `init` for initializing the heuristic value of *a* with *v*
  - `factor` for amplifying the heuristic value of *a* by factor *v*
  - `level` for ranking all atoms; the rank of *a* is *v*
  - `sign` for attributing the sign of *v* as truth value to *a*
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Simple STRIPS planner

```
time(1..t).

holds(P,0) :- init(P).

1 { occurs(A,T) : action(A) } 1 :- time(T).
 :- occurs(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- holds(F,T-1), not nolds(F,T), time(T).
holds(F,T) :- occurs(A,T), add(A,F).
nolds(F,T) :- occurs(A,T), del(A,F).

 :- query(F), not holds(F,t).
```

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 :- query(F), not holds(F,t).

_heuristic(occurs(A,T),factor,2) :- action(A), time(T).
```

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holds(F,T) :- occurs(A,T), add(A,F).
nolds(F,T) :- occurs(A,T), del(A,F).

 :- query(F), not holds(F,t).

_heuristic(occurs(A,T),level,1) :- action(A), time(T).
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_heuristic(A,level,V) :- _heuristic(A,true, V).
_heuristic(A,sign, 1) :- _heuristic(A,true, V).
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_heuristic(A,level,V) :- _heuristic(A,false,V).
_heuristic(A,sign,-1) :- _heuristic(A,false,V).
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holds(F,T) :- occurs(A,T), add(A,F).
nolds(F,T) :- occurs(A,T), del(A,F).

 :- query(F), not holds(F,t).

_heuristic(holds(F,T-1),true, t-T+1) :- holds(F,T).
_heuristic(holds(F,T-1),false,t-T+1) :-
                fluent(F), time(T), not holds(F,T).
```

Heuristic modifications to functions h and s

- $\nu(V_{a,m}(A))$ — "*value for modifier m on atom a wrt assignment A*"
- `init` and

$$d_0(a) = \quad \nu(V_{a,\texttt{init}}(A_0)) + h_0(a)$$

$$d_i(a) = \begin{cases} \nu(V_{a,\texttt{factor}}(A_i)) \times h_i(a) & \text{if } V_{a,\texttt{factor}}(A_i) \neq \emptyset \\ h_i(a) & \text{otherwise} \end{cases}$$

- `sign`

$$t_i(a) = \begin{cases} \mathbf{T} & \text{if } \nu(V_{a,\texttt{sign}}(A_i)) > 0 \\ \mathbf{F} & \text{if } \nu(V_{a,\texttt{sign}}(A_i)) < 0 \\ s_i(a) & \text{otherwise} \end{cases}$$

- `level` $\quad \ell_{A_i}(\mathcal{A}') = argmax_{a \in \mathcal{A}'} \nu(V_{a,\texttt{level}}(A_i)) \qquad \mathcal{A}' \subseteq \mathcal{A}$

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- `init` and `factor`

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$$d_i(a) = \begin{cases} \nu(V_{a,\texttt{factor}}(A_i)) \times h_i(a) & \text{if } V_{a,\texttt{factor}}(A_i) \neq \emptyset \\ h_i(a) & \text{otherwise} \end{cases}$$

- `sign`

$$t_i(a) = \begin{cases} \mathbf{T} & \text{if } \nu(V_{a,\texttt{sign}}(A_i)) > 0 \\ \mathbf{F} & \text{if } \nu(V_{a,\texttt{sign}}(A_i)) < 0 \\ s_i(a) & \text{otherwise} \end{cases}$$

- `level` $\quad \ell_{A_i}(\mathcal{A}') = argmax_{a\in\mathcal{A}'}\nu(V_{a,\texttt{level}}(A_i)) \qquad \mathcal{A}' \subseteq \mathcal{A}$

Potassco

Inside *decide*, heuristically modified

0 $h(a) := d(a)$ for each $a \in \mathcal{A}$

1 $h(a) := \alpha \times h(a) + \beta(a)$ for each $a \in \mathcal{A}$

2 $U := \ell_A(\mathcal{A} \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}}))$

3 $C := argmax_{a \in U} d(a)$

4 $a := \tau(C)$

5 $A := A \cup \{a \mapsto t(a)\}$

Potassco

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Selected high scores from systematic experiments

Setting	Labyrinth	Sokoban	Hanoi Tower
base configuration	9,108*s* (14) 24,545,667	2,844*s* (3) 19,371,267	9,137*s* (11) 41,016,235
a, `init`, 2	95% (12) 94%	91% (1) 84%	85% (9) 89%
a, `factor`, 4	78% (8) 30%	120% (1) 107%	109% (11) 110%
a, `factor`, 16	78% (10) 23%	120% (1) 107%	109% (11) 110%
a, `level`, 1	90% (12) 5%	119% (2) 91%	126% (15) 120%
f, `init`, 2	103% (14) 123%	74% (2) 71%	97% (10) 109%
f, `factor`, 2	98% (12) 49%	116% (3) 134%	55% (6) 70%
f, `sign`, -1	94% (13) 89%	105% (1) 100%	92% (12) 92%

base configuration versus 38 (static) heuristic modifications (action, a, and fluent, f)

Potassco

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base configuration versus 38 (static) heuristic modifications (action, a, and fluent, f)

Potassco

Abductive problems with optimization

Setting		Diagnosis	Expansion	Repair (H)	Repair (S)
base configuration		111.1*s* (115)	161.5*s* (100)	101.3*s* (113)	33.3*s* (27)
`sign,-1`		324.5*s* (407)	7.6*s* (3)	8.4*s* (5)	3.1*s* (0)
`sign,-1`	`factor,2`	310.1*s* (387)	7.4*s* (2)	3.5*s* (0)	3.2*s* (1)
`sign,-1`	`factor,8`	305.9*s* (376)	7.7*s* (2)	3.1*s* (0)	2.9*s* (0)
`sign,-1`	`level,1`	76.1*s* (83)	6.6*s* (2)	0.8*s* (0)	2.2*s* (1)
	`level,1`	77.3*s* (86)	12.9*s* (5)	3.4*s* (0)	2.1*s* (0)

(abducibles subject to optimization)

Potassco

Abductive problems with optimization

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(abducibles subject to optimization)

Potassco

Planning Competition Benchmarks

```
_heuristic(holds(F,T-1),true, t-T+1) :- holds(F,T).
_heuristic(holds(F,T-1),false,t-T+1) :-
                fluent(F), time(T), not holds(F,T).
```

Problem	*base configuration*		_heuristic		*base c. (SAT)*		_heur. *(SAT)*	
Blocks'00	134.4*s*	(180/61)	9.2*s*	(239/3)	163.2*s*	(59)	2.6*s*	(0)
Elevator'00	3.1*s*	(279/0)	0.0*s*	(279/0)	3.4*s*	(0)	0.0*s*	(0)
Freecell'00	288.7*s*	(147/115)	184.2*s*	(194/74)	226.4*s*	(47)	52.0*s*	(0)
Logistics'00	145.8*s*	(148/61)	115.3*s*	(168/52)	113.9*s*	(23)	15.5*s*	(3)
Depots'02	400.3*s*	(51/184)	297.4*s*	(115/135)	389.0*s*	(64)	61.6*s*	(0)
Driverlog'02	308.3*s*	(108/143)	189.6*s*	(169/92)	245.8*s*	(61)	6.1*s*	(0)
Rovers'02	245.8*s*	(138/112)	165.7*s*	(179/79)	162.9*s*	(41)	5.7*s*	(0)
Satellite'02	398.4*s*	(73/186)	229.9*s*	(155/106)	364.6*s*	(82)	30.8*s*	(0)
Zenotravel'02	350.7*s*	(101/169)	239.0*s*	(154/116)	224.5*s*	(53)	6.3*s*	(0)
Total	252.8*s*	(1225/1031)	158.9*s*	(1652/657)	187.2*s*	(430)	17.1*s*	(3)

Potassco

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Potassco

Outline

37 Potassco

38 gringo

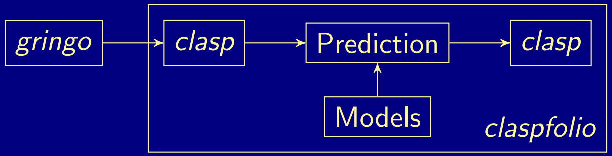
39 clasp

40 Siblings
- hclasp
- claspfolio
- claspD
- clingcon
- iclingo
- oclingo
- clavis

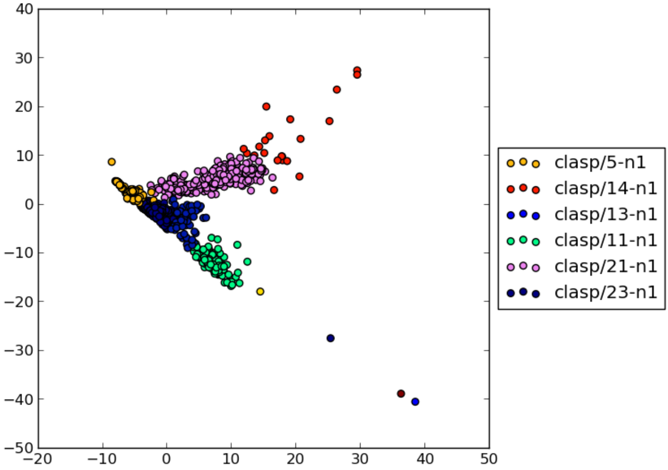
claspfolio

- Automatic selection of some *clasp* configuration among several predefined ones via (learned) classifiers

- Basic architecture of *claspfolio*:



Instance Feature Clusters (after PCA)



Solving with *clasp* (as usual)

```
$ clasp queens500 --quiet

clasp version 2.0.2
Reading from queens500
Solving...
SATISFIABLE

Models      : 1+
Time        : 11.445s (Solving: 10.58s 1st Model: 10.55s Unsat: 0.00s)
CPU Time    : 11.410s
```

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$ clasp queens500 --quiet

clasp version 2.0.2
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Solving...
SATISFIABLE

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Time         : 11.445s (Solving: 10.58s 1st Model: 10.55s Unsat: 0.00s)
CPU Time     : 11.410s
```

Solving with *claspfolio*

```
$ claspfolio queens500 --quiet

PRESOLVING
Reading from queens500
Solving...
claspfolio version 1.0.1 (based on clasp version 2.0.2)
Reading from queens500
Solving...
SATISFIABLE

Models       : 1+
Time         : 4.785s (Solving: 3.96s 1st Model: 3.92s Unsat: 0.00s)
CPU Time     : 4.780s
```

Potassco

Solving with *claspfolio*

