Modeling and Solving in Answer Set Programming

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Outline

1. Motivation
2. Introduction
3. Basic modeling
4. (Language extensions)
5. Solving
6. Advanced modeling
7. Systems
8. Summary

Bibliography

Bias

Focus

- Answer Set Programming as Boolean Constraint Satisfaction Problem
- Answer Set Solving as a Boolean Constraint Solving
- Answer Set Systems at `http://potassco.sourceforge.net`

Further resources

`http://potassco.sourceforge.net/teaching.html`

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Motivation: Overview

1 Motivation

2 Nutshell

3 Shifting paradigms

4 Rooting ASP

5 Problem solving

6 Use

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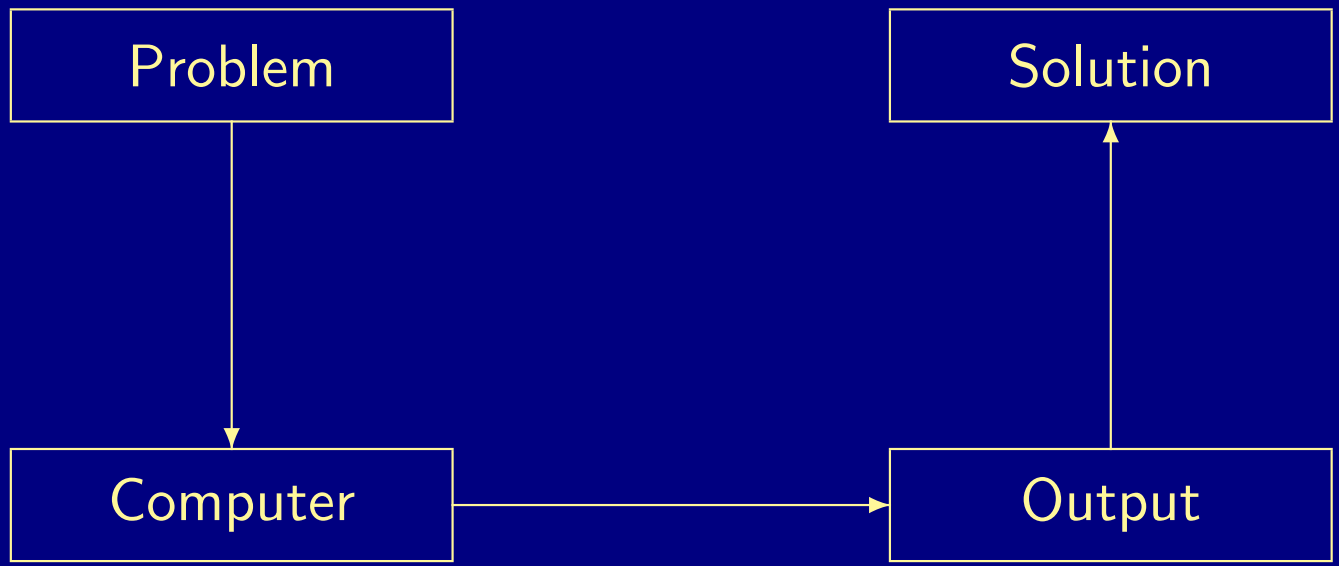
Informatics

"What is the problem?" versus *"How to solve the problem?"*



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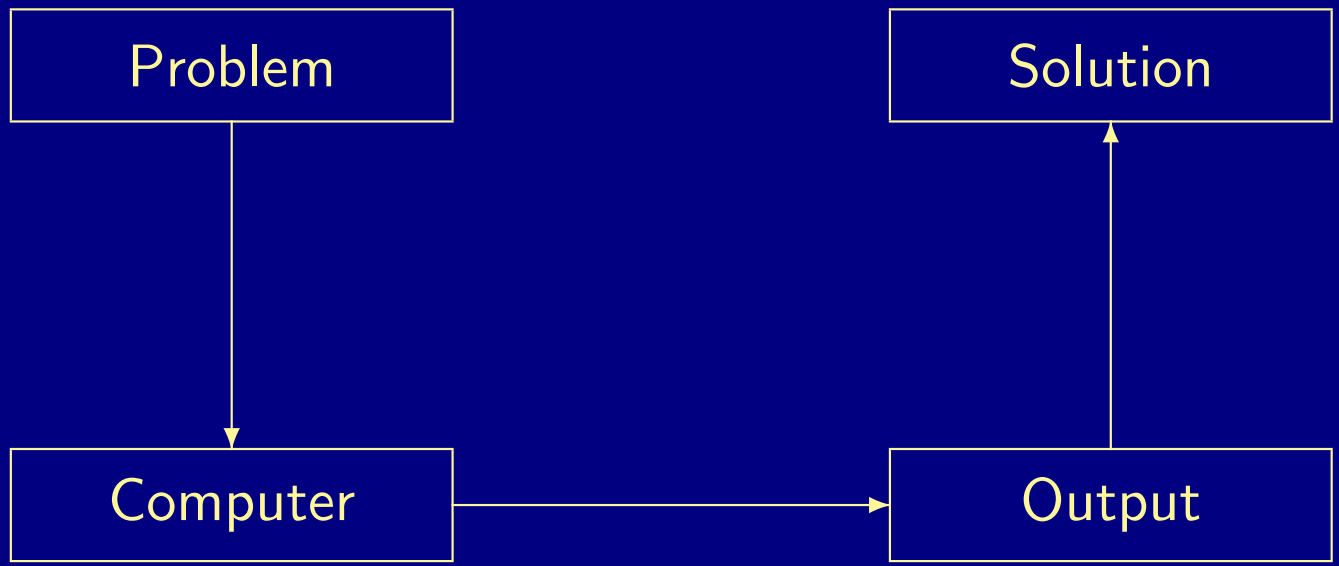

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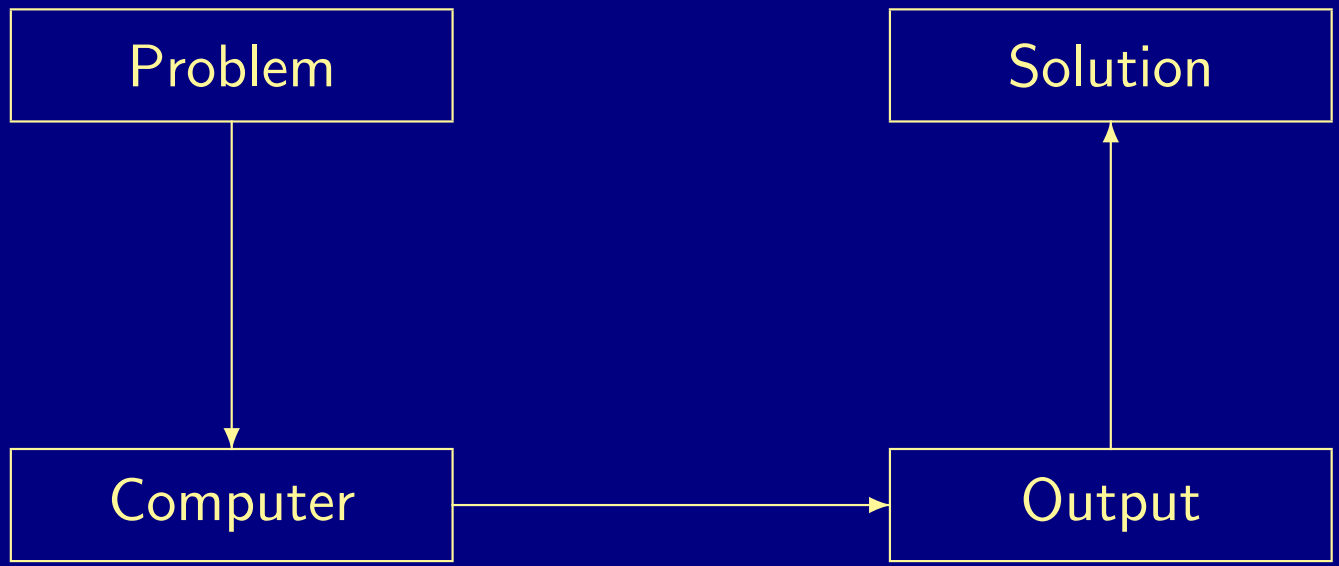

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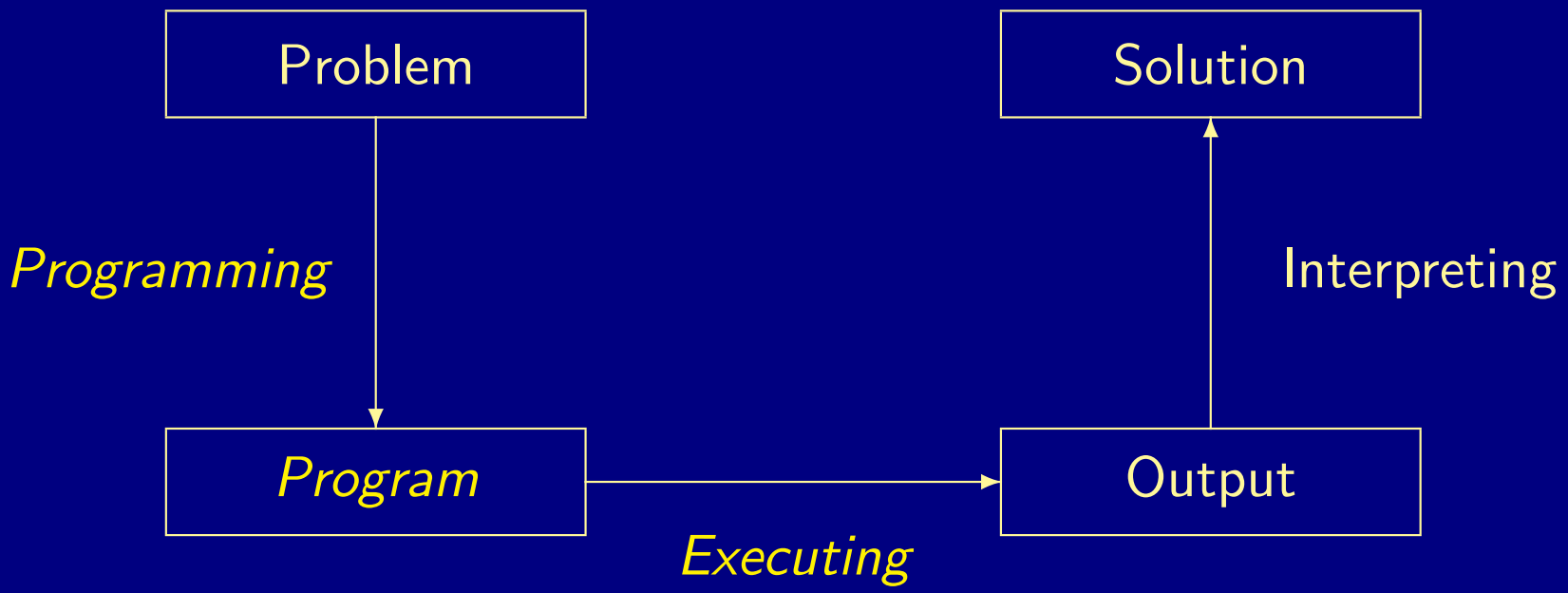

Declarative problem solving

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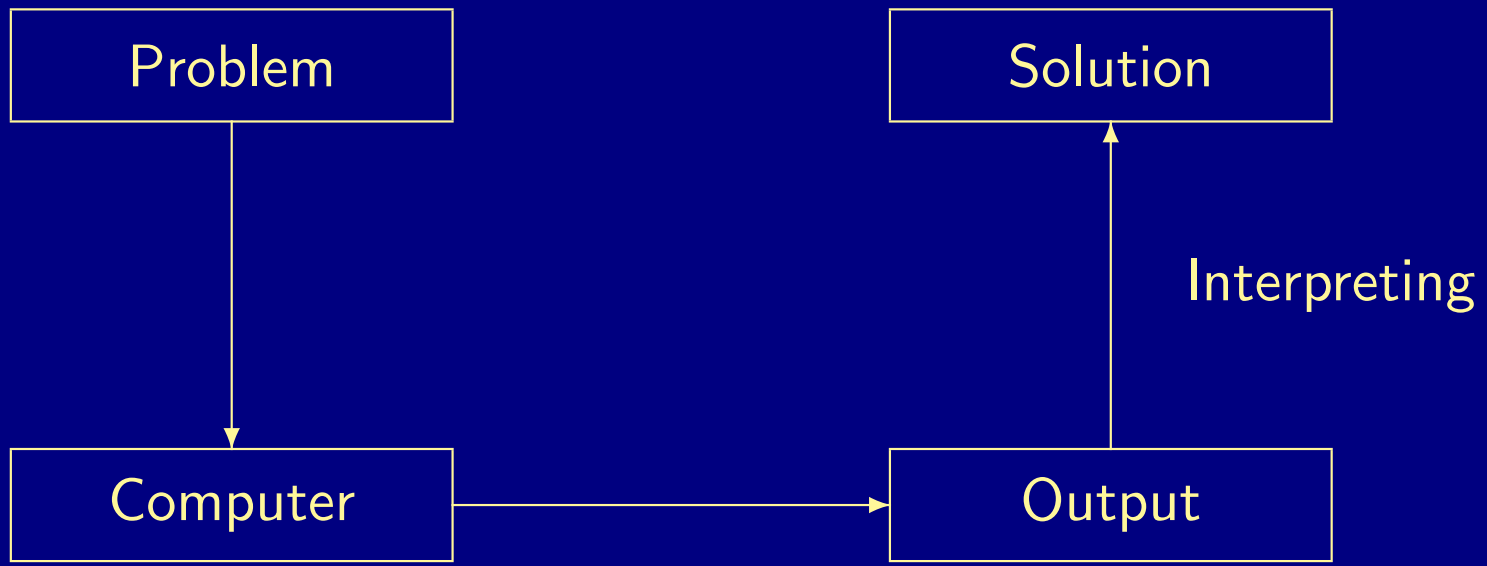

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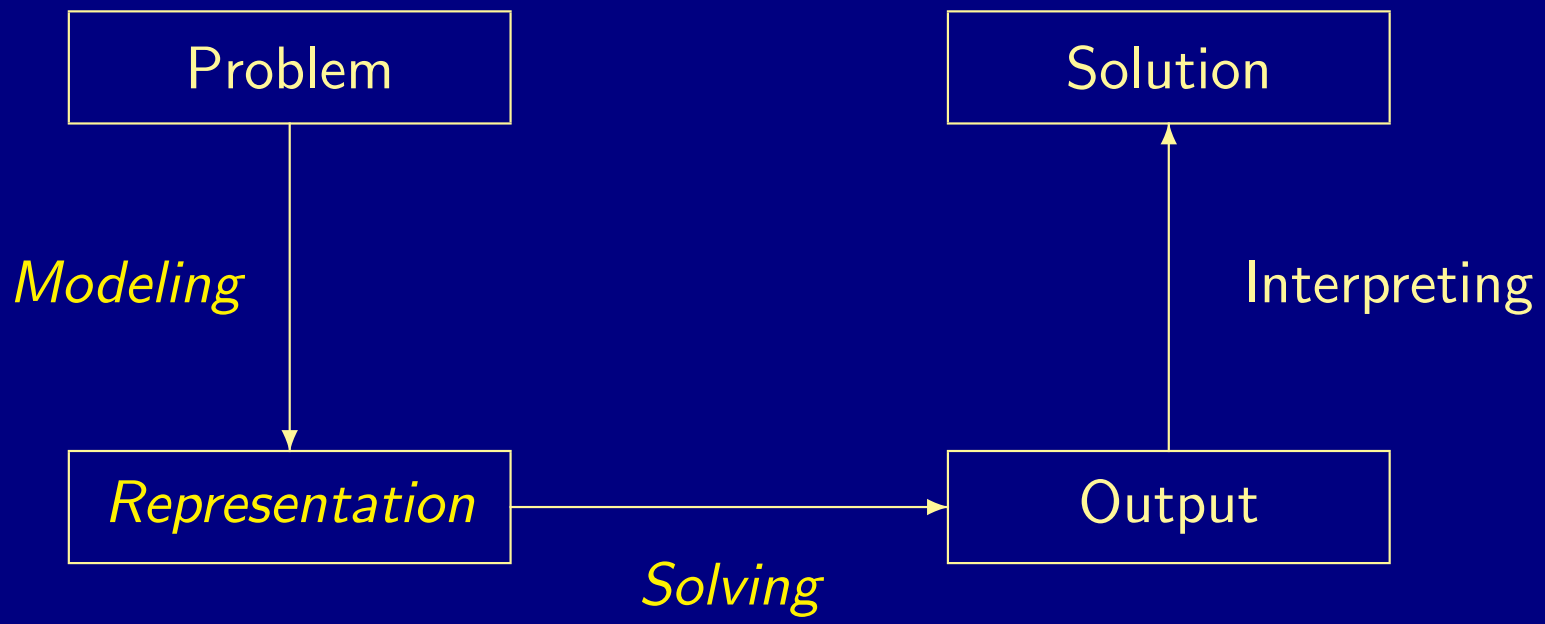

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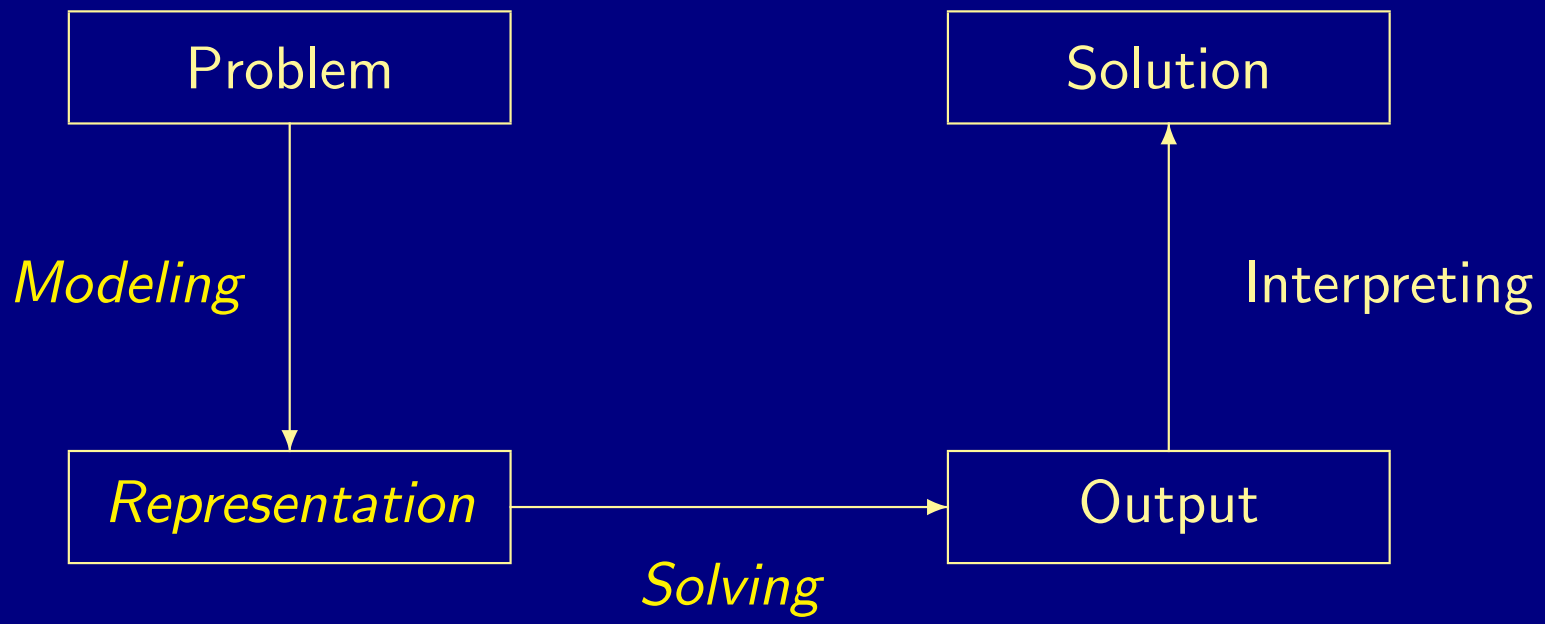

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Answer Set Programming

in a Nutshell

ASP is an approach to declarative problem solving, combining
- a rich yet simple modeling language
- with high-performance solving capacities

ASP has its roots in
- (logic-based) knowledge representation and (nonmonotonic) reasoning
- (deductive) databases
- constraint solving (in particular, SATisfiability testing)
- logic programming (with negation)

ASP allows for solving all search problems in NP (and NP^{NP}) in a uniform way

ASP is versatile as reflected by the ASP solver *clasp*, winning first places at ASP'07/09/11, CASC'11, MISC'11, PB'09/11, and SAT'09/11

ASP embraces many emerging application areas

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ASP = KR+DB+SAT+LP

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KR's shift of paradigm

Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem.
2. A solution is given by a derivation of a query.

Model Generation based approach (eg. SATisfiability testing)

1. Provide a representation of the problem.
2. A solution is given by a model of the representation.

Automated planning, Kautz and Selman (ECAI'92)

Represent planning problems as propositional theories so that models not proofs describe solutions

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Model Generation based Problem Solving

Representation	Solution
constraint satisfaction problem	assignment
propositional horn theories	smallest model
propositional theories	models
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propositional theories	stable models
propositional programs	minimal models
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first-order theories	models
first-order theories	minimal models
first-order theories	stable models
first-order theories	Herbrand models
auto-epistemic theories	expansions
default theories	extensions
⋮	⋮

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Model Generation based Problem Solving

Representation	Solution	
constraint satisfaction problem	assignment	
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propositional theories	models	**SAT**
propositional theories	minimal models	
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⋮	⋮	

LP-style playing with blocks

Prolog program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Prolog queries

```
?- above(a,c).
true.

?- above(c,a).
no.
```

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Another Prolog program

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SAT-style playing with blocks

Formula

$$
\begin{aligned}
& on(a,b) \\
\wedge \quad & on(b,c) \\
\wedge \quad & (on(X,Y) \rightarrow above(X,Y)) \\
\wedge \quad & (on(X,Z) \wedge above(Z,Y) \rightarrow above(X,Y))
\end{aligned}
$$

Herbrand model

$$
\begin{aligned}
\{ & on(b,b), on(a,b), on(b,c), on(a,c), \\
& above(b,b), above(c,b), above(a,b), \\
& above(b,c), above(c,c), above(a,c) \ \}
\end{aligned}
$$

SAT-style playing with blocks

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Herbrand model (among 426!)

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Answer Set Programming *at large*

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Answer Set Programming *commonly*

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Answer Set Programming *in practice*

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Stable Herbrand model

$$\{\ \texttt{on(a,b)}, \texttt{on(b,c)}, \texttt{above(b,c)}, \texttt{above(a,b)}, \texttt{above(a,c)}\ \}$$

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Stable Herbrand model (and no others)

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ASP versus LP

ASP	Prolog
Model generation	Query orientation
Bottom-up	Top-down
Modeling language	Programming language
Rule-based format	
Instantiation Flat terms	Unification Nested terms
(Turing +) $NP(^{NP})$	Turing

ASP versus SAT

ASP	SAT
Model generation	
Bottom-up	
Constructive Logic	Classical Logic
Closed (and open) world reasoning	Open world reasoning
Modeling language	—
Complex reasoning modes	Satisfiability testing
Satisfiability	Satisfiability
Enumeration/Projection	—
Optimization	—
Intersection/Union	—
(Turing +) $NP(^{NP})$	NP

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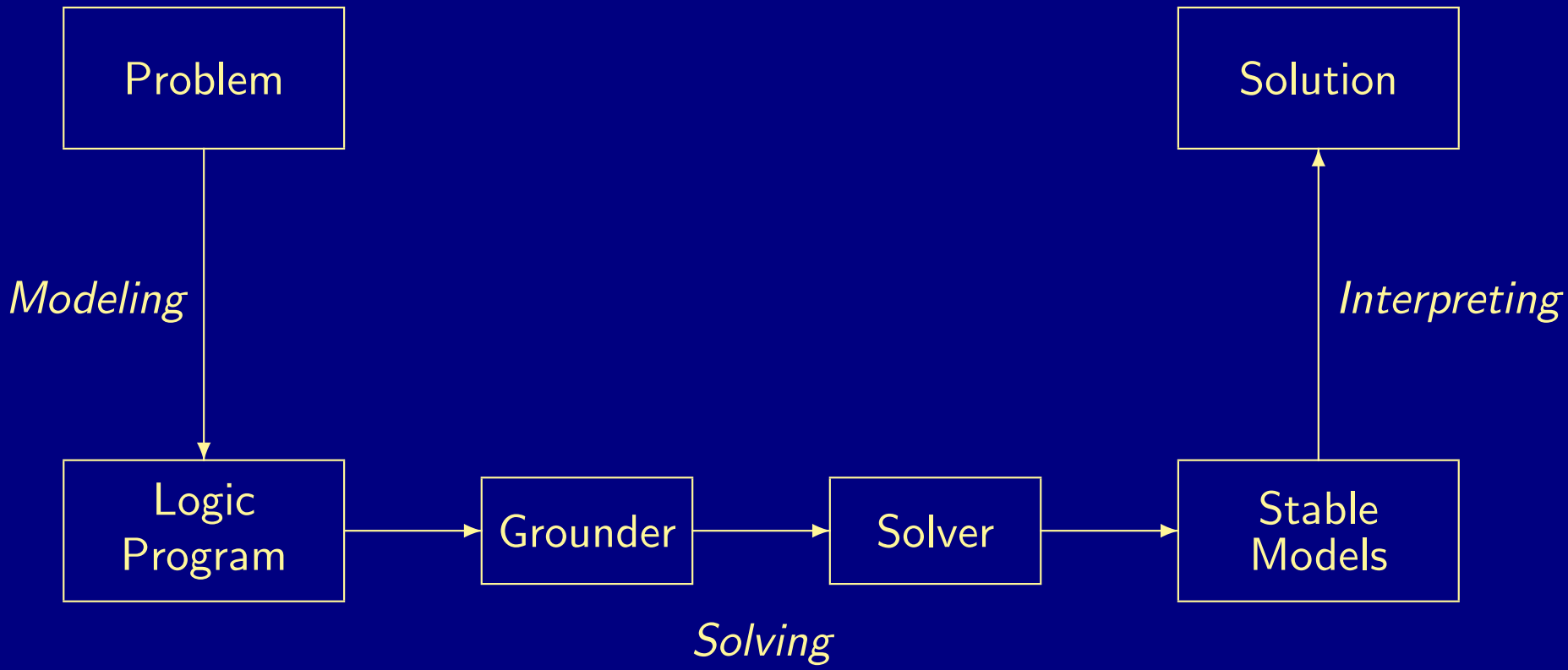
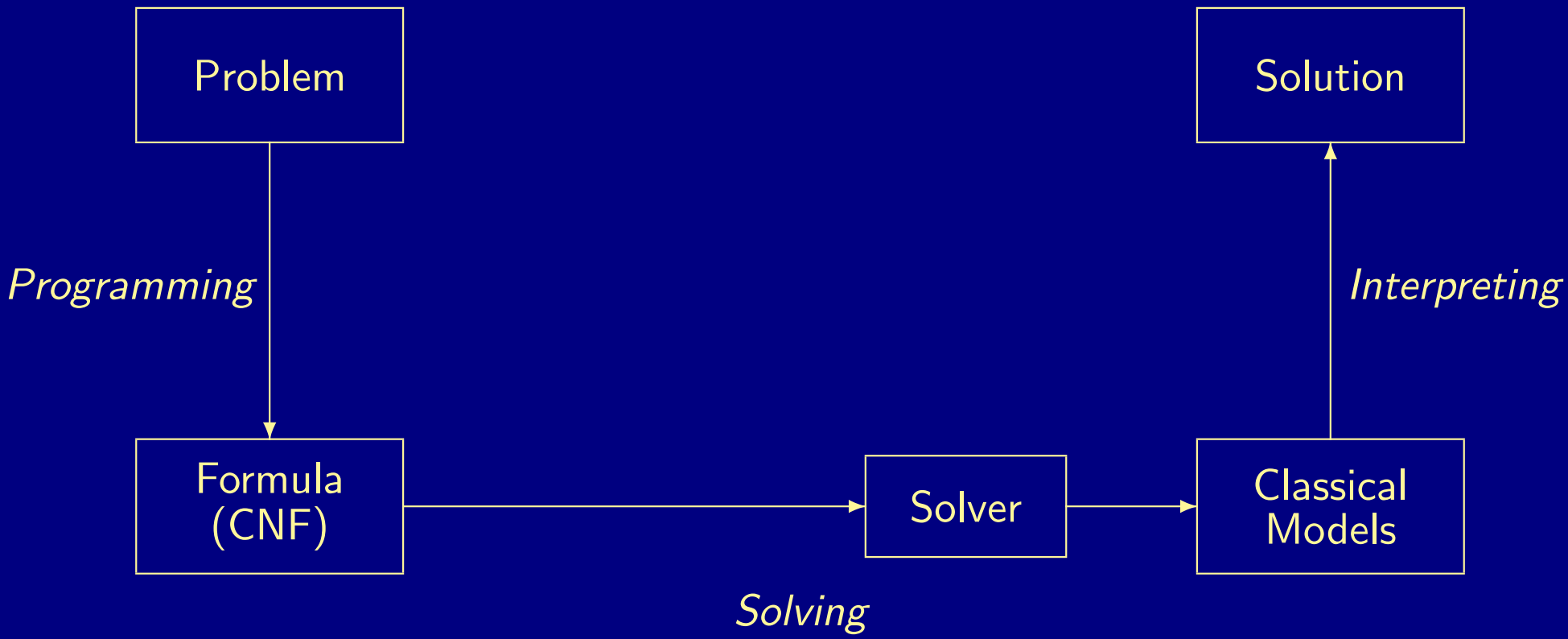
ASP solving


Problem
Solution
Modeling
Interpreting
Logic Program
Grounder
Solver
Stable Models
Solving

SAT solving


Problem
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Formula
(CNF)
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Classical
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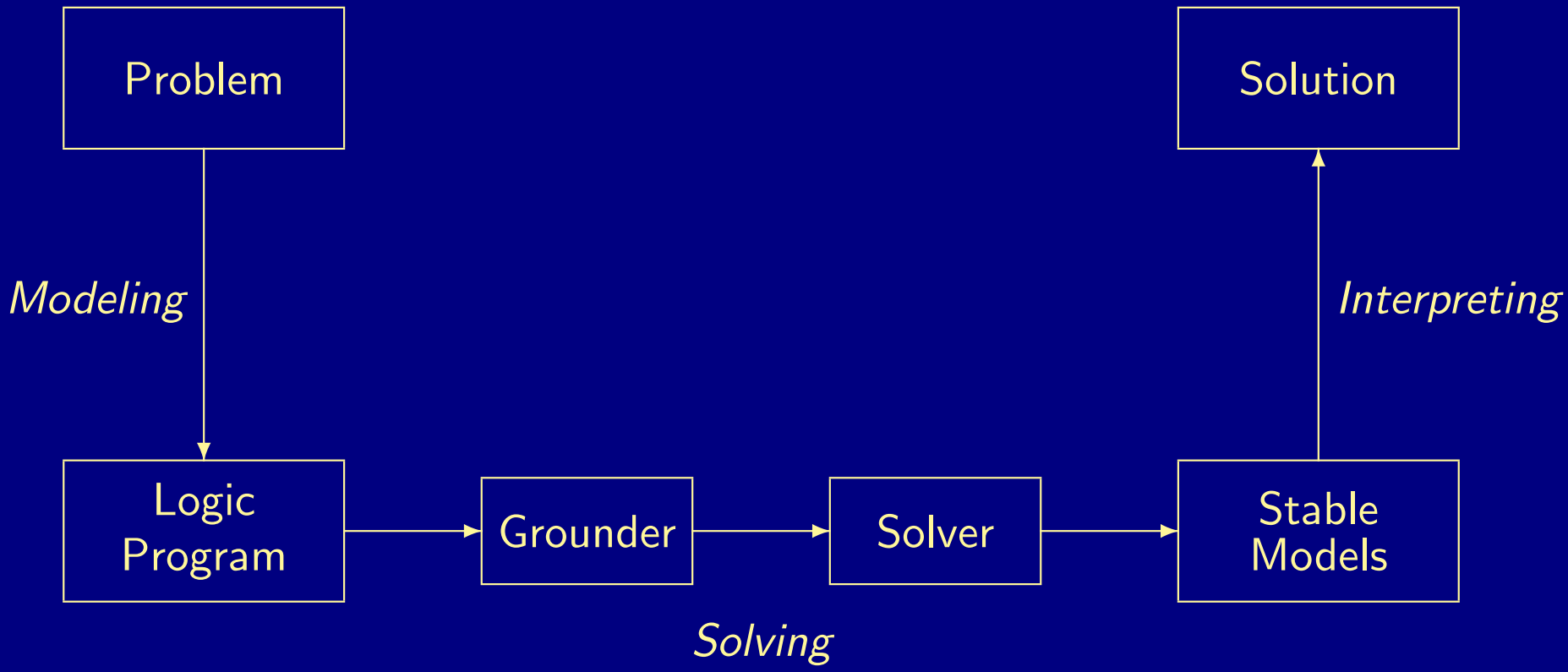
Rooting ASP solving



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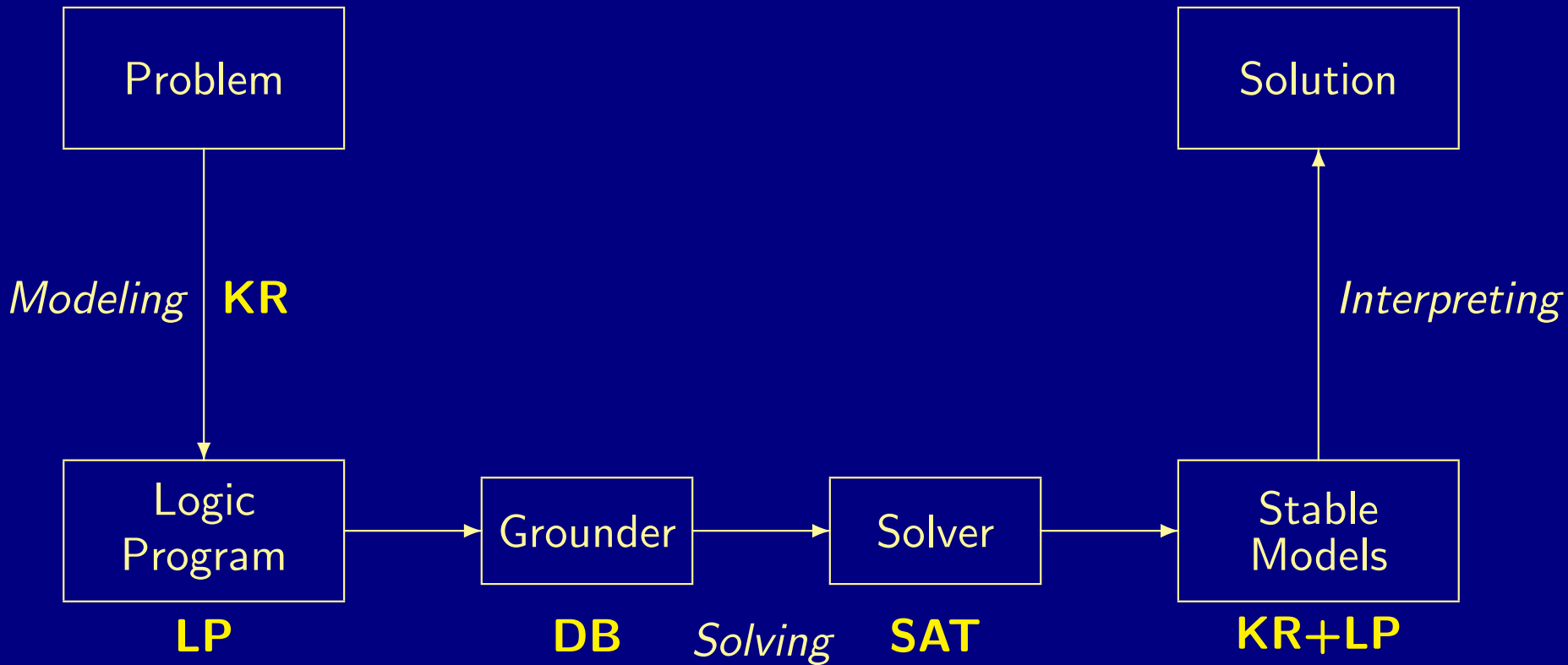

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What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated Planning,
  - Code Optimization,
  - Composition of Renaissance Music,
  - Database Integration,
  - Decision Support for NASA shuttle controllers,
  - Model Checking,
  - Product Configuration,
  - Robotics,
  - System Biology,
  - System Synthesis,
  - (industrial) Team-building,
  - and many many more.

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What does ASP offer?

- Integration of KR, DB, and SAT techniques
- Succinct, elaboration-tolerant problem representations
  - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
  - including: data, frame axioms, exceptions, defaults, closures, etc.

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7 Syntax

8 Semantics

9 Examples

10 Variables

11 Language Constructs

12 Reasoning Modes

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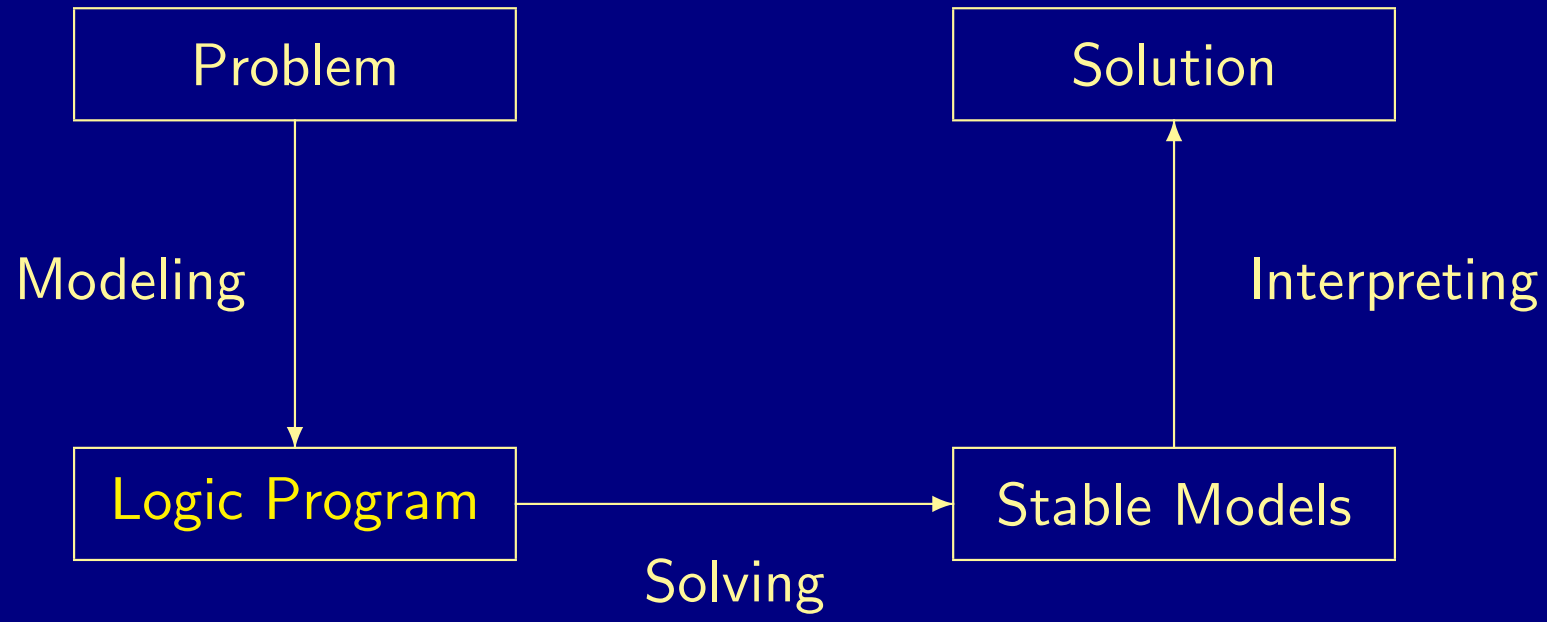
Problem solving in ASP: Syntax



Normal logic programs

- A (normal) rule, r, is an ordered pair of the form

$$A_0 \leftarrow A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n,$$

  where $n \geq m \geq 0$, and each A_i $(0 \leq i \leq n)$ is an atom.
- A (normal) logic program is a finite set of rules.
- Notation

$$\begin{aligned} \mathit{head}(r) &= A_0 \\ \mathit{body}(r) &= \{A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n\} \\ \mathit{body}(r)^+ &= \{A_1, \ldots, A_m\} \\ \mathit{body}(r)^- &= \{A_{m+1}, \ldots, A_n\} \end{aligned}$$

- A program is called positive if $\mathit{body}(r)^- = \emptyset$ for all its rules.

Normal logic programs

- A (normal) rule, r, is an ordered pair of the form

$$A_0 \leftarrow A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n,$$

  where $n \geq m \geq 0$, and each A_i $(0 \leq i \leq n)$ is an atom.
- A (normal) logic program is a finite set of rules.
- Notation

$$\begin{aligned} \mathit{head}(r) &= A_0 \\ \mathit{body}(r) &= \{A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n\} \\ \mathit{body}(r)^+ &= \{A_1, \ldots, A_m\} \\ \mathit{body}(r)^- &= \{A_{m+1}, \ldots, A_n\} \end{aligned}$$

- A program is called positive if $\mathit{body}(r)^- = \emptyset$ for all its rules.

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(Rough) notational convention

We sometimes use the following notation interchangeably in order to stress the respective view:

	if	and	or	negation as failure	classical negation
source code	`:-`	`,`	`|`	`not`	`-`
logic program	$\leftarrow$	,	;	$not/\sim$	$\neg$
formula	$\rightarrow$	$\wedge$	$\vee$	$\sim/(\neg)$	$\neg$

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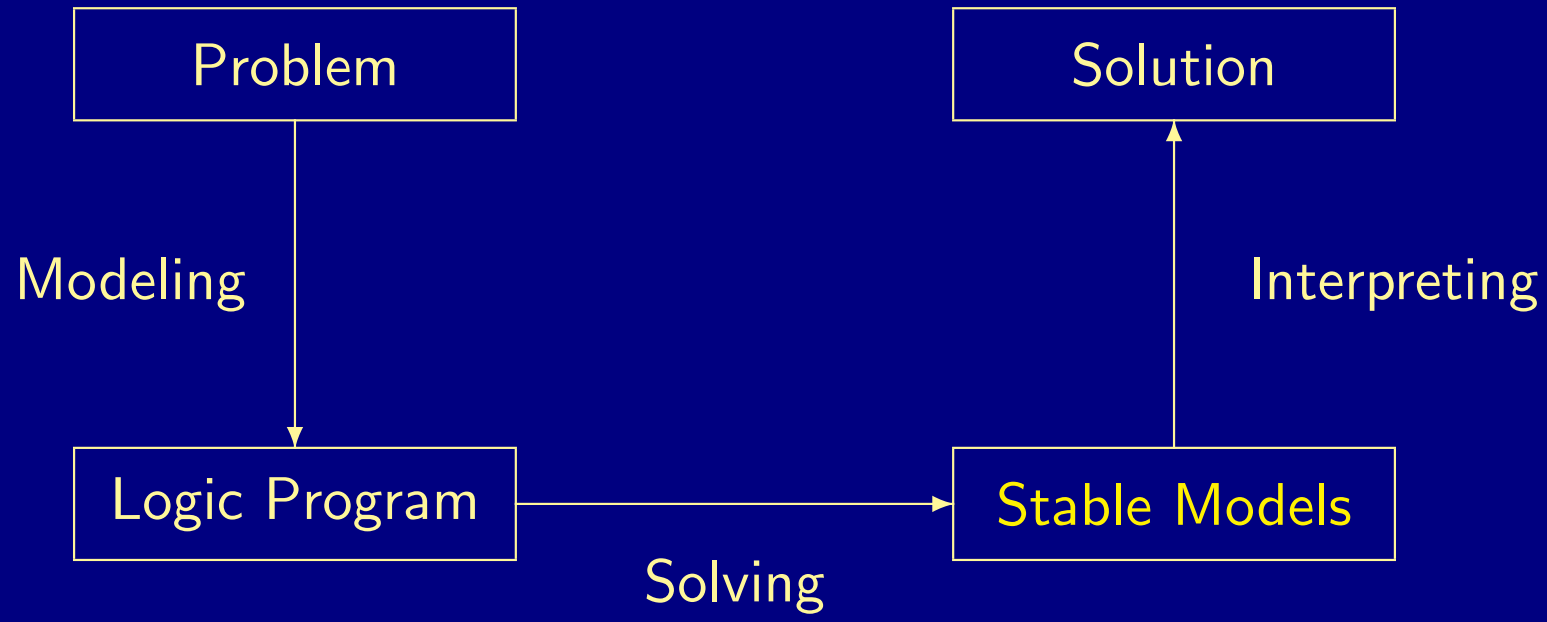
Problem solving in ASP: Semantics



Answer set: Formal Definition

Positive programs

- A set of atoms X is closed under a positive program Π iff for any $r \in \Pi$, $\mathit{head}(r) \in X$ whenever $\mathit{body}(r)^+ \subseteq X$.
  - ➥ X corresponds to a model of Π (seen as a formula).
- The smallest set of atoms which is closed under a positive program Π is denoted by $Cn(\Pi)$.
  - ➥ $Cn(\Pi)$ corresponds to the $\subseteq$-smallest model of Π (ditto).
- The set $Cn(\Pi)$ of atoms is the answer set of a *positive* program Π.