```
$ claspfolio queens500 --quiet

PRESOLVING
Reading from queens500
Solving...
claspfolio version 1.0.1 (based on clasp version 2.0.2)
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Solving...
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```

Feature-extraction with *claspfolio*

```
$ claspfolio --features queens500

PRESOLVING
Reading from queens500
Solving...
UNKNOWN
Features    : 84998,3994,0,250000,1.020,62.594,63.844,21.281,84998, \
 3994,100,250000,1.020,62.594,63.844,21.281,84998,3994,250,250000,  \
 1.020,62.594,63.844,21.281,84998,3994,475,250000,1.020,62.594,     \
 63.844,21.281,757989,757989,0,510983,506992,3990,1,0,127.066,9983, \
 1023958,502993,1994,518971,1,0,0,254994,0,3990,0.100,0.000,99.900, \
 0,270303,812,4,0,812,2223,2223,262,262,2.738,2.738,0.000,812,812,  \
 2270.982,0,0.000

$ claspfolio --list-features

maxLearnt,Constraints,LearntConstraints,FreeVars,Vars/FreeVars, ...
```

Feature-extraction with *claspfolio*

```
$ claspfolio --features queens500

PRESOLVING
Reading from queens500
Solving...
UNKNOWN
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 2270.982,0,0.000

$ claspfolio --list-features

maxLearnt,Constraints,LearntConstraints,FreeVars,Vars/FreeVars, ...
```

Prediction with *claspfolio*

```
$ claspfolio queens500 --decisionvalues

PRESOLVING
Reading from queens500
Solving...

Portfolio Decision Values:
[1] : 3.437538   [10] : 3.639444   [19] : 3.726391
[2] : 3.501728   [11] : 3.483334   [20] : 3.020325
[3] : 3.784733   [12] : 3.271890   [21] : 3.220219
[4] : 3.672955   [13] : 3.344085   [22] : 3.998709
[5] : 3.557408   [14] : 3.315235   [23] : 3.961214
[6] : 3.942037   [15] : 3.620479   [24] : 3.512924
[7] : 3.335304   [16] : 3.396838   [25] : 3.078143
[8] : 3.375315   [17] : 3.238764
[9] : 3.432931   [18] : 3.403484

UNKNOWN
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Prediction with *claspfolio*

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[9] : 3.432931    [18] : 3.403484

UNKNOWN
```

Potassco

Solving with *claspfolio* (slightly verbosely)

```
$ claspfolio queens500 --quiet --autoverbose=1

PRESOLVING
Reading from queens500
Solving...

Chosen configuration: [20]
clasp --configurations=./models/portfolio.txt           \
      --modelpath=./models/                             \
      queens500 --quiet --autoverbose=1                 \
      --heu=VSIDS --sat-pre=20,25,120 --trans-ext=integ

claspfolio version 1.0.1 (based on clasp version 2.0.2)
Reading from queens500
Solving...
SATISFIABLE

Models      : 1+
Time        : 4.783s (Solving: 3.96s 1st Model: 3.93s Unsat: 0.00s)
CPU Time    : 4.760s
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      queens500 --quiet --autoverbose=1                \
      --heu=VSIDS --sat-pre=20,25,120 --trans-ext=integ

claspfolio version 1.0.1 (based on clasp version 2.0.2)
Reading from queens500
Solving...
SATISFIABLE

Models       : 1+
Time         : 4.783s (Solving: 3.96s 1st Model: 3.93s Unsat: 0.00s)
CPU Time     : 4.760s
```

Outline

37 Potassco

38 gringo

39 clasp

40 Siblings

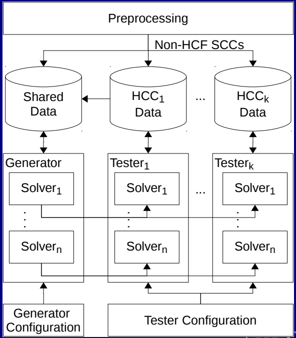
- hclasp
- claspfolio
- claspD
- clingcon
- iclingo
- oclingo
- clavis

claspD

- *claspD* is a multi-threaded solver for disjunctive logic programs
- aiming at an equitable interplay between "generating" and "testing" solver units
- allowing for a bidirectional dynamic information exchange between solver units for orthogonal tasks

claspD

- *claspD* is a multi-threaded solver for disjunctive logic programs
- aiming at an equitable interplay between "generating" and "testing" solver units
- allowing for a bidirectional dynamic information exchange between solver units for orthogonal tasks



Outline

37 Potassco

38 gringo

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40 Siblings
- hclasp
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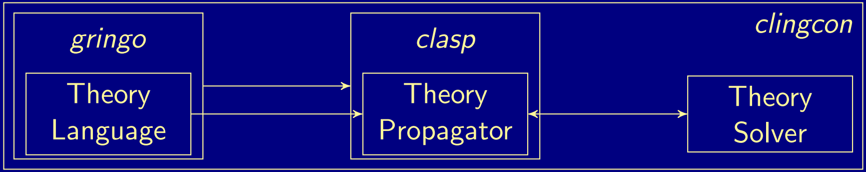
- clavis

Potassco

clingcon

- Hybrid grounding and solving
- Solving in hybrid domains, like Bio-Informatics

- Basic architecture of *clingcon*:



Pouring Water into Buckets on a Scale

```
time(0..t).             $domain(0..500).
bucket(a).              volume(a,0) $== 0.
bucket(b).              volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Potassco

Pouring Water into Buckets on a Scale

```
time(0..t).              $domain(0..500).
bucket(a).               volume(a,0) $== 0.
bucket(b).               volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Potassco

Pouring Water into Buckets on a Scale

```
time(0..t).               $domain(0..500).
bucket(a).                volume(a,0) $== 0.
bucket(b).                volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Potassco

Pouring Water into Buckets on a Scale

```
time(0..t).               $domain(0..500).
bucket(a).                volume(a,0) $== 0.
bucket(b).                volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).              $domain(0..500).
bucket(a).               volume(a,0) $== 0.
bucket(b).               volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, not (1 $<= amount(B,T)).
amount(B,T) $<= 30            :- pour(B,T), T < t.
amount(B,T) $==  0            :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).               $domain(0..500).
bucket(a).                volume(a,0) $== 0.
bucket(b).                volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Potassco

Pouring Water into Buckets on a Scale

```
time(0..t).              $domain(0..500).
bucket(a).               volume(a,0) $== 0.
bucket(b).               volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
 :- pour(B,T), T < t, amount(B,T) $> 30.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Potassco

Pouring Water into Buckets on a Scale

```
time(0..t).                $domain(0..500).
bucket(a).                 volume(a,0) $== 0.
bucket(b).                 volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
 :- pour(B,T), T < t, amount(B,T) $> 30.
 :- not pour(B,T), bucket(B), time(T), T < t, amount(B,T) $!=  0.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).             $domain(0..500).
bucket(a).              volume(a,0) $== 0.
bucket(b).              volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
 :- pour(B,T), T < t, amount(B,T) $> 30.
 :- not pour(B,T), bucket(B), time(T), T < t, amount(B,T) $!=  0.

 :- bucket(B), time(T), T < t, volume(B,T+1) $!= volume(B,T)$+amount(B,T).

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                        $domain(0..500).
bucket(a).                                                    :- volume(a,0) $!= 0.
bucket(b).                                                    :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                           ...  1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                        ...   :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                        ...   :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                       ...   :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                       ...   :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.                   ...   :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.                   ...   :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).       ...   :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).       ...   :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).                ...  down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).                ...  down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).                ...  down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).                ...  down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                               ...  up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                               ...  up(b,4) :- not down(b,4).