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Some "logical" remarks

- Positive rules are also referred to as definite clauses.
  - Definite clauses are disjunctions with exactly one positive atom:

$$A_0 \vee \neg A_1 \vee \cdots \vee \neg A_m$$

  - A set of definite clauses has a (unique) smallest model.
- Horn clauses are clauses with at most one positive atom.
  - Every definite clause is a Horn clause but not vice versa.
  - Non-definite Horn clauses can be regarded as integrity constraints.
  - A set of Horn clauses has a smallest model or none.
- This smallest model is the intended semantics of such set of clauses.
  - Given a positive program Π, $Cn(\Pi)$ corresponds to the smallest model of the set of definite clauses corresponding to Π.

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Answer set: Basic idea

Consider the logical formula Φ and its three (classical) models:

Φ $\quad q \wedge (q \wedge \neg r \rightarrow p)$

$\{p, q\}, \{q, r\},$ and $\{p, q, r\}$

Formula Φ has one stable model, often called answer set:

Π_Φ

$q \leftarrow$

$p \leftarrow q,\ not\ r$

$\{p, q\}$

Informally, a set X of atoms is an answer set of a logic program Π

if X is a (classical) model of Π and

if all atoms in X are justified by some rule in Π

(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))

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$$\boxed{\begin{array}{lcl} p & \mapsto & 1 \\ q & \mapsto & 1 \\ r & \mapsto & 0 \end{array}}$$

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Normal programs

- The reduct, Π^X, of a program Π relative to a set X of atoms is defined by

$$\Pi^X = \{head(r) \leftarrow body(r)^+ \mid r \in \Pi \text{ and } body(r)^- \cap X = \emptyset\}.$$

- A set X of atoms is a stable model of a program Π, if $Cn(\Pi^X) = X$.

- Note: $Cn(\Pi^X)$ is the $\subseteq$–smallest (classical) model of Π^X.
- Note: Every atom in X is justified by an *"applying rule from* Π*"*
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A closer look at Π^X

- In other words, given a set X of atoms from Π,

  Π^X is obtained from Π by deleting
  1. each rule having a *not* A in its body with $A \in X$
     and then
  2. all negative atoms of the form *not* A
     in the bodies of the remaining rules.

- Note: Only negative body literals are evaluated wrt X

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A first example

$\Pi = \{p \leftarrow p,\ \ q \leftarrow not\ p\}$

X	Π^X	$Cn(\Pi^X)$
$\emptyset$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p\}$	$p \leftarrow p$	$\emptyset$
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p, q\}$	$p \leftarrow p$	$\emptyset$

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Answer set: Some properties

- A logic program may have zero, one, or multiple answer sets!
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Programs with Variables

Let Π be a logic program.

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- Ground Instances of $r \in \Pi$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \to \mathcal{T}\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution.

- Ground Instantiation of Π: $ground(\Pi) = \bigcup_{r \in \Pi} ground(r)$

Programs with Variables

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- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$ (also called alphabet or Herbrand base)
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An example

$$\Pi = \{\ r(a,b) \leftarrow,\ r(b,c) \leftarrow,\ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a, b, c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

$$ground(\Pi) = \left\{ \begin{array}{l} r(a,b) \leftarrow\ , \\ r(b,c) \leftarrow\ , \\ t(a,a) \leftarrow r(a,a),\ t(b,a) \leftarrow r(b,a),\ t(c,a) \leftarrow r(c,a), \\ t(a,b) \leftarrow r(a,b),\ t(b,b) \leftarrow r(b,b),\ t(c,b) \leftarrow r(c,b), \\ t(a,c) \leftarrow r(a,c),\ t(b,c) \leftarrow r(b,c),\ t(c,c) \leftarrow r(c,c) \end{array} \right\}$$

Intelligent Grounding aims at reducing the ground instantiation.

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$$\mathcal{T} = \{a, b, c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

$$ground(\Pi) = \left\{ \begin{array}{l} r(a,b) \leftarrow\ , \\ r(b,c) \leftarrow\ , \\ t(a,a) \leftarrow r(a,a),\ t(b,a) \leftarrow r(b,a),\ t(c,a) \leftarrow r(c,a), \\ t(a,b) \leftarrow r(a,b),\ t(b,b) \leftarrow r(b,b),\ t(c,b) \leftarrow r(c,b), \\ t(a,c) \leftarrow r(a,c),\ t(b,c) \leftarrow r(b,c),\ t(c,c) \leftarrow r(c,c) \end{array} \right\}$$

- Intelligent Grounding aims at reducing the ground instantiation.

An example

$$\Pi = \{\ r(a,b) \leftarrow,\ r(b,c) \leftarrow,\ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a, b, c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

$$ground(\Pi) = \left\{ \begin{array}{l} r(a,b) \leftarrow\ , \\ r(b,c) \leftarrow\ , \\ t(a,a) \leftarrow r(a,a),\ t(b,a) \leftarrow r(b,a),\ t(c,a) \leftarrow r(c,a), \\ t(a,b) \leftarrow r(a,b),\ t(b,b) \leftarrow r(b,b),\ t(c,b) \leftarrow r(c,b), \\ t(a,c) \leftarrow r(a,c),\ t(b,c) \leftarrow r(b,c),\ t(c,c) \leftarrow r(c,c) \end{array} \right\}$$

- Intelligent Grounding aims at reducing the ground instantiation.

Answer sets of programs with Variables

Let Π be a normal logic program with variables.

- A set X of (ground) atoms as a stable model of Π, if $Cn(ground(\Pi)^X) = X$.

Answer sets of programs with Variables

Let Π be a normal logic program with variables.

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Introduction: Overview

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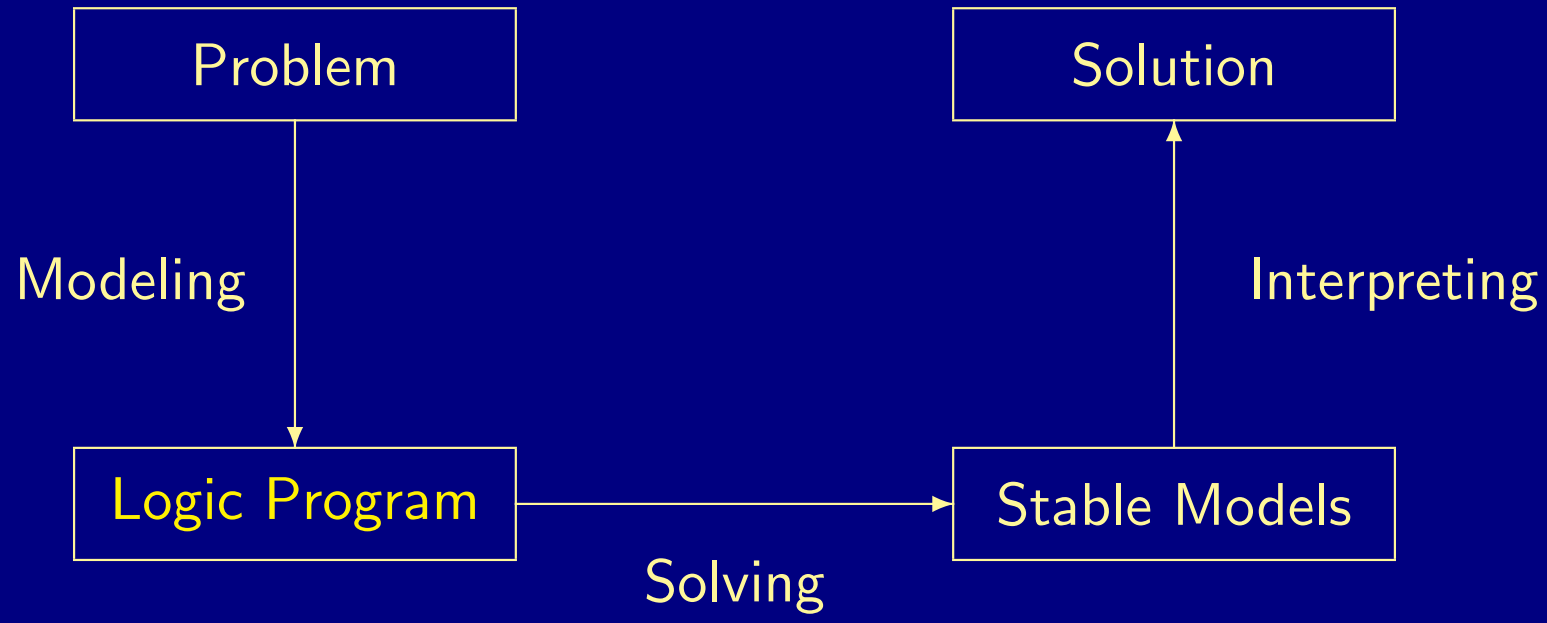
12 Reasoning Modes

Problem solving in ASP: Extended Syntax



Language Constructs

Variables (over the Herbrand Universe)

`p(X) :- q(X)` over constants $\{$`a, b, c`$\}$ stands for

```
p(a) :- q(a), p(b) :- q(b), p(c) :- q(c)
```

Conditional Literals

`p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for

```
p :- q(a), q(b), q(c)
```

Disjunction

```
p(X) | q(X) :- r(X)
```

Integrity Constraints

```
:- q(X), p(X)
```

Choice

```
2 { p(X,Y) : q(X) } 7 :- r(Y)
```

Aggregates

```
s(Y) :- r(Y), 2 #count { p(X,Y) : q(X) } 7
```

also: `#sum`, `#avg`, `#min`, `#max`, `#even`, `#odd`

Language Constructs

- Variables (over the Herbrand Universe)
  - `p(X) :- q(X)` over constants $\{$`a`, `b`, `c`$\}$ stands for
    `p(a) :- q(a), p(b) :- q(b), p(c) :- q(c)`
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  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for
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- Disjunction
  - `p(X) | q(X) :- r(X)`
- Integrity Constraints
  - `:- q(X), p(X)`
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  - `2 { p(X,Y) : q(X) } 7 :- r(Y)`
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  - `p(X) :- q(X)` over constants $\{a, b, c\}$ stands for
    `p(a) :- q(a), p(b) :- q(b), p(c) :- q(c)`
- Conditional Literals
  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for
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- Disjunction
  - `p(X) | q(X) :- r(X)`
- Integrity Constraints
  - `:- q(X), p(X)`
- Choice
  - `2 { p(X,Y) : q(X) } 7 :- r(Y)`
- Aggregates
  - `s(Y) :- r(Y), 2 #count { p(X,Y) : q(X) } 7`
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  - `s(Y) :- r(Y), 2 #count { p(X,Y) : q(X) } 7`
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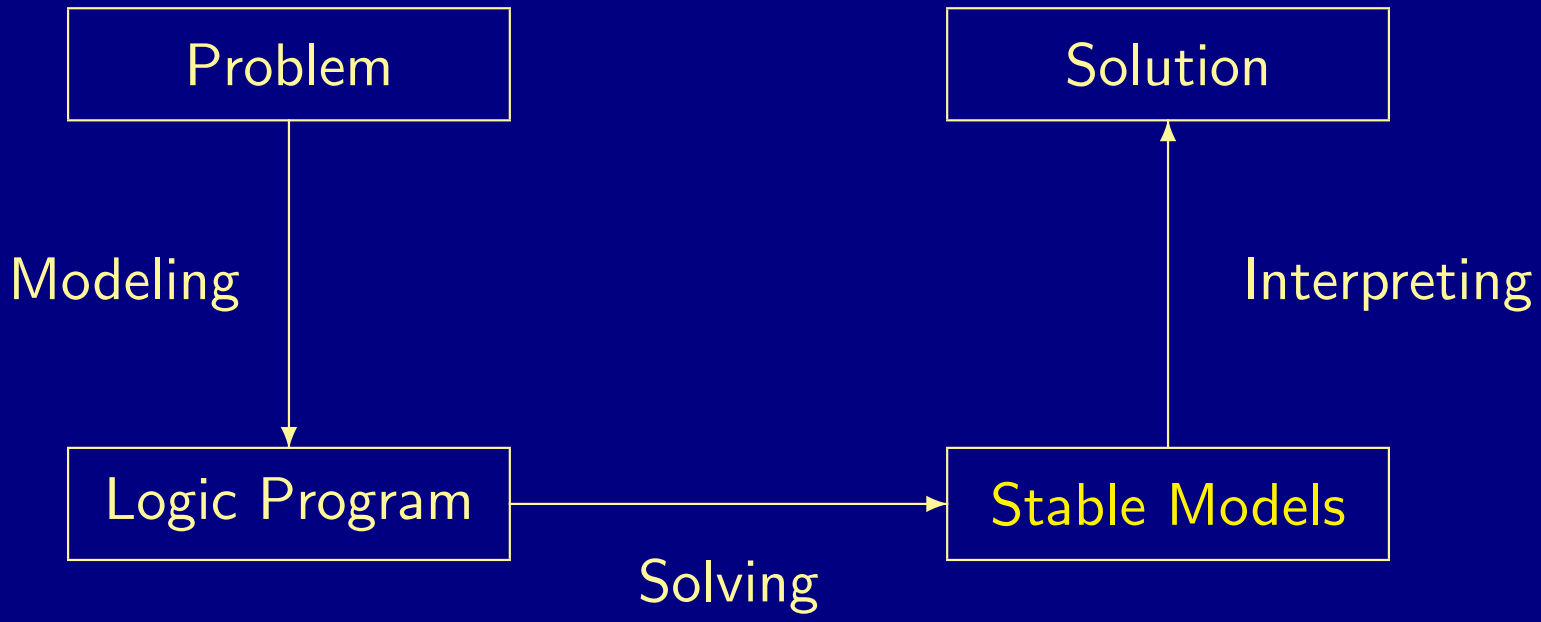
Problem solving in ASP: Reasoning Modes



Reasoning Modes

- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization
- and combinations of them

† without solution recording

‡ without solution enumeration

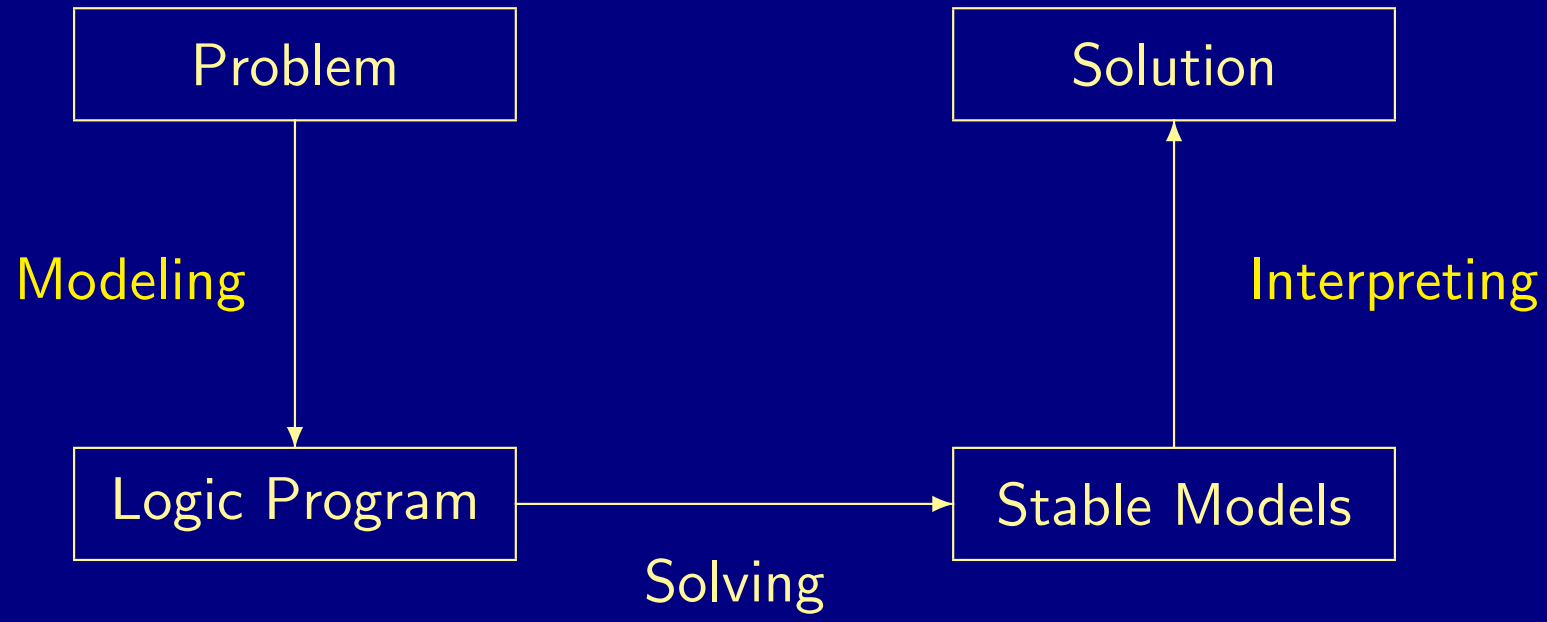
Basic Modeling: Overview

13 ASP Solving Process

14 Problems as Logic Programs
- Graph Coloring

15 Methodology
- Satisfiability
- Queens
- Reviewer Assignment
- Planning

Modeling and Interpreting



Modeling

For solving a problem class P for a problem instance I, encode

1. the problem instance I as a set C(I) of facts and
2. the problem class P as a set C(P) of rules

such that the solutions to P for I can be (polynomially) extracted from the answer sets of $C(I) \cup C(P)$.

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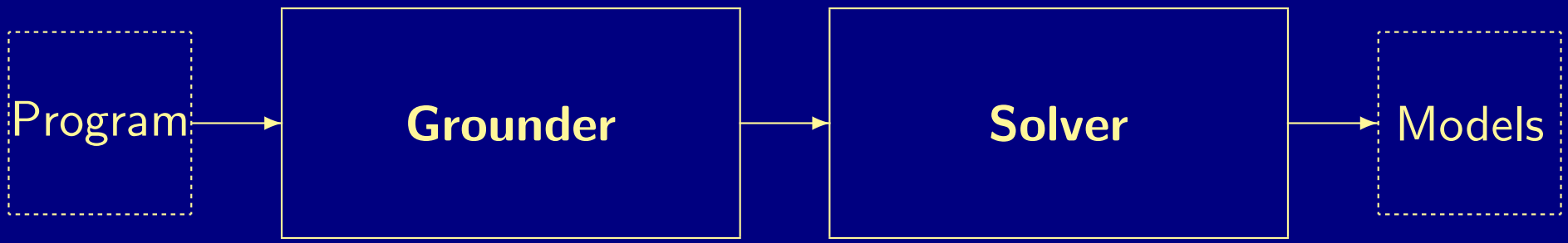
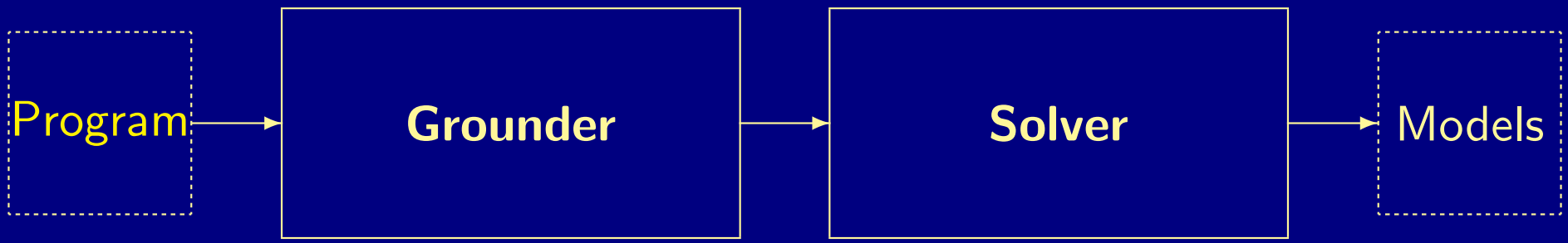
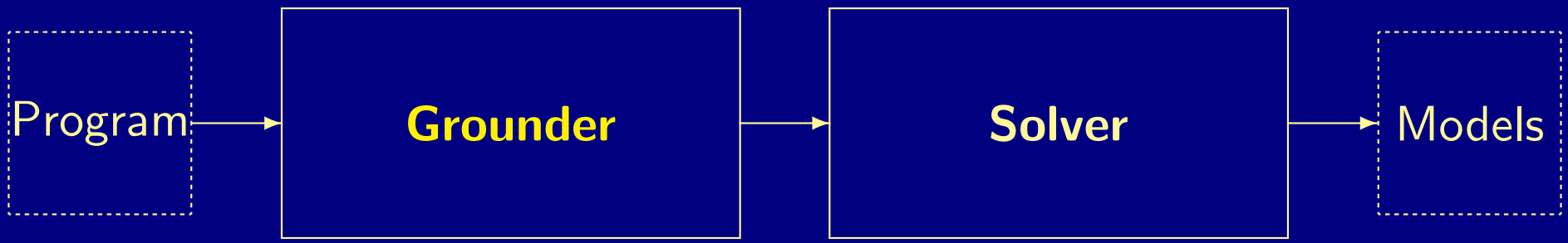
ASP Solving Process



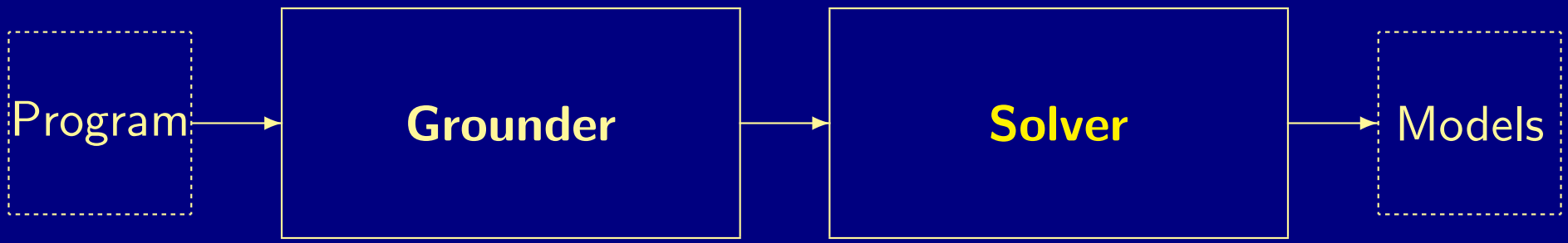
ASP Solving Process



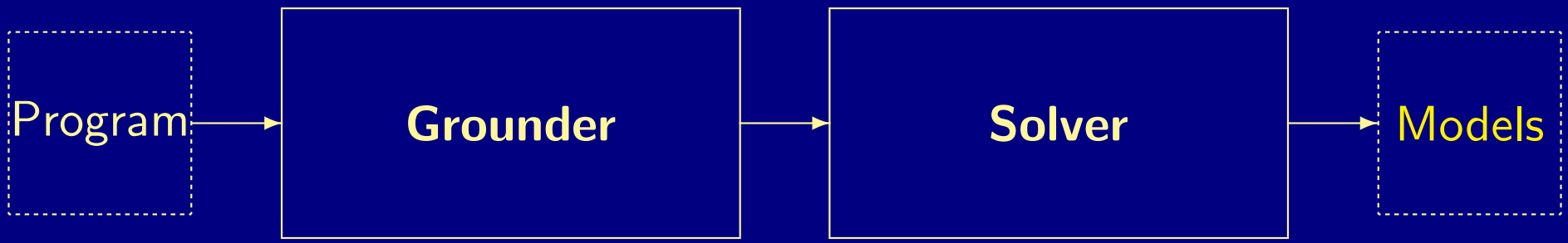
ASP Solving Process



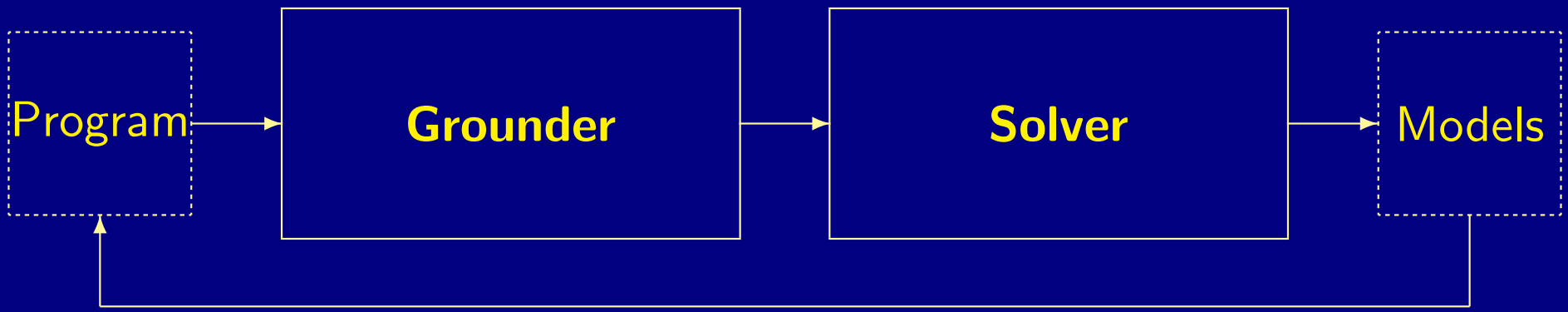
ASP Solving Process



ASP Solving Process



ASP Solving Process



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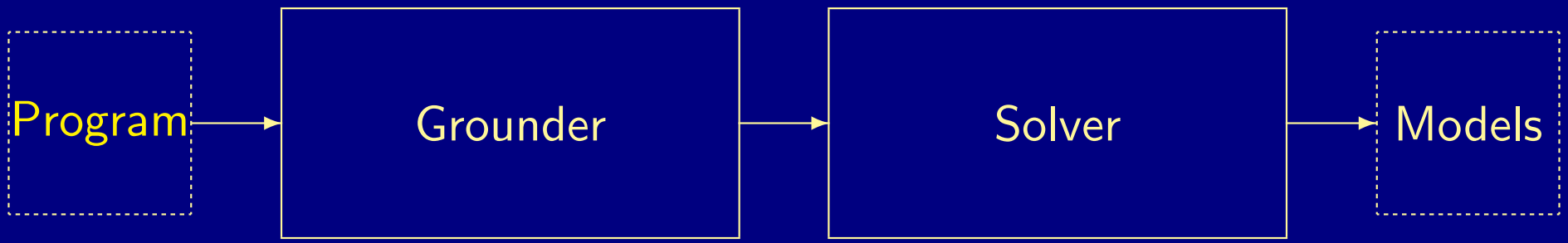
ASP Solving Process



Graph Coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).

1 {color(X,C) : col(C)} 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Graph Coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
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Graph Coloring

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edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).

1 {color(X,C) : col(C)} 1 :- node(X).

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Graph Coloring

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edge(4,1).  edge(4,2).
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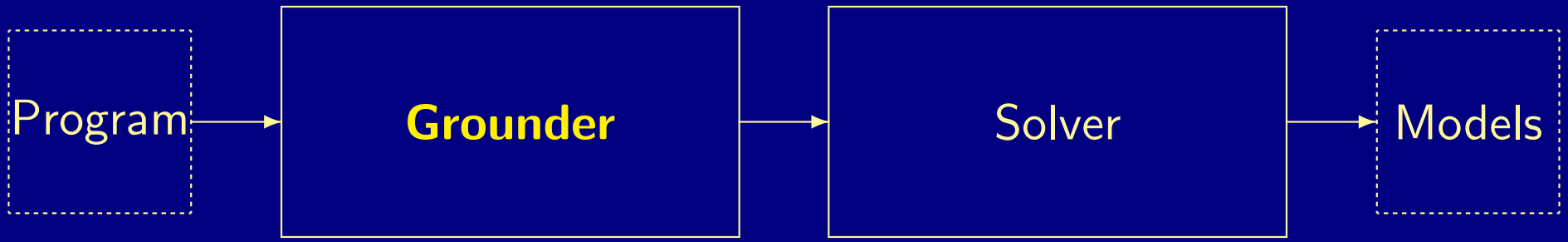
ASP Solving Process



Graph Coloring: Grounding

```
$ gringo -t color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g). ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).      :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).      :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).      :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).      :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).      :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).      :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).      :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).      :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

Graph Coloring: Grounding

```
$ gringo -t color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

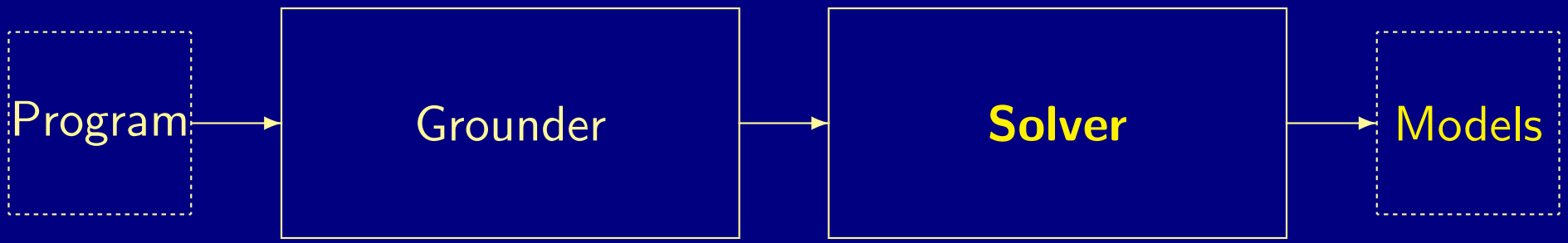
edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g). ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).      :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).      :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).      :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).      :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).      :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).      :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).      :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).      :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

ASP Solving Process



Graph Coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 1.2.1
Reading from stdin
Reading      : Done(0.000s)
Preprocessing: Done(0.000s)
Solving...
Answer: 1
color(1,b) color(2,r) color(3,r) color(4,g) color(5,b) color(6,g) node(1) ... edge(1,2) ... col(r) ...
Answer: 2
color(1,g) color(2,r) color(3,r) color(4,b) color(5,g) color(6,b) node(1) ... edge(1,2) ... col(r) ...
Answer: 3
color(1,b) color(2,g) color(3,g) color(4,r) color(5,b) color(6,r) node(1) ... edge(1,2) ... col(r) ...
Answer: 4
color(1,g) color(2,b) color(3,b) color(4,r) color(5,g) color(6,r) node(1) ... edge(1,2) ... col(r) ...
Answer: 5
color(1,r) color(2,b) color(3,b) color(4,g) color(5,r) color(6,g) node(1) ... edge(1,2) ... col(r) ...
Answer: 6
color(1,r) color(2,g) color(3,g) color(4,b) color(5,r) color(6,b) node(1) ... edge(1,2) ... col(r) ...

Models       : 6
Time         : 0.000  (Solving: 0.000)
```

Graph Coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 1.2.1
Reading from stdin
Reading      : Done(0.000s)
Preprocessing: Done(0.000s)
Solving...
Answer: 1
color(1,b) color(2,r) color(3,r) color(4,g) color(5,b) color(6,g) node(1) ... edge(1,2) ... col(r) ...
Answer: 2
color(1,g) color(2,r) color(3,r) color(4,b) color(5,g) color(6,b) node(1) ... edge(1,2) ... col(r) ...
Answer: 3
color(1,b) color(2,g) color(3,g) color(4,r) color(5,b) color(6,r) node(1) ... edge(1,2) ... col(r) ...
Answer: 4
color(1,g) color(2,b) color(3,b) color(4,r) color(5,g) color(6,r) node(1) ... edge(1,2) ... col(r) ...
Answer: 5
color(1,r) color(2,b) color(3,b) color(4,g) color(5,r) color(6,g) node(1) ... edge(1,2) ... col(r) ...
Answer: 6
color(1,r) color(2,g) color(3,g) color(4,b) color(5,r) color(6,b) node(1) ... edge(1,2) ... col(r) ...

Models       : 6
Time         : 0.000  (Solving: 0.000)
```

Basic Modeling: Overview

13 ASP Solving Process

14 Problems as Logic Programs
- Graph Coloring

15 Methodology
- Satisfiability
- Queens
- Reviewer Assignment
- Planning

Basic Methodology

Generate and Test (or: Guess and Check) approach

Generator Generate potential answer set candidates
(typically through non-deterministic constructs)

Tester Eliminate invalid candidates
(typically through integrity constraints)

Nutshell

$$\text{Logic program} = \text{Data} + \text{Generator} + \text{Tester}$$
$$(+ \text{ Optimizer})$$

Basic Methodology

Generate and Test (or: Guess and Check) approach

Generator	Generate potential answer set candidates (typically through non-deterministic constructs)
Tester	Eliminate invalid candidates (typically through integrity constraints)

Nutshell

$$\text{Logic program} = \text{Data} + \text{Generator} + \text{Tester} \; (+ \text{Optimizer})$$

Satisfiability

- Problem Instance: A propositional formula ϕ in CNF.
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true.

- Example: Consider formula $(a \vee \neg b) \wedge (\neg a \vee b)$.
- Logic Program:

Generator	**Tester**	**Stable models**
{a,b} $\leftarrow$	$\leftarrow$ not a, b	X_1 = {a,b}
	$\leftarrow$ a, not b	X_2 = {}

Satisfiability

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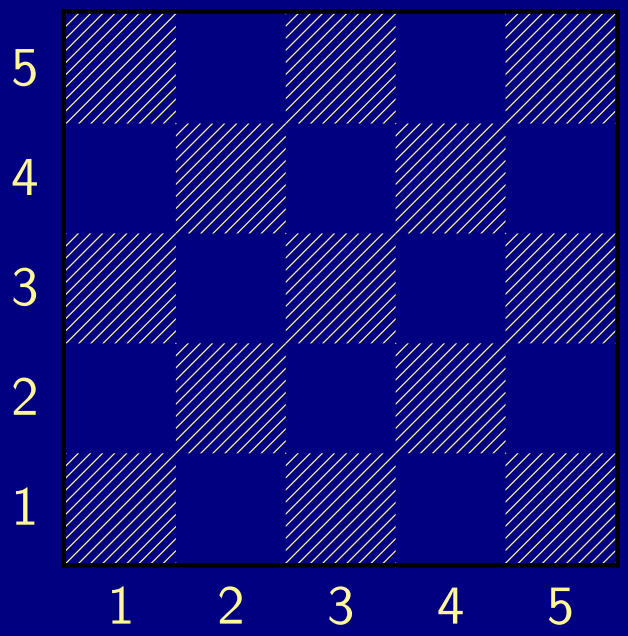
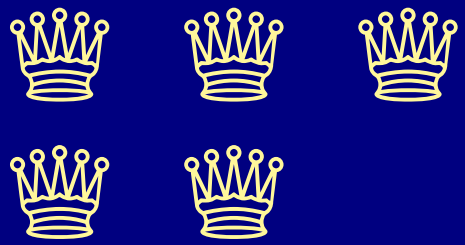
- Logic Program:

Generator	**Tester**	**Stable models**
{a,b} ←	← not a, b	X_1 = {a,b}
	← a, not b	X_2 = {}

The n-Queens Problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another



Defining the Field

queens.lp

```
row(1..n).
col(1..n).
```

- Create file `queens.lp`
- Define the field
  - n rows
  - n columns

Defining the Field

Running ...

```
$ clingo queens.lp -c n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models       : 1
Time         : 0.000
  Prepare    : 0.000
  Prepro.    : 0.000
  Solving    : 0.000
```

Placing some Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
```

- Guess a solution candidate
- Place some queens on the board

Placing some Queens

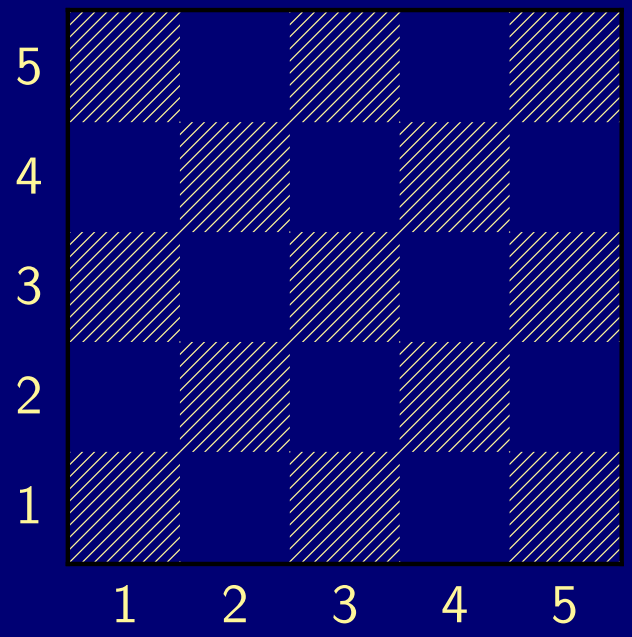
Running ...

```
$ clingo queens.lp -c n=5 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE

Models       : 3+
```

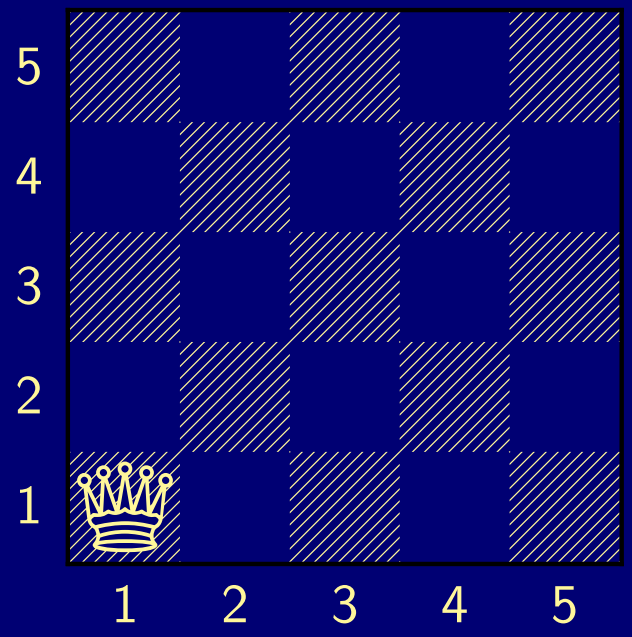
Placing some Queens: Answer 1

Answer 1



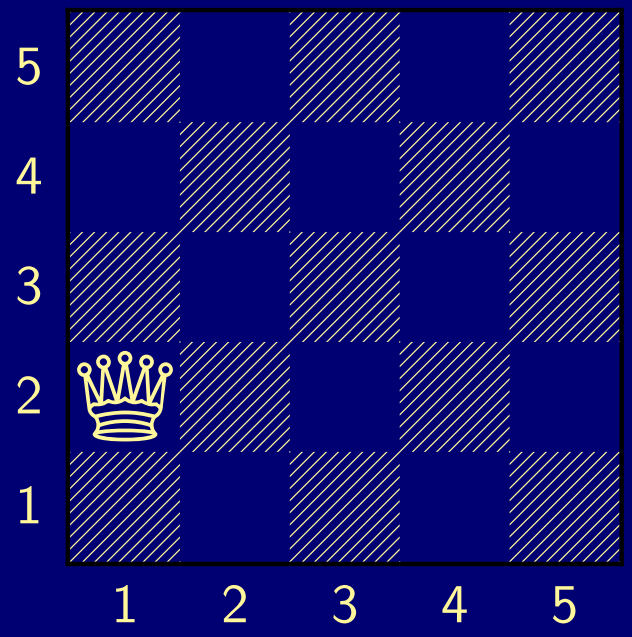
Placing some Queens: Answer 2

Answer 2



Placing some Queens: Answer 3

Answer 3



Placing n Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
```

- Place exactly n queens on the board

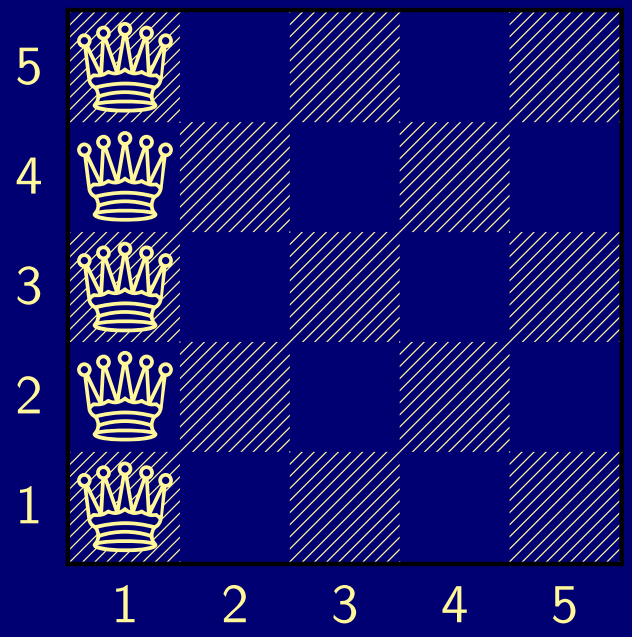
Placing n Queens

Running ...

```
$ clingo queens.lp -c n=5 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
...
```

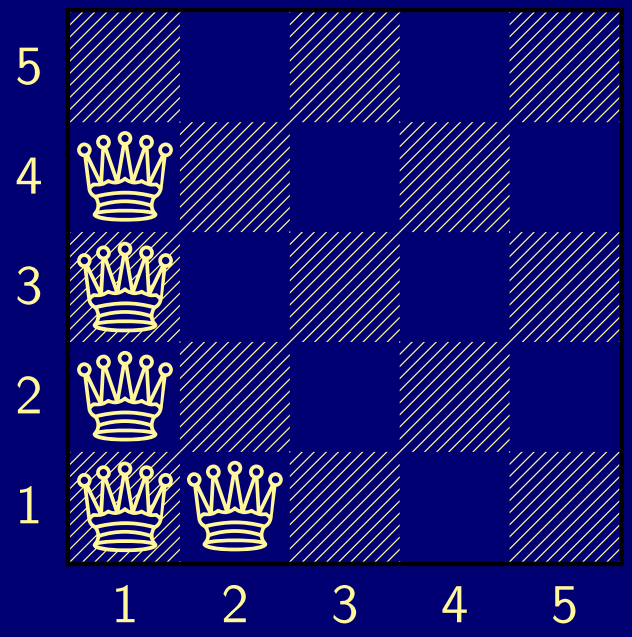
Placing n Queens: Answer 1

Answer 1



Placing n Queens: Answer 2

Answer 2



Horizontal and vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

Horizontal and vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

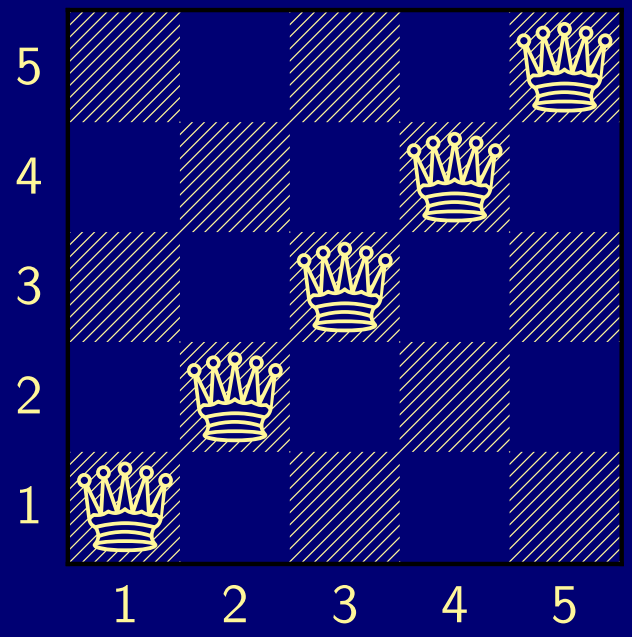
Horizontal and vertical Attack

Running ...

```
$ clingo queens.lp -c n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) \
queen(2,2) queen(1,1)
...
```

Horizontal and vertical Attack: Answer 1

Answer 1



Diagonal Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ),
   I-J == II-JJ.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ),
   I+J == II+JJ.
```

- Forbid diagonal attacks

Diagonal Attack

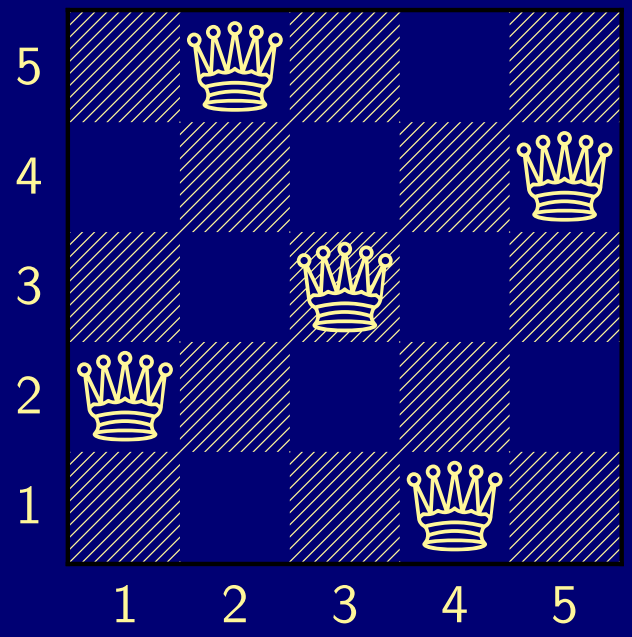
Running ...

```
$ clingo queens.lp -c n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) \
queen(5,2) queen(2,1)
SATISFIABLE

Models       : 1+
Time         : 0.000
  Prepare    : 0.000
  Prepro.    : 0.000
  Solving    : 0.000
```

Diagonal Attack: Answer 1

Answer 1



Optimizing

queens-opt.lp

```
1 { queen(I,1..n) } 1 :- I = 1..n.
1 { queen(1..n,J) } 1 :- J = 1..n.
:- { queen(D-J,J) } 2, D =   2..2*n.
:- { queen(D+J,J) } 2, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

- See Section *Tweaking N-Queens*

Reviewer Assignment

by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
...

3 { assigned(P,R) : reviewer(R) } 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- 9 { assigned(P,R) : paper(P) }  , reviewer(R).
 :-   { assigned(P,R) : paper(P) } 6, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { assignedB(P,R) : paper(P) : reviewer(R) }.
```

Reviewer Assignment

by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
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```

Simplistic STRIPS Planning

```
fluent(p).    fluent(q).    fluent(r).
action(a).    pre(a,p).     add(a,q).     del(a,p).
action(b).    pre(b,q).     add(b,r).     del(b,q).
init(p).      query(r).

time(1..k).   lasttime(T) :- time(T), not time(T+1).

holds(P,0) :- init(P).

1 { occ(A,T) : action(A) } 1 :- time(T).
 :- occ(A,T), pre(A,F), not holds(F,T-1).

ocdel(F,T) :- occ(A,T), del(A,F).
holds(F,T) :- occ(A,T), add(A,F).
holds(F,T) :- holds(F,T-1), not ocdel(F,T), time(T).

 :- query(F), not holds(F,T), lasttime(T).
```

Simplistic STRIPS Planning

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fluent(p).    fluent(q).    fluent(r).
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```

Language Extensions: Overview

16 Motivation

17 Integrity Constraints

18 Choice Rules

19 Cardinality Constraints

20 Weight Constraints

21 Optimization statements

22 Conditional literals

23 smodels format

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Language extensions

- The expressiveness of a language can be enhanced by introducing new constructs
- To this end, we must address the following issues:
  - What is the syntax of the new language construct?
  - What is the semantics of the new language construct?
  - How to implement the new language construct?
- A way of providing semantics is to furnish a translation removing the new constructs, eg. classical negation
- This translation might also be used for implementing the language extension

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Integrity Constraints

- Idea Eliminate unwanted solution candidates
- Syntax An integrity constraint is of the form

$$\leftarrow A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n,$$

  where $n \geq m \geq 1$, and each A_i $(1 \leq i \leq n)$ is a atom
- Example `:- edge(X,Y), color(X,C), color(Y,C).`
- Embedding For a new symbol x, map

$$\begin{array}{rcl} & \leftarrow & A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n \\ \mapsto \quad x & \leftarrow & A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n, \mathit{not}\ x \end{array}$$

- Another example $\Pi = \{p \leftarrow \mathit{not}\ q,\ q \leftarrow \mathit{not}\ p\}$
  versus $\Pi' = \Pi \cup \{\leftarrow p\}$ and $\Pi'' = \Pi \cup \{\leftarrow \mathit{not}\ p\}$

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Choice rules

- Idea Choices over subsets
- Syntax

$$\{A_1, \ldots, A_m\} \leftarrow A_{m+1}, \ldots, A_n, \mathit{not}\ A_{n+1}, \ldots, \mathit{not}\ A_o,$$

- Informal meaning If the body is satisfied in an answer set, then any subset of $\{A_1, \ldots, A_m\}$ can be included in the answer set
- Example `1 {color(X,C) : col(C)} 1 :- node(X).`
- Another Example Program $\Pi = \{\ \{a\} \leftarrow b,\ b \leftarrow\}$ has two answer sets: $\{b\}$ and $\{a, b\}$

Choice rules

- Idea Choices over subsets
- Syntax

$$\{A_1, \ldots, A_m\} \leftarrow A_{m+1}, \ldots, A_n, \mathit{not}\ A_{n+1}, \ldots, \mathit{not}\ A_o,$$

- Informal meaning If the body is satisfied in an answer set, then any subset of $\{A_1, \ldots, A_m\}$ can be included in the answer set
- Example `1 {color(X,C) : col(C)} 1 :- node(X).`
- Another Example Program $\Pi = \{\ \{a\} \leftarrow b,\ b \leftarrow\}$ has two answer sets: $\{b\}$ and $\{a, b\}$

Choice rules

- Idea Choices over subsets
- Syntax

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Embedding in normal logic programs

- A choice rule of form

$$\{A_1, \ldots, A_m\} \leftarrow A_{m+1}, \ldots, A_n, \mathit{not}\ A_{n+1}, \ldots, \mathit{not}\ A_o$$

can be translated into $2m + 1$ rules

$$A \leftarrow A_{m+1}, \ldots, A_n, \mathit{not}\ A_{n+1}, \ldots, \mathit{not}\ A_o$$

$$A_1 \leftarrow A, \mathit{not}\ \overline{A_1} \quad \ldots \quad A_m \leftarrow A, \mathit{not}\ \overline{A_m}$$

$$\overline{A_1} \leftarrow \mathit{not}\ A_1 \quad \ldots \quad \overline{A_m} \leftarrow \mathit{not}\ A_m$$

by introducing new atoms $A, \overline{A_1}, \ldots, \overline{A_m}$

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Cardinality constraints

- Syntax A (positive) cardinality constraint is of the form

$$l\ \{A_1, \ldots, A_m\}\ u$$

- Informal meaning A cardinality constraint is satisfied in an answer set X, if the number of atoms from $\{A_1, \ldots, A_m\}$ satisfied in X is between l and u (inclusive)
  More formally, if $l \leq |\{A_1, \ldots, A_m\} \cap X| \leq u$
- Example 2 {hd(a),. . . ,hd(m)} 4
- Conditions $l\ \{A_1 : B_1, \ldots, A_m : B_m\}\ u$
  where $B_1, \ldots, B_m$ are used for restricting instantiations of variables occurring in $A_1, \ldots, A_m$

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Cardinality rules

- Syntax

$$A_0 \leftarrow l\ \{A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n\}$$

- Informal meaning If at least l elements of the "body" are true in an answer set, then add A_0 to the answer set
  - ➥ l is a lower bound on the "body"
- Example Program $\Pi = \{\ a \leftarrow 1\{b, c\},\ \ b \leftarrow\}$ has answer set $\{a, b\}$

Embedding in normal logic programs

- Replace each cardinality rule

$$A_0 \leftarrow l\ \{A_1, \ldots, A_m\} \qquad \text{by} \qquad A_0 \leftarrow cc(A_1, l)$$

where atom $cc(A_i, j)$ represents the fact that at least j of the atoms in $\{A_i, \ldots, A_m\}$, that is, of the atoms that have an equal or greater index than i, are in a particular answer set

- The definition of $cc(A_i, j)$ is given by the rules

$$\begin{aligned} cc(A_i, j{+}1) &\leftarrow cc(A_{i+1}, j), A_i \\ cc(A_i, j) &\leftarrow cc(A_{i+1}, j) \\ cc(A_{m+1}, 0) &\leftarrow \end{aligned}$$

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. . . and vice versa

- A normal rule

$$A_0 \leftarrow A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n,$$

can be represented by the cardinality rule

$$A_0 \leftarrow n\ \{A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n\}$$

Cardinality rules with upper bounds

- A rule of the form

$$A_0 \leftarrow l\ \{A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n\}\ u$$

stands for

$$\begin{aligned} A_0 &\leftarrow B, not\ C \\ B &\leftarrow l\ \{A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n\} \\ C &\leftarrow u{+}1\ \{A_1, \ldots, A_m, not\ A_{m+1}, \ldots, not\ A_n\} \end{aligned}$$

Cardinality constraints as heads

- A rule of the form

$$I\ \{A_1, \ldots, A_m\}\ u \leftarrow A_{m+1}, \ldots, A_n, \mathit{not}\ A_{n+1}, \ldots, \mathit{not}\ A_o,$$

stands for

$$\begin{aligned} B &\leftarrow A_{m+1}, \ldots, A_n, \mathit{not}\ A_{n+1}, \ldots, \mathit{not}\ A_o \\ \{A_1, \ldots, A_m\} &\leftarrow B \\ C &\leftarrow I\ \{A_1, \ldots, A_m\}\ u \\ &\leftarrow B, \mathit{not}\ C \end{aligned}$$

Full-fledged cardinality rules

- A rule of the form

$$l_0\ S_0\ u_0 \leftarrow l_1\ S_1\ u_1, \ldots, l_n\ S_n\ u_n$$

stands for $0 \leq i \leq n$

$$\begin{aligned} B_i &\leftarrow l_i\ S_i \\ C_i &\leftarrow u_i{+}1\ S_i \\ A &\leftarrow B_1, \ldots, B_n, \mathit{not}\ C_1, \ldots, \mathit{not}\ C_n \\ &\leftarrow A, \mathit{not}\ B_0 \\ &\leftarrow A, C_0 \\ S_0 \cap \mathcal{A} &\leftarrow A \end{aligned}$$

where $\mathcal{A}$ is the underlying alphabet

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Weight constraints

- Syntax $l\ [A_1 = w_1, \ldots, A_m = w_m,$
  $not\ A_{m+1} = w_{m+1}, \ldots, not\ A_n = w_n]\ u$
- Informal meaning A weight constraint is satisfied in an answer set X, if

$$l \leq \left(\sum_{1 \leq i \leq m, A_i \in X} w_i + \sum_{m < i \leq n, A_i \notin X} w_i \right) \leq u$$

  ➥ Generalization of cardinality constraints
- Example 80 [hd(a)=50,. . . ,hd(m)=100] 400

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Optimization statements

- Idea Compute optimal answer sets by minimizing (or maximizing) a weighted sum of given elements
- Syntax
  - $\#minimize\ [A_1 = w_1, \ldots, A_m = w_m, not\ A_{m+1} = w_{m+1}, \ldots, not\ A_n = w_n]$
  - $\#maximize\ [A_1 = w_1, \ldots, A_m = w_m, not\ A_{m+1} = w_{m+1}, \ldots, not\ A_n = w_n]$
- Example
  - #minimize [hd(a)=30,. . . ,hd(m)=50]
  - #minimize [road(X,Y) : length(X,Y,L) = L]

- Multi-criteria optimization can be accomplished by adding priority levels to weighted literals, that is, by replacing $L_i = w_i$ by $L_i = w_i@P_i$

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Conditional literals

- Idea Encode the contents of a (multi-)set without enumerating its elements
- Syntax $A_0 : A_1 : \ldots : A_m : \mathit{not}\ A_{m+1} : \ldots : \mathit{not}\ A_n$
- Informal meaning List all ground instances of A_0 such that corresponding instances of $A_1, \ldots, A_m, \mathit{not}\ A_{m+1}, \ldots, \mathit{not}\ A_n$ are true
- Example Given ‘`p(1). p(2). p(3). q(2).`’ the choice

  ```
  {r(X) : p(X) : not q(X)}.
  ```

  is instantiated to

  ```
  {r(1), r(3)}.
  ```

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smodels format

- Logic programs in *smodels* format consist of
  - normal rules
  - choice rules
  - cardinality rules
  - weight rules
  - optimization statements
- Such a format is obtained by grounders *lparse* and `gringo`

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Motivation

- Goal Approach to computing answer sets of logic programs, based on concepts from
  - Constraint Processing (CP) and
  - Satisfiability Checking (SAT)
- Idea View inferences in ASP as unit propagation on nogoods
- Benefits
  - A uniform constraint-based framework for different kinds of inferences in ASP
  - Advanced techniques from the areas of CP and SAT
  - Highly competitive implementation

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Assignments

- An assignment A over $dom(A) = atom(\Pi) \cup body(\Pi)$ is a sequence

$$(\sigma_1, \ldots, \sigma_n)$$

of signed literals σ_i of form $\mathbf{T}p$ or $\mathbf{F}p$ for $p \in dom(A)$ and $1 \leq i \leq n$.

  - ☞ $\mathbf{T}p$ expresses that p is *true* and $\mathbf{F}p$ that it is *false*.

- The complement, $\overline{\sigma}$, of a literal σ is defined as $\overline{\mathbf{T}p} = \mathbf{F}p$ and $\overline{\mathbf{F}p} = \mathbf{T}p$.
- $A \circ B$ denotes the concatenation of assignments A and B.
- Given $A = (\sigma_1, \ldots, \sigma_{k-1}, \sigma_k, \ldots, \sigma_n)$, we let $A[\sigma_k] = (\sigma_1, \ldots, \sigma_{k-1})$.
- We sometimes identify an assignment with the set of its literals. Given this, we access *true* and *false* propositions in A via

$$A^{\mathbf{T}} = \{p \in dom(A) \mid \mathbf{T}p \in A\} \quad \text{and} \quad A^{\mathbf{F}} = \{p \in dom(A) \mid \mathbf{F}p \in A\}\,.$$

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Nogoods, Solutions, and Unit Propagation

- A nogood is a set $\{\sigma_1, \ldots, \sigma_n\}$ of signed literals, expressing a constraint violated by any assignment containing $\sigma_1, \ldots, \sigma_n$.
- An assignment A such that $A^{\mathbf{T}} \cup A^{\mathbf{F}} = dom(A)$ and $A^{\mathbf{T}} \cap A^{\mathbf{F}} = \emptyset$ is a solution for a set Δ of nogoods, if $\delta \not\subseteq A$ for all $\delta \in \Delta$.
- For a nogood δ, a literal $\sigma \in \delta$, and an assignment A, we say that $\overline{\sigma}$ is unit-resulting for δ wrt A, if
  1. $\delta \setminus A = \{\sigma\}$ and
  2. $\overline{\sigma} \notin A$.
- For a set Δ of nogoods and an assignment A, unit propagation is the iterated process of extending A with unit-resulting literals until no further literal is unit-resulting for any nogood in Δ.

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Nogoods from logic programs

via program completion

The completion of a logic program Π can be defined as follows:

$$\{p_\beta \leftrightarrow p_1 \wedge \cdots \wedge p_m \wedge \neg p_{m+1} \wedge \cdots \wedge \neg p_n \mid \beta \in body(\Pi), \beta = \{p_1, \ldots, p_m, not\ p_{m+1}, \ldots, not\ p_n\}\}$$

$$\cup \quad \{p \leftrightarrow p_{\beta_1} \vee \cdots \vee p_{\beta_k} \mid p \in atom(\Pi), body(p) = \{\beta_1, \ldots, \beta_k\}\} \ ,$$

where $body(p) = \{body(r) \mid r \in \Pi, head(r) = p\}$.

Nogoods from logic programs

via program completion

Let $\beta = \{p_1, \ldots, p_m, \mathit{not}\ p_{m+1}, \ldots, \mathit{not}\ p_n\}$ be a body.

The equivalence

$$p_\beta \leftrightarrow p_1 \wedge \cdots \wedge p_m \wedge \neg p_{m+1} \wedge \cdots \wedge \neg p_n$$

can be decomposed into two implications.

1. We get

$$p_\beta \rightarrow p_1 \wedge \cdots \wedge p_m \wedge \neg p_{m+1} \wedge \cdots \wedge \neg p_n$$

which is equivalent to the conjunction of

$$\neg p_\beta \vee p_1,\ \ldots,\ \neg p_\beta \vee p_m,\ \neg p_\beta \vee \neg p_{m+1},\ \ldots,\ \neg p_\beta \vee \neg p_n$$

This set of clauses expresses the following set of nogoods:

$$\Delta(\beta) = \{\{\mathbf{T}\beta, \mathbf{F}p_1\}, \ldots, \{\mathbf{T}\beta, \mathbf{F}p_m\}, \{\mathbf{T}\beta, \mathbf{T}p_{m+1}\}, \ldots, \{\mathbf{T}\beta, \mathbf{T}p_n\}\}$$

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via program completion

Let $\beta = \{p_1, \ldots, p_m, \mathit{not}\ p_{m+1}, \ldots, \mathit{not}\ p_n\}$ be a body.

The equivalence

$$p_\beta \leftrightarrow p_1 \wedge \cdots \wedge p_m \wedge \neg p_{m+1} \wedge \cdots \wedge \neg p_n$$

can be decomposed into two implications.

2. The converse of the previous implication, viz.

   $$p_1 \wedge \cdots \wedge p_m \wedge \neg p_{m+1} \wedge \cdots \wedge \neg p_n \rightarrow p_\beta$$

   gives rise to the nogood

   $$\delta(\beta) = \{\mathbf{F}\beta, \mathbf{T}p_1, \ldots, \mathbf{T}p_m, \mathbf{F}p_{m+1}, \ldots, \mathbf{F}p_n\}$$

   Intuitively, $\delta(\beta)$ is a constraint enforcing the truth of body β, or the falsity of a contained literal.

Nogoods from logic programs

via program completion

Let $\beta = \{p_1, \ldots, p_m, \mathit{not}\ p_{m+1}, \ldots, \mathit{not}\ p_n\}$ be a body.

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   Intuitively, $\delta(\beta)$ is a constraint enforcing the truth of body β, or the falsity of a contained literal.

Nogoods from logic programs

via program completion

Proceeding analogously with the atom-based equivalences, viz.

$$p \leftrightarrow p_{\beta_1} \vee \cdots \vee p_{\beta_k}$$

we obtain for an atom $p \in \mathit{atom}(\Pi)$ along with its bodies $\mathit{body}(p) = \{\beta_1, \ldots, \beta_k\}$ the nogoods

$$\Delta(p) = \{\, \{\mathbf{F}p, \mathbf{T}\beta_1\}, \ldots, \{\mathbf{F}p, \mathbf{T}\beta_k\} \,\} \text{ and } \delta(p) = \{\mathbf{T}p, \mathbf{F}\beta_1, \ldots, \mathbf{F}\beta_k\}\,.$$

Nogoods from logic programs

atom-oriented nogoods

For an atom p where $body(p) = \{\beta_1, \ldots, \beta_k\}$, recall that

$$\begin{aligned} \delta(p) &= \{\mathbf{T}p, \mathbf{F}\beta_1, \ldots, \mathbf{F}\beta_k\} \\ \Delta(p) &= \{\{\mathbf{F}p, \mathbf{T}\beta_1\}, \ldots, \{\mathbf{F}p, \mathbf{T}\beta_k\}\} \ . \end{aligned}$$

For example, for atom x with $body(x) = \{\{y\}, \{not\ z\}\}$, we obtain

$$\boxed{\begin{array}{lcl} x & \leftarrow & y \\ x & \leftarrow & not\ z \end{array}}$$

$$\begin{aligned} \delta(x) &= \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{not\ z\}\} \\ \Delta(x) &= \{\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{not\ z\}\}\} \end{aligned}$$

For nogood $\delta(x) = \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{not\ z\}\}$, the signed literal

$\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{not\ z\})$ and

$\mathbf{T}\{not\ z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs

atom-oriented nogoods

For an atom p where $body(p) = \{\beta_1, \ldots, \beta_k\}$, recall that

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x	$\leftarrow$	y
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For nogood $\delta(x) = \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{not\ z\}\}$, the signed literal

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Nogoods from logic programs

atom-oriented nogoods

For an atom p where $body(p) = \{\beta_1, \ldots, \beta_k\}$, recall that

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For nogood $\delta(x) = \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{not\ z\}\}$, the signed literal

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Nogoods from logic programs

atom-oriented nogoods

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For nogood $\delta(x) = \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{not\ z\}\}$, the signed literal

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Nogoods from logic programs

atom-oriented nogoods

For an atom p where $body(p) = \{\beta_1, \ldots, \beta_k\}$, recall that

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Nogoods from logic programs

atom-oriented nogoods

For an atom p where $body(p) = \{\beta_1, \ldots, \beta_k\}$, recall that

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Nogoods from logic programs

atom-oriented nogoods

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Nogoods from logic programs

atom-oriented nogoods

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Nogoods from logic programs

atom-oriented nogoods

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Nogoods from logic programs

atom-oriented nogoods

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Nogoods from logic programs

atom-oriented nogoods

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Nogoods from logic programs

atom-oriented nogoods

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Nogoods from logic programs

atom-oriented nogoods

For an atom p where $body(p) = \{\beta_1, \ldots, \beta_k\}$, recall that

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- $\mathbf{T}\{not\ z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs

atom-oriented nogoods

For an atom p where $body(p) = \{\beta_1, \ldots, \beta_k\}$, recall that

$$\begin{aligned} \delta(p) &= \{\mathbf{T}p, \mathbf{F}\beta_1, \ldots, \mathbf{F}\beta_k\} \\ \Delta(p) &= \{\{\mathbf{F}p, \mathbf{T}\beta_1\}, \ldots, \{\mathbf{F}p, \mathbf{T}\beta_k\}\} \ . \end{aligned}$$

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For nogood $\delta(x) = \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{not\ z\}\}$, the signed literal

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- $\mathbf{T}\{not\ z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs

body-oriented nogoods

For a body $\beta = \{p_1, \ldots, p_m, not\ p_{m+1}, \ldots, not\ p_n\}$, recall that

$$\begin{aligned} \delta(\beta) &= \{\mathbf{F}\beta, \mathbf{T}p_1, \ldots, \mathbf{T}p_m, \mathbf{F}p_{m+1}, \ldots, \mathbf{F}p_n\} \\ \Delta(\beta) &= \{\{\mathbf{T}\beta, \mathbf{F}p_1\}, \ldots, \{\mathbf{T}\beta, \mathbf{F}p_m\}, \{\mathbf{T}\beta, \mathbf{T}p_{m+1}\}, \ldots, \{\mathbf{T}\beta, \mathbf{T}p_n\}\}\ . \end{aligned}$$

For example, for body $\{x, not\ y\}$, we obtain

$$\begin{array}{l} \ldots \leftarrow x, not\ y \\ \qquad\vdots \\ \ldots \leftarrow x, not\ y \end{array}$$

$$\begin{aligned} \delta(\{x, not\ y\}) &= \{\mathbf{F}\{x, not\ y\}, \mathbf{T}x, \mathbf{F}y\} \\ \Delta(\{x, not\ y\}) &= \{\{\mathbf{T}\{x, not\ y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, not\ y\}, \mathbf{T}y\}\} \end{aligned}$$

For nogood $\delta(\{x, not\ y\}) = \{\mathbf{F}\{x, not\ y\}, \mathbf{T}x, \mathbf{F}y\}$, the signed literal

- $\mathbf{T}\{x, not\ y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
- $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, not\ y\}, \mathbf{T}x)$.

Nogoods from logic programs

body-oriented nogoods

For a body $\beta = \{p_1, \ldots, p_m, not\ p_{m+1}, \ldots, not\ p_n\}$, recall that

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Nogoods from logic programs

body-oriented nogoods

For a body $\beta = \{p_1, \ldots, p_m, not\ p_{m+1}, \ldots, not\ p_n\}$, recall that

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Nogoods from logic programs

body-oriented nogoods

For a body $\beta = \{p_1, \ldots, p_m, not\ p_{m+1}, \ldots, not\ p_n\}$, recall that

$$\begin{aligned} \delta(\beta) &= \{\mathbf{F}\beta, \mathbf{T}p_1, \ldots, \mathbf{T}p_m, \mathbf{F}p_{m+1}, \ldots, \mathbf{F}p_n\} \\ \Delta(\beta) &= \{\{\mathbf{T}\beta, \mathbf{F}p_1\}, \ldots, \{\mathbf{T}\beta, \mathbf{F}p_m\}, \{\mathbf{T}\beta, \mathbf{T}p_{m+1}\}, \ldots, \{\mathbf{T}\beta, \mathbf{T}p_n\}\}\ . \end{aligned}$$

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- $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, not\ y\}, \mathbf{T}x)$.

Nogoods from logic programs

body-oriented nogoods

For a body $\beta = \{p_1, \ldots, p_m, \mathit{not}\ p_{m+1}, \ldots, \mathit{not}\ p_n\}$, recall that

$$\begin{aligned} \delta(\beta) &= \{\mathbf{F}\beta, \mathbf{T}p_1, \ldots, \mathbf{T}p_m, \mathbf{F}p_{m+1}, \ldots, \mathbf{F}p_n\} \\ \Delta(\beta) &= \{\{\mathbf{T}\beta, \mathbf{F}p_1\}, \ldots, \{\mathbf{T}\beta, \mathbf{F}p_m\}, \{\mathbf{T}\beta, \mathbf{T}p_{m+1}\}, \ldots, \{\mathbf{T}\beta, \mathbf{T}p_n\}\}\ . \end{aligned}$$

For example, for body $\{x, \mathit{not}\ y\}$, we obtain

$$\begin{array}{|c|} \hline \ldots \leftarrow x, \mathit{not}\ y \\ \vdots \\ \ldots \leftarrow x, \mathit{not}\ y \\ \hline \end{array}$$

$$\begin{aligned} \delta(\{x, \mathit{not}\ y\}) &= \{\mathbf{F}\{x, \mathit{not}\ y\}, \mathbf{T}x, \mathbf{F}y\} \\ \Delta(\{x, \mathit{not}\ y\}) &= \{\{\mathbf{T}\{x, \mathit{not}\ y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, \mathit{not}\ y\}, \mathbf{T}y\}\} \end{aligned}$$

For nogood $\delta(\{x, \mathit{not}\ y\}) = \{\mathbf{F}\{x, \mathit{not}\ y\}, \mathbf{T}x, \mathbf{F}y\}$, the signed literal

- $\mathbf{T}\{x, \mathit{not}\ y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
- $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, \mathit{not}\ y\}, \mathbf{T}x)$.

Characterization of answer sets

for tight logic programs

Let Π be a logic program and

$$\begin{aligned}\Delta_\Pi &= \{\delta(p) \mid p \in \mathit{atom}(\Pi)\} \cup \{\delta \in \Delta(p) \mid p \in \mathit{atom}(\Pi)\} \\ &\cup \{\delta(\beta) \mid \beta \in \mathit{body}(\Pi)\} \cup \{\delta \in \Delta(\beta) \mid \beta \in \mathit{body}(\Pi)\}\ .\end{aligned}$$

Theorem

Let Π be a tight logic program. Then,
$X \subseteq \mathit{atom}(\Pi)$ is an answer set of Π iff
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Characterization of answer sets

for tight logic programs, ie. free of positive recursion

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Nogoods from logic programs

via loop formulas

Let Π be a normal logic program and recall that:

- For $L \subseteq atom(\Pi)$, the external supports of L for Π are
$$ES_\Pi(L) = \{r \in \Pi \mid head(r) \in L, body(r)^+ \cap L = \emptyset\}.$$
- The (disjunctive) loop formula of L for Π is
$$LF_\Pi(L) = (\bigvee_{A \in L} A) \rightarrow (\bigvee_{r \in ES_\Pi(L)} body(r))$$
$$\equiv (\bigwedge_{r \in ES_\Pi(L)} \neg body(r)) \rightarrow (\bigwedge_{A \in L} \neg A)$$
  - ☞ The loop formula of L enforces all atoms in L to be *false* whenever L is not externally supported.
- The external bodies of L for Π are
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Nogoods from logic programs

loop nogoods

- For a logic program Π and some $\emptyset \subset U \subseteq atom(\Pi)$, define the loop nogood of an atom $p \in U$ as

$$\lambda(p, U) \quad = \quad \{\mathbf{T}p, \mathbf{F}\beta_1, \ldots, \mathbf{F}\beta_k\}$$

  where $EB(U) = \{\beta_1, \ldots, \beta_k\}$.
- In all, we get the following set of loop nogoods for Π:

$$\Lambda_\Pi \quad = \quad \bigcup_{\emptyset \subset U \subseteq atom(\Pi)} \{\lambda(p, U) \mid p \in U\}$$

- The set Λ_Π of loop nogoods denies cyclic support among *true* atoms.

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Example

Consider

$$\Pi = \left\{ \begin{array}{ll} x \leftarrow not\ y & u \leftarrow x \\ y \leftarrow not\ x & u \leftarrow v \\ & v \leftarrow u, y \end{array} \right\}$$

For u in the set $\{u, v\}$, we obtain the loop nogood:

$$\lambda(u, \{u, v\}) \quad = \quad \{\mathbf{T}u, \mathbf{F}\{x\}\}$$

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Characterization of answer sets

Theorem

Let Π be a logic program. Then,
$X \subseteq atom(\Pi)$ is an answer set of Π iff
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Some remarks

- Nogoods in Λ_Π augment Δ_Π with conditions checking for unfounded sets, in particular, those being loops.
- While $|\Delta_\Pi|$ is linear in the size of Π, Λ_Π may contain exponentially many (non-redundant) loop nogoods

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Conflict-Driven Answer Set Solving: Overview

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25 Boolean Constraints

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- Nogoods from program completion
- Nogoods from loop formulas

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- CDNL-ASP Algorithm
- Nogood Propagation
- Conflict Analysis

Conflict-driven search

Boolean constraint solving algorithms pioneered for SAT led to:

- Traditional DPLL-style approach
  - (Unit) propagation
  - (Chronological) backtracking
- Modern CDCL-style approach
  - (Unit) propagation
  - Conflict analysis (via resolution)
  - Learning + Backjumping + Assertion

DPLL-style solving

loop

propagate // compute deterministic consequences

if no conflict **then**

if all variables assigned **then return** variable assignment

else *decide* // non-deterministically assign some literal

else

if top-level conflict **then return** unsatisfiable

else

backtrack // undo assignments made after last decision

flip // assign complement of last decision literal

CDCL-style solving

loop

propagate // compute deterministic consequences

if no conflict **then**

if all variables assigned **then return** variable assignment

else *decide* // non-deterministically assign some literal

else

if top-level conflict **then return** unsatisfiable

else

analyze // analyze conflict and add a conflict constraint

backjump // undo assignments until conflict constraint is unit

Outline of CDNL-ASP algorithm

- Keep track of deterministic consequences by unit propagation on:
  - Program completion $[\Delta_\Pi]$
  - Loop nogoods, determined and recorded on demand $[\Lambda_\Pi]$
    - ☞ Dedicated unfounded set detection !
  - Dynamic nogoods, derived from conflicts and unfounded sets $[\nabla]$
- When a nogood in $\Delta_\Pi \cup \nabla$ becomes violated:
  - Analyze the conflict by resolution (until reaching a Unique Implication Point, short: UIP)
  - Learn the derived conflict nogood δ
  - Backjump to the earliest (heuristic) choice such that the complement of the UIP is unit-resulting for δ
  - Assert the complement of the UIP and proceed (by unit propagation)
- Terminate when either:
  - Finding an answer set (a solution for $\Delta_\Pi \cup \Lambda_\Pi$)
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Algorithm 1: CDNL-ASP

Input : A logic program Π.
Output : An answer set of Π or "no answer set".

$A \leftarrow \emptyset$ // *assignment over* $atom(\Pi) \cup body(\Pi)$
$\nabla \leftarrow \emptyset$ // *set of (dynamic) nogoods*
$dl \leftarrow 0$ // *decision level*
loop
  $(A, \nabla) \leftarrow \text{NogoodPropagation}(\Pi, \nabla, A)$
  if $\varepsilon \subseteq A$ for some $\varepsilon \in \Delta_\Pi \cup \nabla$ **then**
    if $dl = 0$ **then return** no answer set
    $(\delta, k) \leftarrow \text{ConflictAnalysis}(\varepsilon, \Pi, \nabla, A)$
    $\nabla \leftarrow \nabla \cup \{\delta\}$ // *learning*
    $A \leftarrow (A \setminus \{\sigma \in A \mid k < dl(\sigma)\})$ // *backjumping*
    $dl \leftarrow k$
  else if $A^{\mathbf{T}} \cup A^{\mathbf{F}} = atom(\Pi) \cup body(\Pi)$ **then**
    return $A^{\mathbf{T}} \cap atom(\Pi)$ // *answer set*
  else
    $\sigma_d \leftarrow \text{Select}(\Pi, \nabla, A)$ // *heuristic choice of* $\sigma_d \notin A$
    $dl \leftarrow dl + 1$
    $A \leftarrow A \circ (\sigma_d)$ // $dl(\sigma_d) = dl$

Observations

- Decision level dl, initially set to 0, is used to count the number of heuristically chosen literals in assignment A.
- For a heuristically chosen literal $\sigma_d = \mathbf{T}p$ or $\sigma_d = \mathbf{F}p$, respectively, we require $p \in (atom(\Pi) \cup body(\Pi)) \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}})$.
- For any literal $\sigma \in A$, $dl(\sigma)$ denotes the decision level of σ, viz. the value dl had when σ was assigned.
- A conflict is detected from violation of a nogood $\varepsilon \subseteq \Delta_\Pi \cup \nabla$.
- A conflict at decision level 0 (where A contains no heuristically chosen literals) indicates non-existence of answer sets.
- A nogood δ derived by conflict analysis is asserting, that is, some literal is unit-resulting for δ at a decision level $k < dl$.
  - After learning δ and backjumping to decision level k, at least one literal is newly derivable by unit propagation.
  - No explicit flipping of heuristically chosen literals !

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Example: CDNL-ASP

Consider

$$\Pi = \left\{ \begin{array}{llll} x \leftarrow not\ y & u \leftarrow x, y & v \leftarrow x & w \leftarrow not\ x, not\ y \\ y \leftarrow not\ x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ
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Example: CDNL-ASP

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Outline of NogoodPropagation

- Derive deterministic consequences via:
  - Unit propagation on Δ_Π and ∇;
  - Unfounded sets $U \subseteq atom(\Pi)$.
- Note that U is unfounded if $EB(U) \subseteq A^{\mathbf{F}}$.
  - ☞ For any $p \in U$, we have $(\lambda(p, U) \setminus \{\mathbf{T}p\}) \subseteq A$.
- An "interesting" unfounded set U satisfies:

$$\emptyset \subset U \subseteq (atom(\Pi) \setminus A^{\mathbf{F}}) \ .$$

- Wrt a fixpoint of unit propagation, such an unfounded set contains some loop of Π.
  - ➥ Tight programs do not yield "interesting" unfounded sets !
- Given an unfounded set U and some $p \in U$, adding $\lambda(p, U)$ to ∇ triggers a conflict or further derivations by unit propagation.
  - ☞ Add loop nogoods atom by atom to eventually falsify all $p \in U$.

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Algorithm 2: NogoodPropagation

Input : A logic program Π, a set ∇ of nogoods, and an assignment A.
Output : An extended assignment and set of nogoods.

$U \leftarrow \emptyset$ *// set of unfounded atoms*
loop
  repeat
    if $\delta \subseteq A$ for some $\delta \in \Delta_\Pi \cup \nabla$ **then return** (A, ∇) *// conflict*
    $\Sigma \leftarrow \{\delta \in \Delta_\Pi \cup \nabla \mid (\delta \setminus A) = \{\sigma\}, \overline{\sigma} \notin A\}$ *// unit-resulting nogoods*
    if $\Sigma \neq \emptyset$ **then**
      let $\sigma \in (\delta \setminus A)$ for some $\delta \in \Sigma$ **in**
        $A \leftarrow A \circ (\overline{\sigma})$ *// $dl(\overline{\sigma}) = max(\{dl(\rho) \mid \rho \in (\delta \setminus \{\sigma\})\} \cup \{0\})$*
  until $\Sigma = \emptyset$

  if Π is tight **then return** (A, ∇) *// no unfounded set $\emptyset \subset U \subseteq (atom(\Pi) \setminus A^{\mathbf{F}})$*
  else
    $U \leftarrow (U \setminus A^{\mathbf{F}})$
    if $U = \emptyset$ **then** $U \leftarrow$ UnfoundedSet(Π, A)
    if $U = \emptyset$ **then return** (A, ∇)*// no unfounded set $\emptyset \subset U \subseteq (atom(\Pi) \setminus A^{\mathbf{F}})$*
    let $p \in U$ **in**
      $\nabla \leftarrow \nabla \cup \{\lambda(p, U)\}$ *// record unit-resulting or violated loop nogood*

Requirements for UnfoundedSet

- Implementations of UnfoundedSet must guarantee the following for a result U
  1. $U \subseteq (atom(\Pi) \setminus A^{\mathbf{F}})$
  2. $EB(U) \subseteq A^{\mathbf{F}}$
  3. $U = \emptyset$ iff there is no nonempty unfounded subset of $(atom(\Pi) \setminus A^{\mathbf{F}})$
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Outline of ConflictAnalysis

- Conflict analysis is triggered whenever some nogood $\delta \in \Delta_\Pi \cup \nabla$ becomes violated, viz. $\delta \subseteq A$, at a decision level $dl > 0$.
  - Note that all but the first literal assigned at dl have been unit-resulting for nogoods $\varepsilon \in \Delta_\Pi \cup \nabla$.
  - If $\sigma \in \delta$ has been unit-resulting for ε, we obtain a new violated nogood by resolving δ and ε as follows:

$$(\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\}) \ .$$

- Resolution is directed by resolving first over the literal $\sigma \in \delta$ derived last, viz. $(\delta \setminus A[\sigma]) = \{\sigma\}$.
  - Iterated resolution progresses in inverse order of assignment.
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Algorithm 3: ConflictAnalysis

Input : A violated nogood δ, a logic program Π, a set ∇ of nogoods, and an assignment A.

Output : A derived nogood and a decision level.

loop

  let $\sigma \in \delta$ such that $(\delta \setminus A[\sigma]) = \{\sigma\}$ **in**

    $k \leftarrow max(\{dl(\rho) \mid \rho \in \delta \setminus \{\sigma\}\} \cup \{0\})$

    if $k = dl(\sigma)$ **then**

      let $\varepsilon \in \Delta_\Pi \cup \nabla$ such that $(\varepsilon \setminus A[\sigma]) = \{\overline{\sigma}\}$ **in**

        $\delta \leftarrow (\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$ *// resolution*

    else return (δ, k)

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		$\mathbf{F}\{x, y\}$	$\{\mathbf{T}\{x, y\}, \mathbf{F}x\} \in \Delta(\{x, y\})$	$\{\mathbf{T}u, \mathbf{F}x, \mathbf{F}\{x\}\}$
		$\mathbf{T}\{not\ x\}$	$\{\mathbf{F}\{not\ x\}, \mathbf{F}x\} = \delta(\{not\ x\})$	
		$\mathbf{T}y$	$\{\mathbf{F}\{not\ y\}, \mathbf{F}y\} = \delta(\{not\ y\})$	
		$\mathbf{T}\{v\}$	$\{\mathbf{T}u, \mathbf{F}\{x, y\}, \mathbf{F}\{v\}\} = \delta(u)$	
		$\mathbf{T}\{u, y\}$	$\{\mathbf{F}\{u, y\}, \mathbf{T}u, \mathbf{T}y\} = \delta(\{u, y\})$	
		$\mathbf{T}v$	$\{\mathbf{F}v, \mathbf{T}\{u, y\}\} \in \Delta(v)$	
			$\{\mathbf{T}u, \mathbf{F}\{x\}, \mathbf{F}\{x, y\}\} = \lambda(u, \{u, v\})$	✘

Example: ConflictAnalysis

Consider

$$\Pi = \left\{ \begin{array}{llll} x \leftarrow not\ y & u \leftarrow x, y & v \leftarrow x & w \leftarrow not\ x, not\ y \\ y \leftarrow not\ x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ	
1	$\mathbf{T}u$			
2	$\mathbf{F}\{not\ x, not\ y\}$			
		$\mathbf{F}w$	$\{\mathbf{T}w, \mathbf{F}\{not\ x, not\ y\}\} = \delta(w)$	
3	$\mathbf{F}\{not\ y\}$			
		$\mathbf{F}x$	$\{\mathbf{T}x, \mathbf{F}\{not\ y\}\} = \delta(x)$	
		$\mathbf{F}\{x\}$	$\{\mathbf{T}\{x\}, \mathbf{F}x\} \in \Delta(\{x\})$	$\{\mathbf{T}u, \mathbf{F}x\}$
		$\mathbf{F}\{x, y\}$	$\{\mathbf{T}\{x, y\}, \mathbf{F}x\} \in \Delta(\{x, y\})$	$\{\mathbf{T}u, \mathbf{F}x, \mathbf{F}\{x\}\}$
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Remarks

- There always is a First-UIP at which conflict analysis terminates.
  - In the worst, resolution stops at the heuristically chosen literal assigned at decision level dl.
- The nogood δ containing First-UIP σ is violated by A, viz. $\delta \subseteq A$.
- We have $k = max(\{dl(\rho) \mid \rho \in \delta \setminus \{\sigma\}\} \cup \{0\}) < dl$.
  - After recording δ in ∇ and backjumping to decision level k, $\overline{\sigma}$ is unit-resulting for δ !
  - Such a nogood δ is called asserting.
- Asserting nogoods direct conflict-driven search into a different region of the search space than traversed before, without explicitly flipping any heuristically chosen literal !

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- Graph Coloring
- Hamiltonian Cycle
- Traveling Salesperson

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- Tweaking N-Queens
- Do's and Dont's

30 Hints

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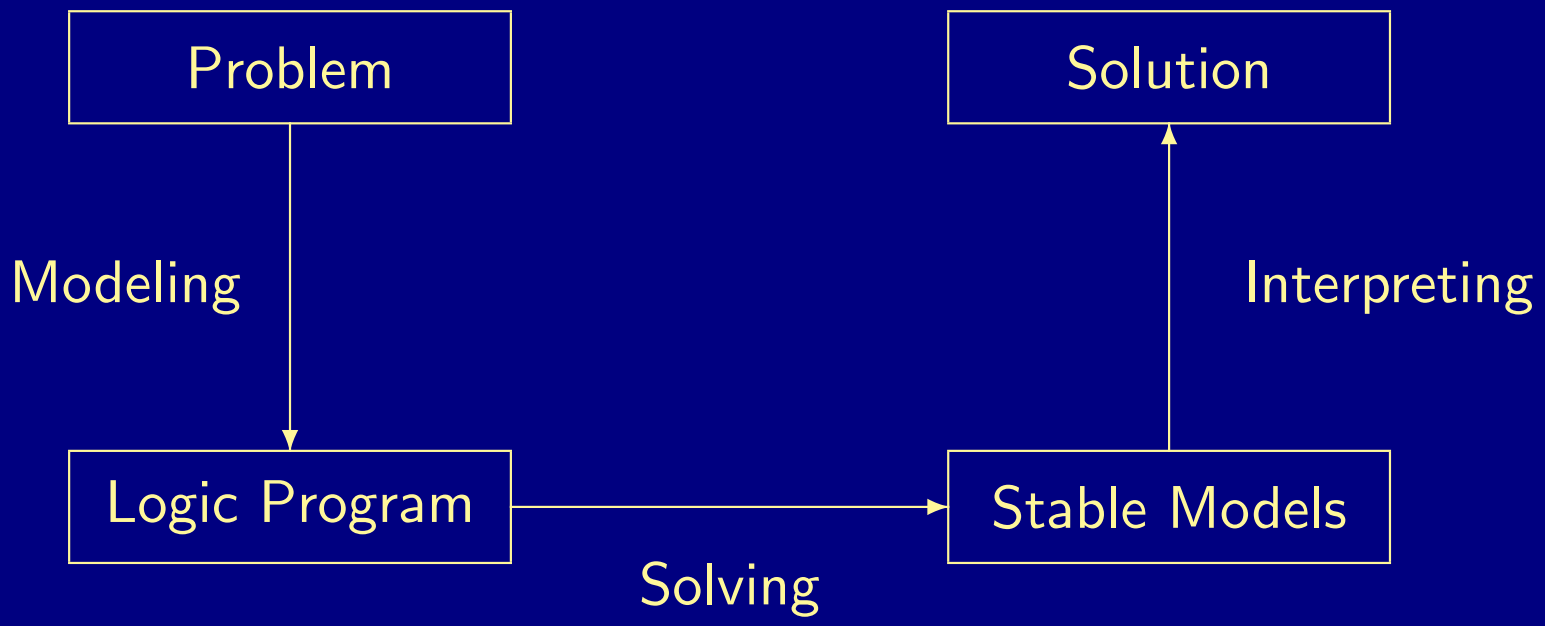
Modeling and Interpreting



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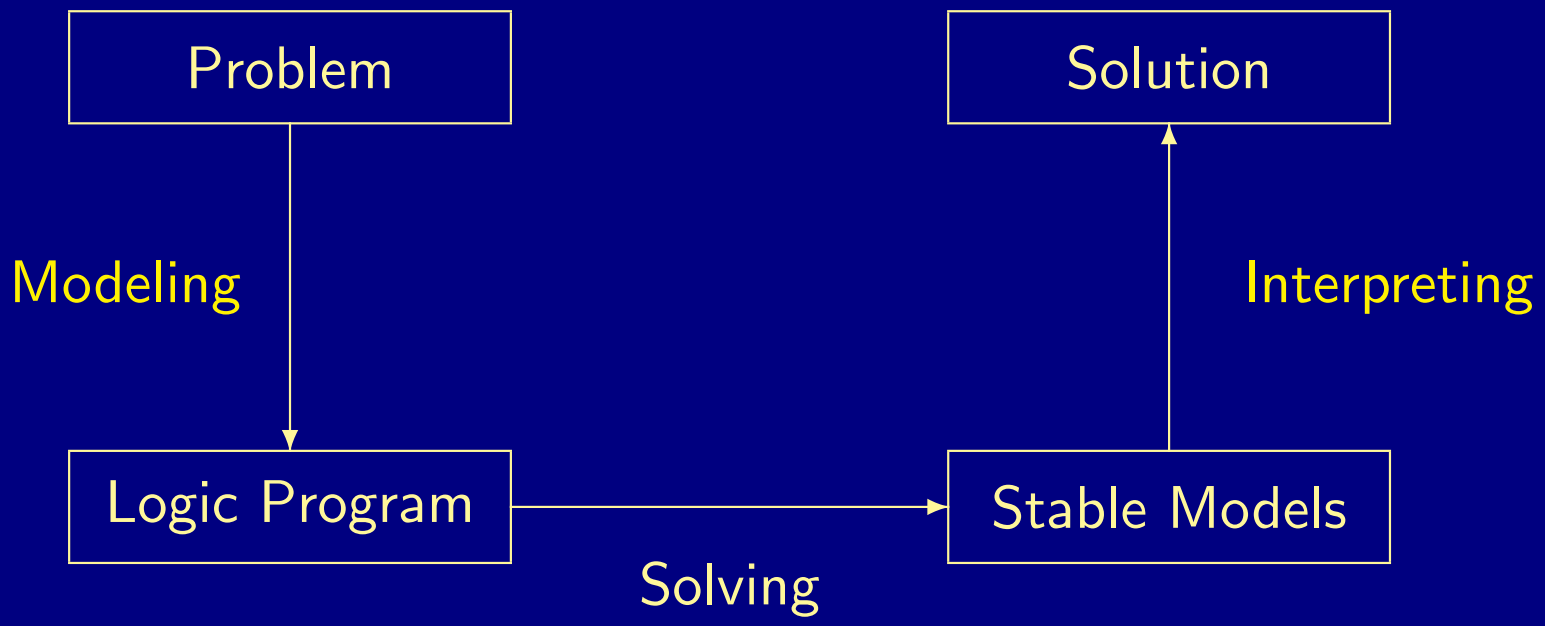

Problems as Logic Program

For solving a problem class P for a problem instance I,
encode

1. the problem instance I as a set C(I) of facts and
2. the problem class P as a set C(P) of rules

such that the solutions to P for I can be (polynomially) extracted from the answer sets of $C(I) \cup C(P)$.

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A uniform encoding C(P) is a first-order logic program, encoding the solutions to P for any set C(I) of facts.

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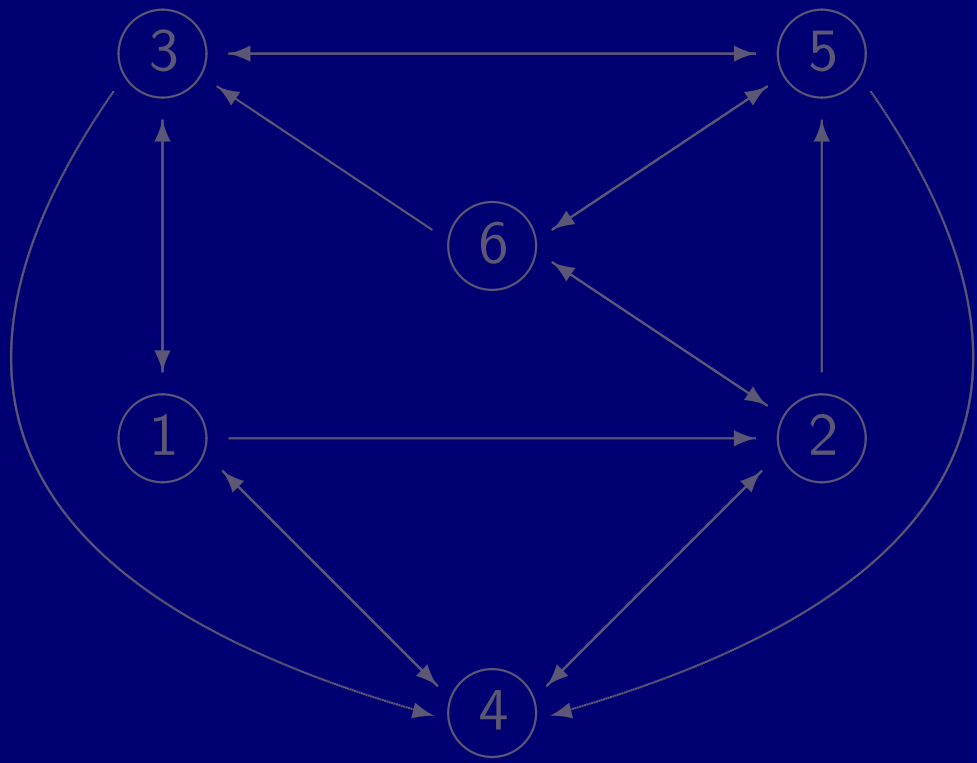
N-Colorability

Problem Instance as Facts

Given: a (directed) graph G

$$G = \begin{pmatrix} V = \{1,2,3,4,5,6\}, \\ E = \{(1,2),(1,3),(1,4), \\ \quad (2,4),(2,5),(2,6), \\ \quad (3,1),(3,4),(3,5), \\ \quad (4,1),(4,2), \\ \quad (5,3),(5,4),(5,6), \\ \quad (6,2),(6,3),(6,5)\} \end{pmatrix}$$



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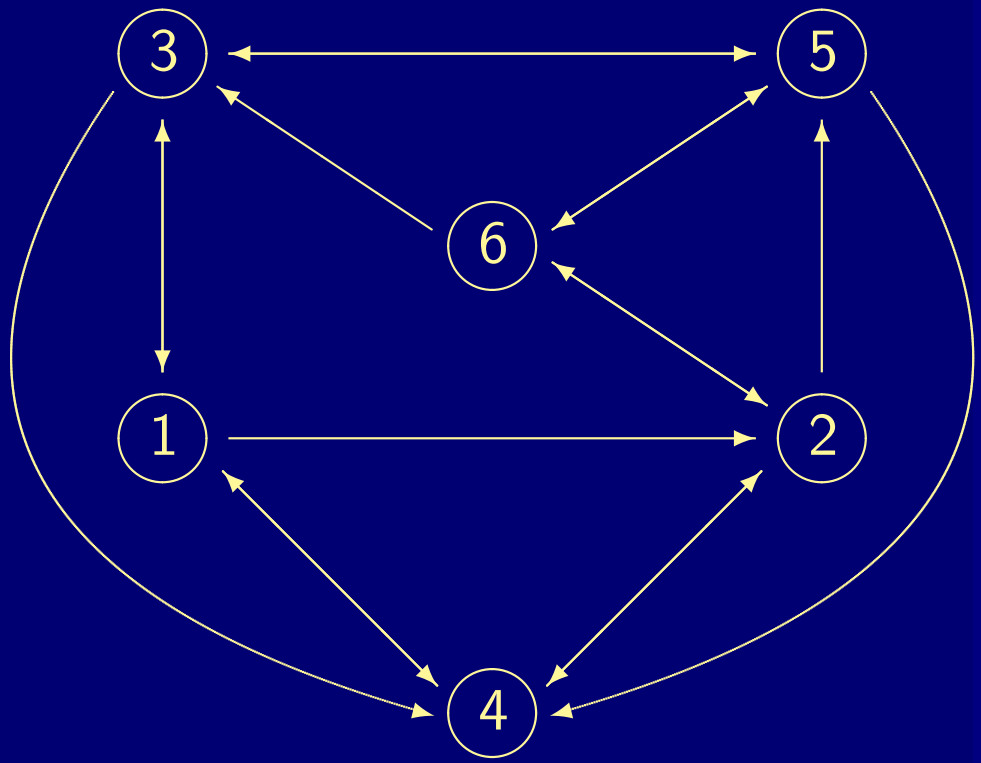

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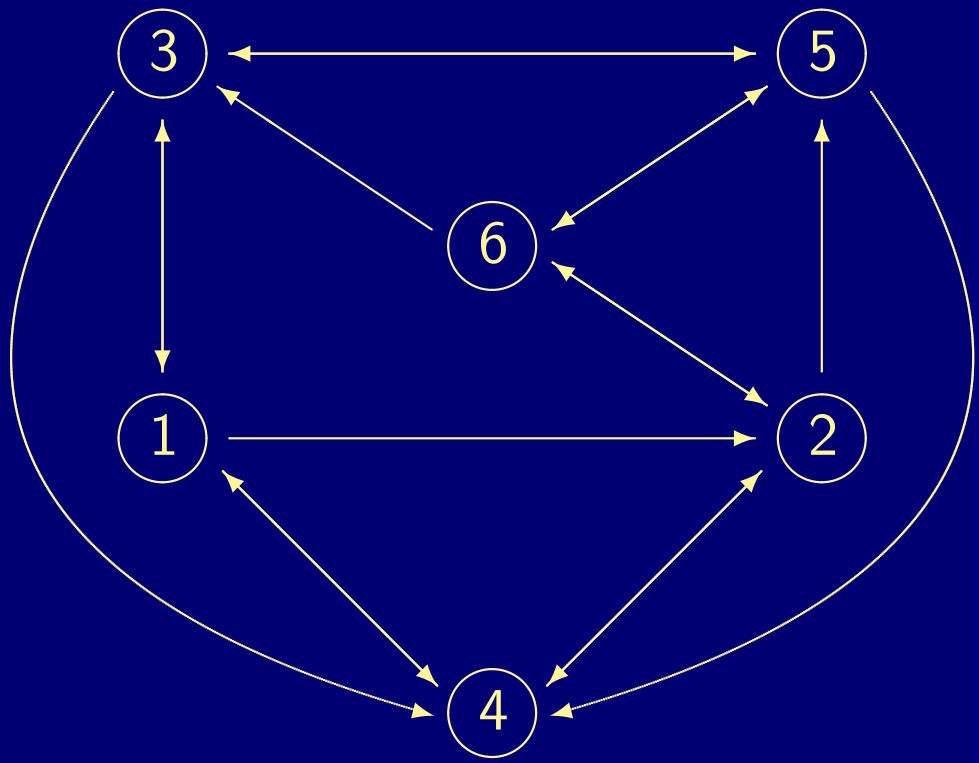

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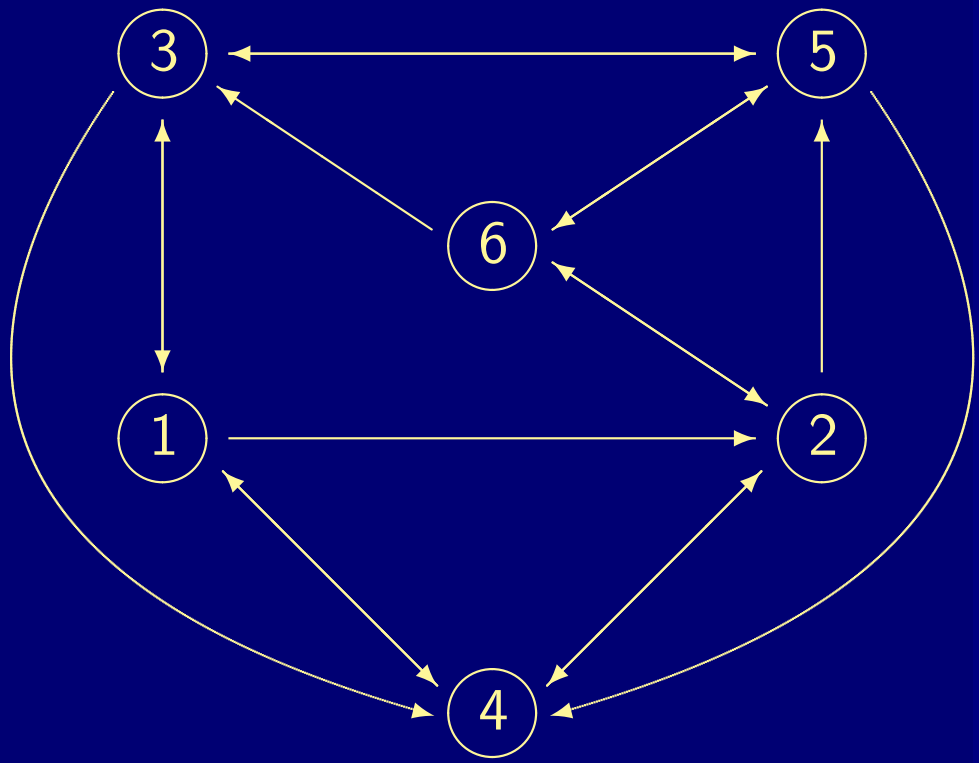

N-Colorability

(Extended) Problem Encoding

Natural Language

1. Each node has a unique color.
2. Any two connected nodes must not have the same color.
3. Let there be three colors.
4. A solution is a coloring.

Logical Language

1. ```
   color(X,C) :- iscol(C),
   node(X), not other(X,C).

   other(X,C) :- iscol(C),
   color(X,D), D != C.
   ```
2. ```
   :- color(X,C), color(Y,C),
      edge(X,Y).
   ```
3. ```
   #const n=3.
   iscol(1..n).
   ```
4. ```
   #hide.
   #show color/2.
   ```

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Logical Language

1. ```
   1 #count{ color(X,C) :
                 iscol(C) } 1
             :- node(X).
   ```
2. ```
   :- color(X,C), color(Y,C),
      edge(X,Y).
   ```
3. ```
   #const n=3.
   iscol(1..n).
   ```
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N-Colorability