 :- up(a,4).
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                              $domain(0..500).
bucket(a).                                          :- volume(a,0) $!= 0.
bucket(b).                                          :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                  ...  1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).               ...   :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).               ...   :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.              ...   :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.              ...   :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.          ...   :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.          ...   :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).  ...   :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).  ...   :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).       ...  down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).       ...  down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).       ...  down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).       ...  down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                      ...  up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                      ...  up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                    $domain(0..500).
bucket(a).                                                :- volume(a,0) $!= 0.
bucket(b).                                                :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                         ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                      ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                      ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                     ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                     ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.                 ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.                 ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).     ...  :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).     ...  :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).              ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).              ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).              ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).              ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                             ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                             ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                $domain(0..500).
bucket(a).                                            :- volume(a,0) $!= 0.
bucket(b).                                            :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                    ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                 ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                 ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.            ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.            ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)). ...  :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)). ...  :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).         ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).         ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).         ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).         ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                        ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                        ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                  $domain(0..500).
bucket(a).                                              :- volume(a,0) $!= 0.
bucket(b).                                              :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                      ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                   ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                   ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                  ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                  ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.              ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.              ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).  ...  :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).  ...  :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).           ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).           ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).           ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).           ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                          ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                          ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models       : 1
Time         : 0.000
```

Boolean variables

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Non-Boolean variables

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11       amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30       amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30       amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30       amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0        volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=11       volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=41       volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=71       volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=101      volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Potassco

Outline

37 Potassco

38 gringo

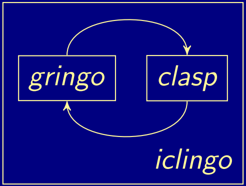
39 clasp

40 Siblings
- hclasp
- claspfolio
- claspD
- clingcon
- iclingo
- oclingo
- clavis

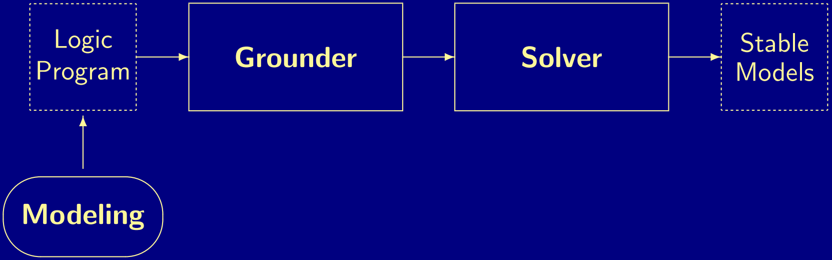
iclingo

- Incremental grounding and solving
- Offline solving in dynamic domains, like Automated Planning

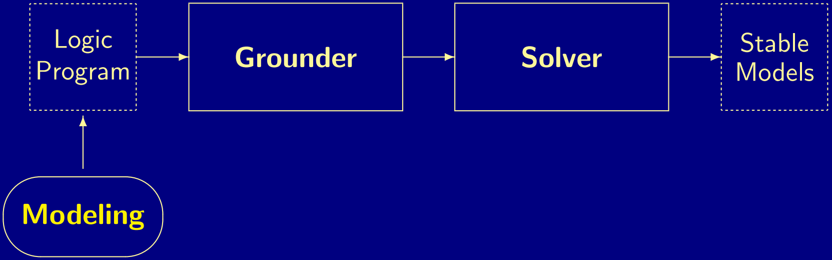
- Basic architecture of *iclingo*:



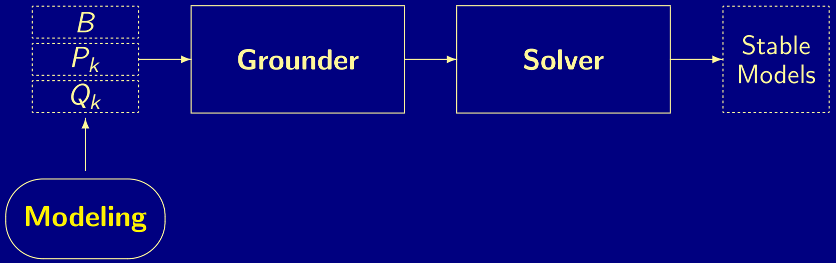
Incremental ASP Solving Process



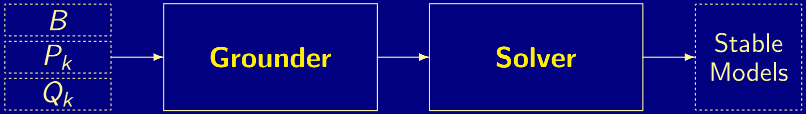
Incremental ASP Solving Process



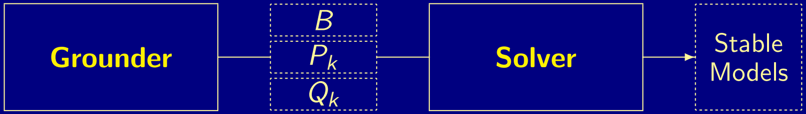
Incremental ASP Solving Process



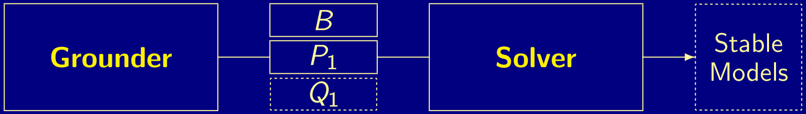
Incremental ASP Solving Process



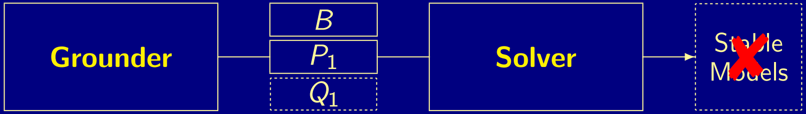
Incremental ASP Solving Process



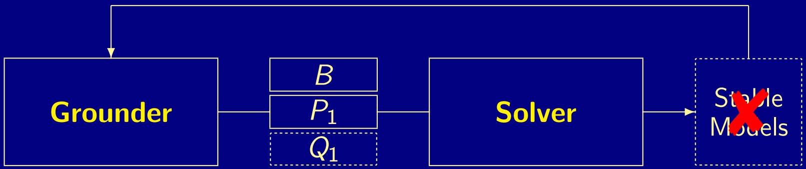
Incremental ASP Solving Process



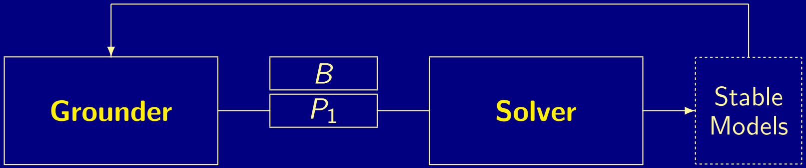
Incremental ASP Solving Process



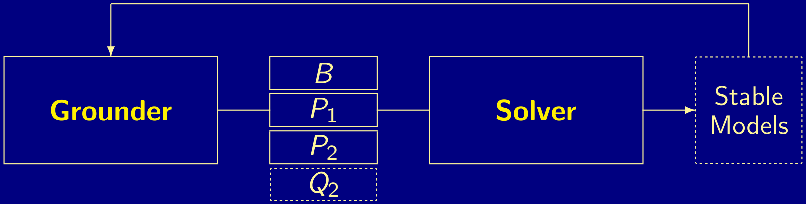
Incremental ASP Solving Process



Incremental ASP Solving Process

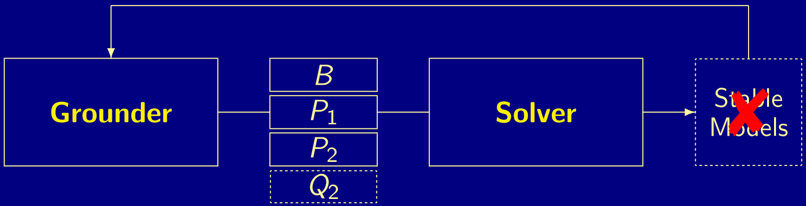


Incremental ASP Solving Process

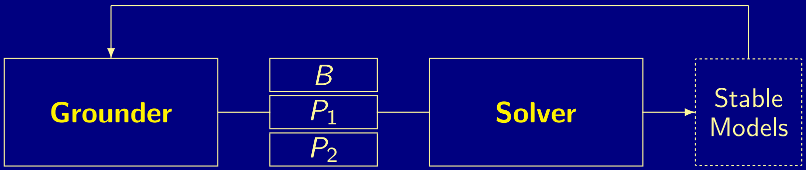


Potassco

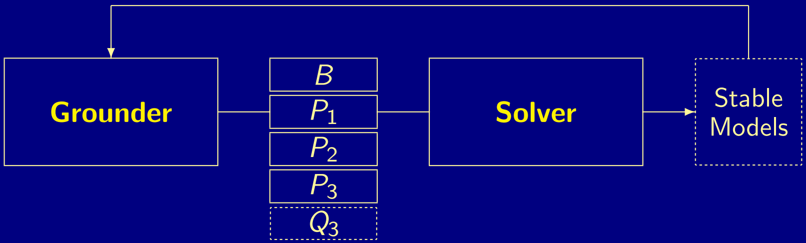
Incremental ASP Solving Process



Incremental ASP Solving Process

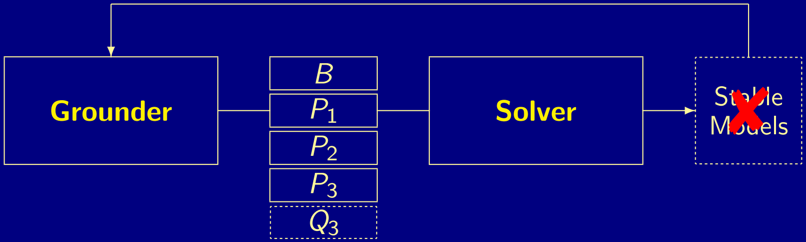


Incremental ASP Solving Process

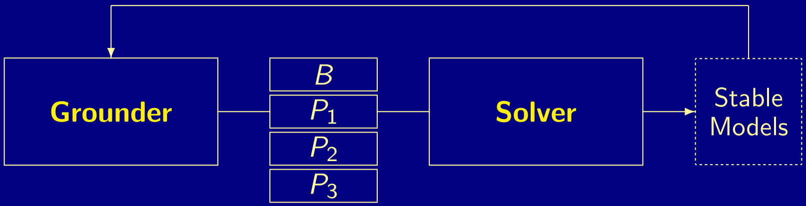


Potassco

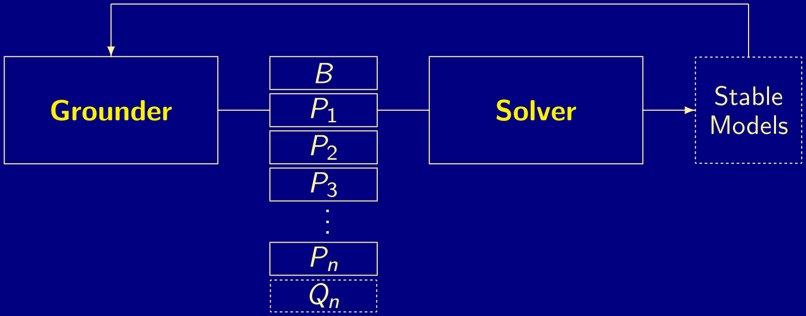
Incremental ASP Solving Process



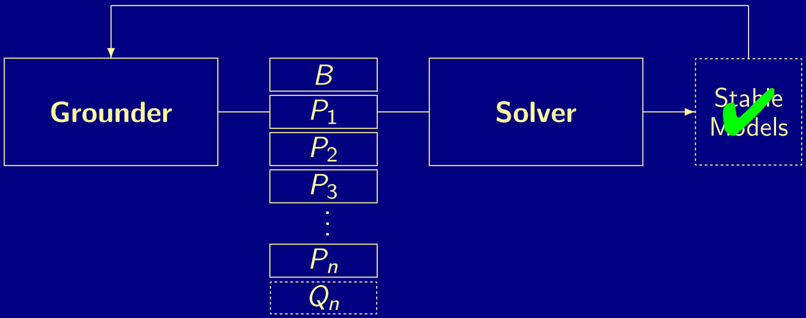
Incremental ASP Solving Process



Incremental ASP Solving Process



Incremental ASP Solving Process



Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).    query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