Recapitulation I

Instance as Facts (in `graph.lp`)

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edge(6,2). edge(6,3). edge(6,5).
```

N-Colorability

Recapitulation II

Uniform Encoding (in `color.lp`)

```
% DOMAIN
#const n=3. iscol(1..n).

% GENERATE
1 #count{ color(X,C) : iscol(C) } 1 :- node(X).
% color(X,C) :- iscol(C), node(X), not other(X,C).
% other(X,C) :- iscol(C), color(X,D), D != C.

% TEST
:- color(X,C), color(Y,C), edge(X,Y).

% DISPLAY
#hide. #show color/2.
```

N-Colorability

Let's Run it!

```
gringo graph.lp color.lp | clasp 0
```

```
clasp version 2.0.2
Reading from stdin
Solving...
Answer: 1
color(6,2) color(5,3) color(4,2) color(3,1) color(2,1) color(1,3)
Answer: 2
color(6,1) color(5,3) color(4,1) color(3,2) color(2,2) color(1,3)
Answer: 3
color(6,3) color(5,2) color(4,3) color(3,1) color(2,1) color(1,2)
Answer: 4
color(6,1) color(5,2) color(4,1) color(3,3) color(2,3) color(1,2)
Answer: 5
color(6,3) color(5,1) color(4,3) color(3,2) color(2,2) color(1,1)
Answer: 6
color(6,2) color(5,1) color(4,2) color(3,3) color(2,3) color(1,1)
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```

N-Colorability

Let's Interpret it!

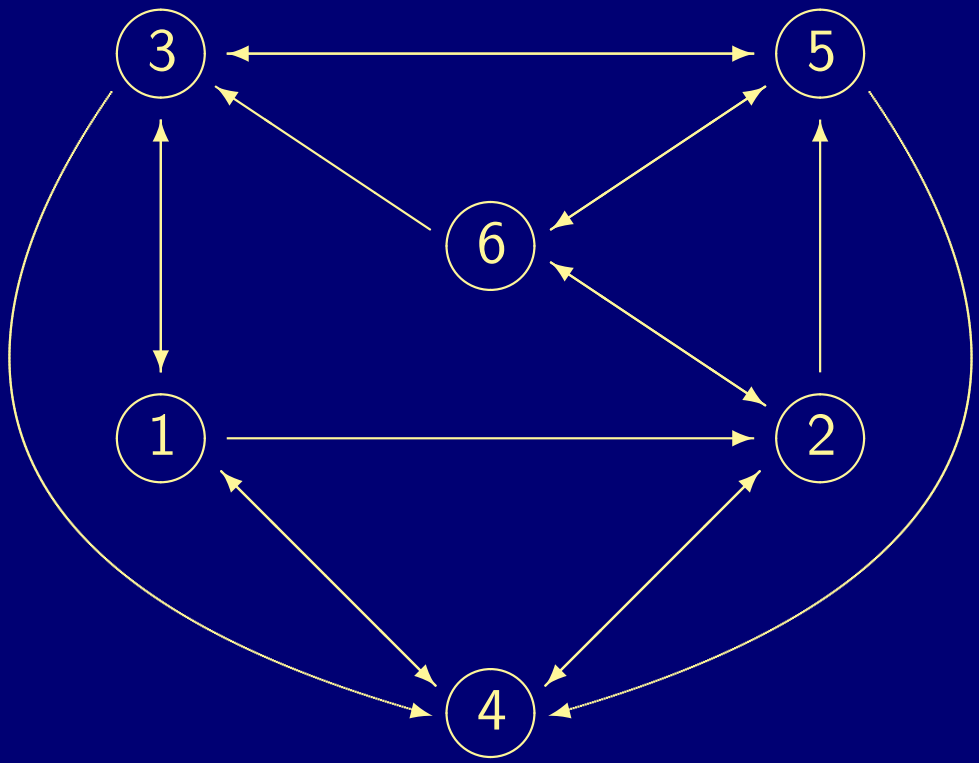
Found: 3-coloring(s)

```
Answer: 1

color(1,3) color(5,3)

color(2,1) color(3,1)

color(4,2) color(6,2)
```



N-Colorability

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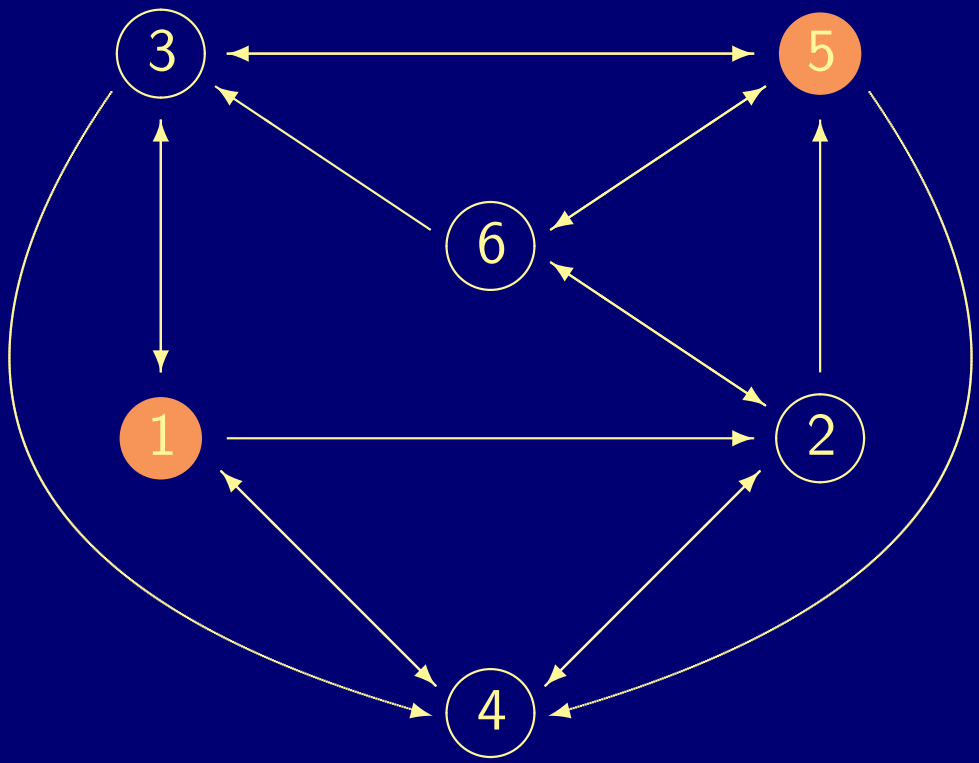

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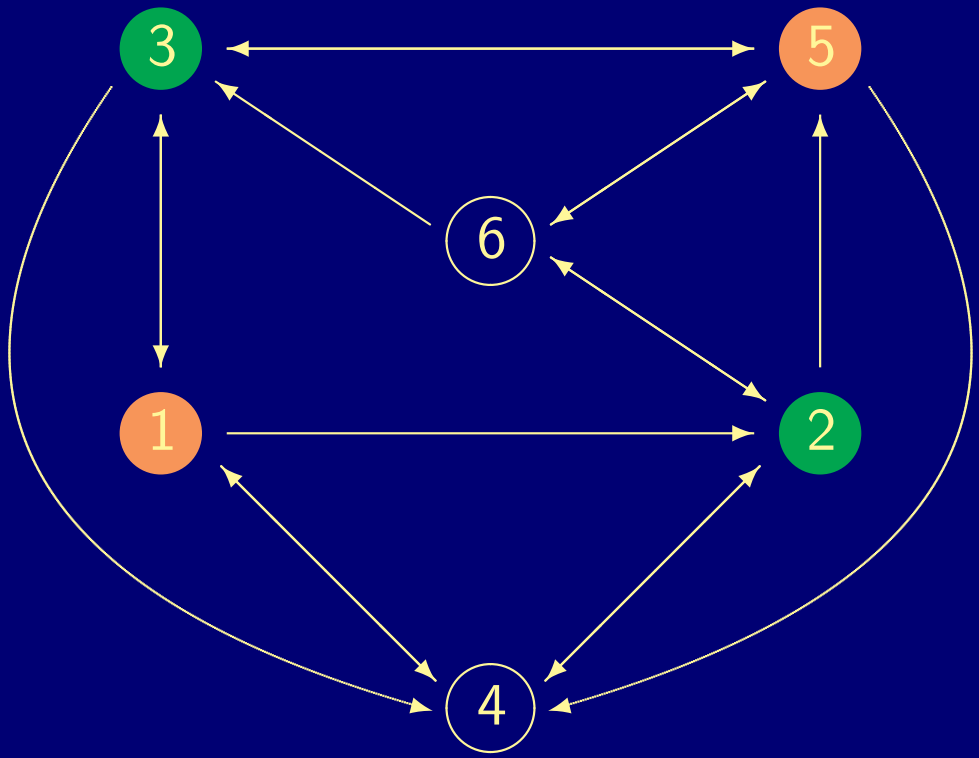
color(4,2) color(6,2)
```



N-Colorability

Let's Interpret it!

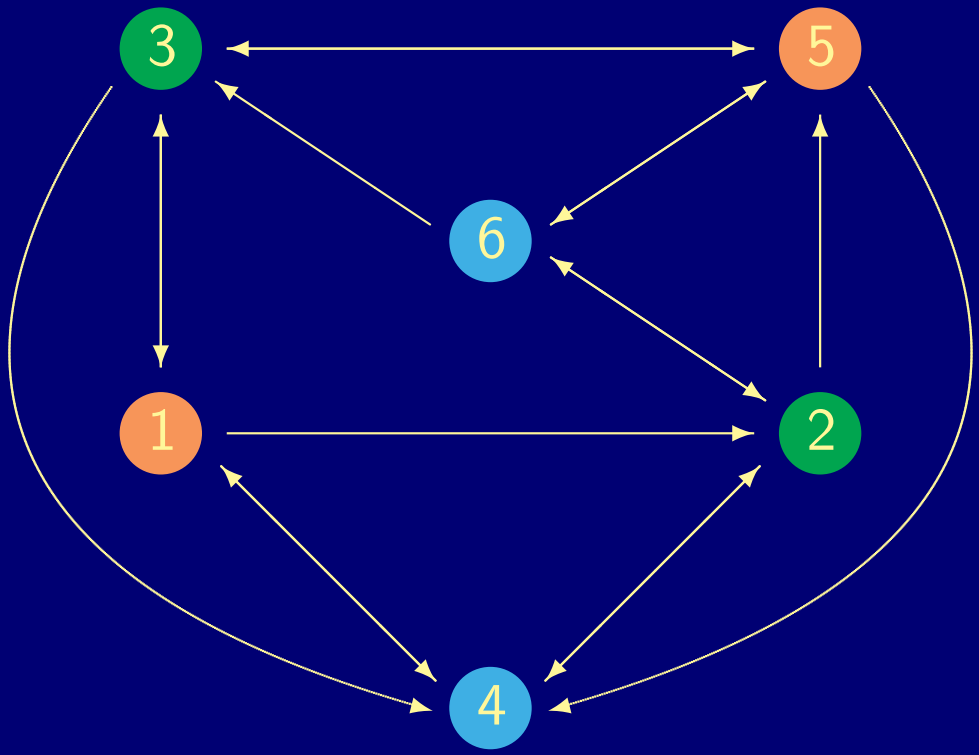
Found: 3-coloring(s)

```
Answer: 1

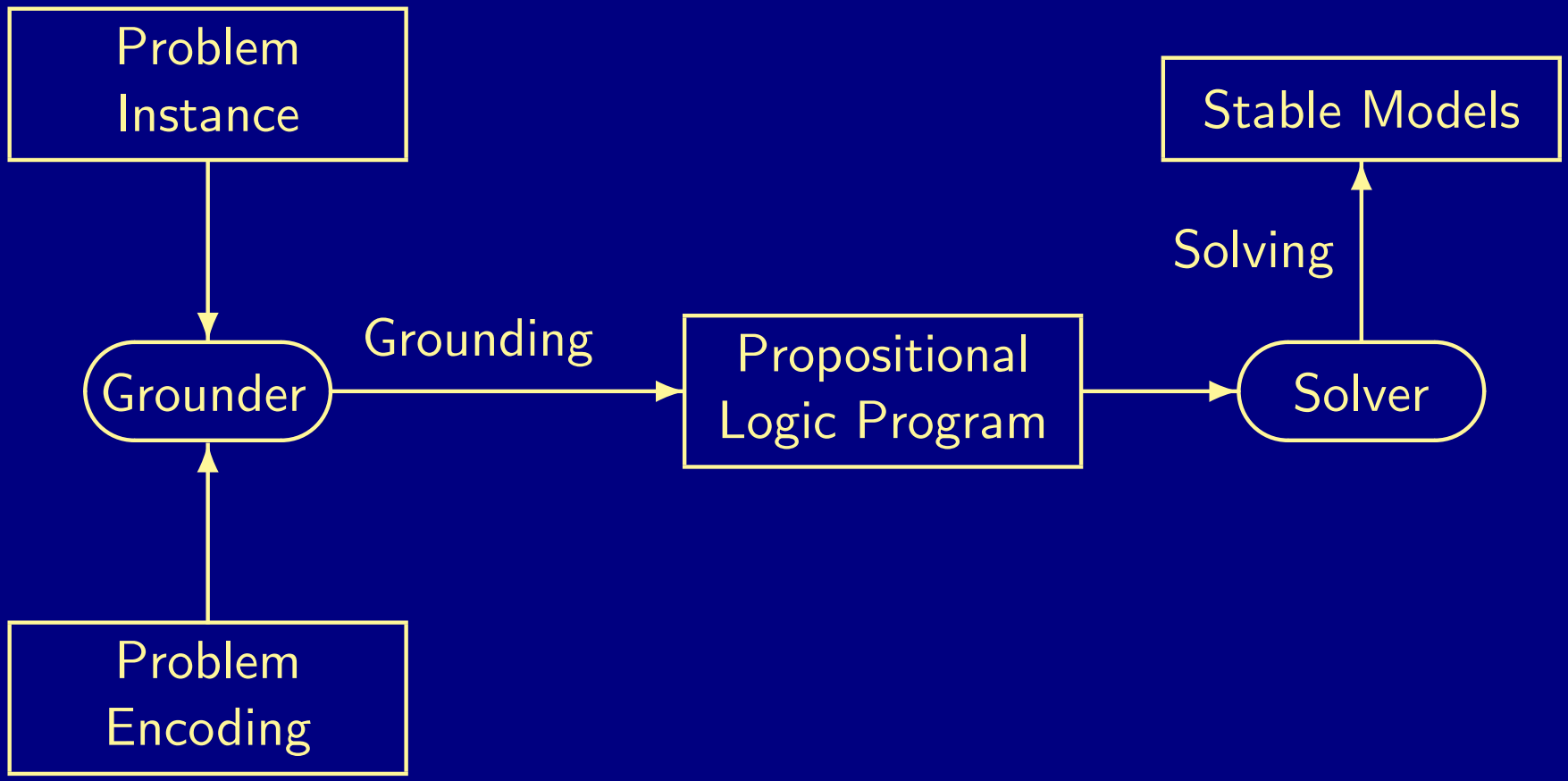
color(1,3) color(5,3)

color(2,1) color(3,1)

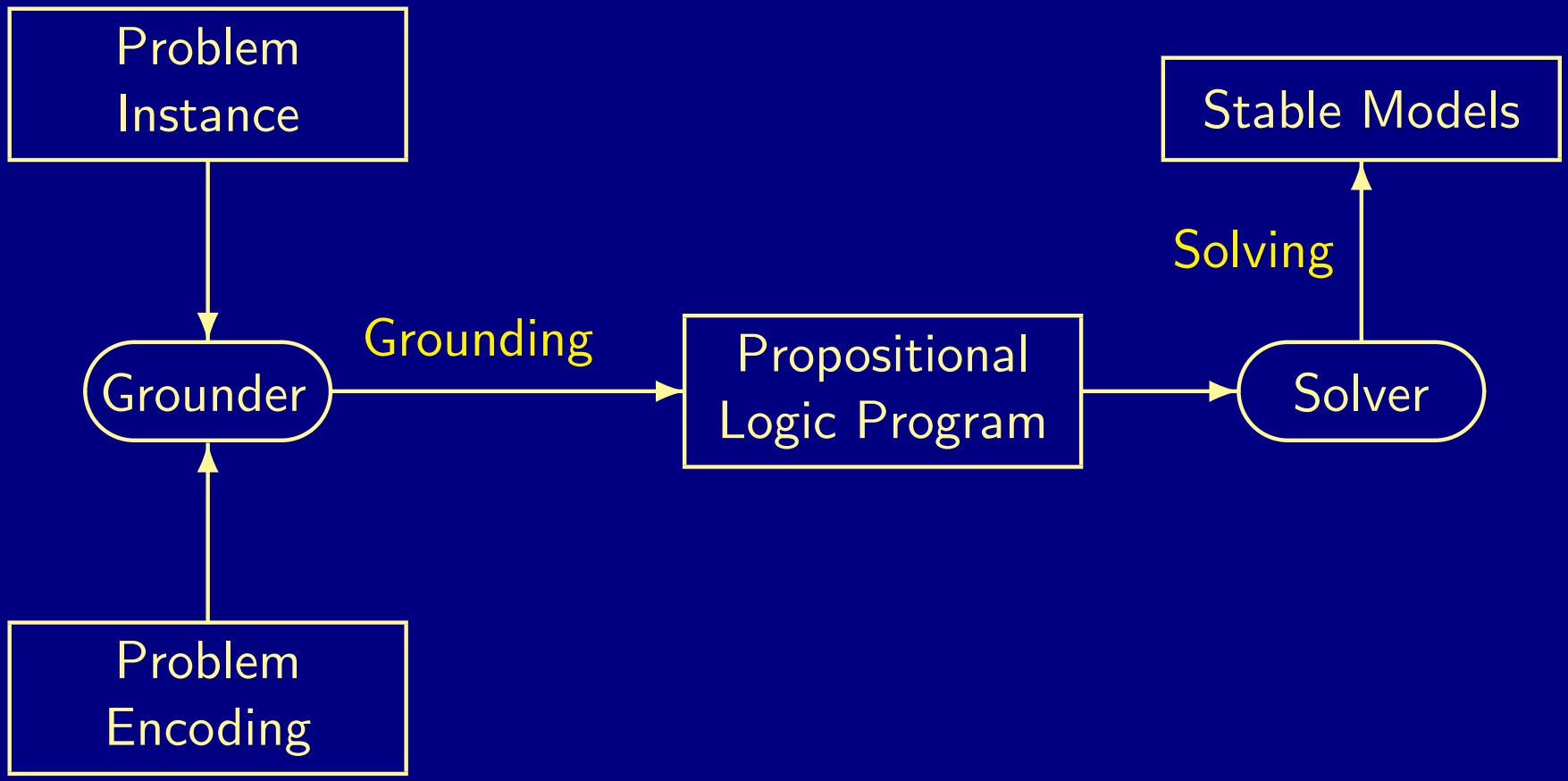
color(4,2) color(6,2)
```



Interlude: Answer Set(s) Computation



Interlude: Answer Set(s) Computation



N-Colorability

Grounding

```
gringo -t graph.lp color.lp
```

```
node(1).   node(2).   node(3).   node(4).   node(5).   node(6).
edge(1,2). edge(1,3). edge(1,4). edge(2,4). edge(2,5). ...

iscol(1).  iscol(2).  iscol(3).

1 #count{ color(1,1), color(1,2), color(1,3) } 1.
1 #count{ color(2,1), color(2,2), color(2,3) } 1.
1 #count{ color(3,1), color(3,2), color(3,3) } 1.
1 #count{ color(4,1), color(4,2), color(4,3) } 1.
1 #count{ color(5,1), color(5,2), color(5,3) } 1.
1 #count{ color(6,1), color(6,2), color(6,3) } 1.

:- color(1,1), color(2,1).
:- color(1,2), color(2,2).
:- color(1,3), color(2,3). ...
```

N-Colorability

Grounding

```
gringo -t graph.lp color.lp
```

```
node(1).   node(2).   node(3).   node(4).   node(5).   node(6).
edge(1,2). edge(1,3). edge(1,4). edge(2,4). edge(2,5). ...

iscol(1).  iscol(2).  iscol(3).

1 #count{ color(1,1), color(1,2), color(1,3) } 1.
1 #count{ color(2,1), color(2,2), color(2,3) } 1.
1 #count{ color(3,1), color(3,2), color(3,3) } 1.
1 #count{ color(4,1), color(4,2), color(4,3) } 1.
1 #count{ color(5,1), color(5,2), color(5,3) } 1.
1 #count{ color(6,1), color(6,2), color(6,3) } 1.

:- color(1,1), color(2,1).
:- color(1,2), color(2,2).
:- color(1,3), color(2,3). ...
```

N-Colorability

Solving

```
gringo graph.lp color.lp | clasp --stats 0
```

```
...
Models       : 6
Time         : 0.001s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
Choices      : 5
Conflicts    : 0
Restarts     : 0

Atoms        : 63
Rules        : 113    (1: 95 2: 12 3: 6)
Bodies       : 64
Equivalences: 106    (Atom=Atom: 31 Body=Body: 6 Other: 69)
Tight        : Yes

Variables    : 63     (Eliminated: 0 Frozen:   30)
Constraints  : 45     (Binary: 73.3% Ternary:  0.0% Other: 26.7%)
Lemmas       : 0      (Binary:  0.0% Ternary:  0.0% Other:  0.0%)
```

N-Colorability

Solving

```
gringo graph.lp color.lp | clasp --stats 0
```

```
...
Models       : 6
Time         : 0.001s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
Choices      : 5
Conflicts    : 0
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Atoms        : 63
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```

N-Colorability

Solving

```
gringo graph.lp color.lp | clasp --stats 0
```

```
...
Models       : 6
Time         : 0.001s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
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Choices      : 5
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Constraints  : 45     (Binary: 73.3% Ternary:  0.0% Other: 26.7%)
Lemmas       : 0      (Binary:  0.0% Ternary:  0.0% Other:  0.0%)
```

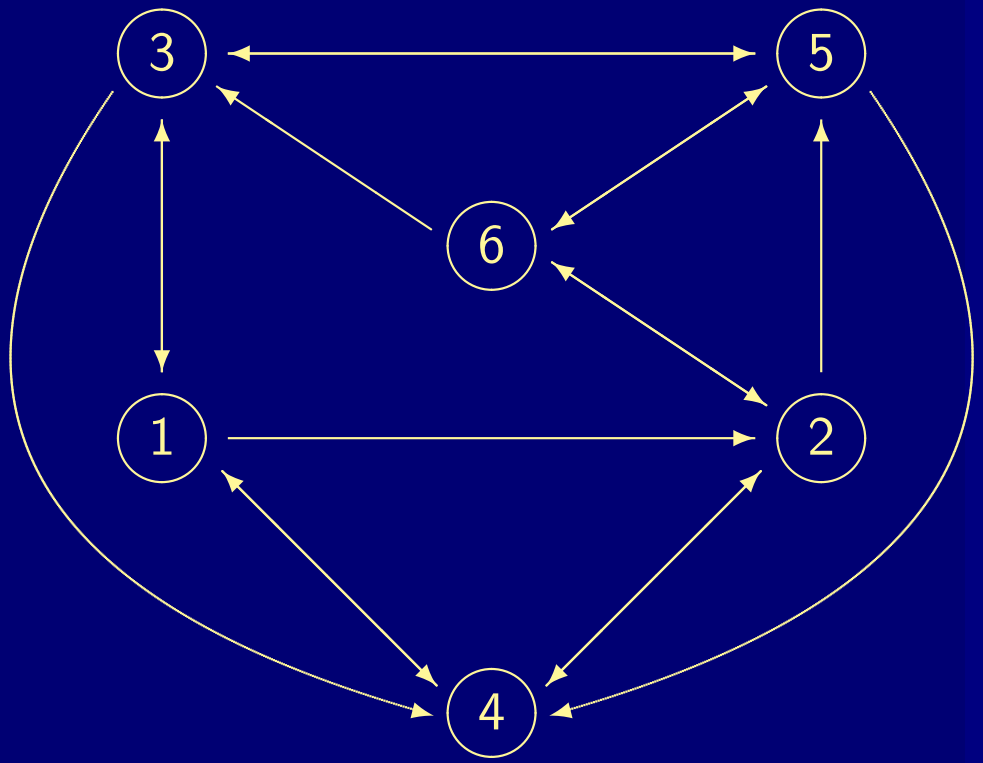
Hamiltonian Cycle

Problem Instance as Facts

Recall: a directed graph G

```
node(1).  node(2).  node(3).
node(4).  node(5).  node(6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2). edge(5,3).
edge(5,4). edge(5,6). edge(6,2).
edge(6,3). edge(6,5).
```



Hamiltonian Cycle

Engineering an Encoding

Problem Specification

A (directed) graph $G = (V, E)$ is Hamiltonian if it contains a cycle C that visits every node of V exactly once.

- ☞ C traverses exactly one incoming and one outgoing edge per node.
- ☞ C traverses every node of V (starting from an arbitrary node in V).

Problem Encoding

```
1 #count{ cycle(X,Y) : edge(X,Y) } 1 :- node(Y).
1 #count{ cycle(X,Y) : edge(X,Y) } 1 :- node(X).
```

Hamiltonian Cycle

Engineering an Encoding

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A (directed) graph $G = (V, E)$ is Hamiltonian if it contains a cycle C that visits every node of V exactly once.

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1 #count{ cycle(X,Y) : edge(X,Y) } 1 :- node(X).
```

Hamiltonian Cycle

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- ☞ C traverses exactly one incoming and one outgoing edge per node.
- ☞ C traverses every node of V (starting from an arbitrary node in V).

Problem Encoding

```
reach(X) :- first(X).
reach(Y) :- reach(X), cycle(X,Y).
```

Hamiltonian Cycle

Engineering an Encoding

Problem Specification

A (directed) graph $G = (V, E)$ is Hamiltonian if it contains a cycle C that visits every node of V exactly once.

- ☞ C traverses exactly one incoming and one outgoing edge per node.
- ☞ C traverses every node of V (starting from an arbitrary node in V).

Problem Encoding

```
reach(X) :- first(X).
reach(Y) :- reach(X), cycle(X,Y).
```

- ☞ The definition of `reach` is recursive!

Hamiltonian Cycle

Engineering an Encoding

Problem Specification

A (directed) graph $G = (V, E)$ is Hamiltonian if it contains a cycle C that visits every node of V exactly once.

- ☞ C traverses exactly one incoming and one outgoing edge per node.
- ☞ C traverses every node of V (starting from an arbitrary node in V).

Problem Encoding

```
reach(X) :- first(X).
reach(Y) :- reach(X), cycle(X,Y).

first(X) :- X = #min[ node(Y) = Y ].
```

Hamiltonian Cycle

Engineering an Encoding

Problem Specification

A (directed) graph $G = (V, E)$ is Hamiltonian if it contains a cycle C that visits every node of V exactly once.

- ☞ C traverses exactly one incoming and one outgoing edge per node.
- ☞ C traverses every node of V (starting from an arbitrary node in V).

Problem Encoding

```
reach(X) :- first(X).
reach(Y) :- reach(X), cycle(X,Y).

:- node(Y), not reach(Y).
```

Hamiltonian Cycle

The Complete Picture

Uniform Encoding (in `cycle.lp`)

```
% DOMAIN
first(X) :- X = #min[ node(Y) = Y ].

% GENERATE
1 #count{ cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 #count{ cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

% DEFINE
reach(X) :- first(X).
reach(Y) :- reach(X), cycle(X,Y).

% TEST
:- node(Y), not reach(Y).

% DISPLAY
#hide. #show cycle/2.
```

Hamiltonian Cycle

Let's Run it!

```
gringo graph.lp cycle.lp | clasp --stats
```

```
Answer: 1
cycle(6,5) cycle(5,3) cycle(4,2) cycle(3,1) cycle(2,6) cycle(1,4)
SATISFIABLE

Models       : 1+
Time         : 0.001s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
Choices      : 3
Conflicts    : 0
Restarts     : 0

Atoms        : 84
Rules        : 117    (1: 84 2: 21 3: 12)
Bodies       : 81
Equivalences: 174    (Atom=Atom: 36 Body=Body: 12 Other: 126)
Tight        : No     (SCCs: 1 Nodes: 20)
```

Hamiltonian Cycle

Let's Run it!

```
gringo graph.lp cycle.lp | clasp --stats
```

```
Answer: 1
cycle(6,5) cycle(5,3) cycle(4,2) cycle(3,1) cycle(2,6) cycle(1,4)
SATISFIABLE

Models       : 1+
Time         : 0.001s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
Choices      : 3
Conflicts    : 0
Restarts     : 0

Atoms        : 84
Rules        : 117    (1: 84 2: 21 3: 12)
Bodies       : 81
Equivalences: 174    (Atom=Atom: 36 Body=Body: 12 Other: 126)
Tight        : No     (SCCs: 1 Nodes: 20)
```

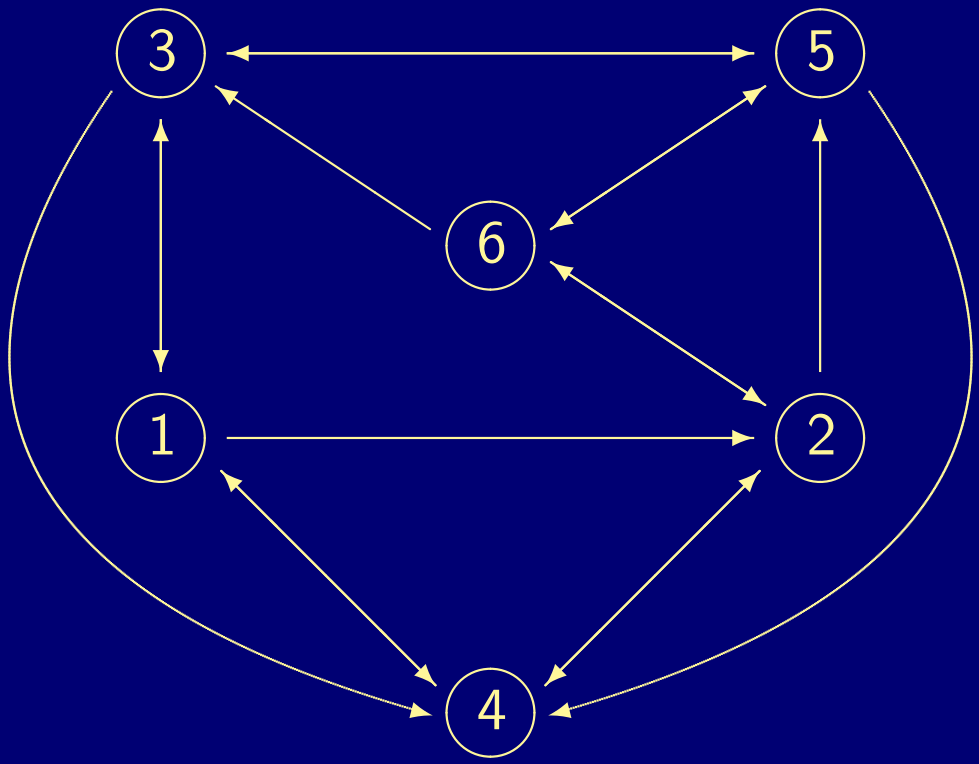
Hamiltonian Cycle

Let's Interpret it!

Found: Hamiltonian cycle

```
Answer: 1

cycle(1,4)
cycle(4,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,1)
```



Hamiltonian Cycle

Let's Interpret it!

Found: Hamiltonian cycle

```
Answer: 1

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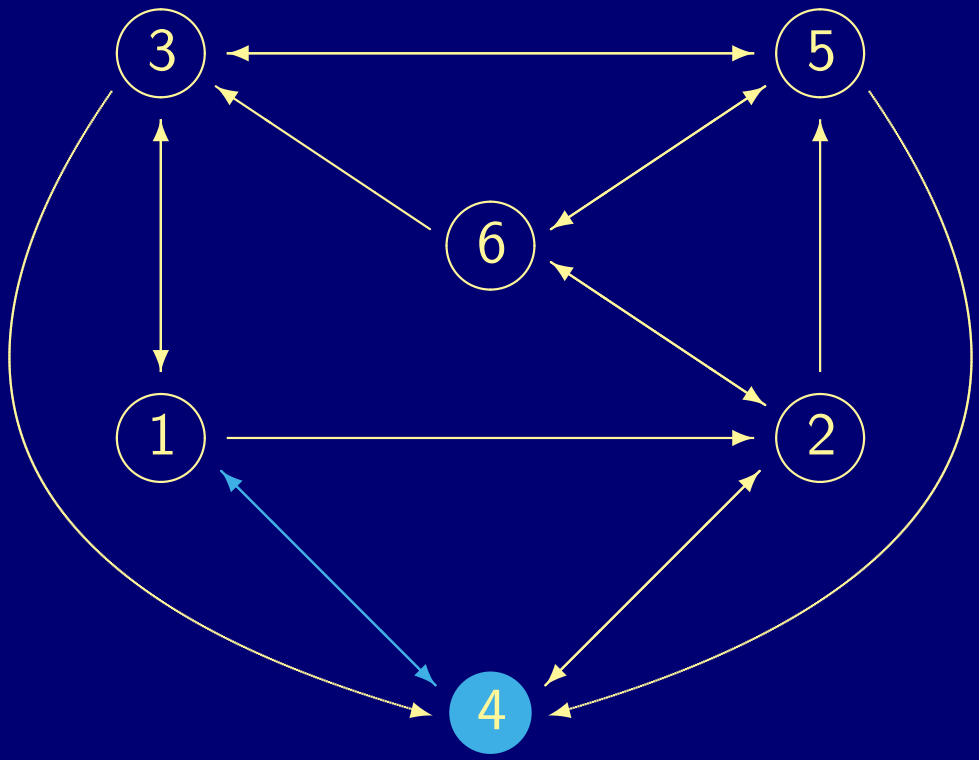
cycle(4,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,1)
```



Hamiltonian Cycle

Let's Interpret it!

Found: Hamiltonian cycle

```
Answer: 1

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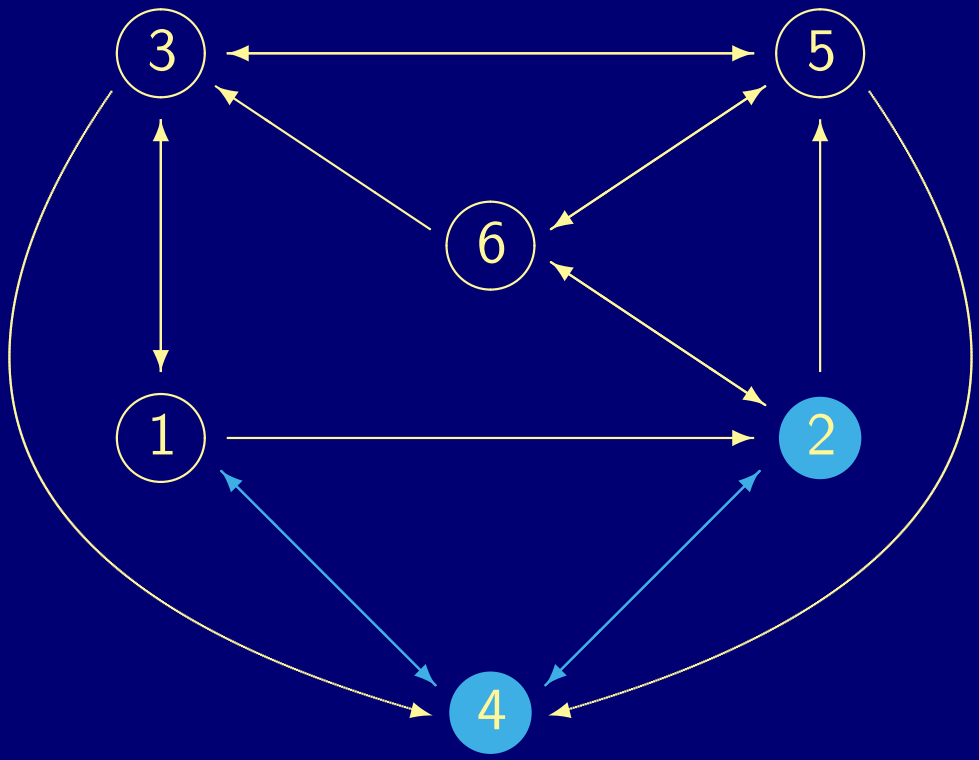
cycle(4,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,1)
```



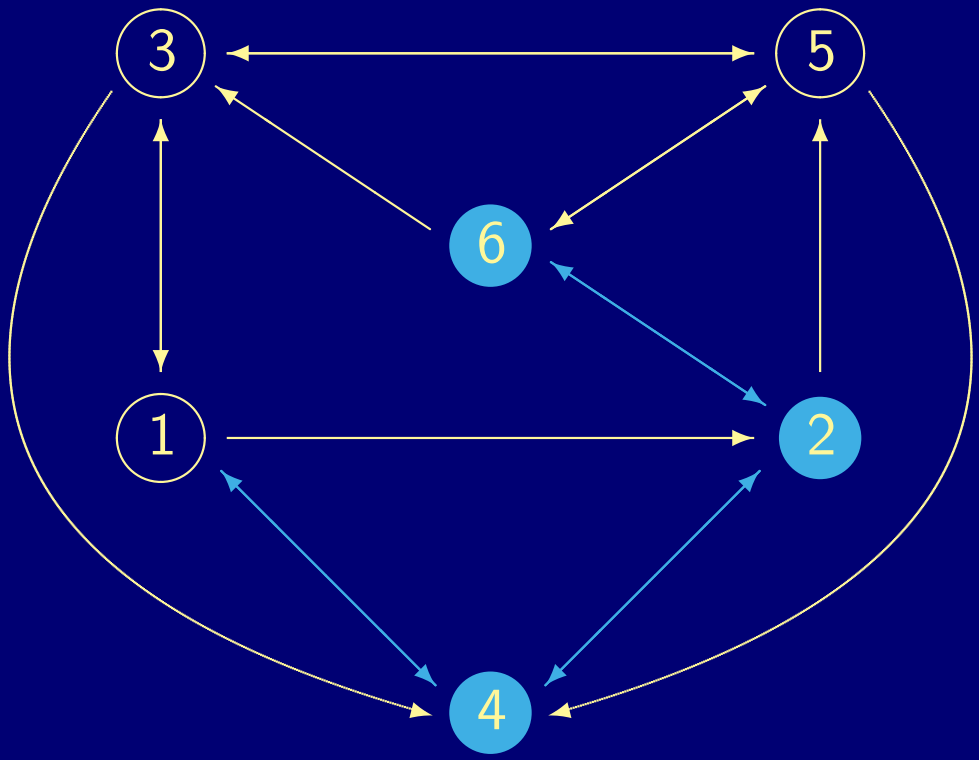
Hamiltonian Cycle

Let's Interpret it!

Found: Hamiltonian cycle

```
Answer: 1

cycle(1,4)
cycle(4,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,1)
```



Hamiltonian Cycle

Let's Interpret it!

Found: Hamiltonian cycle

```
Answer: 1

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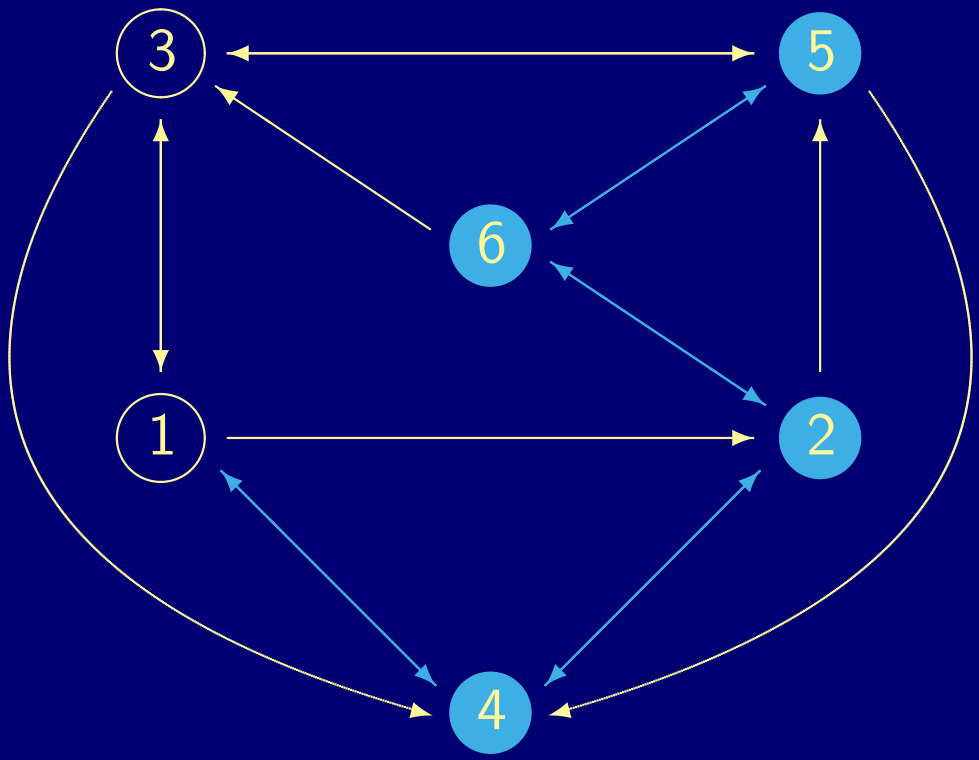
cycle(4,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,1)
```



Hamiltonian Cycle

Let's Interpret it!

Found: Hamiltonian cycle

```
Answer: 1

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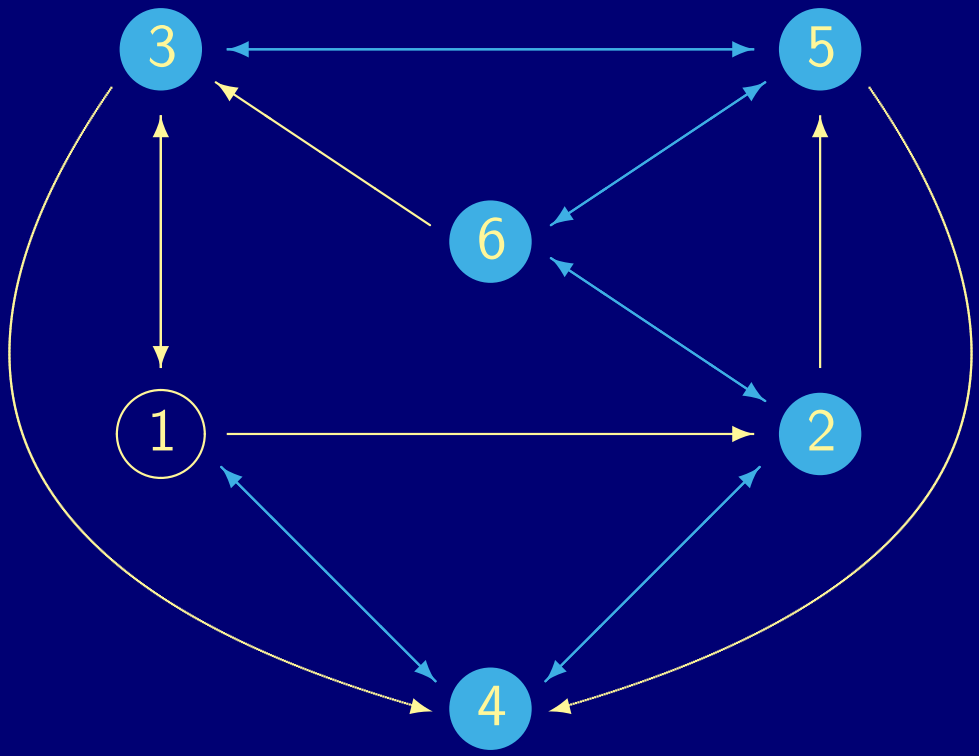
cycle(4,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,1)
```



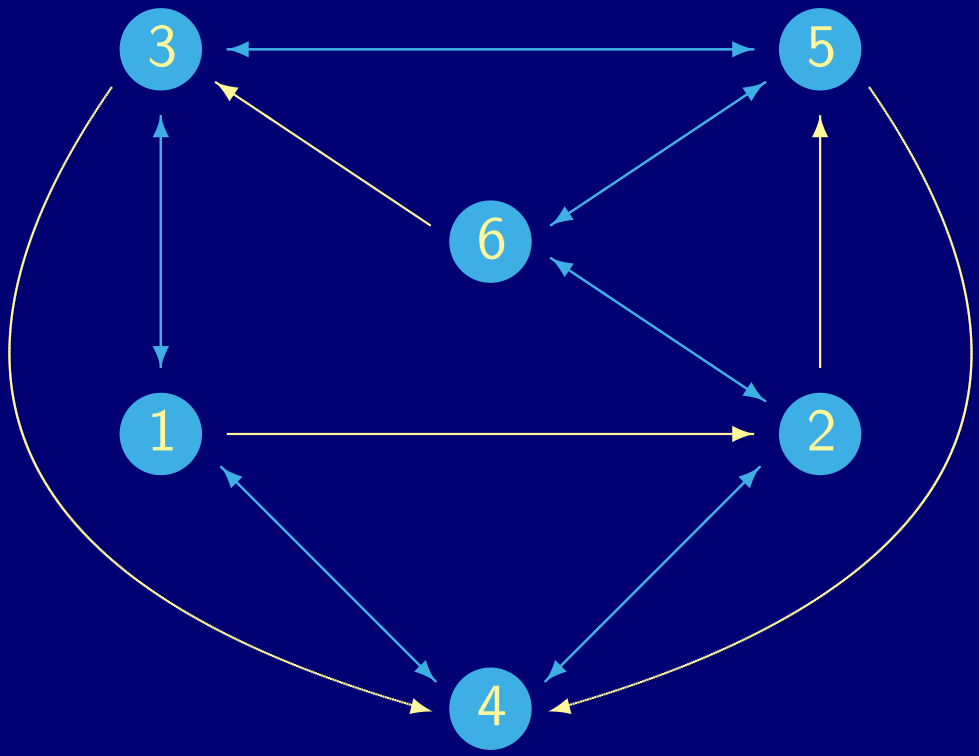
Hamiltonian Cycle

Let's Interpret it!

Found: Hamiltonian cycle

```
Answer: 1

cycle(1,4)
cycle(4,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,1)
```



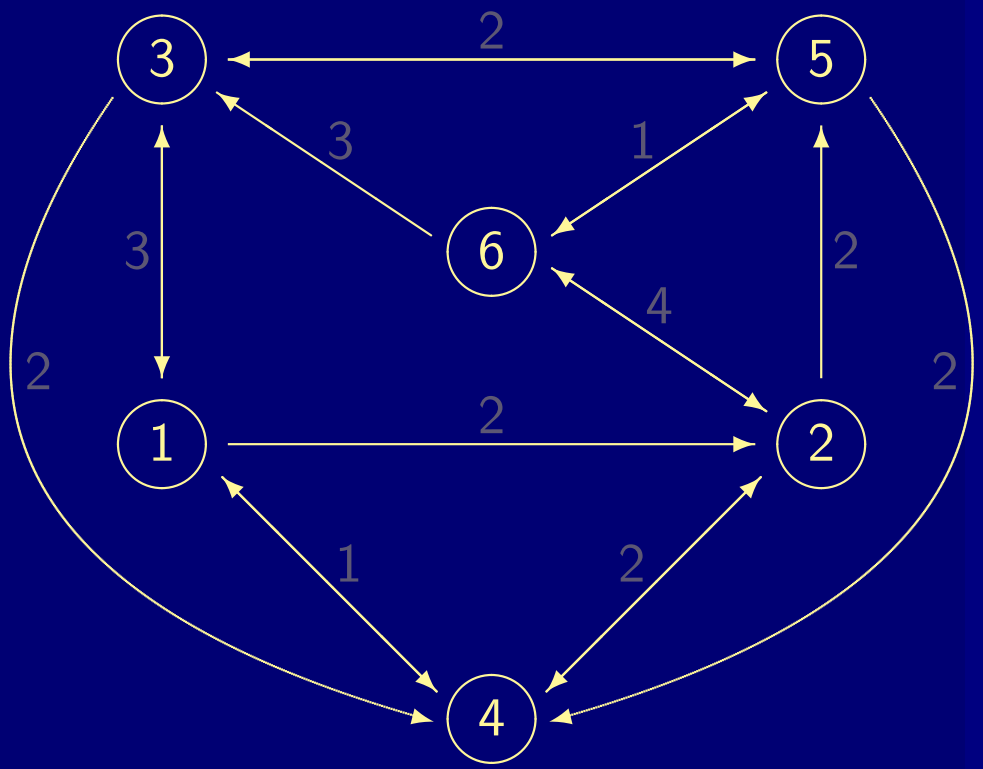
Mr Hamilton as Traveling Salesperson

Problem Instance as Facts

Given: a directed graph G plus edge costs

```
node(1).  node(2).  node(3).
node(4).  node(5).  node(6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2). edge(5,3).
edge(5,4). edge(5,6). edge(6,2).
edge(6,3). edge(6,5).
```

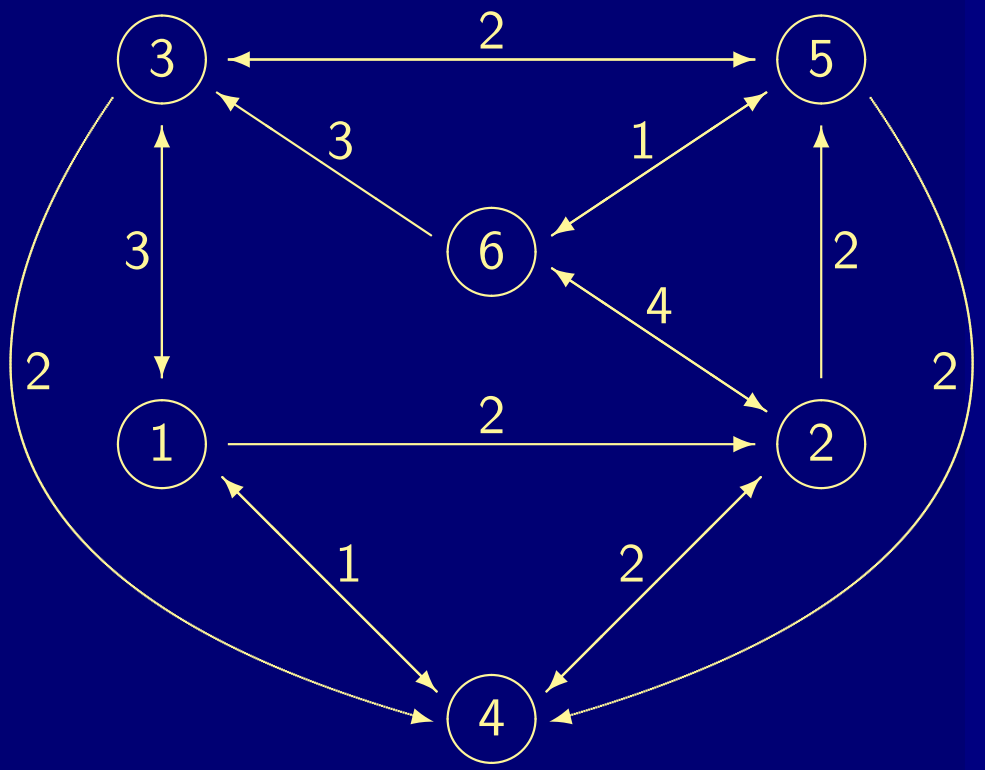


Mr Hamilton as Traveling Salesperson

Problem Instance as Facts

Given: a directed graph G plus edge costs

```
cost(1,2,2).
cost(1,3,3). cost(3,1,3).
cost(1,4,1). cost(4,1,1).
cost(2,4,2). cost(4,2,2).
cost(2,5,2).
cost(2,6,4). cost(6,2,4).
cost(3,4,2).
cost(3,5,2). cost(5,3,2).
cost(5,4,2).
cost(5,6,1). cost(6,5,1).
cost(6,3,3).
```



Mr Hamilton as Traveling Salesperson

Solution Optimization

Optimization Objective

A Hamiltonian cycle is optimal if its accumulated edge costs are minimal.

- ☞ Use `#minimize` (and/or `#maximize`) to associate each answer set with objective value(s).

Optimization Encoding

```
% OPTIMIZE
#minimize[ cycle(X,Y) : cost(X,Y,C) = C@1 ].
```

Target: minimal sum of costs `C` (at priority level `1`) associated with instances of `cycle` in an answer set

Mr Hamilton as Traveling Salesperson

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Mr Hamilton as Traveling Salesperson

Let's Run it!

```
gringo graph.lp costs.lp cycle.lp price.lp | clasp --stats 0
```

```
Answer: 1
cycle(6,5) cycle(5,3) cycle(4,2) cycle(3,1) cycle(2,6) cycle(1,4)
Optimization: 13
Answer: 2
cycle(6,5) cycle(5,3) cycle(4,1) cycle(3,4) cycle(2,6) cycle(1,2)
Optimization: 12
Answer: 3
cycle(6,3) cycle(5,6) cycle(4,1) cycle(3,4) cycle(2,5) cycle(1,2)
Optimization: 11
OPTIMUM FOUND

Models       : 1
  Enumerated: 3
  Optimum    : yes
Optimization: 11
Time         : 0.004s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
```

Mr Hamilton as Traveling Salesperson

Let's Run it!

```
gringo graph.lp costs.lp cycle.lp price.lp | clasp --stats 0
```

```
Answer: 1
cycle(6,5) cycle(5,3) cycle(4,2) cycle(3,1) cycle(2,6) cycle(1,4)
Optimization: 13
Answer: 2
cycle(6,5) cycle(5,3) cycle(4,1) cycle(3,4) cycle(2,6) cycle(1,2)
Optimization: 12
Answer: 3
cycle(6,3) cycle(5,6) cycle(4,1) cycle(3,4) cycle(2,5) cycle(1,2)
Optimization: 11
OPTIMUM FOUND

Models       : 1
  Enumerated: 3
  Optimum    : yes
Optimization: 11
Time         : 0.004s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
```

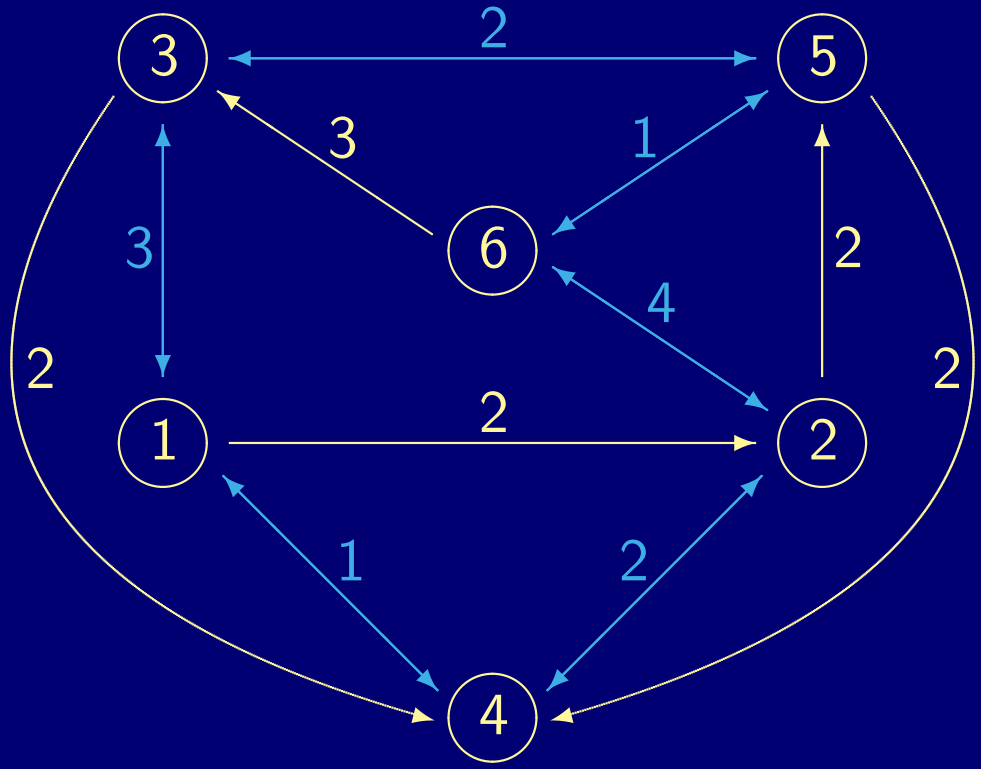
Mr Hamilton as Traveling Salesperson

Let's Interpret it!

Found: optimal Hamiltonian cycle

```
Answer: 1

cycle(1,4)
cycle(4,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,1)
```



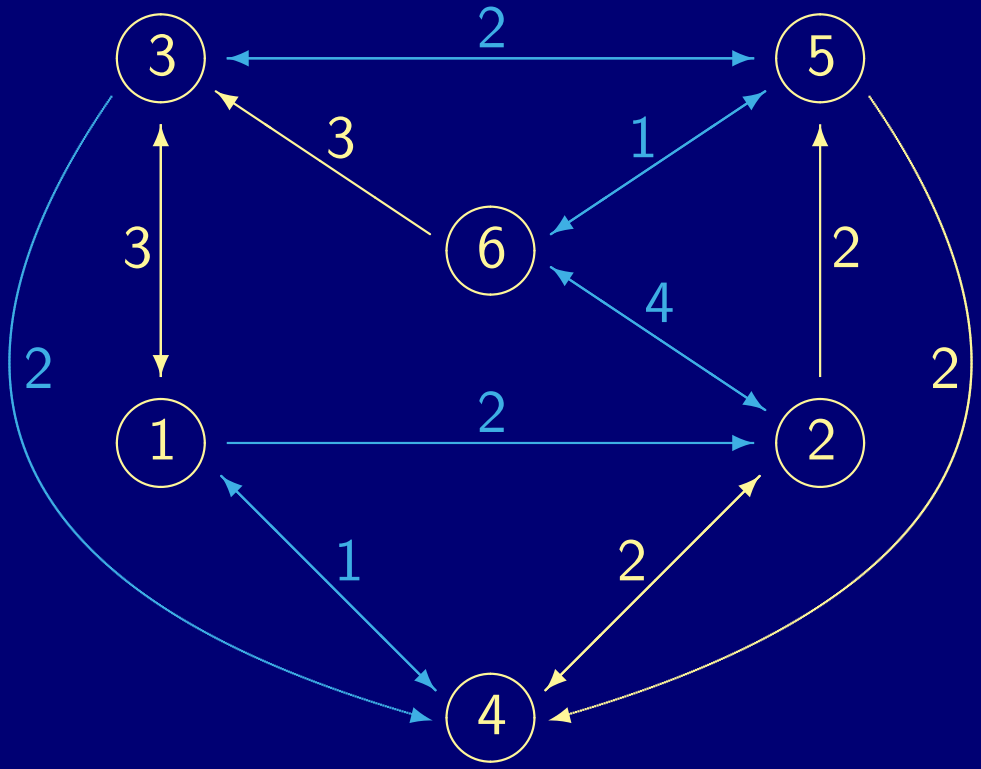
Mr Hamilton as Traveling Salesperson

Let's Interpret it!

Found: optimal Hamiltonian cycle

```
Answer: 2

cycle(1,2)
cycle(2,6)
cycle(6,5)
cycle(5,3)
cycle(3,4)
cycle(4,1)
```



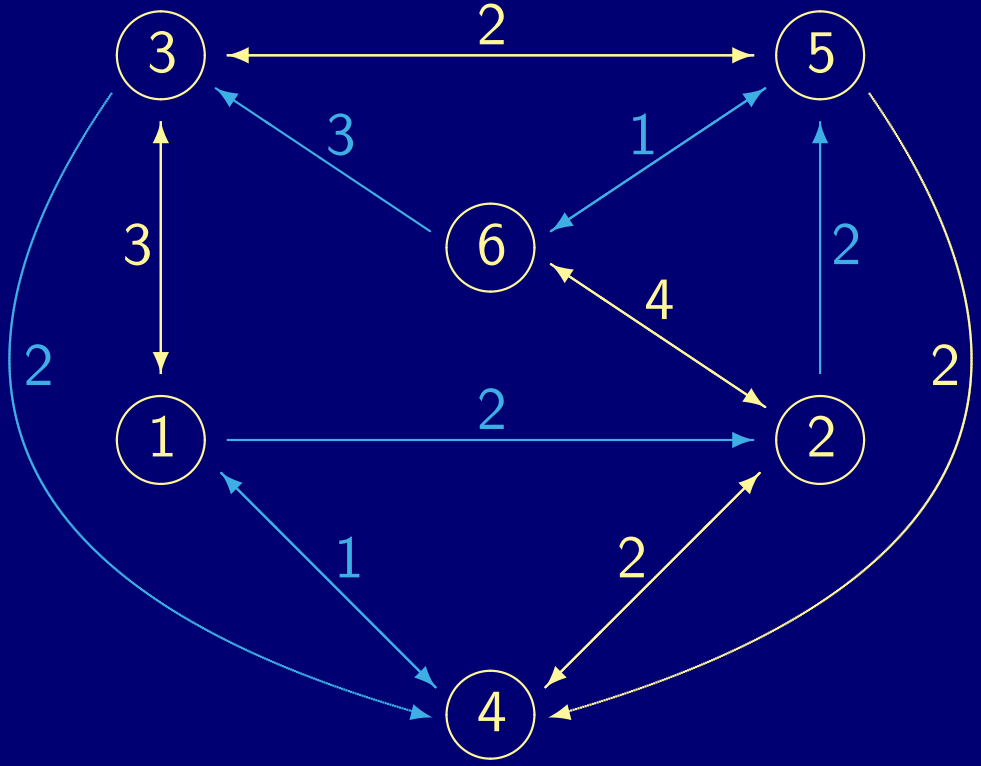
Mr Hamilton as Traveling Salesperson

Let's Interpret it!

Found: optimal Hamiltonian cycle

```
Answer: 3

cycle(1,2)
cycle(2,5)
cycle(5,6)
cycle(6,3)
cycle(3,4)
cycle(4,1)
```



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Logic Program $\subseteq$ (Data + Deduction) + (Generation + Analysis) + Selection + Projection [+ Optimization]

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Effective Modeling: Overview

28 Problems as Logic Programs (Revisited)
- Graph Coloring
- Hamiltonian Cycle
- Traveling Salesperson

29 Encoding Methodology
- Tweaking *N*-Queens
- Do's and Dont's

30 Hints

The Camel through the Eye of a Needle

ASP offers

1. rich yet easy modeling languages
2. efficient instantiation procedures
3. powerful search engines

Question: Anything left to worry about?

Answer: Yes! (unfortunately)

☞ Even in declarative programming, the problem encoding matters.

Consider sorting $[4, 7, 2, 5, 1, 8, 6, 3]$

- divide-and-conquer (Quicksort) $\sim 8(\log_2 8) = 16$ "operations"
- permutation guessing $\sim 8!/2 = 20,160$ "operations"

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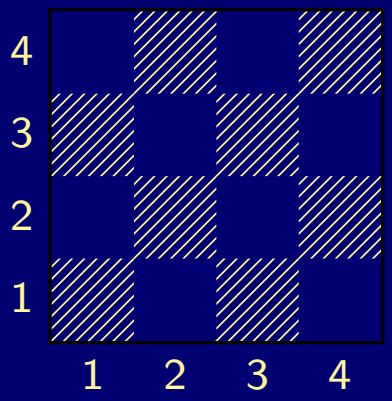
N-Queens Problem

Problem Specification

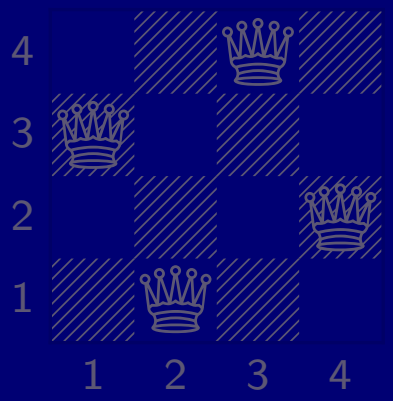
Given an $N \times N$ chessboard,
place N queens such that they do not attack each other
(neither horizontally, vertically, nor diagonally).

$N = 4$

Chessboard



Placement



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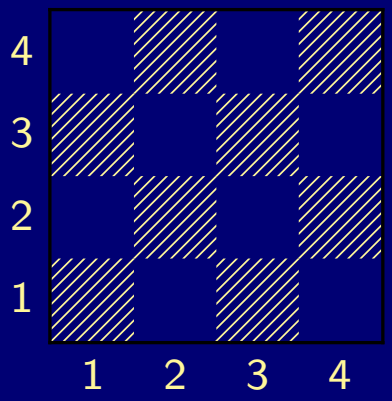

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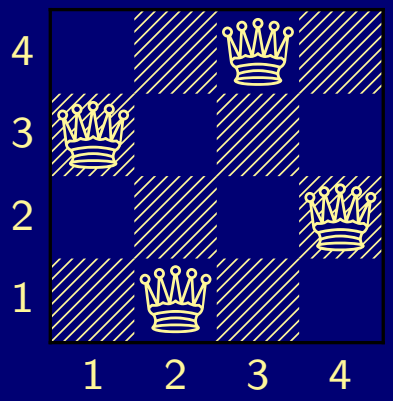

A First Encoding

1. Each square may host a queen.
2. No row, column, or diagonal hosts two queens.
3. A placement is given by instances of `queen` in an answer set.

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
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% TEST
:- queen(X1,Y1), queen(X1,Y2), Y1 < Y2.

% DISPLAY
#hide. #show queen/2.
```

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% TEST
[...]

% DISPLAY
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```

Anything missing?

A First Encoding

1. Each square may host a queen.
2. No row, column, or diagonal hosts two queens.
3. A placement is given by instances of `queen` in an answer set.
4. We have to place (at least) N queens.

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
[...]
:- not n #count{ queen(X,Y) }.

% DISPLAY
#hide. #show queen/2.
```

A First Encoding

Let's Place 8 Queens!

```
gringo -c n=8 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,6) queen(2,3) queen(3,1) queen(4,7)
queen(5,5) queen(6,8) queen(7,2) queen(8,4)
SATISFIABLE

Models       : 1+
Time         : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.000s
Choices      : 18
Conflicts    : 13
Restarts     : 0

Variables    : 793
Constraints  : 729
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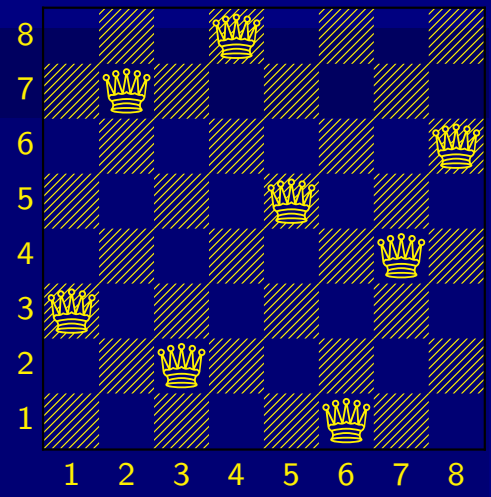


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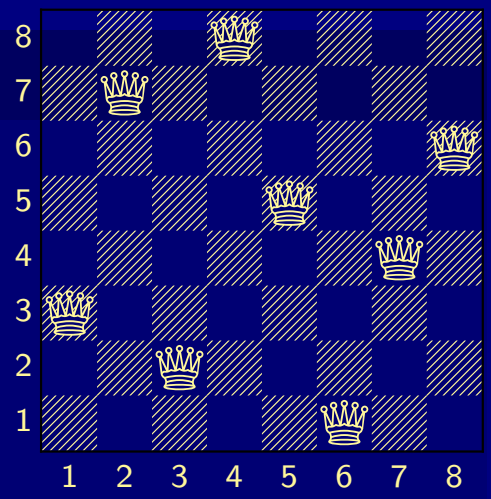


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A First Encoding

Let's Place 22 Queens!

```
gringo -c n=22 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,10) queen(2,6) queen(3,16) queen(4,14) queen(5,8) ...
SATISFIABLE

Models       : 1+
Time         : 150.531s (Solving: 150.37s 1st Model: 150.34s Unsat: 0.00s)
CPU Time     : 147.480s
Choices      : 594960
Conflicts    : 574565
Restarts     : 19

Variables    : 17271
Constraints  : 16787
```

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At least *N* queens? Exactly one queen per row and column!

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A First Refinement

At least N queens?