holds(F,t) :- holds(F,t-1), not nolds(F,t).
holds(F,t) :- occ(A,t), add(A,F).
nolds(F,t) :- occ(A,t), del(A,F).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).    query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

holds(F,t) :- holds(F,t-1), not nolds(F,t).
holds(F,t) :- occ(A,t), add(A,F).
nolds(F,t) :- occ(A,t), del(A,F).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).    query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

holds(F,t) :- holds(F,t-1), not nolds(F,t).
holds(F,t) :- occ(A,t), add(A,F).
nolds(F,t) :- occ(A,t), del(A,F).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).      query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

holds(F,t) :- holds(F,t-1), not nolds(F,t).
holds(F,t) :- occ(A,t), add(A,F).
nolds(F,t) :- occ(A,t), del(A,F).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp

Answer: 1
occ(a,1) occ(b,2)
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp

Answer: 1
occ(a,1) occ(b,2)
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Potassco

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp --istats

=============== step 1 ===============

Models   : 0
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 27
Choices  : 0
Conflicts: 0
=============== step 2 ===============
Answer: 1
occ(a,1) occ(b,2)

Models   : 1
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 16
Choices  : 0
Conflicts: 0
=============== Summary ===============
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp --istats

=============== step 1 ===============

Models   : 0
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 27
Choices  : 0
Conflicts: 0
=============== step 2 ===============
Answer: 1
occ(a,1) occ(b,2)

Models   : 1
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 16
Choices  : 0
Conflicts: 0
=============== Summary ===============
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Outline

37 Potassco

38 gringo

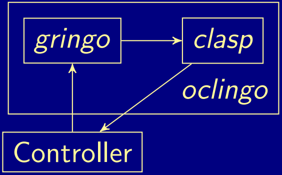
39 clasp

40 Siblings
- hclasp
- claspfolio
- claspD
- clingcon
- iclingo
- oclingo
- clavis

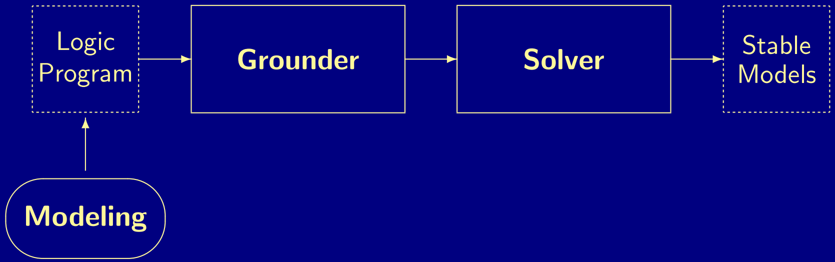
oclingo

- Reactive grounding and solving
- Online solving in dynamic domains, like Robotics

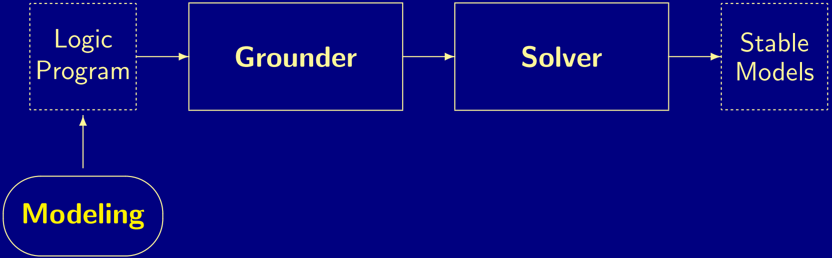
- Basic architecture of *oclingo*:



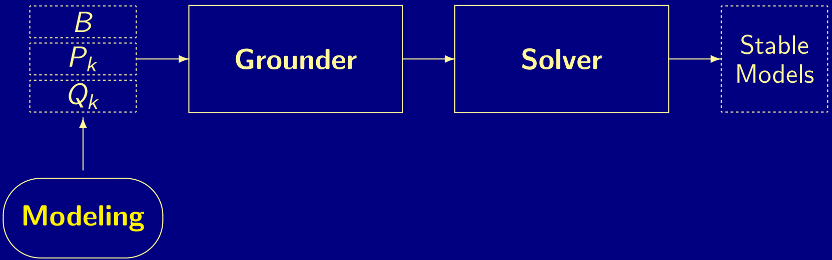
Reactive ASP Solving Process



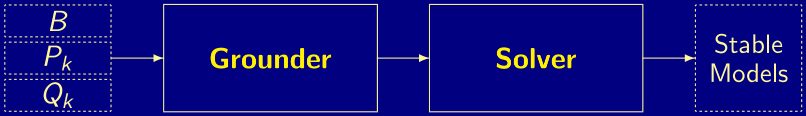
Reactive ASP Solving Process



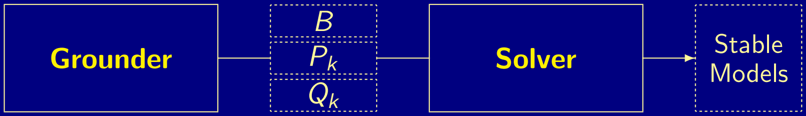
Reactive ASP Solving Process



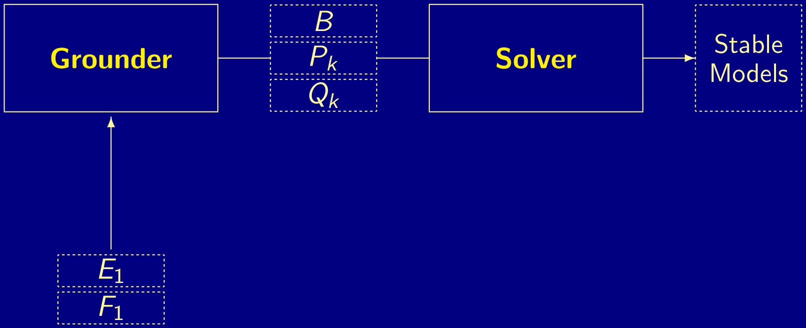
Reactive ASP Solving Process



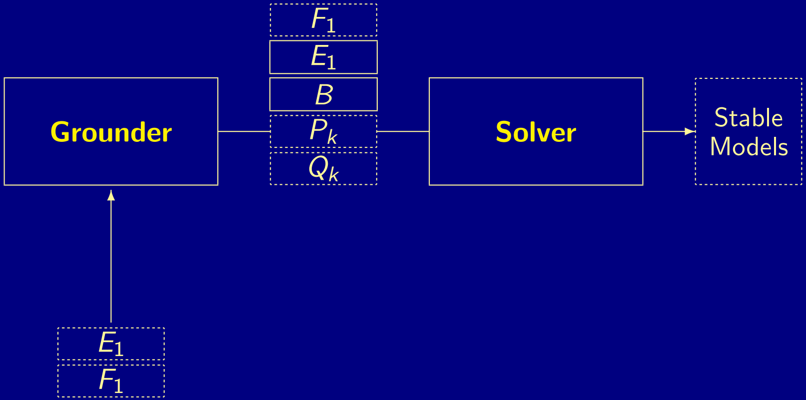
Reactive ASP Solving Process



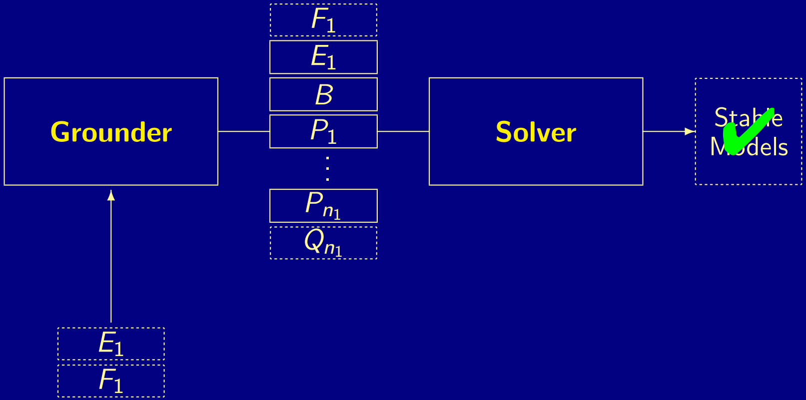
Reactive ASP Solving Process



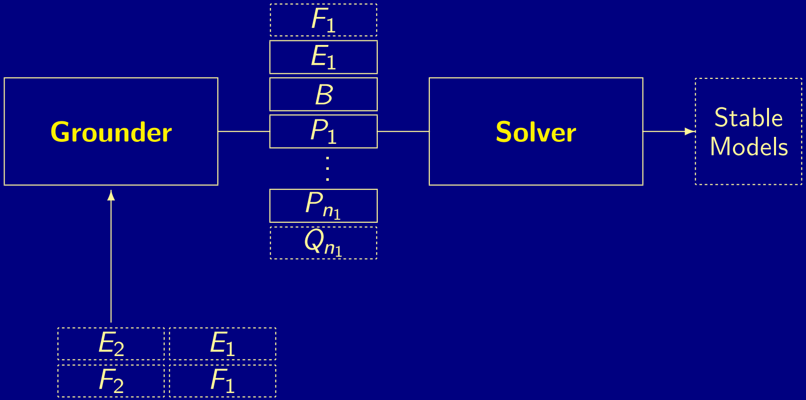
Reactive ASP Solving Process



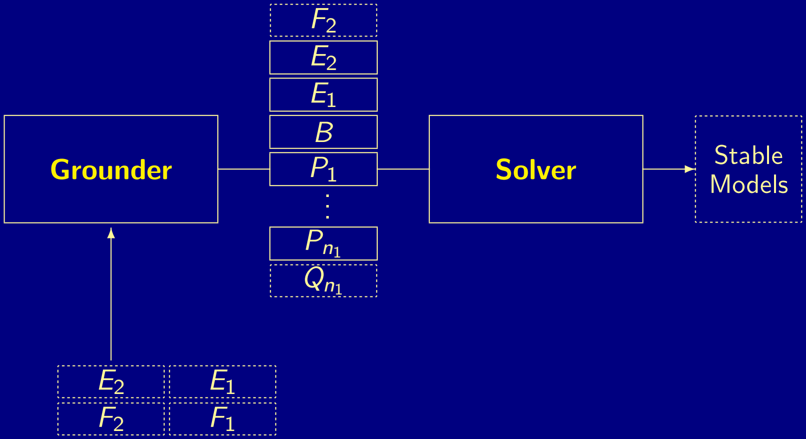
Reactive ASP Solving Process



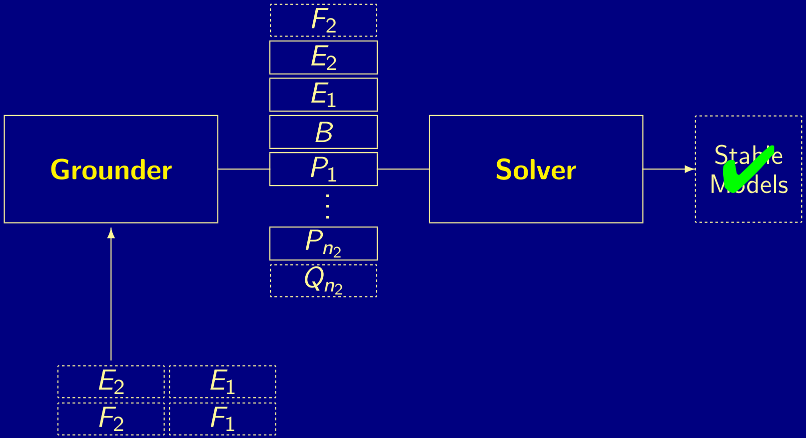
Reactive ASP Solving Process



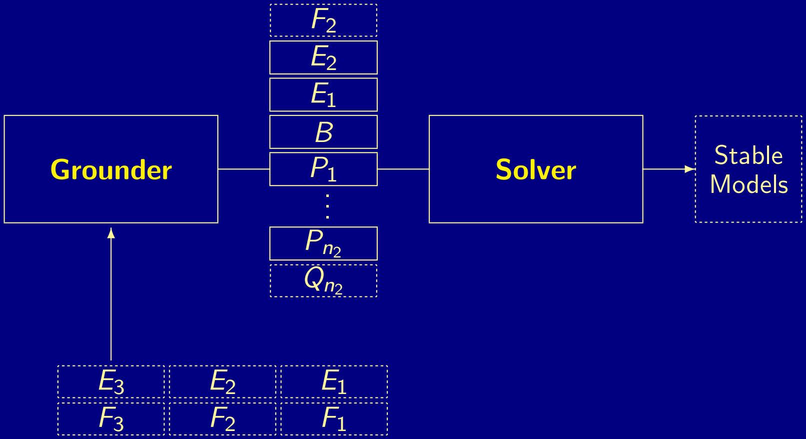
Reactive ASP Solving Process



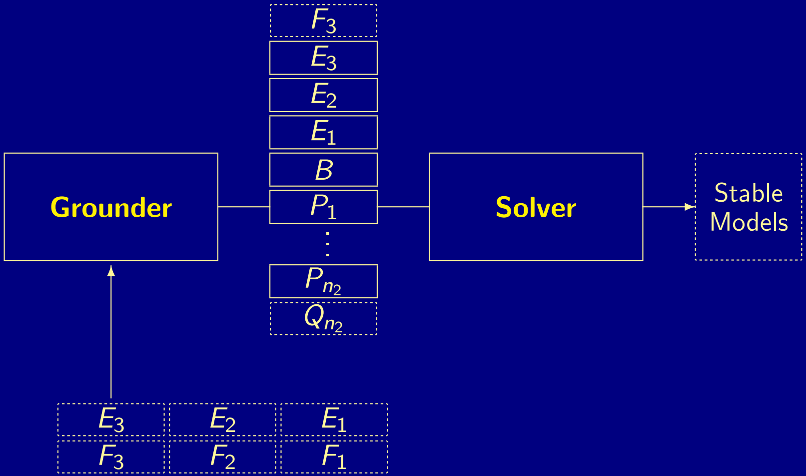
Reactive ASP Solving Process



Reactive ASP Solving Process



Reactive ASP Solving Process



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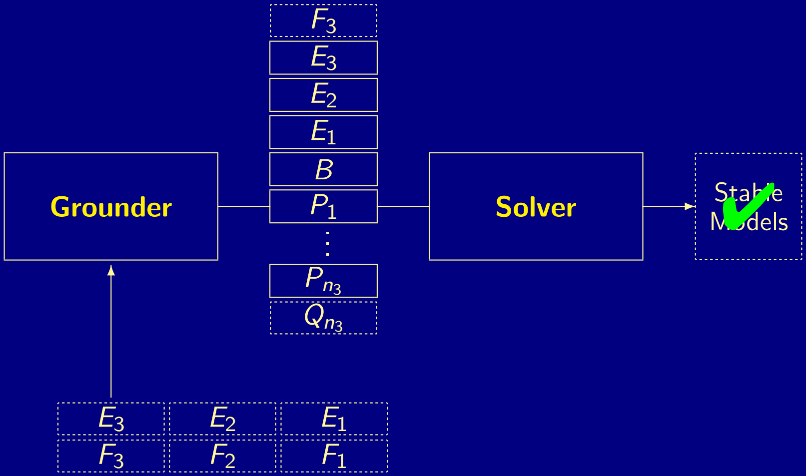

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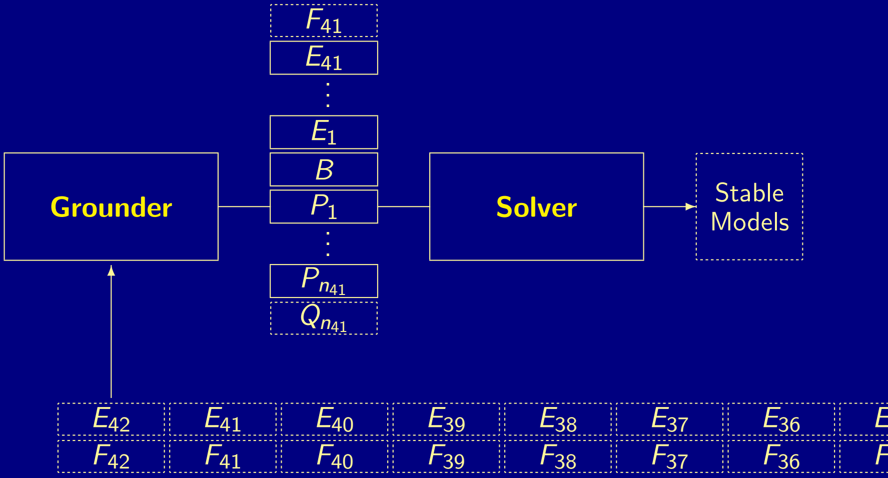

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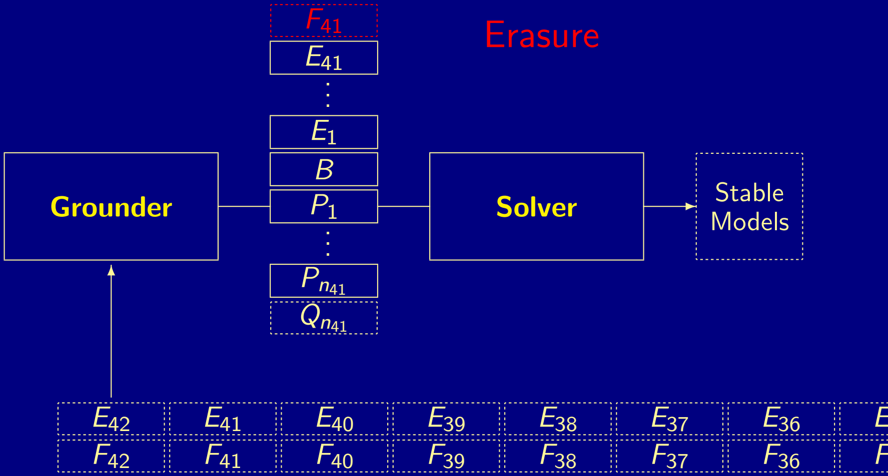

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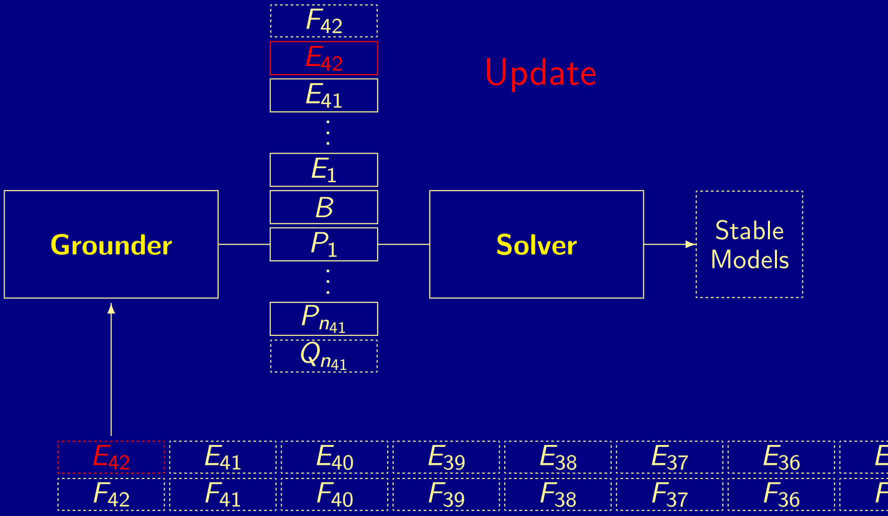

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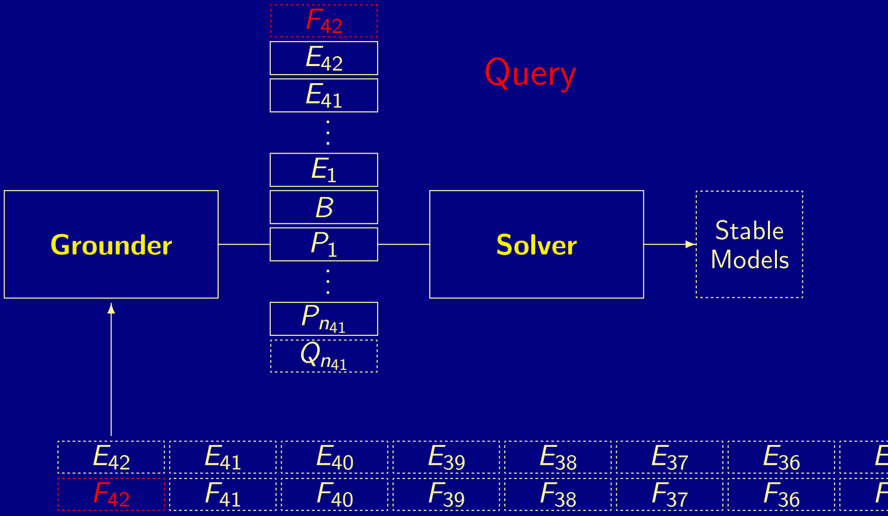

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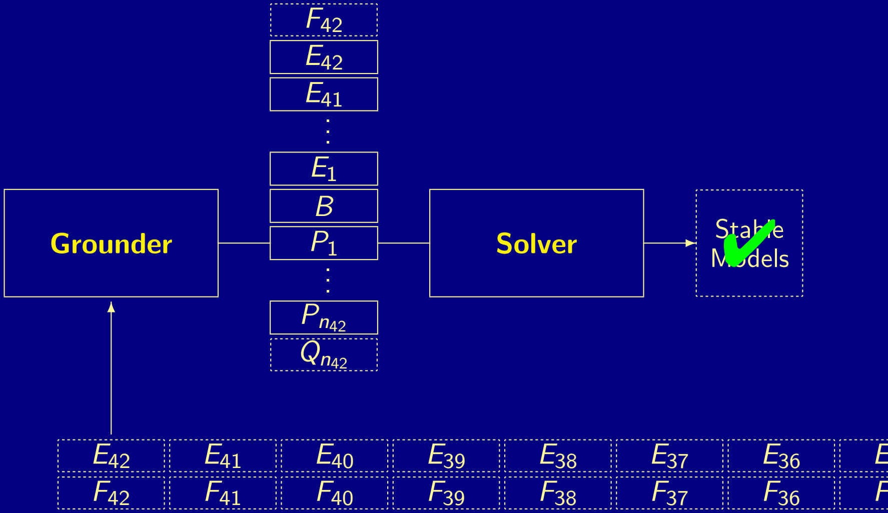

Elevator Control