Exactly one queen per row and column!

queens_1.lp

```
% DOMAIN
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% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.


% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Let's Place 22 Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,18) queen(2,10) queen(3,21) queen(4,3) queen(5,5) ...
SATISFIABLE

Models       : 1+
Time         : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.020s
Choices      : 132
Conflicts    : 105
Restarts     : 1

Variables    : 7238
Constraints  : 6710
```

A First Refinement

Let's Place 22 Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

```
Answer: 1
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SATISFIABLE

Models       : 1+
Time         : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.020s
Choices      : 132
Conflicts    : 105
Restarts     : 1

Variables    : 7238
Constraints  : 6710
```

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models       : 1+
Time         : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time     : 6.930s
Choices      : 1373
Conflicts    : 845
Restarts     : 4

Variables    : 1211338
Constraints  : 1196210
```

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models       : 1+
Time         : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time     : 6.930s
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Models       : 1+
Time         : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time     : 6.930s
Choices      : 1373
Conflicts    : 845
Restarts     : 4

Variables    : 1211338
Constraints  : 1196210
```

A First Refinement

Where Time Has Gone

```
time( gringo -c n=122 queens_1.lp | clasp --stats
```

```
1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

☞ Grounding causes the problem!

A First Refinement

Where Time Has Gone

```
time( gringo -c n=122 queens_1.lp | wc)
```

```
1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

☞ Grounding causes the problem!

A First Refinement

Where Time Has Gone

```
time( gringo -c n=122 queens_1.lp | wc)
```

```
1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

Just kidding :)

☞ Grounding causes the problem!

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)
```

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1241358 7402724 24950848

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Where Time Has Gone

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real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

☞ Grounding causes the problem!

A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                  O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                        O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.       O(n²×n²)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                  O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                        O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.       O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                    O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                          O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                        O(n×n)
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.                        O(n×n)
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A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

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#const n=4. square(1..n,1..n).                                    O(n×n)

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0 #count{ queen(X,Y) } 1 :- square(X,Y).                          O(n×n)

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A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

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% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.       O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                    O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                          O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                        O(n×n)
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.                        O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.         O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                    O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                          O(n×n)

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A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

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#const n=4. square(1..n,1..n).                                    O(n×n)

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0 #count{ queen(X,Y) } 1 :- square(X,Y).                          O(n×n)

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% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                  O(n×n)

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).                        O(n×n)

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
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:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.       O(n^2×n^2)

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A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

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#const n=4. square(1..n,1..n).                                  O(n×n)

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A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

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% DOMAIN
#const n=4. square(1..n,1..n).                                    O(n×n)

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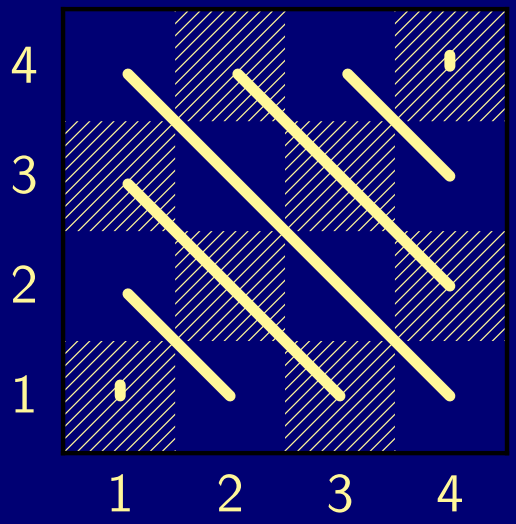
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.         O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

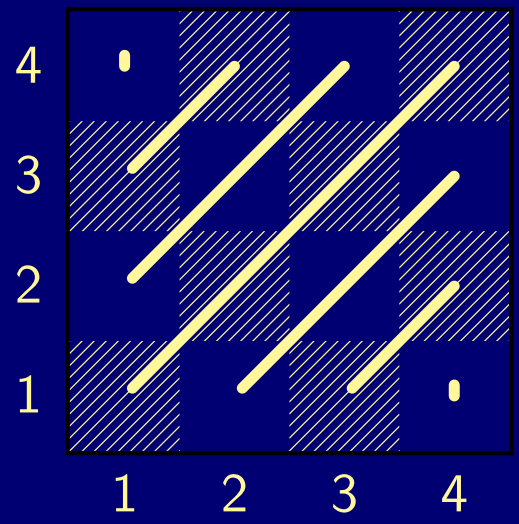
Diagonals cause trouble!

A Nomenclature for Diagonals

$N = 4$



$\#\text{diagonal}_1 = (\#\text{row} + \#\text{column}) - 1$

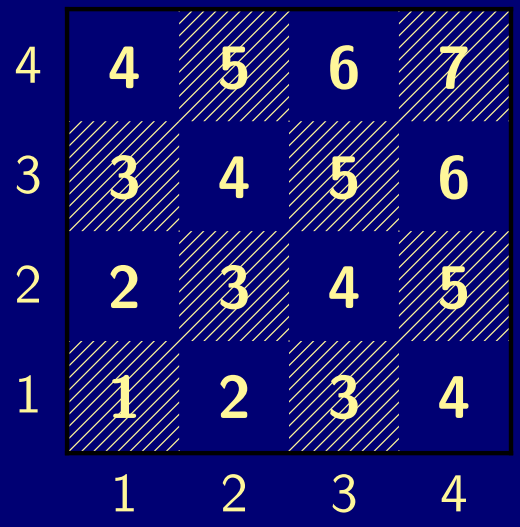


$\#\text{diagonal}_2 = (\#\text{row} - \#\text{column}) + N$

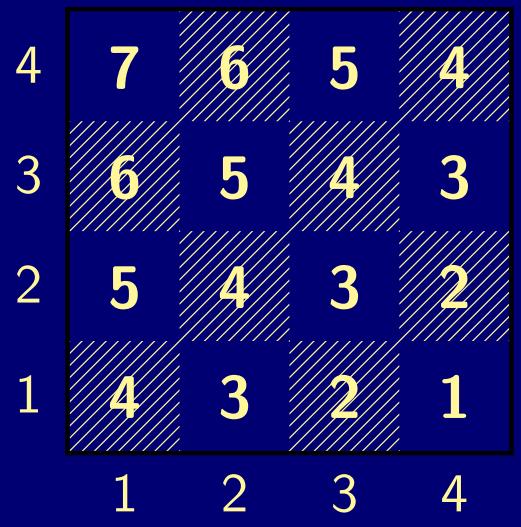
☞ $\#\text{diagonal}_{1/2}$ can be determined in this way for arbitrary N.

A Nomenclature for Diagonals

$N = 4$



$\#\text{diagonal}_1 = (\#\text{row} + \#\text{column}) - 1$

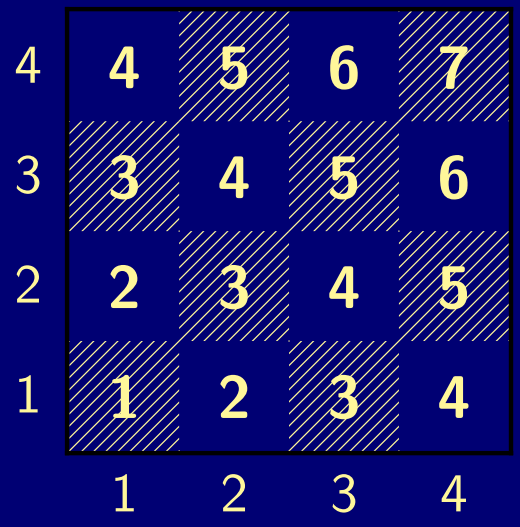


$\#\text{diagonal}_2 = (\#\text{row} - \#\text{column}) + N$

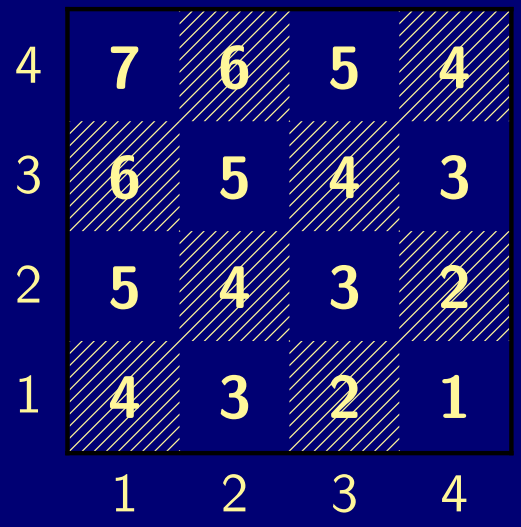
☞ $\#\text{diagonal}_{1/2}$ can be determined in this way for arbitrary N.

A Nomenclature for Diagonals

$N = 4$



$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$



$\#\text{diagonal}_2 =$
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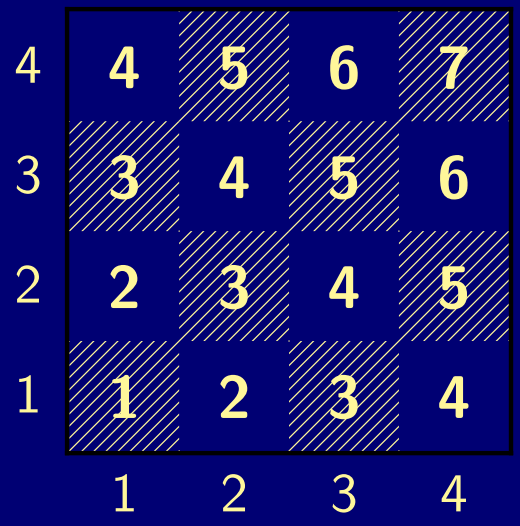
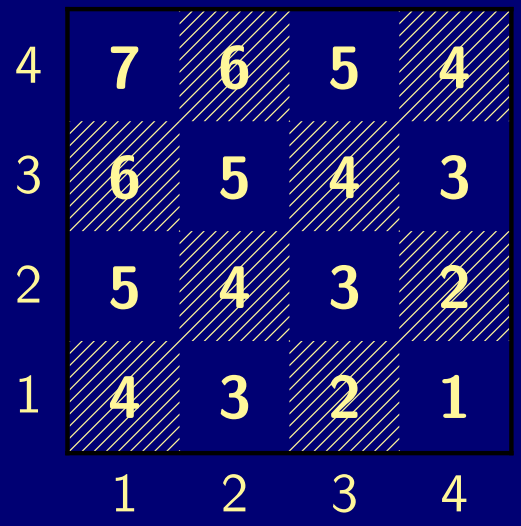
A Nomenclature for Diagonals

$N = 4$



$\#\text{diagonal}_1 = (\#\text{row} + \#\text{column}) - 1$



$\#\text{diagonal}_2 = (\#\text{row} - \#\text{column}) + N$

☞ $\#\text{diagonal}_{1/2}$ can be determined in this way for arbitrary N.

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.


% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
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:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1


% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_1.lp

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#const n=4. square(1..n,1..n).

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:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models       : 1+
Time         : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time     : 0.210s
Choices      : 11036
Conflicts    : 499
Restarts     : 3

Variables    : 16098
Constraints  : 970
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
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Time         : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
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A Second Refinement

Let's Place 122 Queens!

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gringo -c n=122 queens_2.lp | clasp --stats
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```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
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Models       : 1+
Time         : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time     : 0.210s
Choices      : 11036
Conflicts    : 499
Restarts     : 3

Variables    : 16098
Constraints  : 970
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models       : 1+
Time         : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time     : 7.250s
Choices      : 141445
Conflicts    : 7488
Restarts     : 9

Variables    : 92994
Constraints  : 2394
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models       : 1+
Time         : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time     : 7.250s
Choices      : 141445
Conflicts    : 7488
Restarts     : 9

Variables    : 92994
Constraints  : 2394
```

A Second Refinement

Let's Place 300 Queens!

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gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models       : 1+
Time         : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time     : 7.250s
Choices      : 141445
Conflicts    : 7488
Restarts     : 9

Variables    : 92994
Constraints  : 2394
```

A Third Refinement

Let's Precompute Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precompute Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precompute Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precompute Diagonals!

queens_3.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models       : 1+
Time         : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time     : 7.320s
Choices      : 141445
Conflicts    : 7488
Restarts     : 9

Variables    : 92994
Constraints  : 2394
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models       : 1+
Time         : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time     : 7.320s
Choices      : 141445
Conflicts    : 7488
Restarts     : 9

Variables    : 92994
Constraints  : 2394
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models       : 1+
Time         : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time     : 7.320s
Choices      : 141445
Conflicts    : 7488
Restarts     : 9

Variables    : 92994
Constraints  : 2394
```

A Third Refinement

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models       : 1+
Time         : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time     : 68.620s
Choices      : 869379
Conflicts    : 25746
Restarts     : 12

Variables    : 365994
Constraints  : 4794
```

A Third Refinement

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models       : 1+
Time         : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time     : 68.620s
Choices      : 869379
Conflicts    : 25746
Restarts     : 12

Variables    : 365994
Constraints  : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models       : 1+
Time         : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time     : 68.620s
Choices      : 869379
Conflicts    : 25746
Restarts     : 12

Variables    : 365994
Constraints  : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
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Models       : 1+
Time         : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time     : 68.620s
Choices      : 869379
Conflicts    : 25746
Restarts     : 12

Variables    : 365994
Constraints  : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...
SATISFIABLE

Models       : 1+
Time         : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)
CPU Time     : 29.580s
Choices      : 961315
Conflicts    : 3222
Restarts     : 7

Variables    : 365994
Constraints  : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...
SATISFIABLE

Models       : 1+
Time         : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)
CPU Time     : 29.580s
Choices      : 961315
Conflicts    : 3222
Restarts     : 7

Variables    : 365994
Constraints  : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,90) queen(2,452) queen(3,494) queen(4,145) queen(5,84) ...
SATISFIABLE

Models       : 1+
Time         : 22.654s (Solving: 10.53s 1st Model: 10.47s Unsat: 0.00s)
CPU Time     : 15.750s
Choices      : 1058729
Conflicts    : 2128
Restarts     : 6

Variables    : 403123
Constraints  : 49636
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change
2. use variable-sized conjunction (via ':') ... adapts to changing facts
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).       veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap).
pro(asparagus,fresh). pro(cucumber,fresh).
pro(asparagus,tasty). pro(cucumber,tasty).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts
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Example: vegetables to buy

```
veg(asparagus).       veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap).
pro(asparagus,fresh). pro(cucumber,fresh).
pro(asparagus,tasty). pro(cucumber,tasty).


buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty).
```

Implementing Universal Quantification

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Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).
pro(asparagus,clean).

buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty), pro(X,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✘
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).
pro(asparagus,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✘
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).  pre(cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).  pre(fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).  pre(tasty).
pro(asparagus,clean).

buy(X) :- veg(X), pro(X,P) : pre(P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✘
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).    pro(cucumber,cheap).    pre(cheap).
pro(asparagus,fresh).    pro(cucumber,fresh).    pre(fresh).
pro(asparagus,tasty).    pro(cucumber,tasty).    pre(tasty).
pro(asparagus,clean).                            pre(clean).

buy(X) :- veg(X), pro(X,P) : pre(P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✘
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).  pre(cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).  pre(fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).  pre(tasty).
pro(asparagus,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✓
3. use negation of complement ... adapts to changing facts ✓

Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).  pre(cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).  pre(fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).  pre(tasty).
pro(asparagus,clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✓
3. use negation of complement ... adapts to changing facts ✓

Example: vegetables to buy

```
veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).    pro(cucumber,cheap).    pre(cheap).
pro(asparagus,fresh).    pro(cucumber,fresh).    pre(fresh).
pro(asparagus,tasty).    pro(cucumber,tasty).    pre(tasty).
pro(asparagus,clean).                            pre(clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✘
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

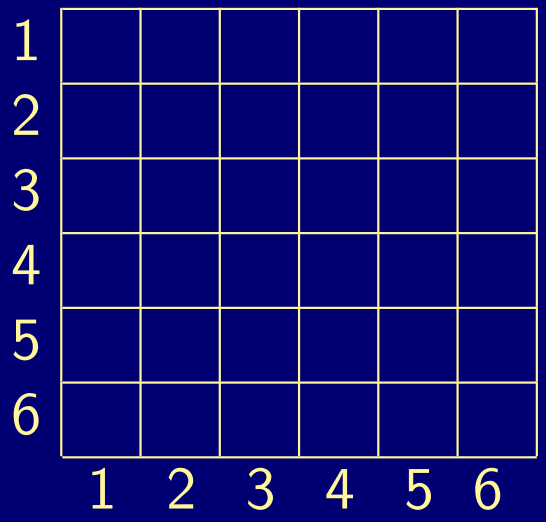
Example: vegetables to buy

```
veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).    pro(cucumber,cheap).    pre(cheap).
pro(asparagus,fresh).    pro(cucumber,fresh).    pre(fresh).
pro(asparagus,tasty).    pro(cucumber,tasty).    pre(tasty).
pro(asparagus,clean).                            pre(clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Running Example: Latin Square

Given: an $N \times N$ board



represented by facts:

```
square(1,1). ... square(1,6).
square(2,1). ... square(2,6).
square(3,1). ... square(3,6).
square(4,1). ... square(4,6).
square(5,1). ... square(5,6).
square(6,1). ... square(6,6).
```

Wanted: assignment of $1, \ldots, N$

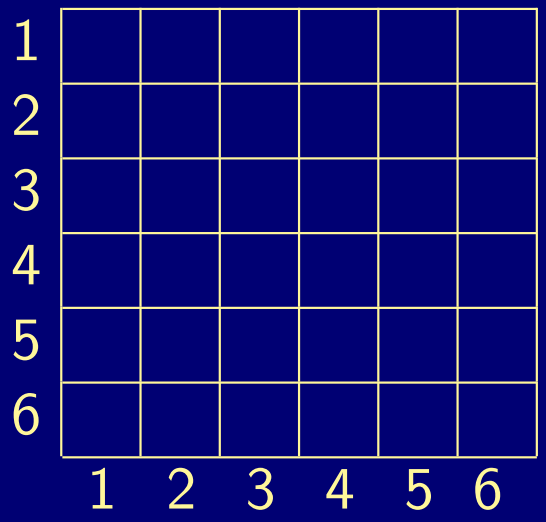
	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	1	2
4	4	5	6	1	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5

represented by atoms:

```
num(1,1,1) num(1,2,2) ... num(1,6,6)
num(2,1,2) num(2,2,3) ... num(2,6,1)
num(3,1,3) num(3,2,4) ... num(3,6,2)
num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6.5)
```

Running Example: Latin Square

Given: an $N \times N$ board



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square(1,1). ... square(1,6).
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square(3,1). ... square(3,6).
square(4,1). ... square(4,6).
square(5,1). ... square(5,6).
square(6,1). ... square(6,6).
```

Wanted: assignment of $1, \ldots, N$

1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	1	2
4	4	5	6	1	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5
	1	2	3	4	5	6

represented by atoms:

```
num(1,1,1) num(1,2,2) ... num(1,6,6)
num(2,1,2) num(2,2,3) ... num(2,6,1)
num(3,1,3) num(3,2,4) ... num(3,6,2)
num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6.5)
```

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).


% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

☞ unreused "singleton variables"

```
gringo latin_0.lp | wc
105480 2558984 14005258
```

```
gringo latin_1.lp | wc
42056 273672 1690522
```

Projecting Irrelevant Details Out

A Latin square encoding

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% DOMAIN
#const n=32. square(1..n,1..n).


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:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

☞ unreused "singleton variables"

```
gringo latin_0.lp | wc
```

```
105480 2558984 14005258
```

```
gringo latin_1.lp | wc
```

```
42056 273672 1690522
```

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
squareX(X) :- square(X,Y).      squareY(Y) :- square(X,Y).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- squareX(X1)   , N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- squareY(Y1)   , N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

☞ unreused "singleton variables"

```
gringo latin_0.lp | wc
```

```
105480 2558984 14005258
```

```
gringo latin_1.lp | wc
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Projecting Irrelevant Details Out

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:- squareY(Y1)   , N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

☞ unreused "singleton variables"

```
gringo latin_0.lp | wc
```

```
105480 2558984 14005258
```

```
gringo latin_1.lp | wc
```

```
42056 273672 1690522
```

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

☞ duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

```
gringo latin_2.lp | wc
2071560 12389384 40906946
```

```
gringo latin_3.lp | wc
1055752 6294536 21099558
```

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

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2071560 12389384 40906946
```

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gringo latin_3.lp | wc
1055752 6294536 21099558
```

Unraveling Symmetric Inequalities

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1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

☞ duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

```
gringo latin_2.lp | wc
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2071560 12389384 40906946
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gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 <  Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 <  X2.
```

☞ duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

```
gringo latin_2.lp | wc
```

```
2071560 12389384 40906946
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```
1055752 6294536 21099558
```

Unraveling Symmetric Inequalities

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% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 <  Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 <  X2.
```

☞ duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

```
gringo latin_2.lp | wc
```

```
2071560 12389384 40906946
```

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

☞ uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
1055752 6294536 21099558
```

```
gringo latin_4.lp | wc
228360 1205256 4780744
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

☞ uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc                gringo latin_4.lp | wc
1055752 6294536 21099558              228360 1205256 4780744
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

☞ uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

```
gringo latin_4.lp | wc
```

```
228360 1205256 4780744
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

☞ uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

```
gringo latin_4.lp | wc
```

```
228360 1205256 4780744
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
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228360 1205256 4780744
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Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc                gringo latin_6.lp | wc
204136 5778440 30252505                48136 373768 2185042
```

Assigning Aggregate Values

Yet another Latin square encoding

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% DOMAIN
#const n=32. square(1..n,1..n).
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gringo latin_5.lp | wc gringo latin_6.lp | wc

204136 5778440 30252505 48136 373768 2185042

Assigning Aggregate Values

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% DISPLAY
#hide. #show num/3. #show sigma/1.
```

gringo latin_5.lp | wc gringo latin_6.lp | wc

204136 5778440 30252505 48136 272768 2185042

Assigning Aggregate Values

Yet another Latin square encoding

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% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

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1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
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% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C #count{ num(X,Y,N) } C, C = 0..n.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C #count{ num(X,Y,N) } C, C = 0..n.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

☞ internal transformation by gringo

Assigning Aggregate Values

Yet another Latin square encoding

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% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.        ✗
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.        ✗
:- occX(X,N,C), C != 1.   :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

gringo latin_5.lp | wc gringo latin_6.lp | wc

Assigning Aggregate Values

Yet another Latin square encoding

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```
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% DISPLAY
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```

```
gringo latin_5.lp | wc
```

```
304136 5778440 30252505
```

```
gringo latin_6.lp | wc
```

```
48136 373768 2185042
```

Assigning Aggregate Values

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```
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#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
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48136 373768 2185042
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Breaking Symmetries

The ultimate Latin square encoding?

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#const n=32. square(1..n,1..n).

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% DISPLAY
#hide. #show num/3.
```

☞ many symmetric solutions (mirroring, rotation, value permutation)

Breaking Symmetries

The ultimate Latin square encoding?

```
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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

☞ easy and safe to fix a full row/column!

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The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

☞ easy and safe to fix a full row/column!

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

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% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
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:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

☞ Let's compare enumeration speed!

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

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1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
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% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

```
Models : 161280    Time : 2.078s
```

Breaking Symmetries

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#const n=32. square(1..n,1..n).

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 161280    Time : 2.078s
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 1344      Time : 0.024s
```

Effective Modeling: Overview

28 Problems as Logic Programs (Revisited)
- Graph Coloring
- Hamiltonian Cycle
- Traveling Salesperson

29 Encoding Methodology
- Tweaking N-Queens
- Do's and Dont's

30 Hints

Encode With Care!

1. Create a working encoding
   - Q1: Do you need to modify the encoding if the facts change?
   - Q2: Are all variables significant (or statically functionally dependent)?
   - Q3: Can there be (many) identic ground rules?
   - Q4: Do you enumerate pairs of values (to test uniqueness)?
   - Q5: Do you assign dynamic aggregate values (to check a fixed bound)?
   - Q6: Do you admit (obvious) symmetric solutions?
   - Q7: Do you have additional domain knowledge simplifying the problem?
   - Q8: Are you aware of anything else that, if encoded, would reduce grounding and/or solving efforts?
2. Revise until no "Yes" answer!
   - ☞ If the format of facts makes encoding painful (for instance, abusing grounding for "scientific calculations"), revise the fact format as well.

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Some Hints on (Preventing) Debugging

Kinds of errors

- syntactic ... follow error messages by the grounder
- semantic ... (most likely) encoding/intention mismatch

Ways to identify semantic errors (early)

develop and test incrementally
- prepare toy instances with "interesting features"
- build the encoding bottom-up and verify additions (eg. new predicates)

compare the encoded to the intended meaning
- check whether the grounding fits (use `gringo -t`)
- if answer sets are unintended, investigate conditions that fail to hold
- if answer sets are missing, examine integrity constraints (add heads)

ask tools (eg. `http://www.kr.tuwien.ac.at/research/projects/mmdasp/`)

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Overcoming Performance Bottlenecks

Grounding

- monitor time spent by and output size of `gringo`
  1. system tools (eg. `time(gringo [...] | wc)`)
  2. profiling info (eg. `gringo --gstats --verbose=3 [...] > /dev/null`)

☞ once identified, reformulate "critical" logic program parts

Solving

- check solving statistics (use `clasp --stats`)

  if great search efforts (`Conflicts`/`Choices`/`Restarts`), then

  try auto-configuration (offered by `claspfolio`)
  try manual fine-tuning (requires expert knowledge!)
  if possible, reformulate the problem or add domain knowledge ("redundant" constraints) to help the solver

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  - if possible, reformulate the problem or add domain knowledge ("redundant" constraints) to help the solver

Overcoming Performance Bottlenecks

Grounding

- monitor time spent by and output size of `gringo`
  1. system tools (eg. `time(gringo [...] | wc)`)
  2. profiling info (eg. `gringo --gstats --verbose=3 [...] > /dev/null`)

☞ once identified, reformulate "critical" logic program parts

Solving

- check solving statistics (use `clasp --stats`)

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Systems: Overview

31 Potassco

32 gringo

33 clasp

34 Siblings
- claspfolio
- clingcon
- iclingo
- oclingo

35 Book

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http://potassco.sourceforge.net

Potassco, the Potsdam Answer Set Solving Collection,
bundles tools for ASP developed at the University of Potsdam,
for instance:

- *Grounder*: gringo, pyngo
- *Solver*: clasp, claspd, claspar
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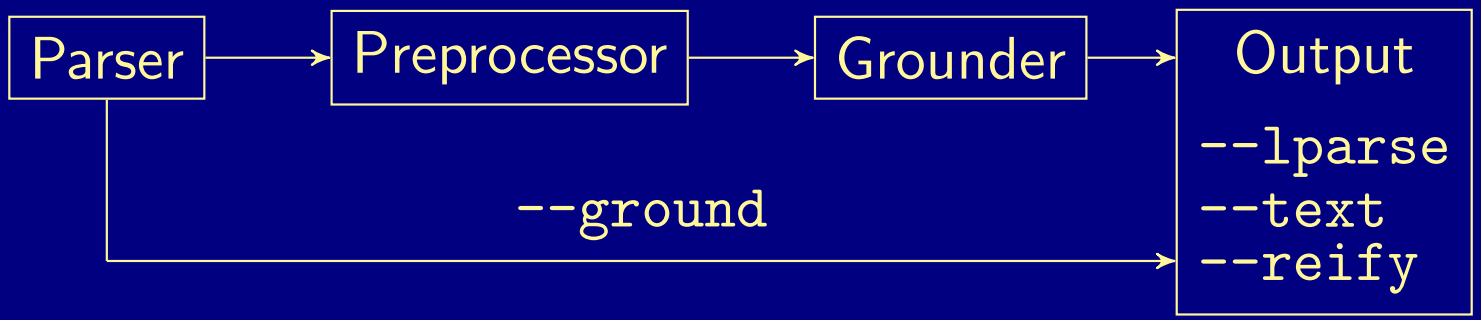
gringo

- Accepts safe programs with aggregates
- Tolerates unrestricted use of function symbols
  (as long as it yields a finite ground instantiation :)
- Expressive power of a Turing machine

- Basic architecture of *gringo*:



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clasp

- Native ASP solver combining conflict-driven search with sophisticated reasoning techniques:
  - Advanced preprocessing including, e.g., equivalence reasoning
  - Lookback-based decision heuristics
  - Restart policies
  - Nogood deletion
  - Progress saving
  - Dedicated data structures for binary and ternary nogoods
  - Lazy data structures (watched literals) for long nogoods
  - Dedicated data structures for cardinality and weight constraints
  - Lazy unfounded set checking based on "source pointers"
  - Tight integration of unit propagation and unfounded set checking
  - Reasoning modes
  - Multi-threaded search
  - ...

Reasoning modes of *clasp*

Beyond deciding answer set existence, *clasp* allows for:

- Optimization
- Enumeration [without solution recording]
- Projective Enumeration [without solution recording]
- Brave and Cautious Reasoning determining the
  - union or
  - intersection

  of all answer sets by computing only linearly many of them
- and combinations thereof

clasp also allows for solving

- propositional CNF formulas (extended *dimacs*)
- pseudo-Boolean formulas (*opb* and *wbo*)

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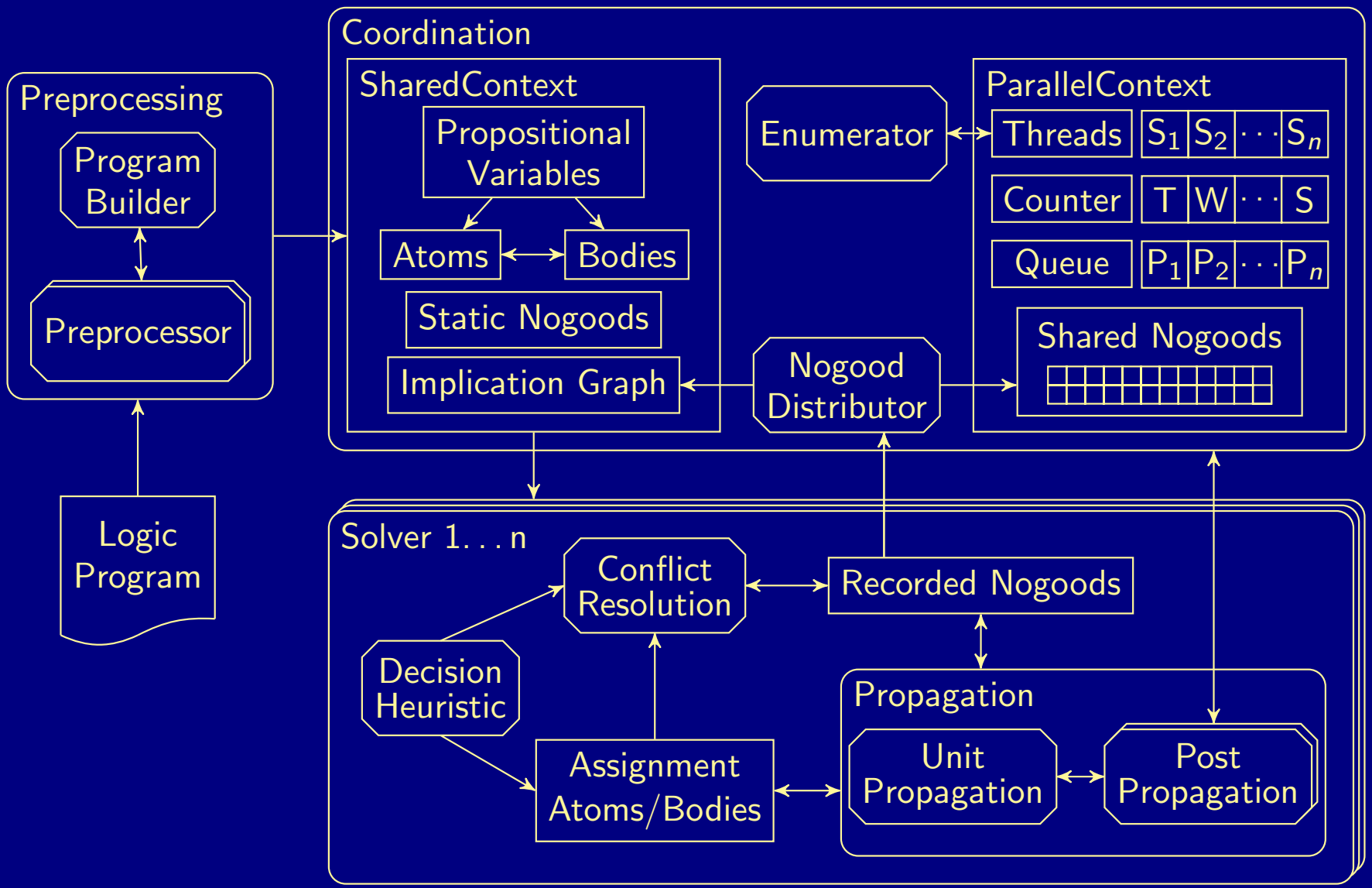
Multi-threaded architecture of *clasp*



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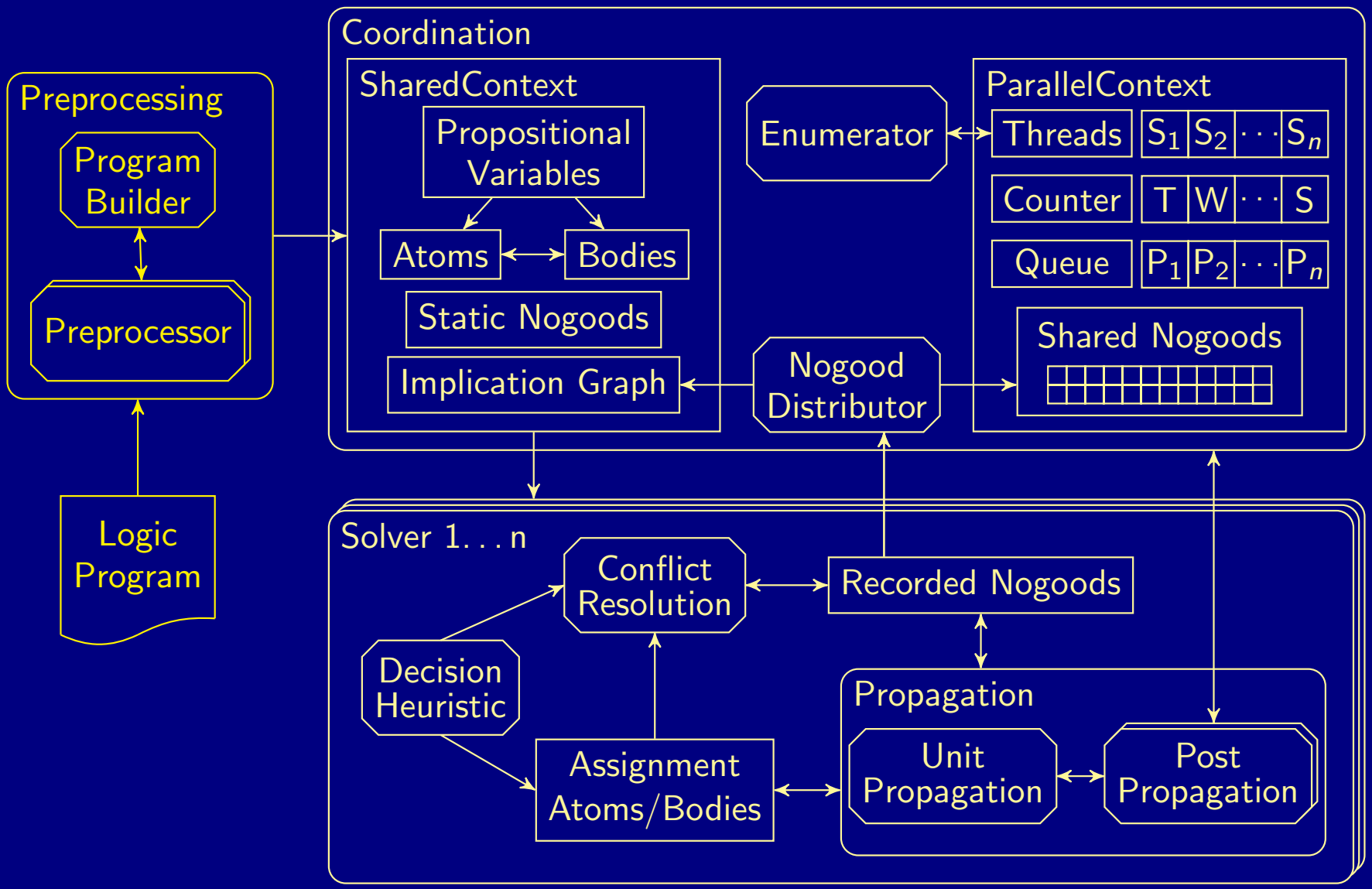

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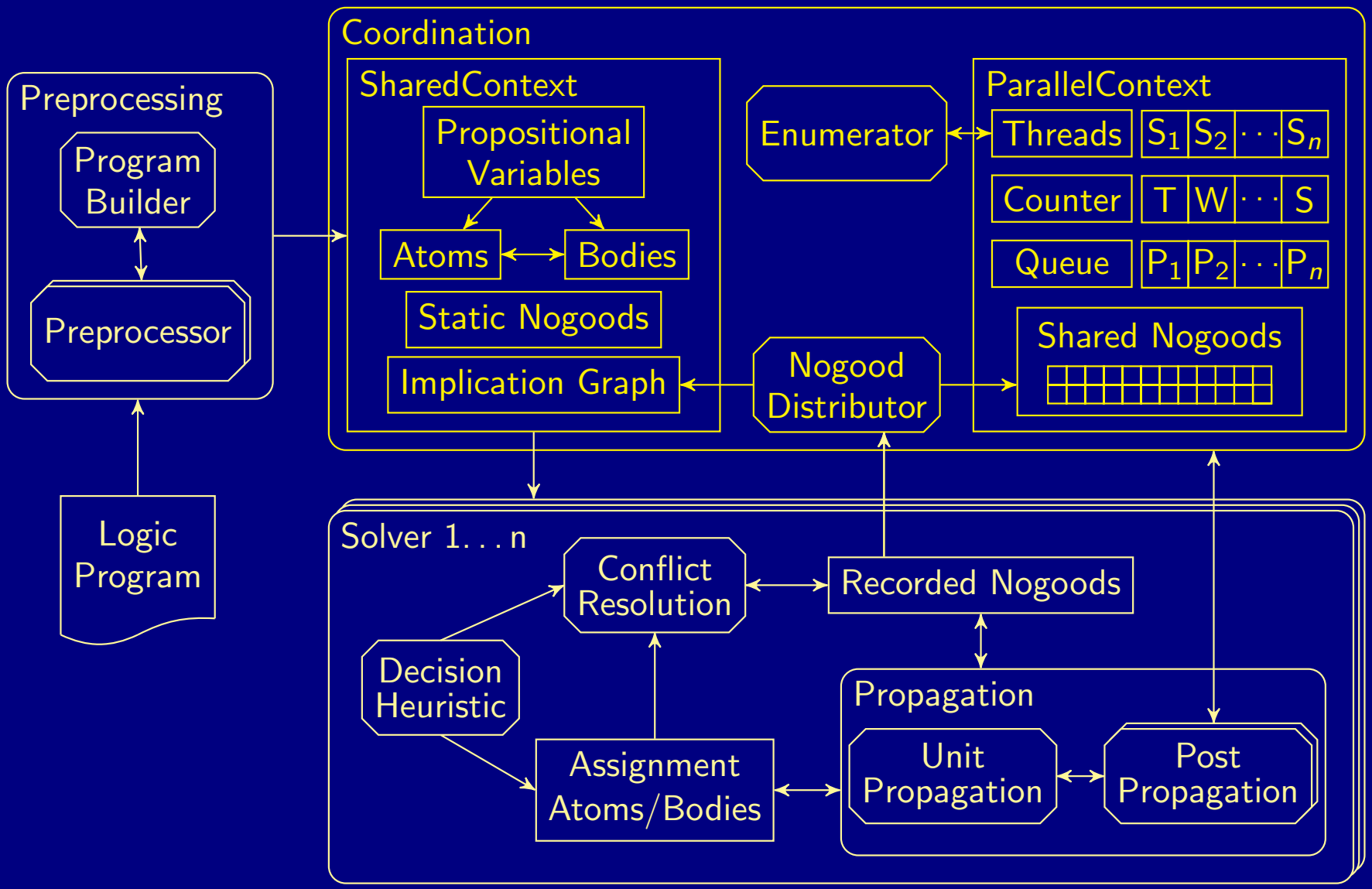

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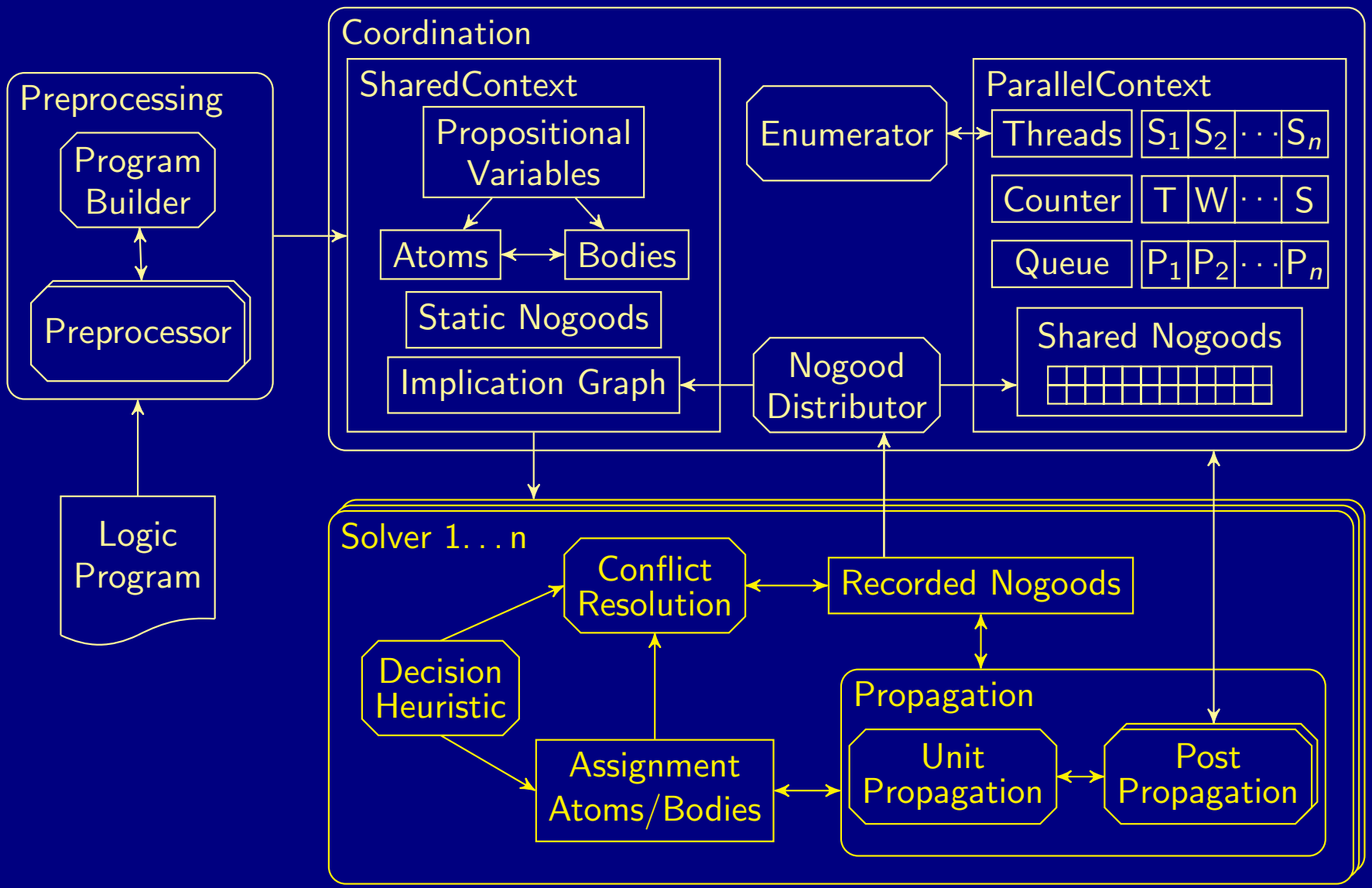

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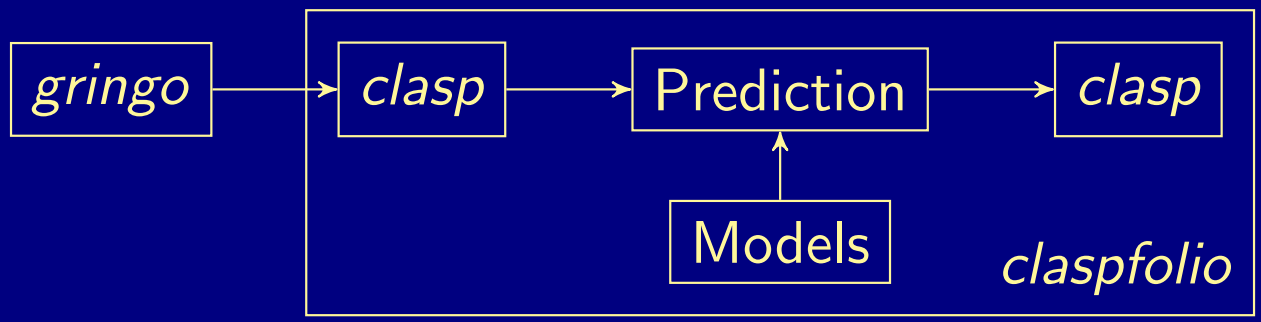
claspfolio

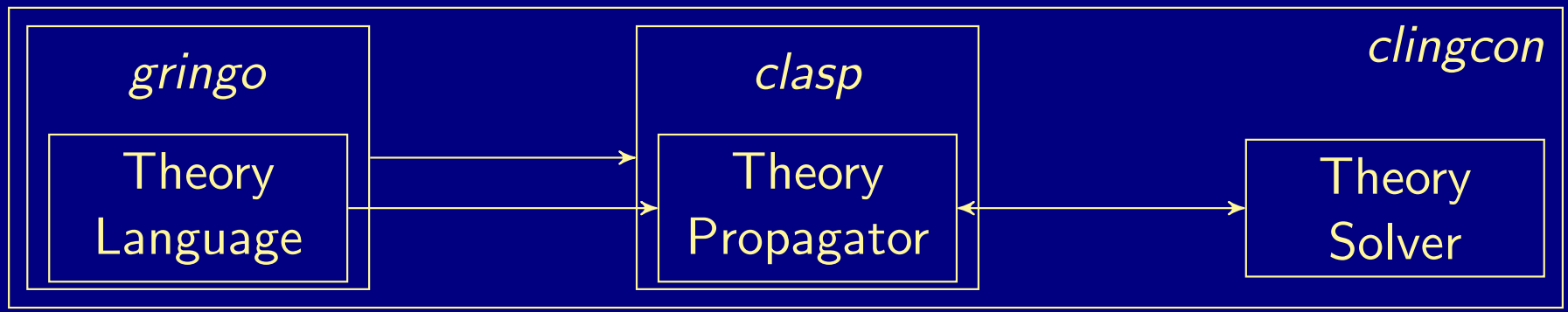
- Automatic selection of *clasp* configuration among 22 configuration via (learned) classifiers
- Basic architecture of *claspfolio*:



clingcon

- Hybrid grounding and solving
- Solving in hybrid domains, like Bio-Informatics

- Basic architecture of *clingcon*:



Pouring Water into Buckets on a Scale

```
time(0..t).                $domain(0..500).
bucket(a).                 volume(a,0) $== 0.
bucket(b).                 volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $== 0           :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) + amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
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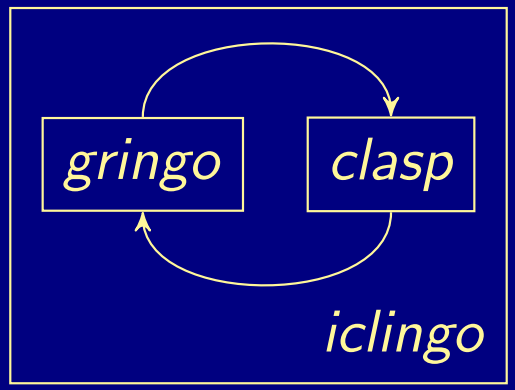
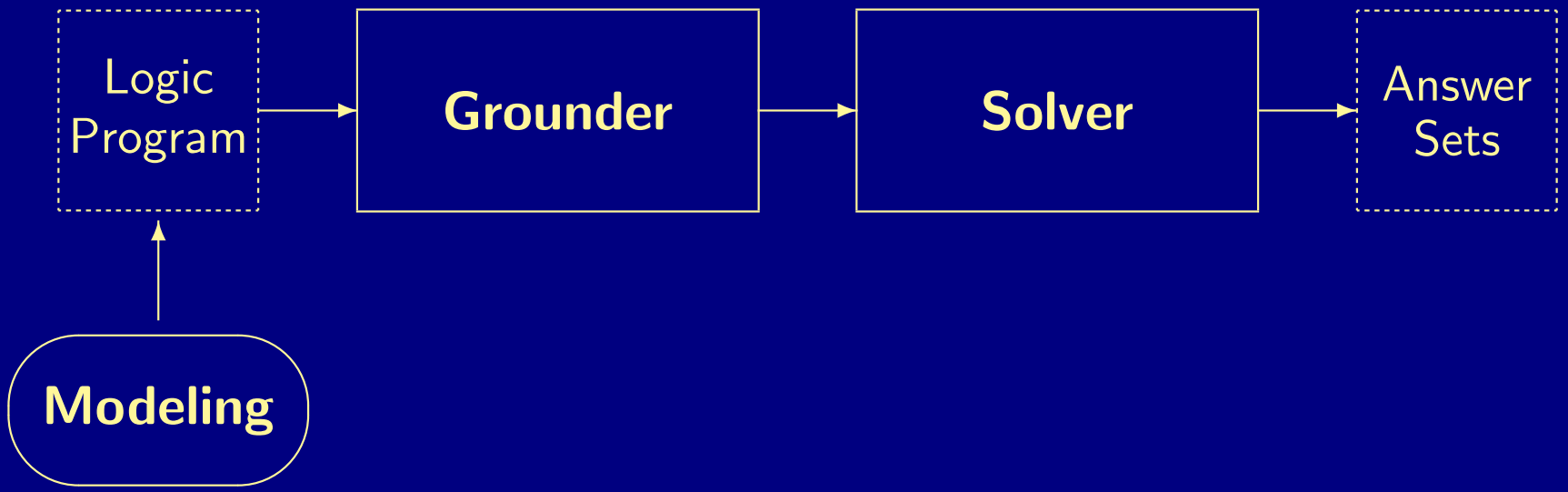
iclingo

- Incremental grounding and solving
- Offline solving in dynamic domains, like Automated Planning

- Basic architecture of *iclingo*:



Incremental ASP Solving Process



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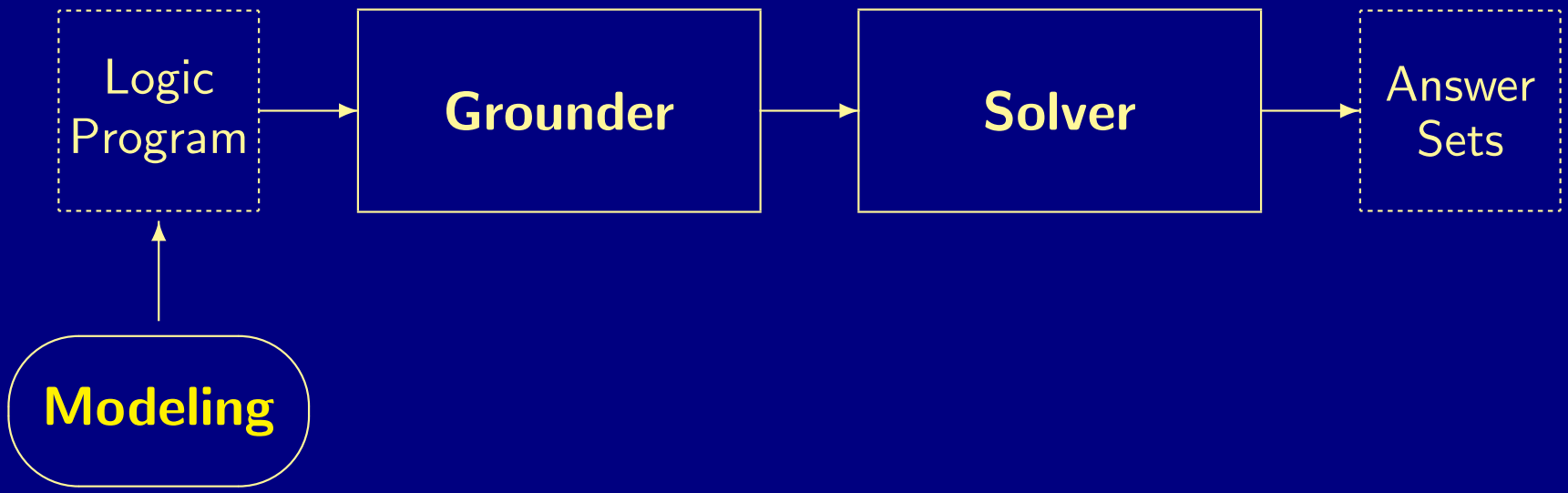

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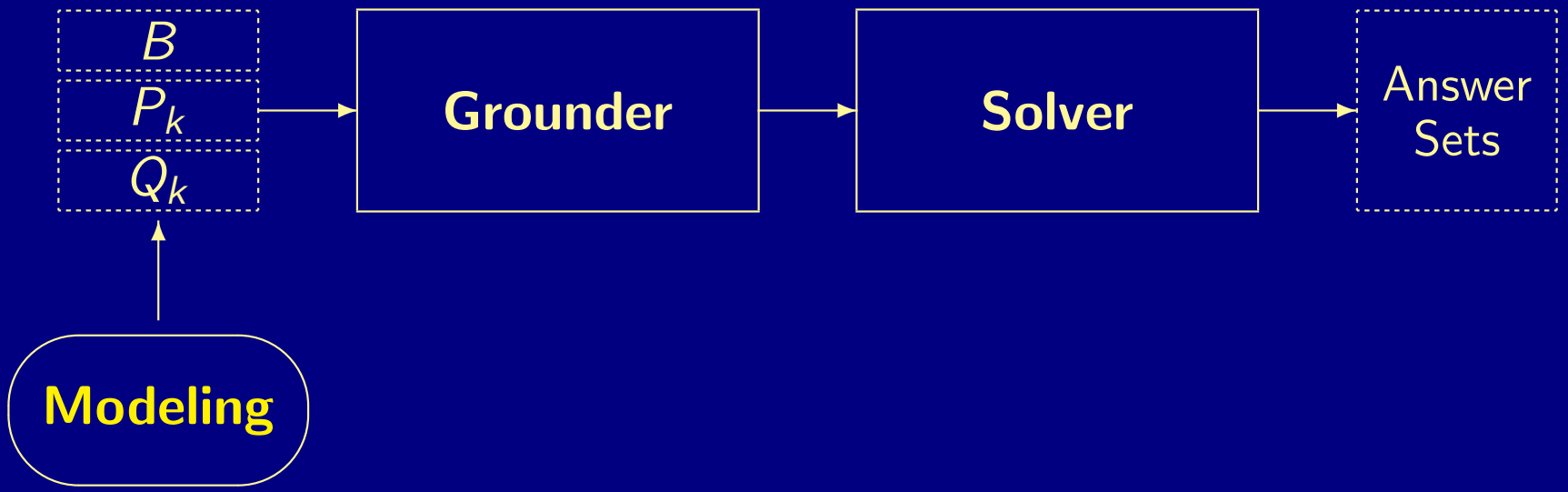

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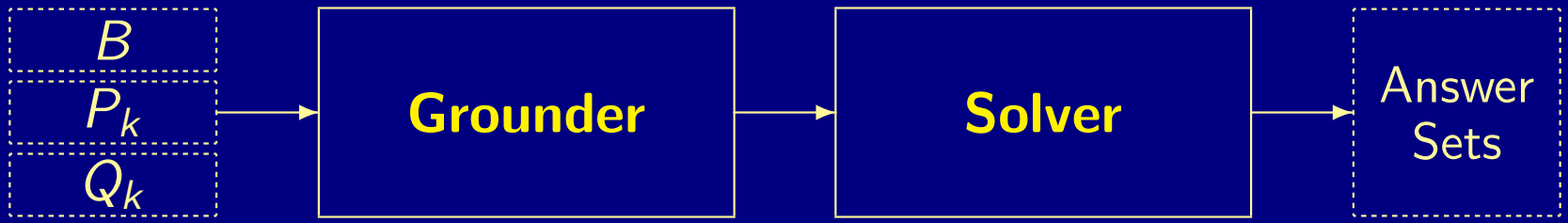

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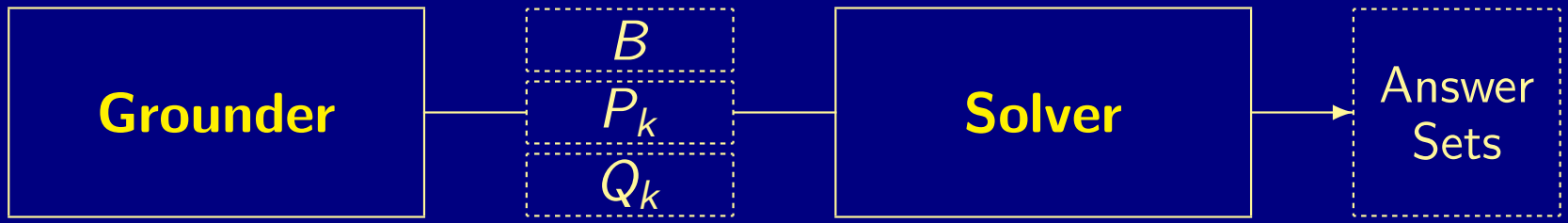

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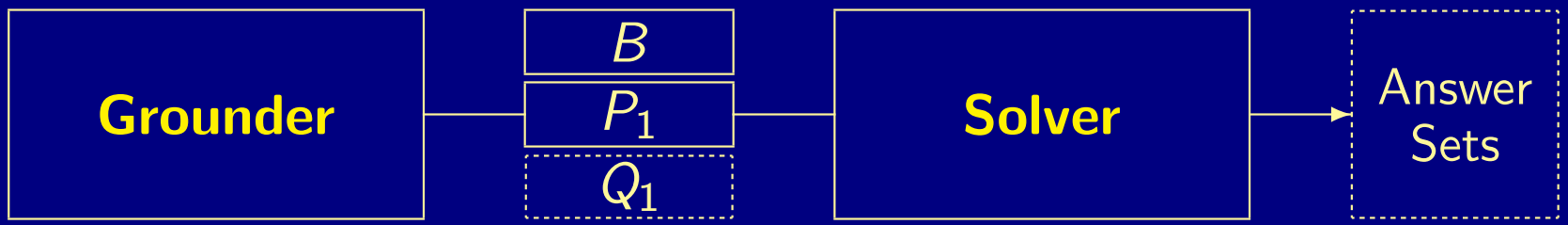

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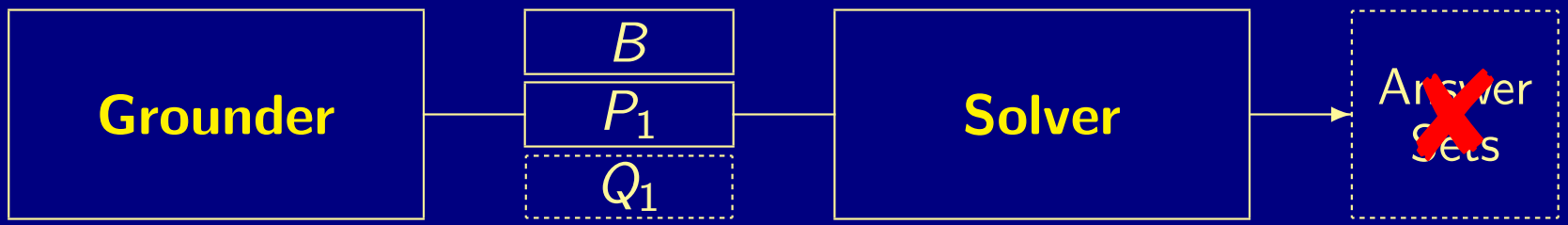

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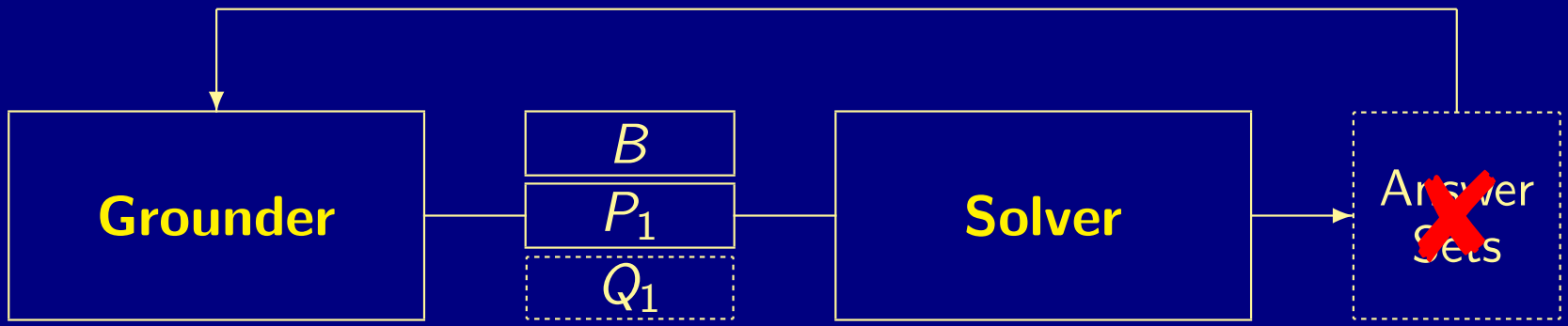

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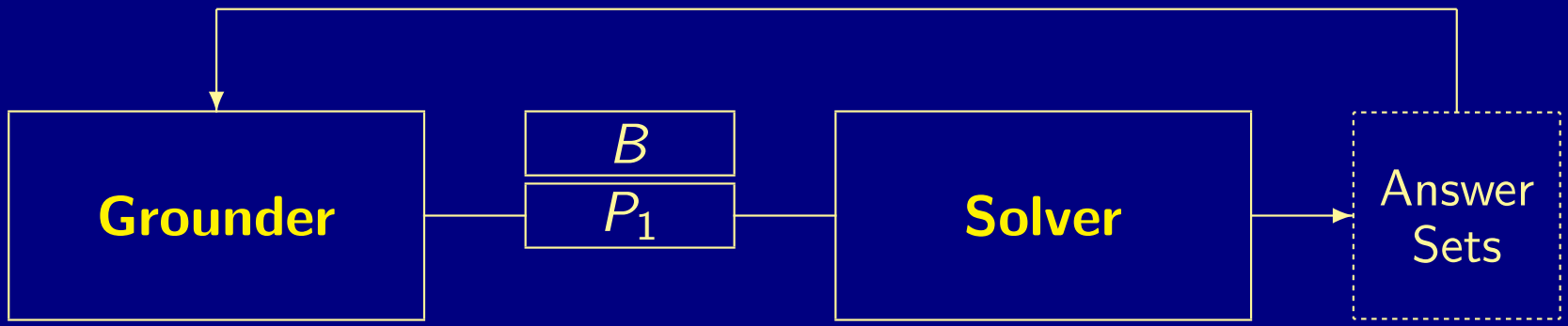

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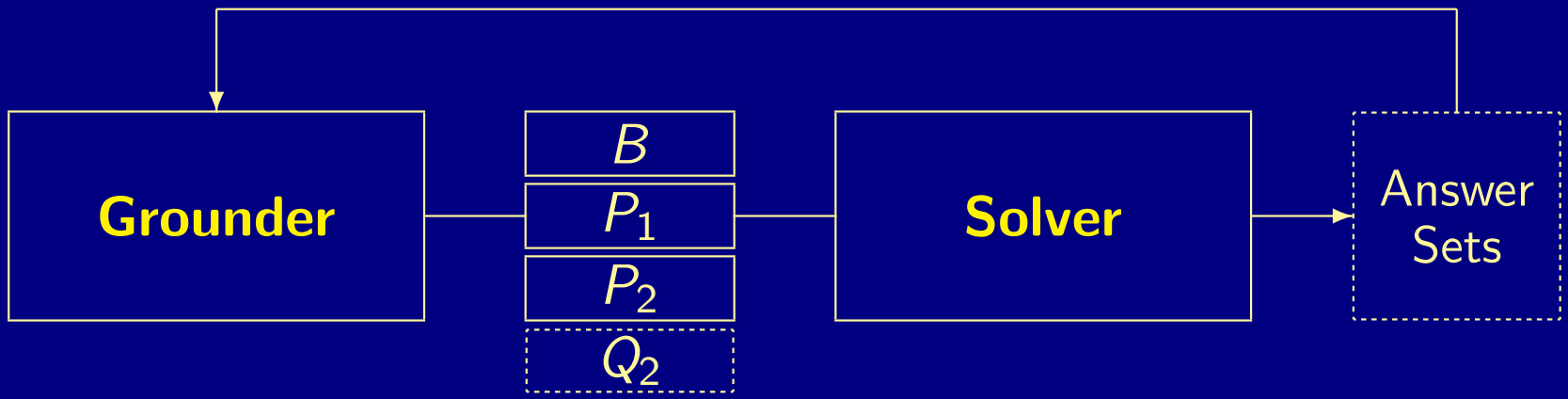

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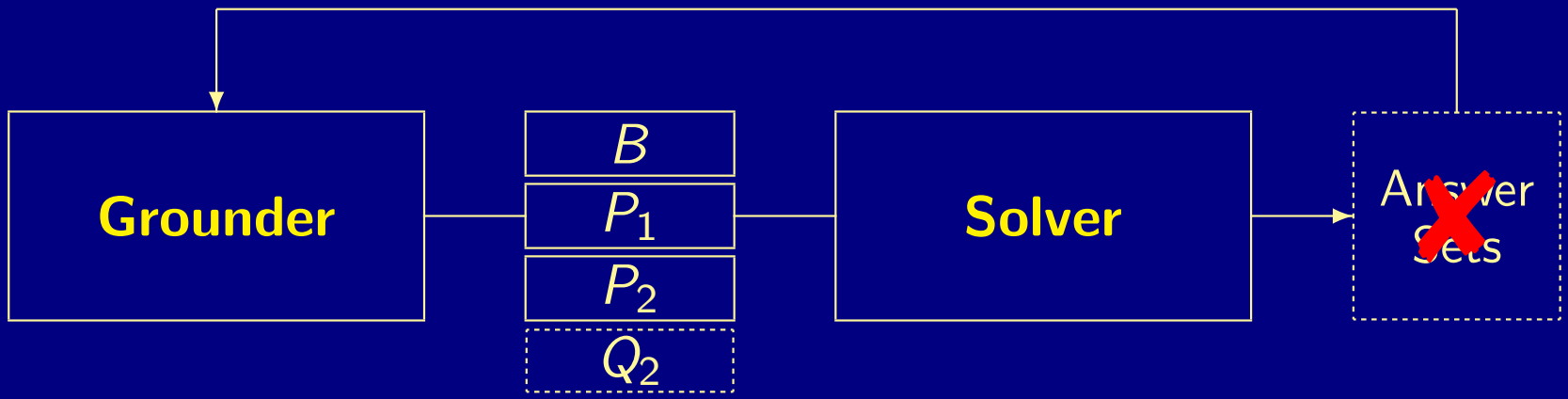

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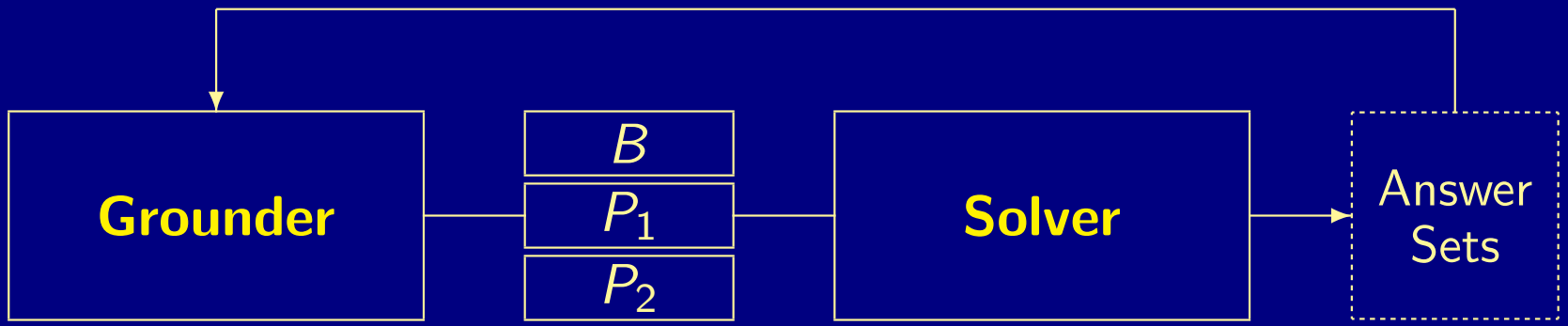

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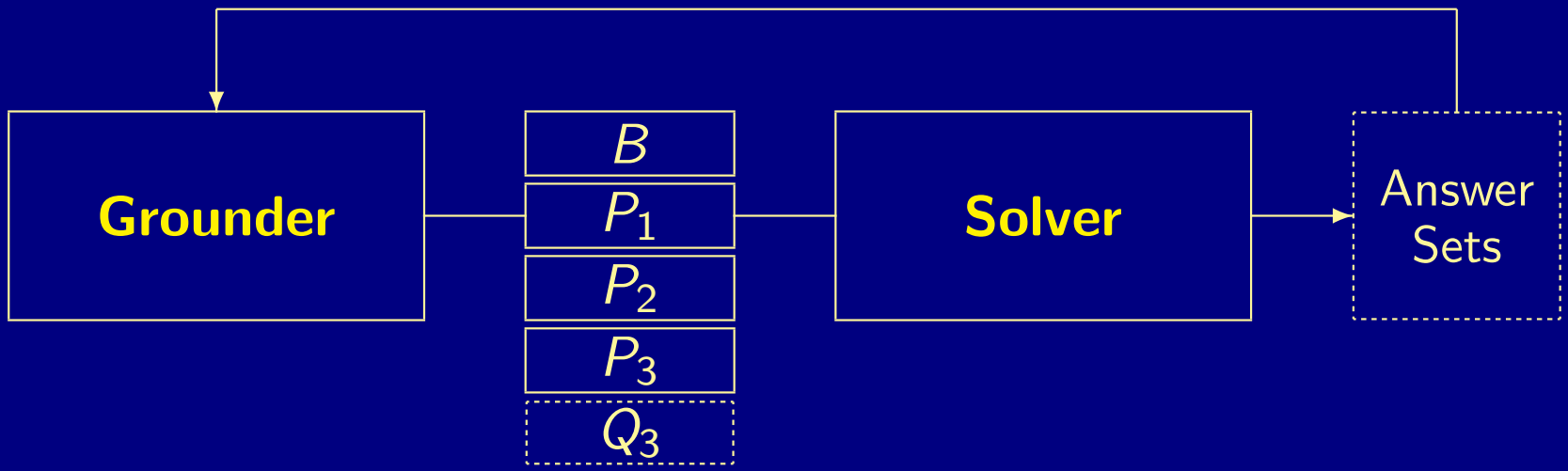

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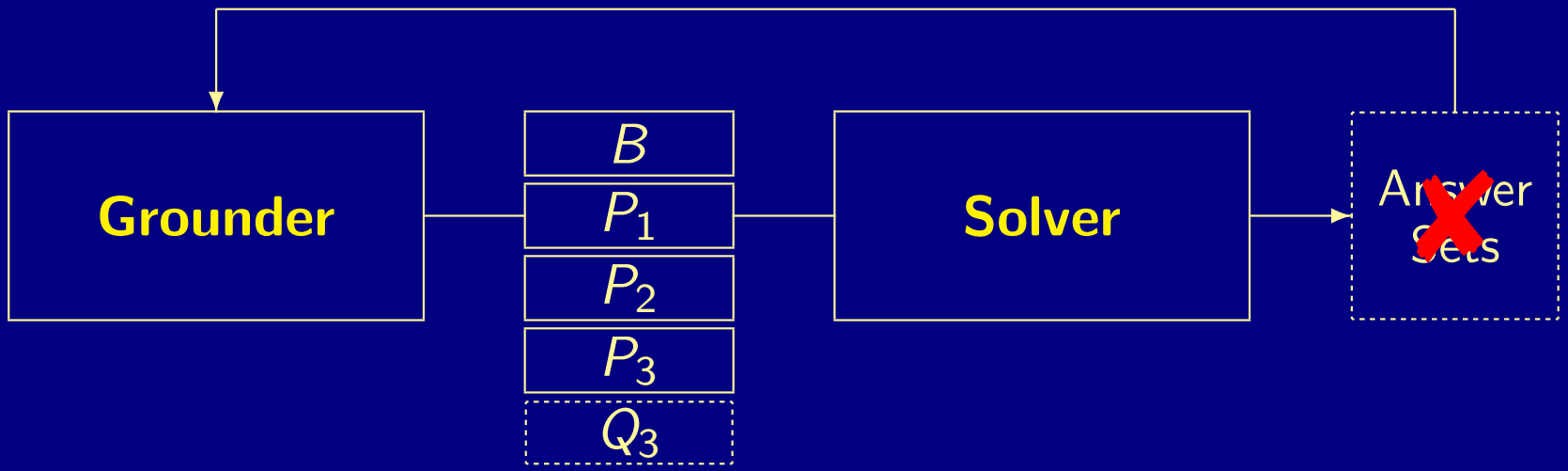

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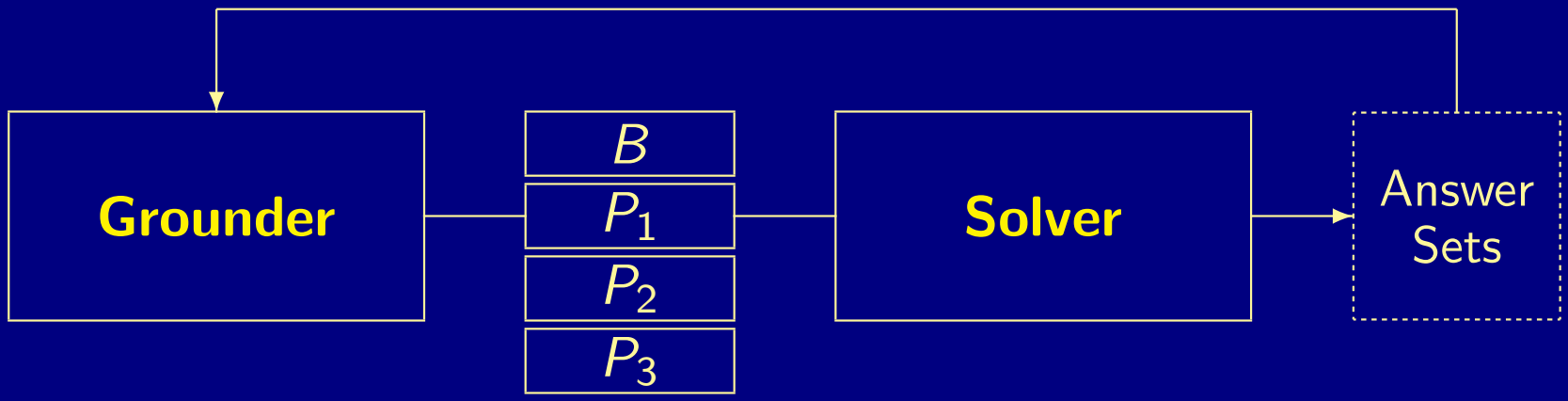

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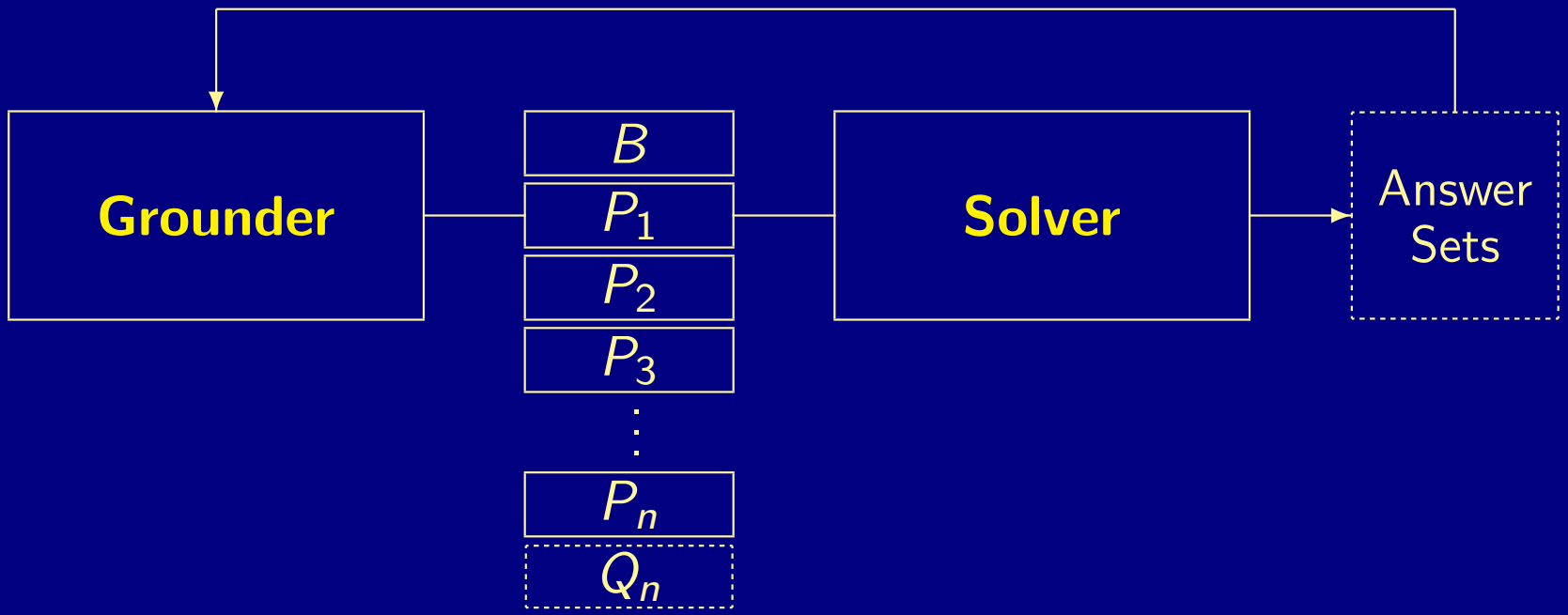

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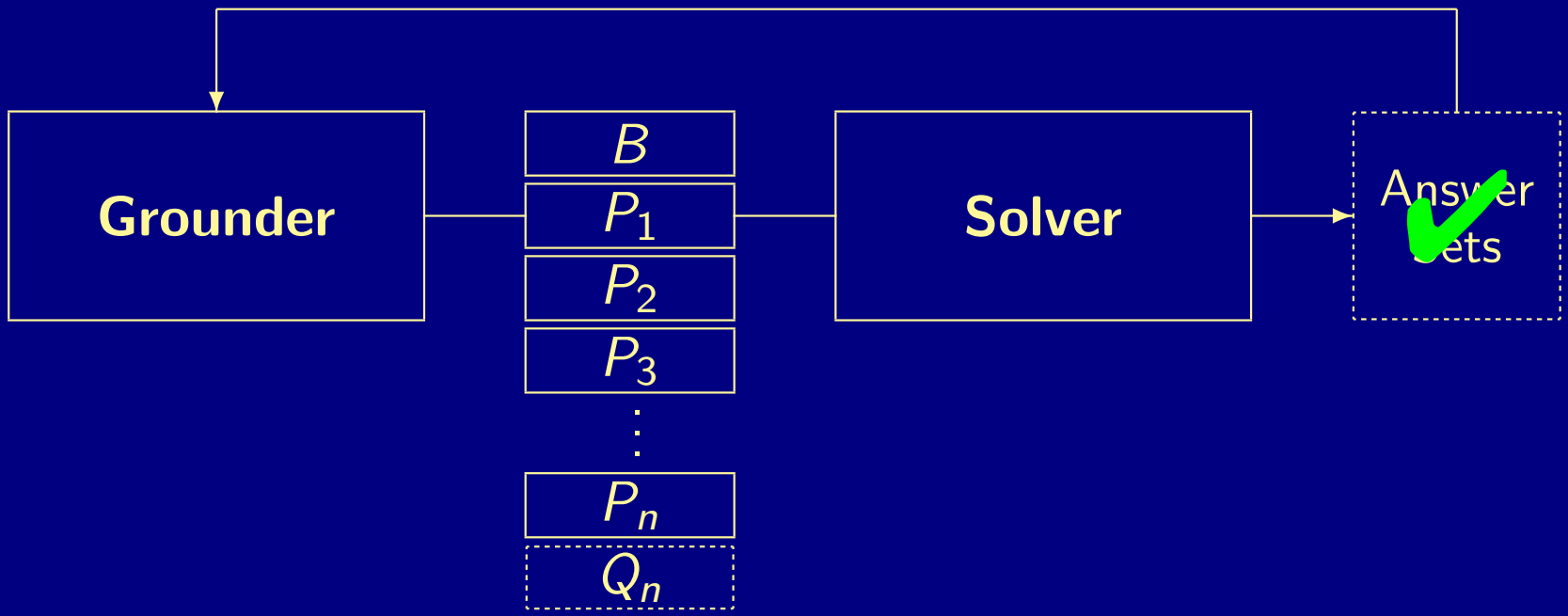

Simplistic STRIPS Planning

```
#base.

fluent(p).     fluent(q).    fluent(r).
action(a).     pre(a,p).     add(a,q).     del(a,p).
action(b).     pre(b,q).     add(b,r).     del(b,q).
init(p).       query(r).

holds(P,0) :- init(P).

#cumulative t.

1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

ocdel(F,t) :- occ(A,t), del(A,F).
holds(F,t) :- occ(A,t), add(A,F).
holds(F,t) :- holds(F,t-1), not ocdel(F,t).

#volatile t.