```
#base.
floor(1..3).
atFloor(1,0).

#cumulative t.
#external request(F,t) : floor(F).
1 { atFloor(F-1;F+1,t) } 1 :- atFloor(F,t-1), floor(F).
:- atFloor(F,t), not floor(F).
requested(F,t) :- request(F,t),     floor(F), not atFloor(F,t).
requested(F,t) :- requested(F,t-1), floor(F), not atFloor(F,t).
goal(t) :- not requested(F,t) : floor(F).

#volatile t.
:- not goal(t).
```

Pushing a button

- oClingo acts as a server listening on a port waiting for client requests
- To issue such requests, a separate controller program sends online progressions using network sockets
- For instance,

```
#step 1.
request(3,1).
#endstep.
```

- This process terminates when the client sends

```
#stop.
```

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Potassco

Outline

37 Potassco

38 gringo

39 clasp

40 Siblings
- hclasp
- claspfolio
- claspD
- clingcon
- iclingo
- oclingo
- clavis

clavis

- Analysis and visualization toolchain for clasp
- *clavis*
  - Event logger integrated in clasp
  - Records CDCL events like propagation, conflicts, restarts, ...
  - Generated logfiles readable with different backends
  - Easily configurable
  - Applicable to clasp variants like hclasp
- *insight*
  - Visualization backend for clavis
  - Combines information about problem structure and solving process
  - Networks for structural and aggregated information
  - Plots for temporal information and navigation

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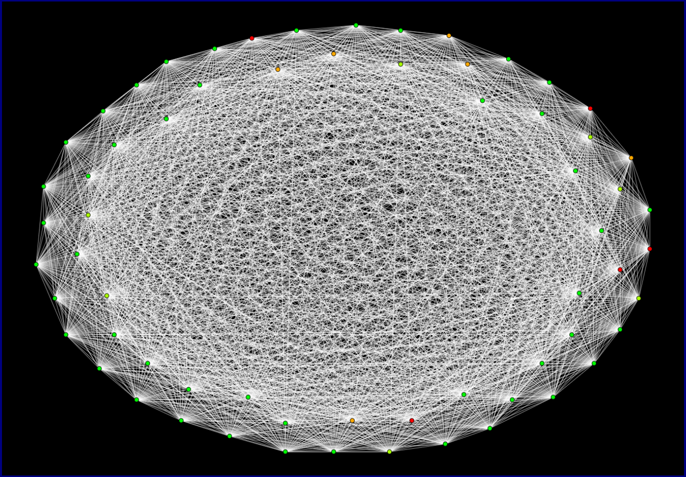
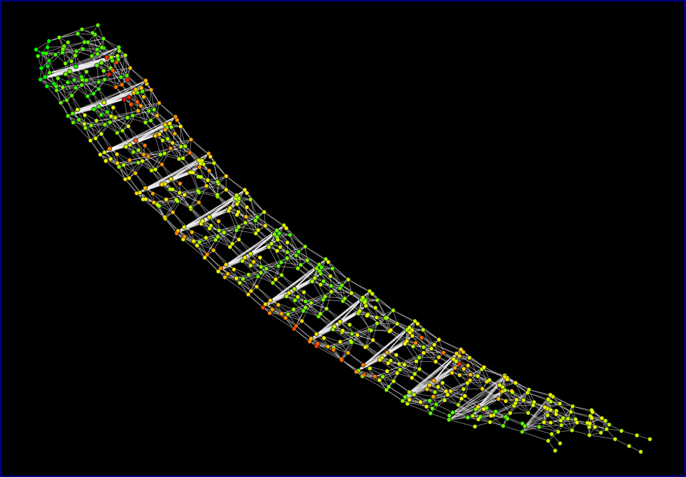
Visualization Examples

8-Queens: program interaction graph



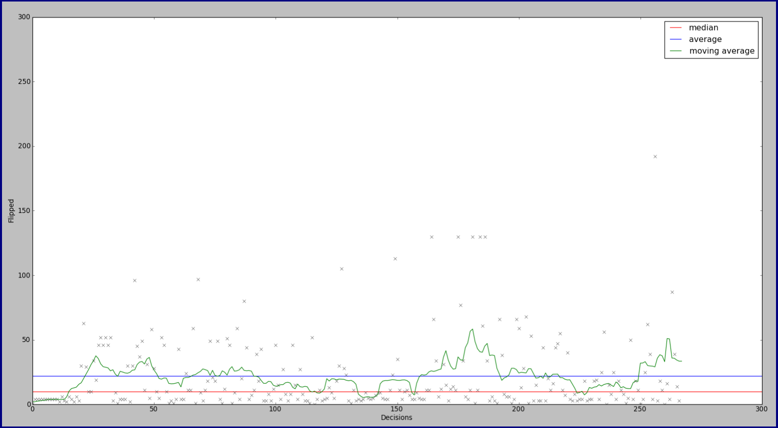
Visualization Examples

Towers of Hanoi: program interaction graph
Colors showing flipped assignments



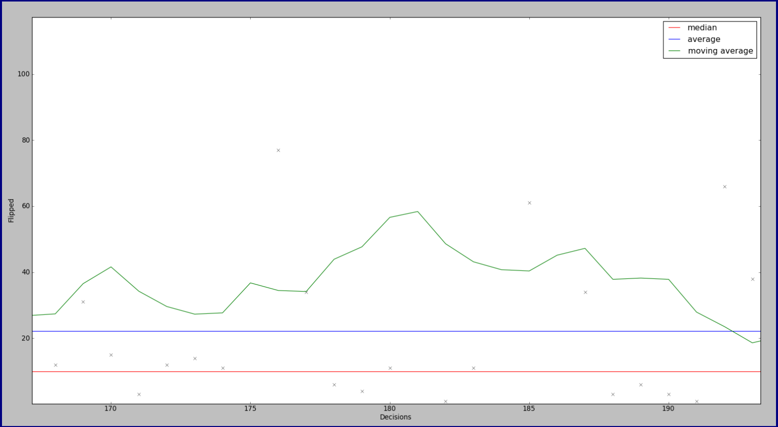
Visualization Examples

Towers of Hanoi: flipped assignments between decisions



Visualization Examples

Towers of Hanoi: flipped assignments between decisions (zoomed in)



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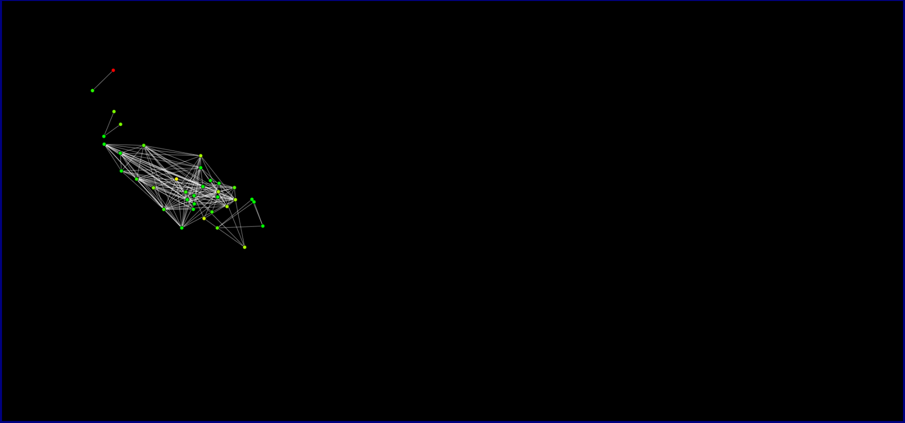
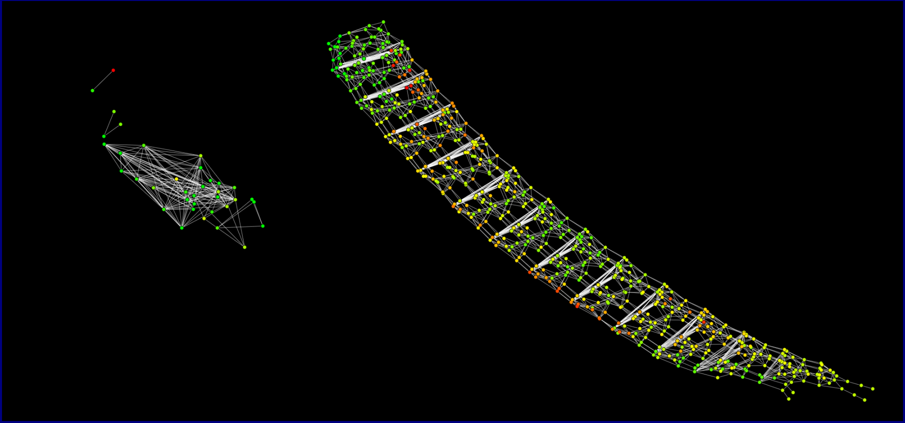
Visualization Examples

Towers of Hanoi: learned nogoods during zoomed in segment projected onto program interaction graph layout

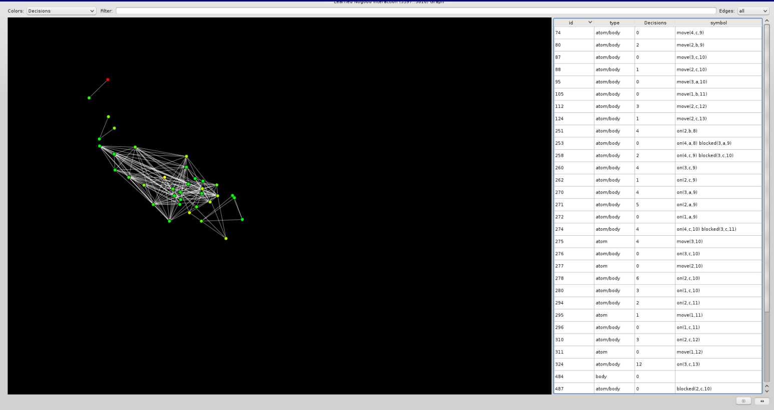


Visualization Examples

Towers of Hanoi: learned nogoods during zoomed in segment compared to program interaction graph

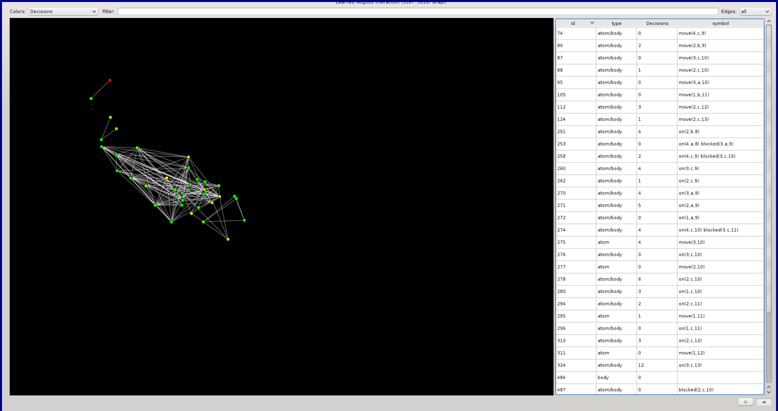


Interactive View



- Symbol table shows additional information about variables
- Search bar and symbol table allow for dynamic change of the view

Interactive View



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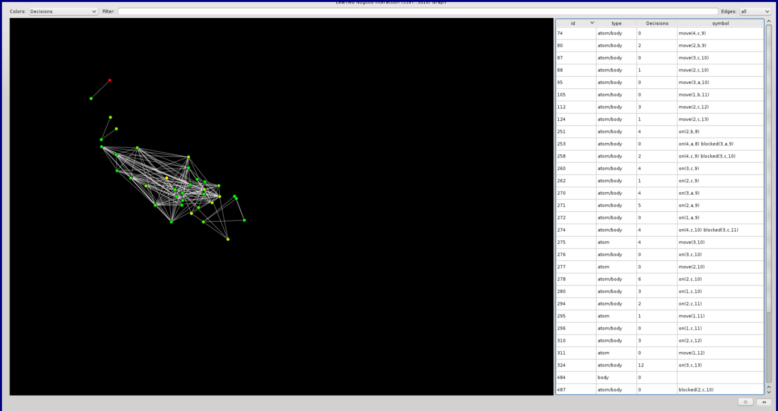

id	type	Decisions	symbol
74	atom/body	0	move(4,c,9)
80	atom/body	2	move(2,b,9)
87	atom/body	0	move(3,c,10)
88	atom/body	1	move(2,c,10)
95	atom/body	0	move(3,a,10)
105	atom/body	0	move(1,b,11)
112	atom/body	3	move(2,c,12)
124	atom/body	1	move(2,c,13)
251	atom/body	4	on(2,b,8)
253	atom/body	0	on(4,a,8) blocked(3,a,9)
258	atom/body	2	on(4,c,9) blocked(3,c,10)
260	atom/body	4	on(3,c,9)
262	atom/body	1	on(2,c,9)
270	atom/body	4	on(3,a,9)
271	atom/body	5	on(2,a,9)
272	atom/body	0	on(1,a,9)
274	atom/body	4	on(4,c,10) blocked(3,c,11)
275	atom	4	move(3,10)
276	atom/body	0	on(3,c,10)
277	atom	0	move(2,10)
278	atom/body	6	on(2,c,10)
280	atom/body	3	on(1,c,10)
294	atom/body	2	on(2,c,11)
295	atom	1	move(1,11)
296	atom/body	0	on(1,c,11)
310	atom/body	3	on(2,c,12)
311	atom	0	move(1,12)
324	atom/body	12	on(3,c,13)
484	body	0	
487	atom/body	0	blocked(2,c,10)

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Advanced Modeling: Overview

41 Tweaking *N*-Queens

42 Do's and Dont's

43 Hints

Anything left to worry about?

- ASP offers
  - rich yet easy modeling languages
  - efficient instantiation procedures
  - powerful search engines

- BUT The problem encoding (still) matters!

- Example Sort a list with 8 elements
  - divide-and-conquer $\sim$ $8(\log_2 8) = 16$ "operations"
  - permutation guessing $\sim$ $8!/2 = 20160$ "operations"

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Outline

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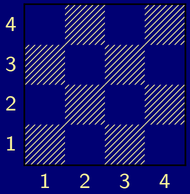
N-Queens Problem

Problem Specification

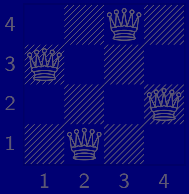
Given an $N \times N$ chessboard,
place N queens such that they do not attack each other
(neither horizontally, vertically, nor diagonally)

$N = 4$

Chessboard



Placement



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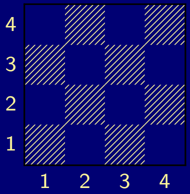

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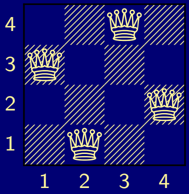

A First Encoding

1. Each square may host a queen
2. No row, column, or diagonal hosts two queens
3. A placement is given by instances of queen in a stable model

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- queen(X1,Y1), queen(X1,Y2), Y1 < Y2.

% DISPLAY
#hide. #show queen/2.
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Anything missing?

Potassco

A First Encoding

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2. No row, column, or diagonal hosts two queens
3. A placement is given by instances of `queen` in a stable model
4. We have to place (at least) N queens

queens_0.lp

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% TEST
[...]
:- not n #count{ queen(X,Y) }.

% DISPLAY
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```

A First Encoding

Let's Place 8 Queens!

```
gringo -c n=8 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,6) queen(2,3) queen(3,1) queen(4,7)
queen(5,5) queen(6,8) queen(7,2) queen(8,4)
SATISFIABLE

Models      : 1+
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 18
Conflicts   : 13
Restarts    : 0

Variables   : 793
Constraints : 729
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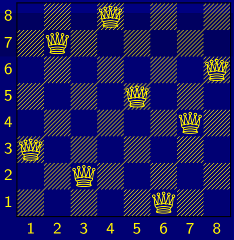


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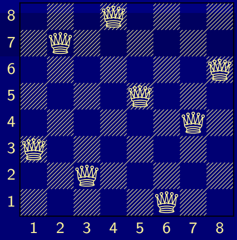

A First Encoding

Let's Place 22 Queens!

```
gringo -c n=22 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,10) queen(2,6) queen(3,16) queen(4,14) queen(5,8) ...
SATISFIABLE

Models      : 1+
Time        : 150.531s (Solving: 150.37s 1st Model: 150.34s Unsat: 0.00s)
CPU Time    : 147.480s
Choices     : 594960
Conflicts   : 574565
Restarts    : 19

Variables   : 17271
Constraints : 16787
```

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A First Encoding

Let's Place 22 Queens!

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Conflicts   : 574565
Restarts    : 19

Variables   : 17271
Constraints : 16787
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A First Refinement

At least *N* queens?

Exactly one queen per row and column!

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A First Refinement

At least *N* queens? Exactly one queen per row and column!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.


% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Let's Place 22 Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,18) queen(2,10) queen(3,21) queen(4,3) queen(5,5) ...
SATISFIABLE

Models      : 1+
Time        : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.020s
Choices     : 132
Conflicts   : 105
Restarts    : 1

Variables   : 7238
Constraints : 6710
```

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A First Refinement

Let's Place 22 Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,18) queen(2,10) queen(3,21) queen(4,3) queen(5,5) ...
SATISFIABLE

Models      : 1+
Time        : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.020s
Choices     : 132
Conflicts   : 105
Restarts    : 1

Variables   : 7238
Constraints : 6710
```

Potassco

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models      : 1+
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time    : 6.930s
Choices     : 1373
Conflicts   : 845
Restarts    : 4

Variables   : 1211338
Constraints : 1196210
```

Potassco

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models      : 1+
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time    : 6.930s
Choices     : 1373
Conflicts   : 845
Restarts    : 4

Variables   : 1211338
Constraints : 1196210
```

Potassco

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models      : 1+
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time    : 6.930s
Choices     : 1373
Conflicts   : 845
Restarts    : 4

Variables   : 1211338
Constraints : 1196210
```

Potassco

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | clasp --stats

1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

Potassco

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)

1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

Potassco

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)
```

```
1241358 7402724 24950848

real  1m15.468s
user  1m15.980s
sys   0m0.090s
```

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)
```

```
1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                        O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                        O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.             O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                   O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                   O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.        O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                      O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                       O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                          O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                          O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.            O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                      O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                      O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                         O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                         O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.           O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                    O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                    O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.         O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Refinement

Grounding Time ~ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                        O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                        O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.             O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                   O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                  O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                     O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                     O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.       O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Refinement

Grounding Time ~ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                   O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                   O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.        O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                    O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                    O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.     O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                        O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                        O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.             O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

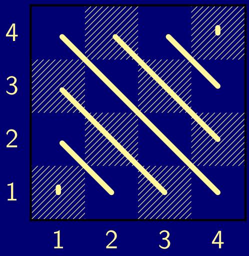
queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                   O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                    O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.         O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

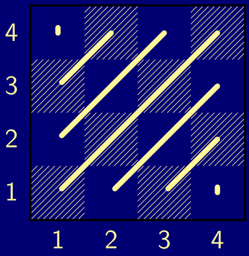
Diagonals make trouble!

Enumerating Diagonals

$N = 4$



$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$



$\#\text{diagonal}_2 =$
$(\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2*N)-1$ refer to squares on $\#\text{diagonal}_{1/2}$

Enumerating Diagonals

$N = 4$

4	**4**	**5**	**6**	**7**
3	**3**	**4**	**5**	**6**
2	**2**	**3**	**4**	**5**
1	**1**	**2**	**3**	**4**
	1	2	3	4

$\#\text{diagonal}_1 = (\#\text{row} + \#\text{column}) - 1$

4	**7**	**6**	**5**	**4**
3	**6**	**5**	**4**	**3**
2	**5**	**4**	**3**	**2**
1	**4**	**3**	**2**	**1**
	1	2	3	4

$\#\text{diagonal}_2 = (\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2{*}N){-}1$ refer to squares on $\#\text{diagonal}_{1/2}$

Potassco

Enumerating Diagonals

$N = 4$

4	**4**	**5**	**6**	**7**
3	**3**	**4**	**5**	**6**
2	**2**	**3**	**4**	**5**
1	**1**	**2**	**3**	**4**
	1	2	3	4

$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$

4	**7**	**6**	**5**	**4**
3	**6**	**5**	**4**	**3**
2	**5**	**4**	**3**	**2**
1	**4**	**3**	**2**	**1**
	1	2	3	4

$\#\text{diagonal}_2 =$
$(\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2*N)-1$ refer to squares on $\#\text{diagonal}_{1/2}$

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Enumerating Diagonals

$N = 4$

4	**4**	**5**	**6**	**7**
3	**3**	**4**	**5**	**6**
2	**2**	**3**	**4**	**5**
1	**1**	**2**	**3**	**4**
	1	2	3	4

$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$

4	**7**	**6**	**5**	**4**
3	**6**	**5**	**4**	**3**
2	**5**	**4**	**3**	**2**
1	**4**	**3**	**2**	**1**
	1	2	3	4

$\#\text{diagonal}_2 =$
$(\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2{*}N){-}1$ refer to squares on $\#\text{diagonal}_{1/2}$

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.


% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1


% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

Potassco

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

Potassco

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time    : 7.250s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time    : 7.250s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time    : 7.250s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Precalculate Indexes!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

queens_3.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

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Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
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Variables   : 365994
Constraints : 4794
```

A Case for Oracles

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```

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Choices     : 869379
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Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...
SATISFIABLE

Models      : 1+
Time        : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)
CPU Time    : 29.580s
Choices     : 961315
Conflicts   : 3222
Restarts    : 7

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
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```

```
Answer: 1
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...
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Time        : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)
CPU Time    : 29.580s
Choices     : 961315
Conflicts   : 3222
Restarts    : 7

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,90) queen(2,452) queen(3,494) queen(4,145) queen(5,84) ...
SATISFIABLE

Models      : 1+
Time        : 22.654s (Solving: 10.53s 1st Model: 10.47s Unsat: 0.00s)
CPU Time    : 15.750s
Choices     : 1058729
Conflicts   : 2128
Restarts    : 6

Variables   : 403123
Constraints : 49636
```

Outline

41 Tweaking *N*-Queens

42 Do's and Dont's

43 Hints

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change
2. use variable-sized conjunction (via ':') ... adapts to changing facts
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).
```

Potassco

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change ✗
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Example: vegetables to buy

```
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pro(asparagus,cheap).  pro(cucumber,cheap).
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buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty).
```

Potassco

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pro(asparagus,tasty).  pro(cucumber,tasty).
pro(asparagus,clean).

buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty), pro(X,clean).
```

Potassco

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Example: vegetables to buy

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pro(asparagus,tasty).   pro(cucumber,tasty).
pro(asparagus,clean).
```

Potassco

Implementing Universal Quantification

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Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).   pre(cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).   pre(fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).   pre(tasty).
pro(asparagus,clean).

buy(X) :- veg(X), pro(X,P) : pre(P).
```

Potassco

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

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Example: vegetables to buy

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veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).  pre(cheap).
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pro(asparagus,tasty).  pro(cucumber,tasty).  pre(tasty).
pro(asparagus,clean).                         pre(clean).

buy(X) :- veg(X), pro(X,P) : pre(P).
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Potassco

Implementing Universal Quantification

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veg(asparagus).         veg(cucumber).
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pro(asparagus,fresh).   pro(cucumber,fresh).   pre(fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).   pre(tasty).
pro(asparagus,clean).
```

Potassco

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
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Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap). pre(cheap).
pro(asparagus,fresh). pro(cucumber,fresh). pre(fresh).
pro(asparagus,tasty). pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

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pro(asparagus,clean).                      pre(clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Potassco

Implementing Universal Quantification

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Example: vegetables to buy

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veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).    pro(cucumber,cheap).   pre(cheap).
pro(asparagus,fresh).    pro(cucumber,fresh).   pre(fresh).
pro(asparagus,tasty).    pro(cucumber,tasty).   pre(tasty).
pro(asparagus,clean).                           pre(clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Potassco

Running Example: Latin Square

Given: an $N \times N$ board

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

represented by facts:

```
square(1,1). ... square(1,6).
square(2,1). ... square(2,6).
square(3,1). ... square(3,6).
square(4,1). ... square(4,6).
square(5,1). ... square(5,6).
square(6,1). ... square(6,6).
```

Wanted: assignment of $1, \ldots, N$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	1	2
4	4	5	6	1	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5

represented by atoms:

```
num(1,1,1) num(1,2,2) ... num(1,6,6)
num(2,1,2) num(2,2,3) ... num(2,6,1)
num(3,1,3) num(3,2,4) ... num(3,6,2)
num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6.5)
```

Running Example: Latin Square

Given: an $N \times N$ board

1						
2						
3						
4						
5						
6						
	1	2	3	4	5	6

represented by facts:

```
square(1,1). ... square(1,6).
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square(3,1). ... square(3,6).
square(4,1). ... square(4,6).
square(5,1). ... square(5,6).
square(6,1). ... square(6,6).
```

Wanted: assignment of $1, \ldots, N$

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6	6	1	2	3	4	5
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num(3,1,3) num(3,2,4) ... num(3,6,2)
num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6.5)
```

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).



% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

Projecting Irrelevant Details Out

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```
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```

42056 273672 1690522

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
squareX(X) :- square(X,Y).       squareY(Y) :- square(X,Y).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- squareX(X1)  , N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- squareY(Y1)  , N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

```
gringo latin_0.lp | wc
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:- squareY(Y1)  , N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

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```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

- Note duplicate ground rules (swapping Y1/Y2 and X1/X2 gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

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:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

- Note duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

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gringo latin_2.lp | wc
```

2071560 12389384 40906946

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gringo latin_3.lp | wc
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1055752 6294536 21099558

Unraveling Symmetric Inequalities

Another Latin square encoding

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% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
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- Note duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

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```

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Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 <  Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 <  X2.
```

- Note duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

```
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2071560 12389384 40906946

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1055752 6294536 21099558

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% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 <  Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 <  X2.
```

- Note duplicate ground rules (swapping Y1/Y2 and X1/X2 gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

- Note uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

co

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.     gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.     gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

co

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc          gringo latin_6.lp | wc

304136 5778440 30252505         48136 373768 2185042
```

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% DISPLAY
#hide. #show num/3. #show sigma/1.
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gringo latin_5.lp | wc
304136 5778440 30252505
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48136 373768 2185042
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Assigning Aggregate Values

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% DOMAIN
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sigma(S) :- S = #sum[ square(X,n) = X ].                    ✔

% GENERATE
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occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
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```

```
gringo latin_5.lp | wc                gringo latin_6.lp | wc

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% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C #count{ num(X,Y,N) } C, C = 0..n.
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:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

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```

- Note internal transformation by gringo

Assigning Aggregate Values

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% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.      ✗
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:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.



% DISPLAY
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```
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```

Potassco

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Breaking Symmetries

The ultimate Latin square encoding?

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% DISPLAY
#hide. #show num/3.
```

- Note many symmetric solutions (mirroring, rotation, value permutation)

Potassco

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

- Note easy and safe to fix a full row/column!

Potassco

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

- Note easy and safe to fix a full row/column!

Potassco

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

- Note Let's compare enumeration speed!

Breaking Symmetries

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% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

Breaking Symmetries

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:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.



% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

```
Models : 161280    Time : 2.078s
```

Breaking Symmetries

The ultimate Latin square encoding?

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 161280    Time : 2.078s
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 1344      Time : 0.024s
```

Outline

41 Tweaking N-Queens

42 Do's and Dont's

43 Hints

Encode With Care!

1. Create a working encoding
   - Q1: Do you need to modify the encoding if the facts change?
   - Q2: Are all variables significant (or statically functionally dependent)?
   - Q3: Can there be (many) identic ground rules?
   - Q4: Do you enumerate pairs of values (to test uniqueness)?
   - Q5: Do you assign dynamic aggregate values (to check a fixed bound)?
   - Q6: Do you admit (obvious) symmetric solutions?
   - Q7: Do you have additional domain knowledge simplifying the problem?
   - Q8: Are you aware of anything else that, if encoded, would reduce grounding and/or solving efforts?
2. Revise until no "Yes" answer!
   - Note If the format of facts makes encoding painful (for instance, abusing grounding for "scientific calculations"), revise the fact format as well.

Potassco

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Some Hints on (Preventing) Debugging

Kinds of errors

- syntactic ... follow error messages by the grounder
- semantic ... (most likely) encoding/intention mismatch

Ways to identify semantic errors (early)

- develop and test incrementally
  - prepare toy instances with "interesting features"
  - build the encoding bottom-up and verify additions (eg. new predicates)
- compare the encoded to the intended meaning
  - check whether the grounding fits (use `gringo -t`)
  - if stable models are unintended, investigate conditions that fail to hold
  - if stable models are missing, examine integrity constraints (add heads)
- ask tools (eg. `http://www.kr.tuwien.ac.at/research/projects/mmdasp/`)

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Overcoming Performance Bottlenecks

Grounding

- monitor time spent by and output size of `gringo`
  1. system tools (eg. `time(gringo [...] | wc)`)
  2. profiling info (eg. `gringo --gstats --verbose=3 [...] > /dev/null`)
- Note once identified, reformulate "critical" logic program parts

Solving

- check solving statistics (use `clasp --stats`)

  if great search efforts (`Conflicts`/`Choices`/`Restarts`), then

  try prefabricated settings (using clasp option '`--configuration`')

  try auto-configuration (offered by `claspfolio`)

  try manual fine-tuning (requires expert knowledge!)

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- Note if great search efforts (`Conflicts`/`Choices`/`Restarts`), then
  1. try prefabricated settings (using `clasp` option '`--configuration`')
  2. try auto-configuration (offered by `claspfolio`)
  3. try manual fine-tuning (requires expert knowledge!)
  4. if possible, reformulate the problem or add domain knowledge ("redundant" constraints) to help the solver

Overcoming Performance Bottlenecks

Grounding

- monitor time spent by and output size of `gringo`
  1. system tools (eg. `time(gringo [...] | wc)`)
  2. profiling info (eg. `gringo --gstats --verbose=3 [...] > /dev/null`)
- Note once identified, reformulate "critical" logic program parts

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Outline

44 Summary

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- ASP is a viable tool for Knowledge Representation and Reasoning
- ASP offers efficient and versatile off-the-shelf solving technology
- ASP offers an expanding functionality and ease of use
  - Rapid application development tool
- ASP has a growing range of applications

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`http://potassco.sourceforge.net`

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