 :- query(F), not holds(F,t).
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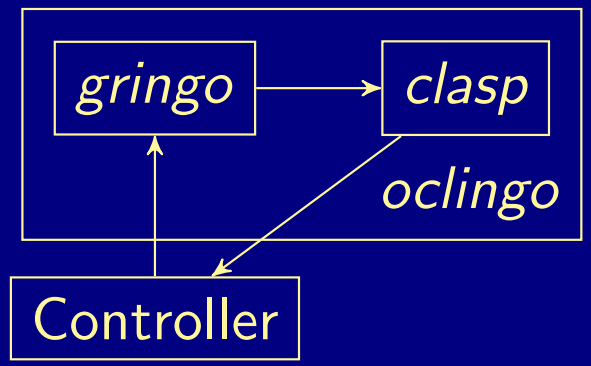
oclingo

- Reactive grounding and solving
- Online solving in dynamic domains, like Robotics

- Basic architecture of *oclingo*:



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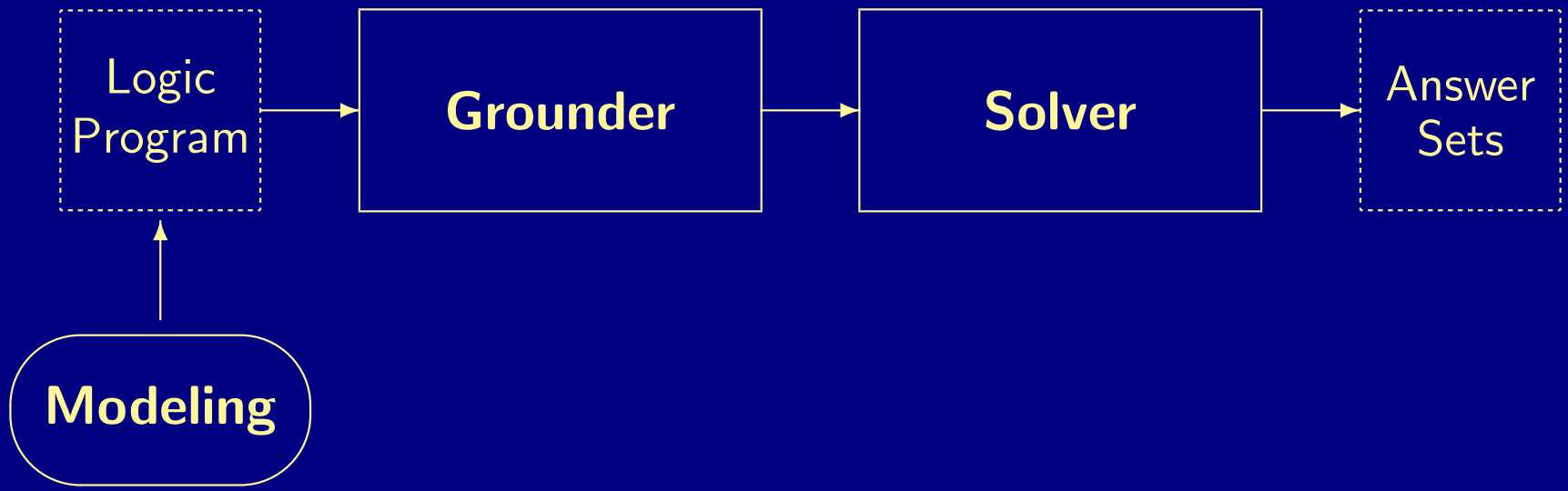

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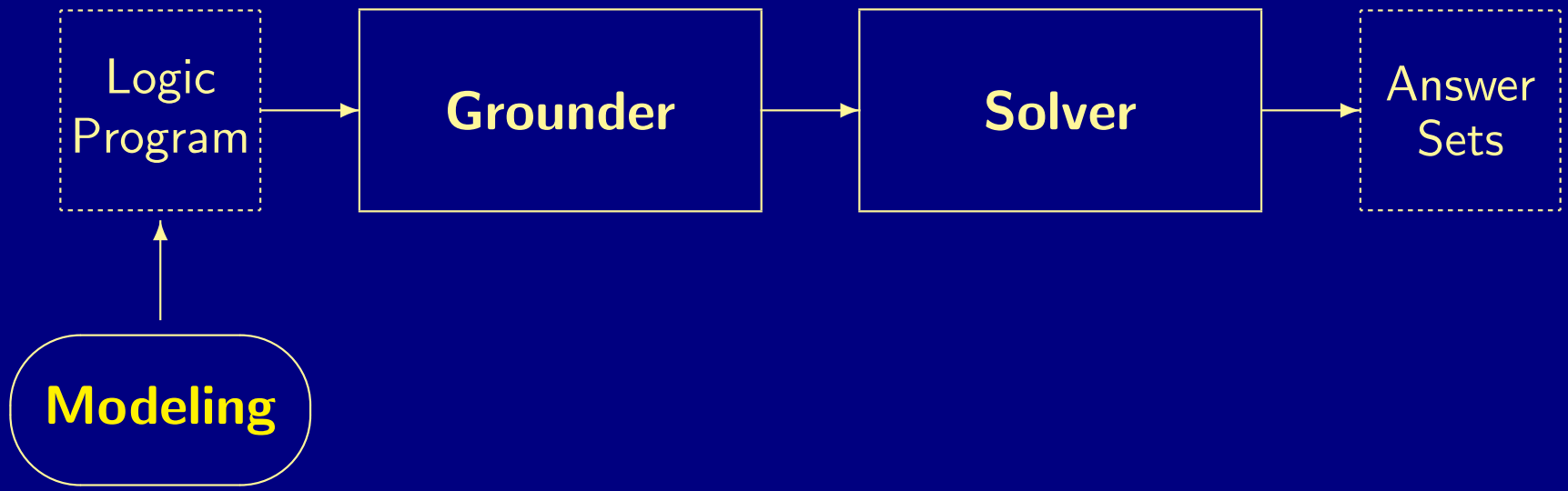

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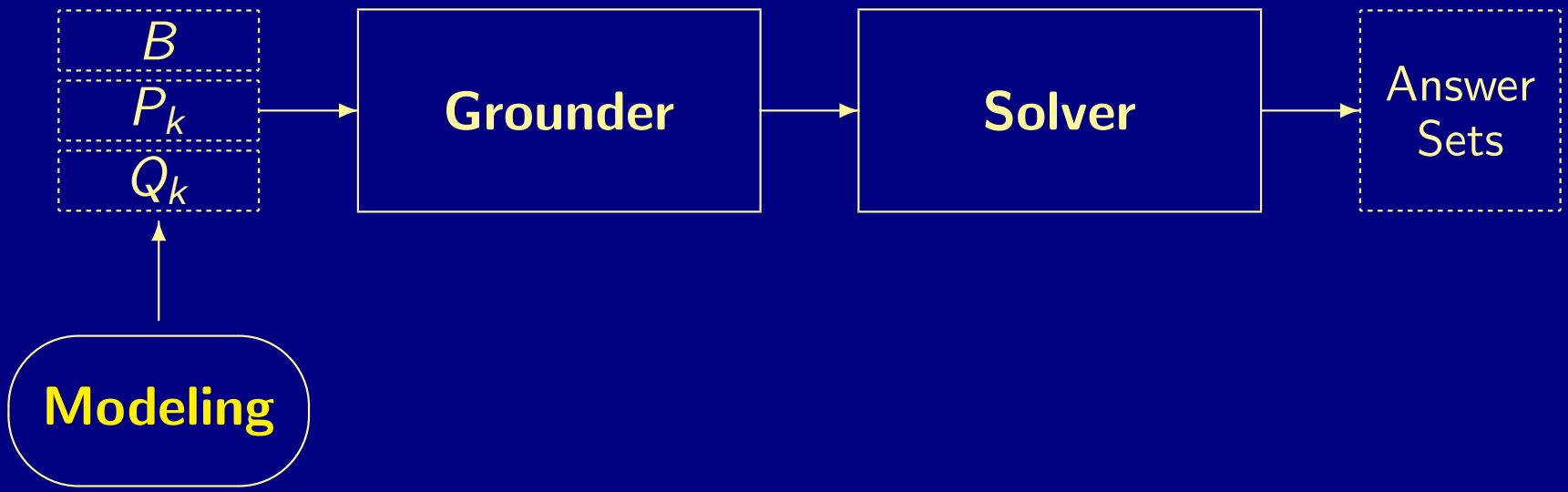

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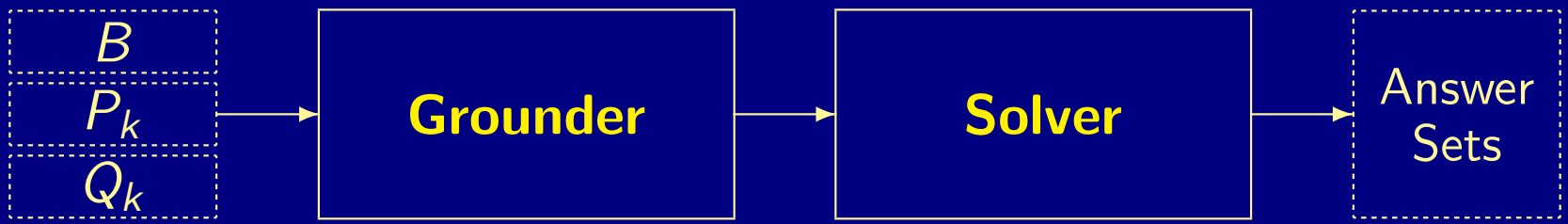

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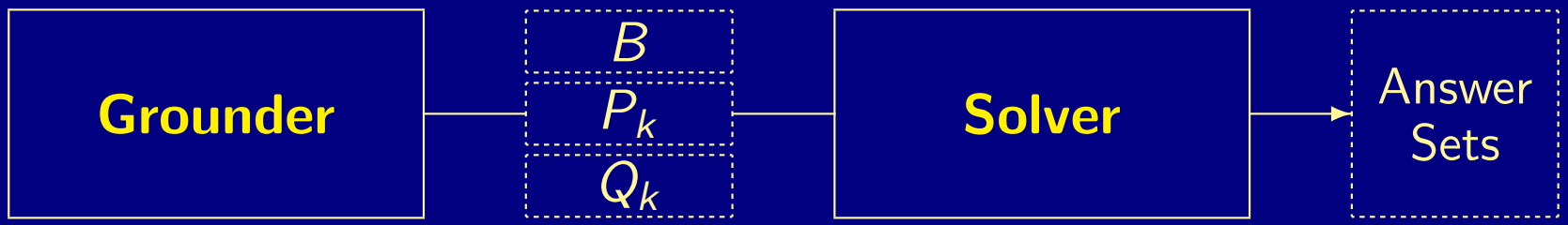
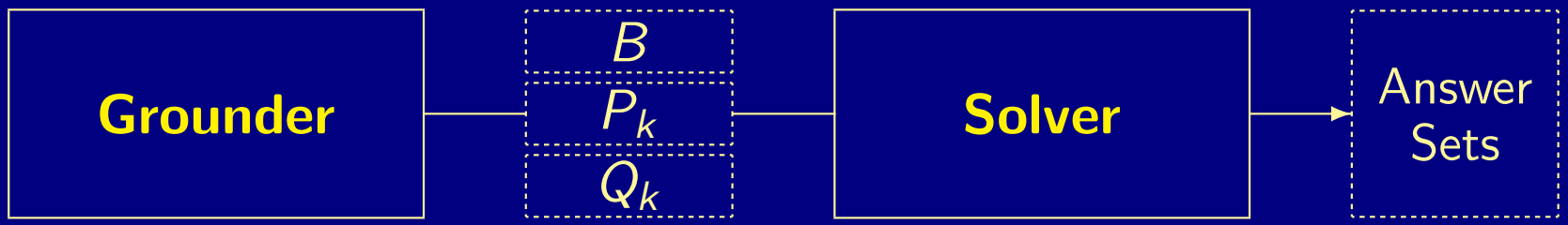


Reactive ASP Solving Process



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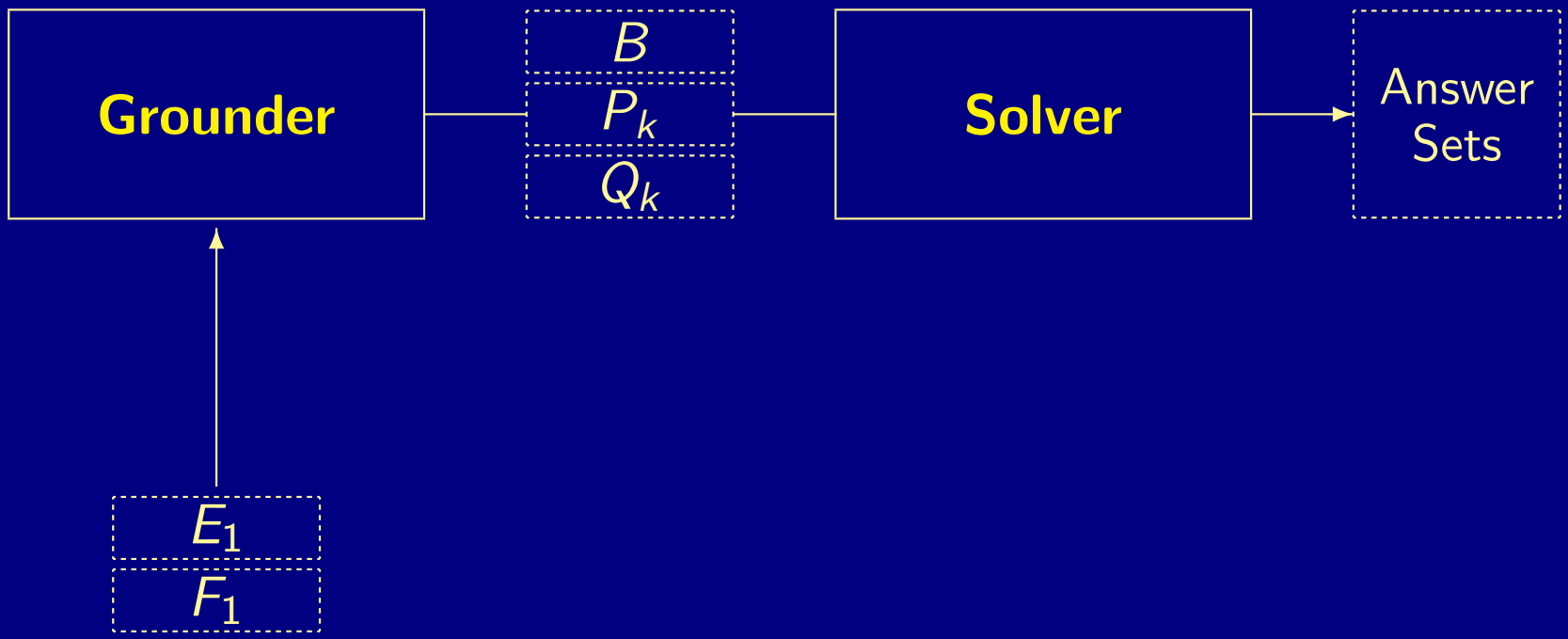

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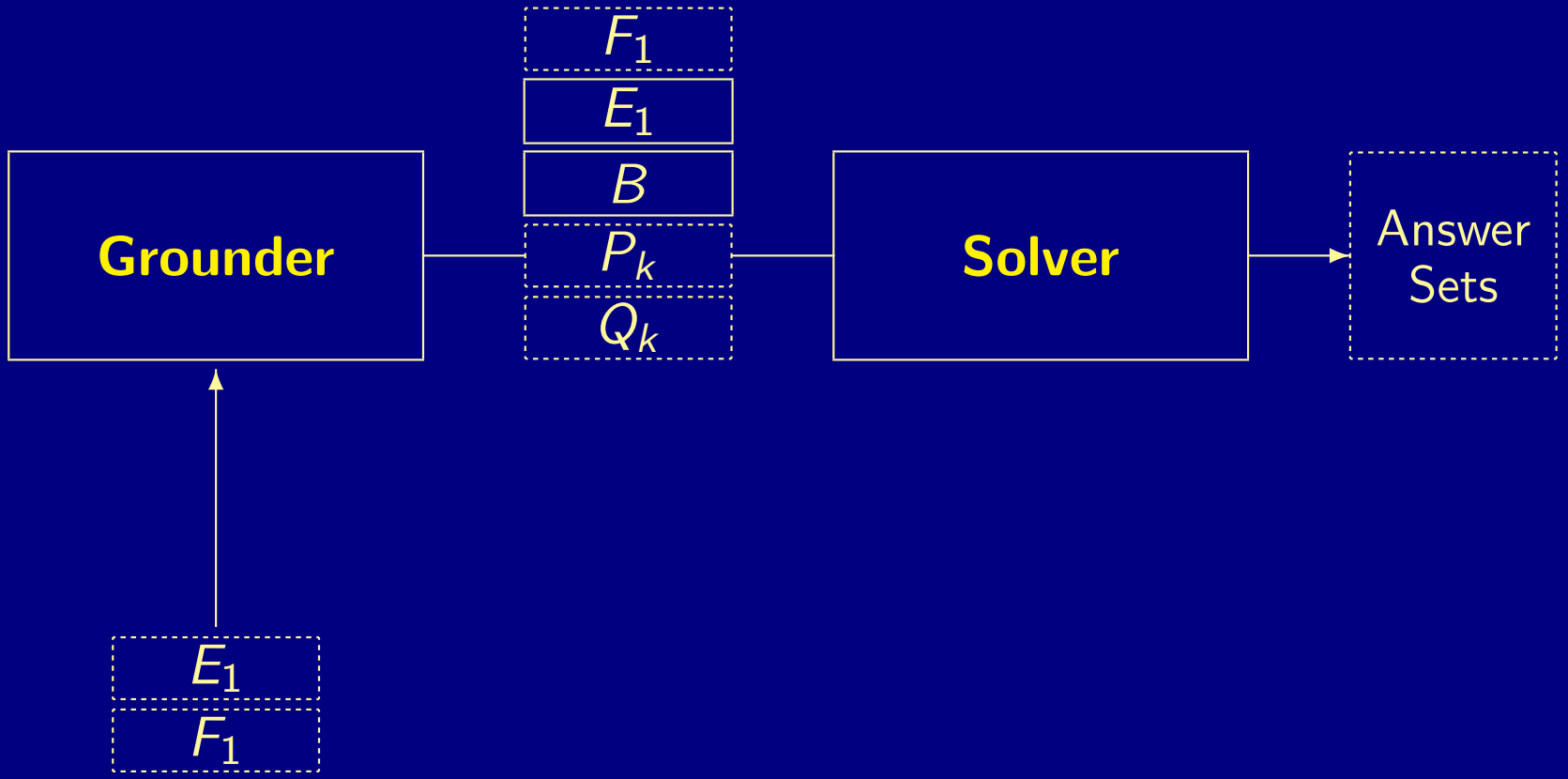

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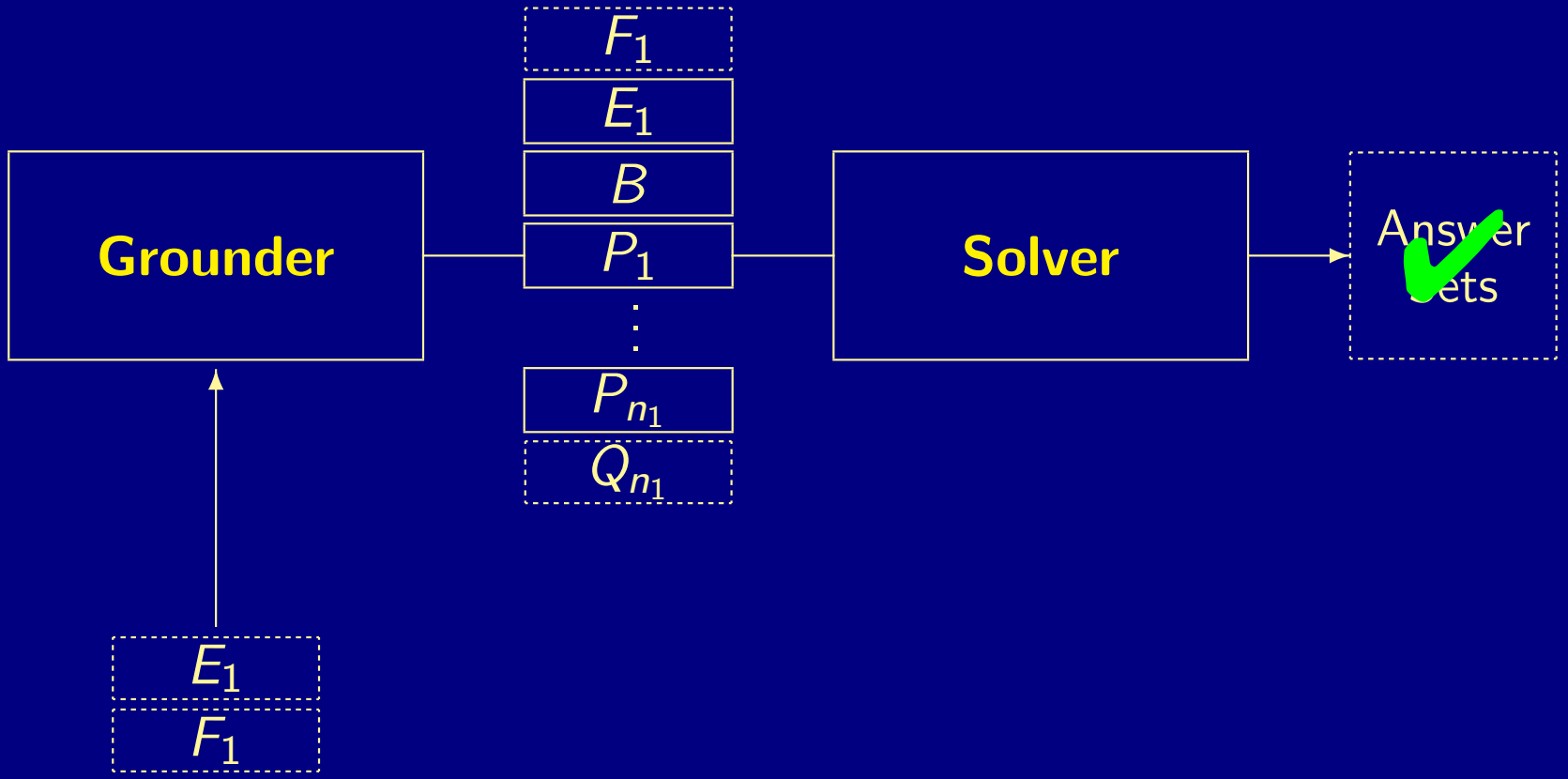

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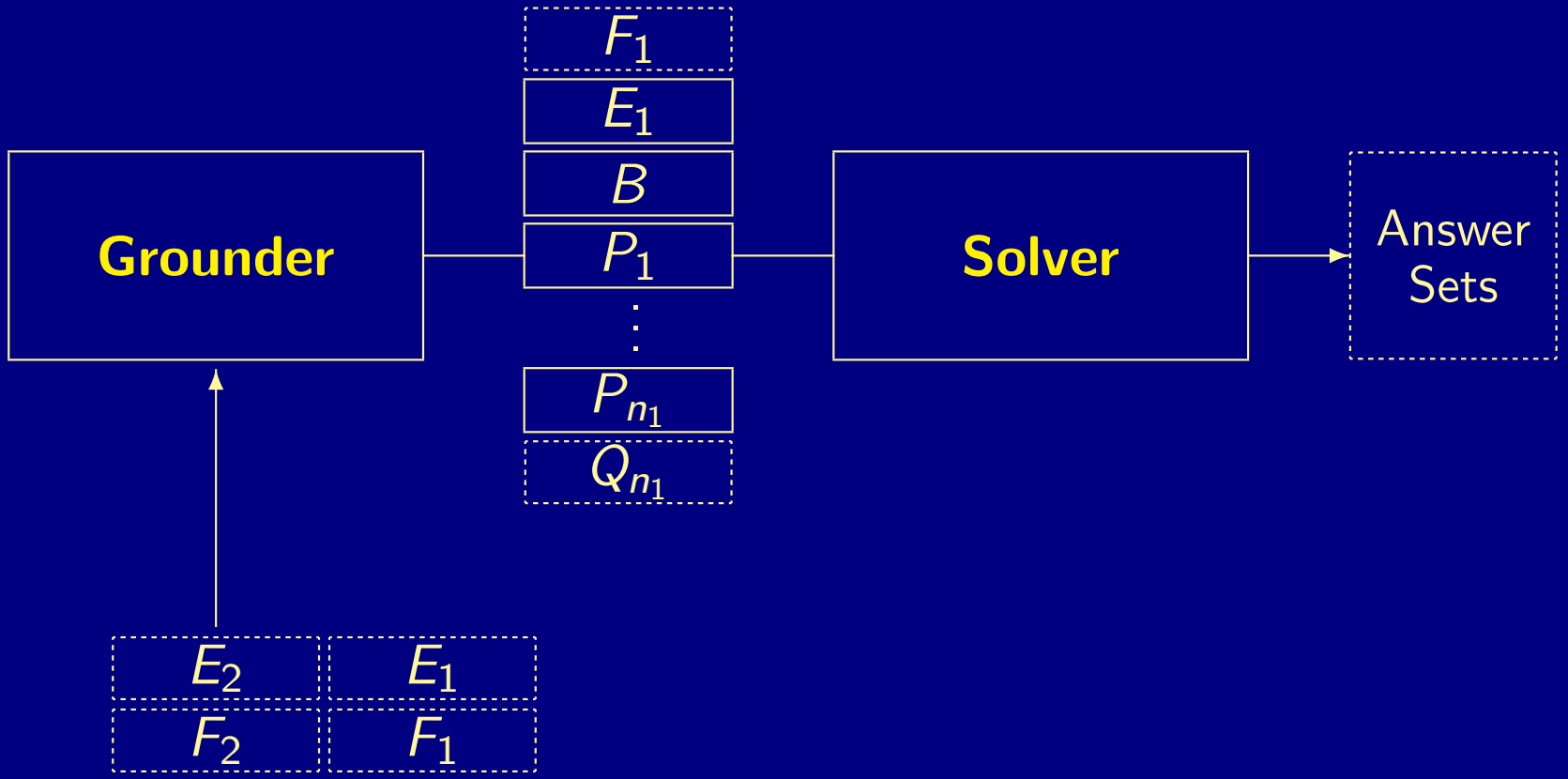

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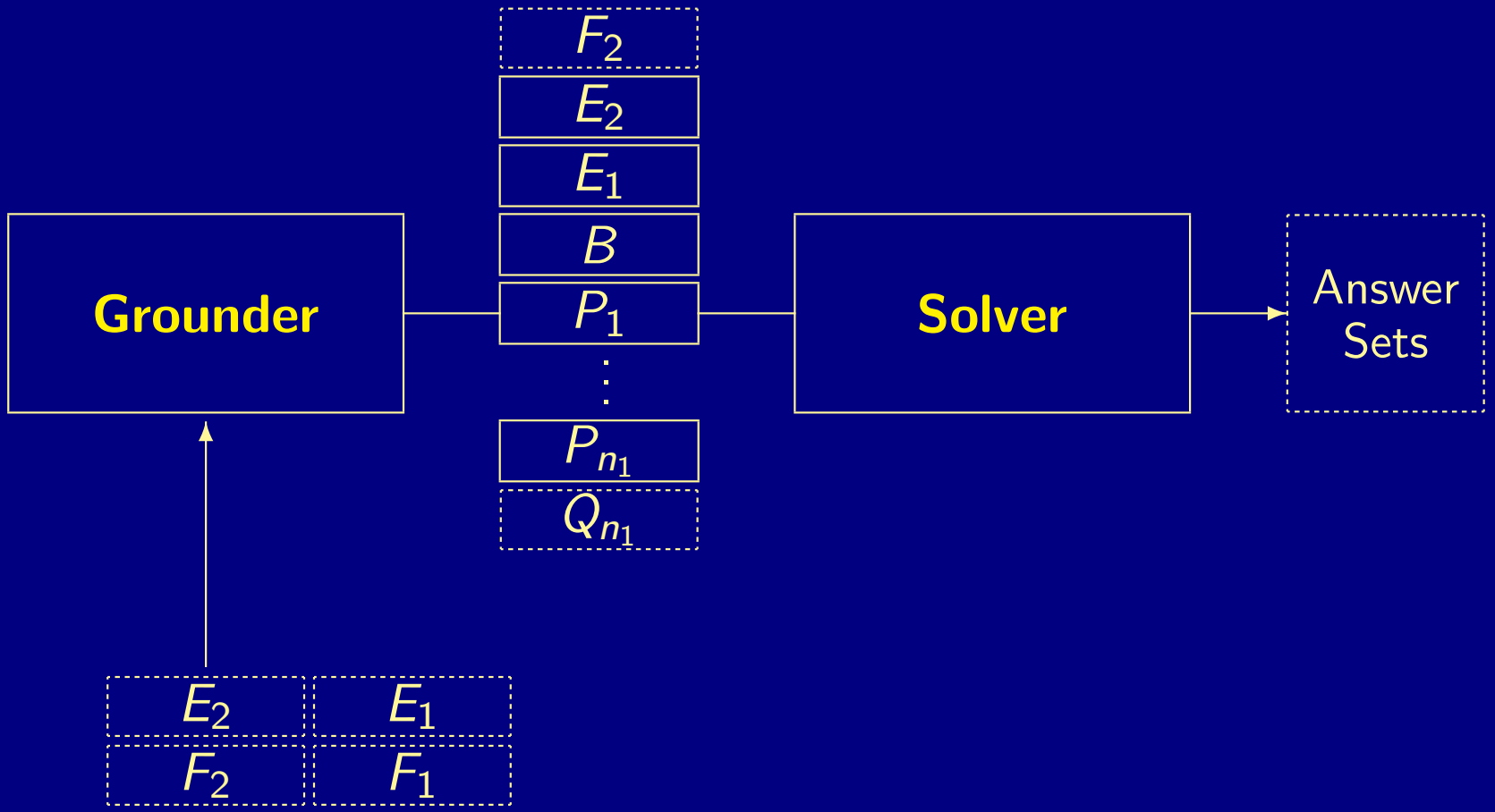

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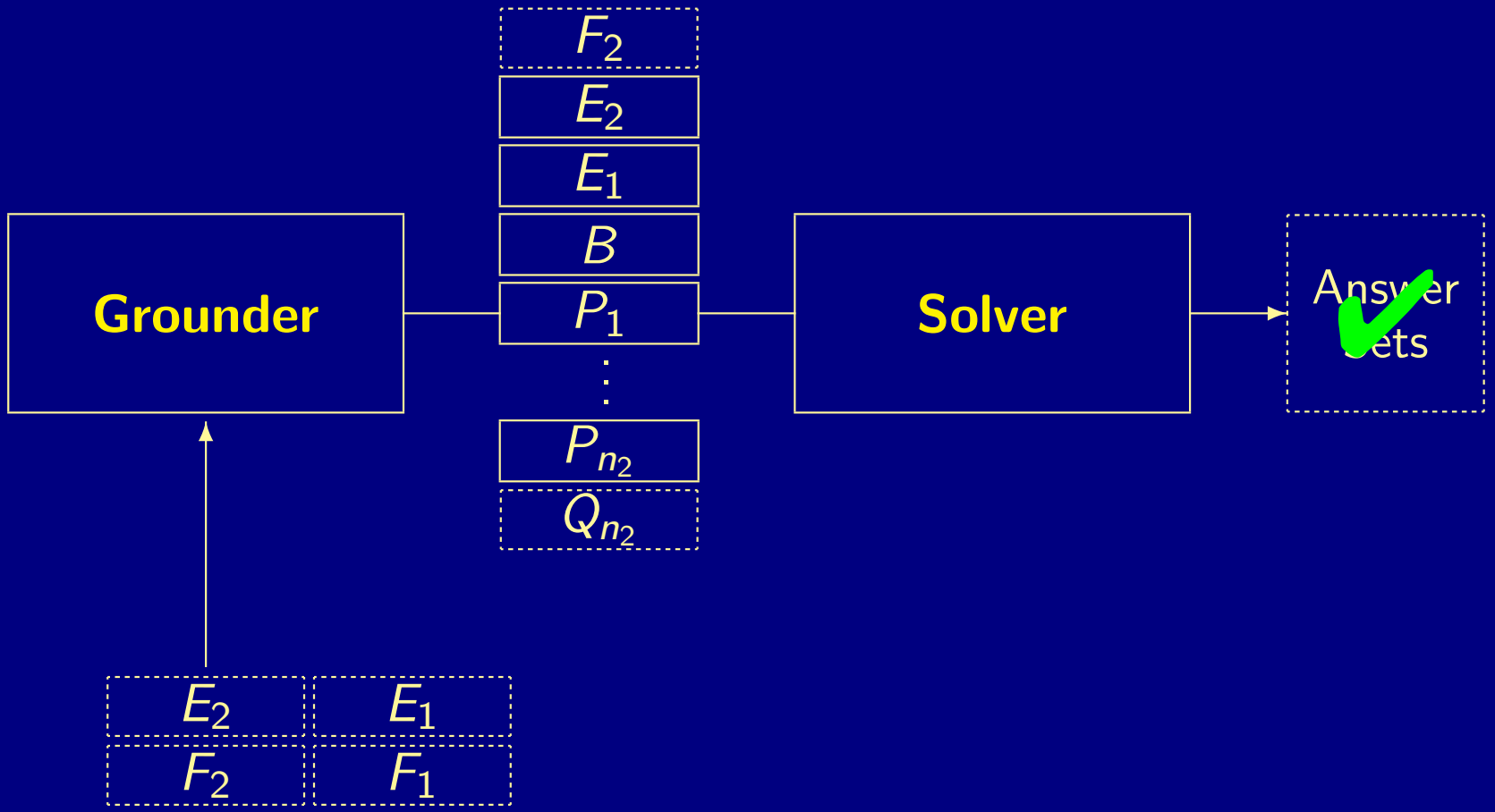

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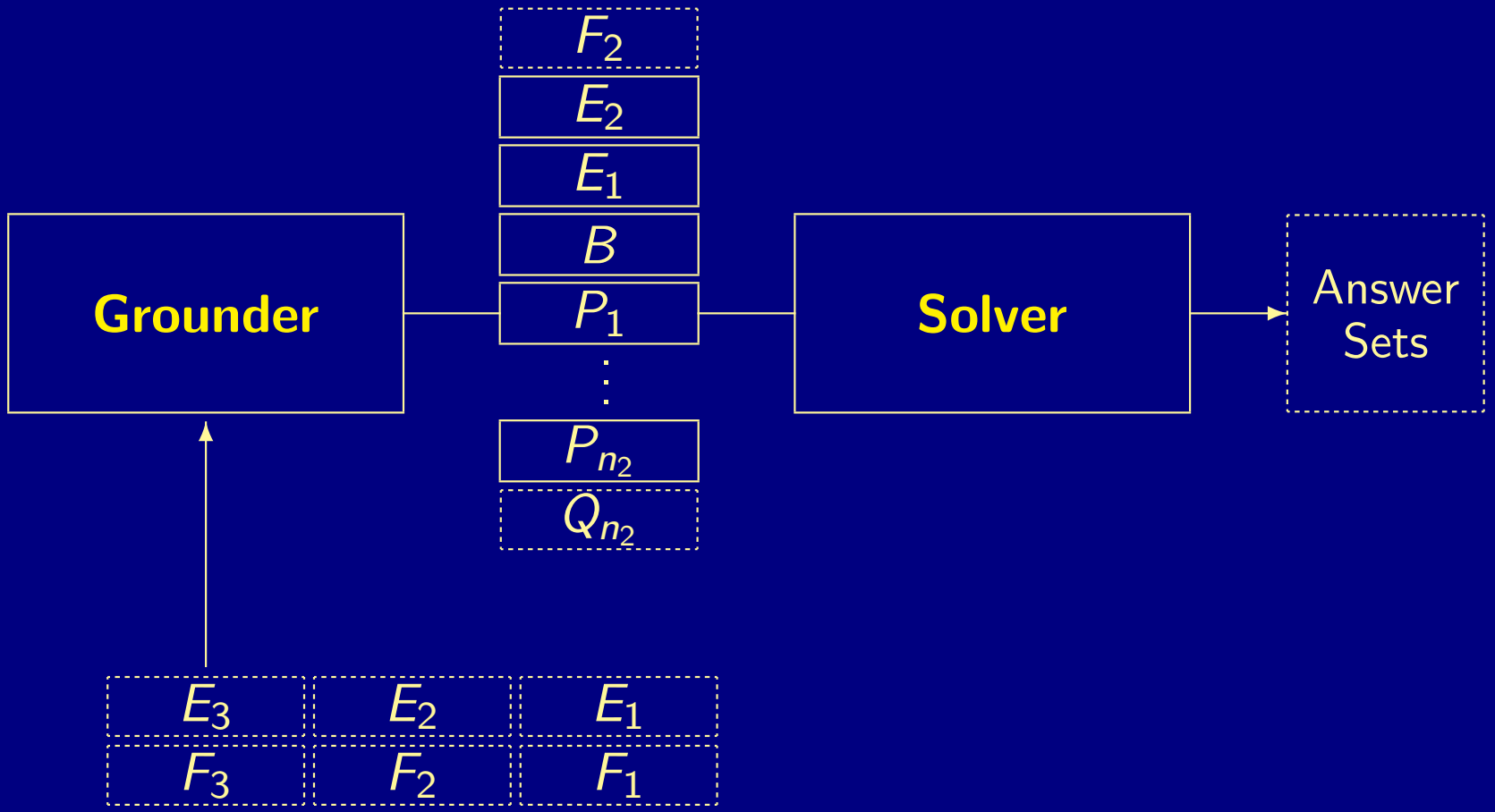

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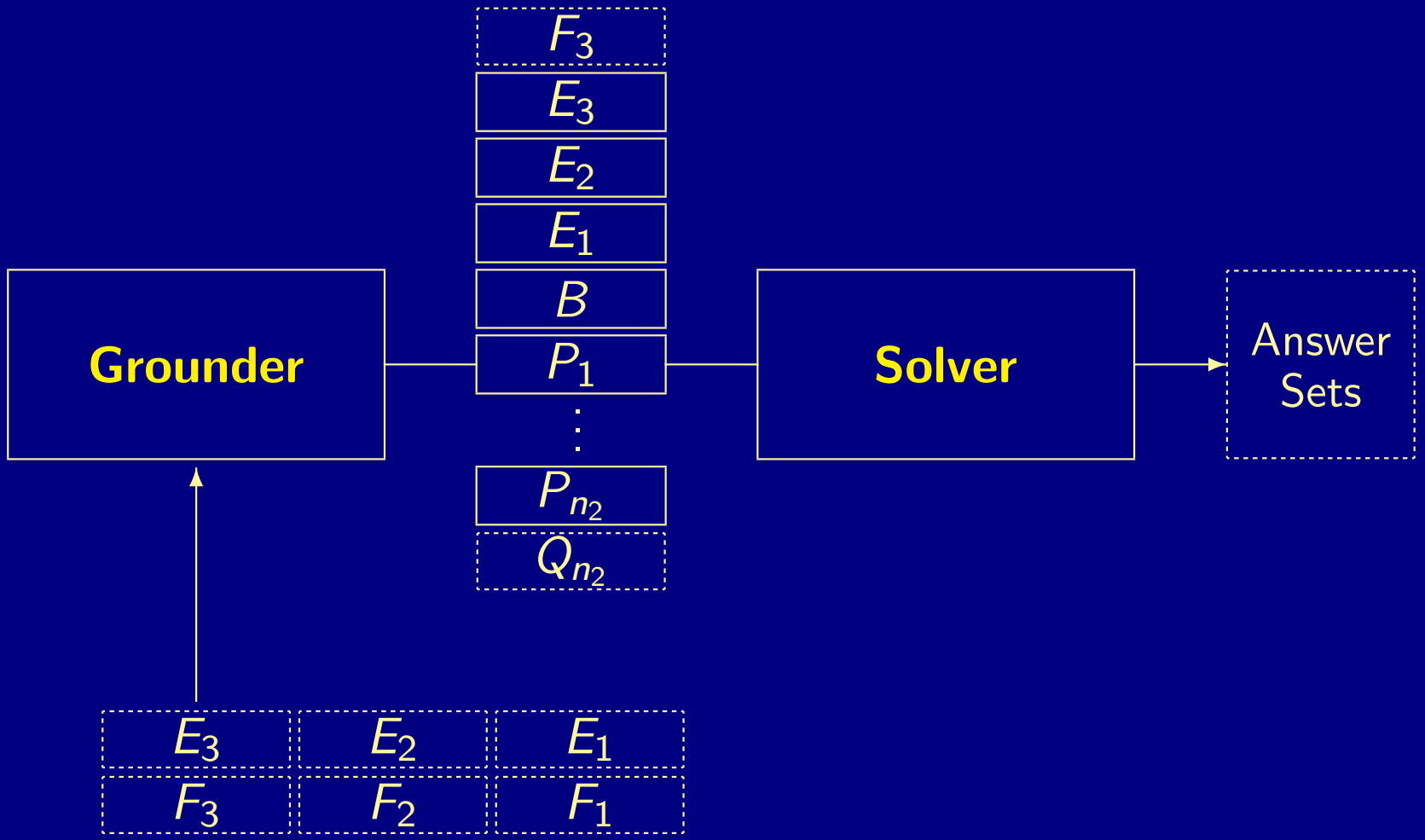

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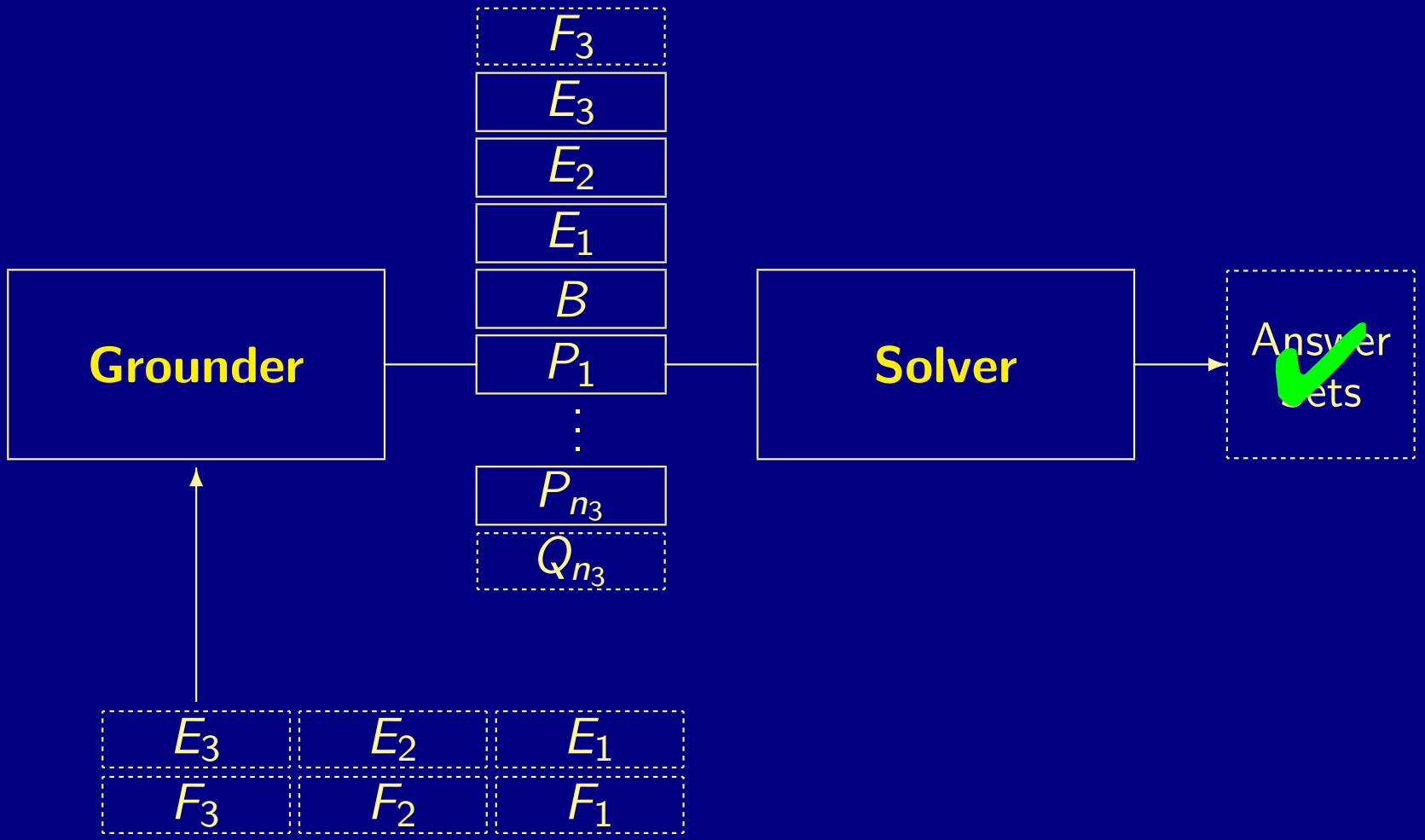

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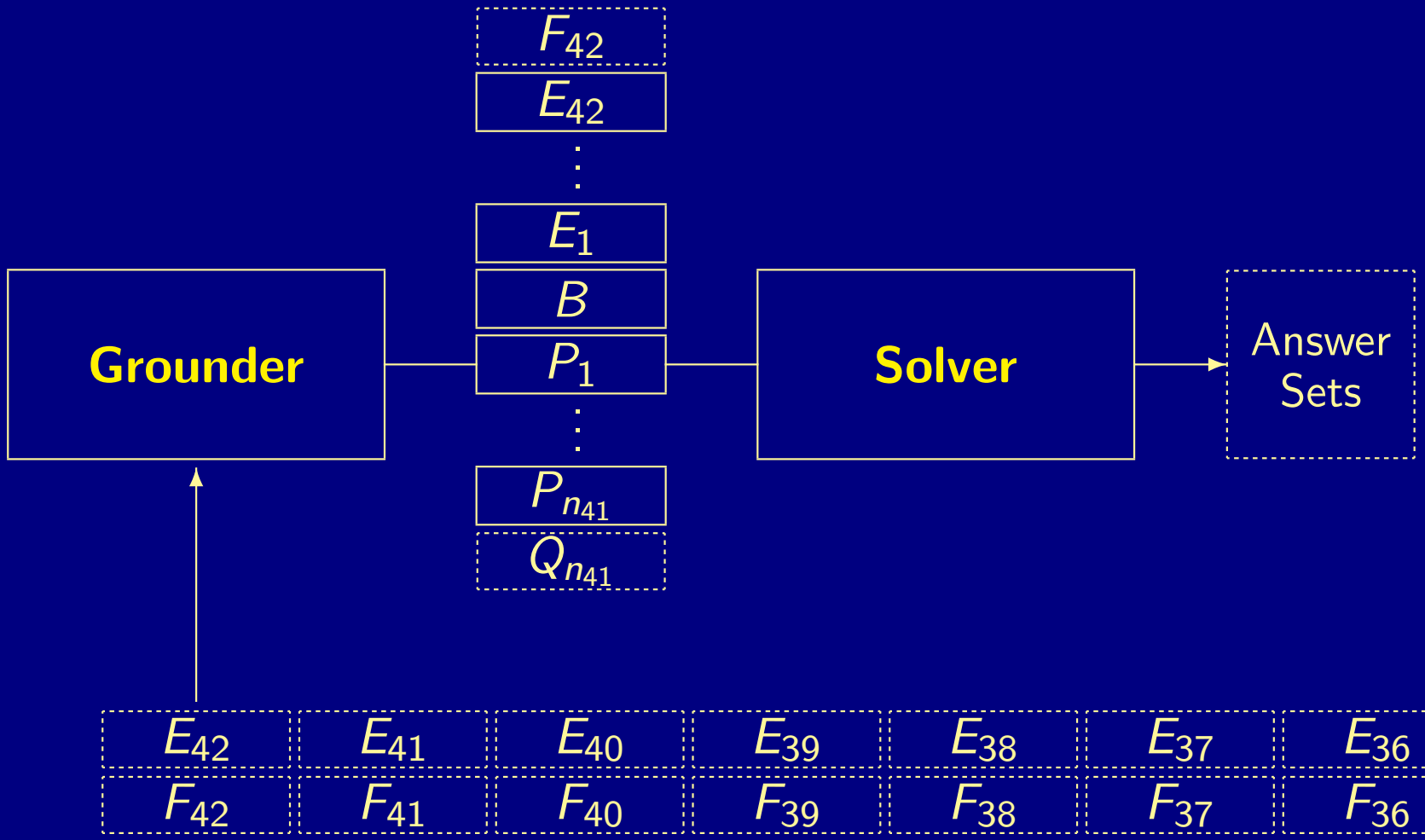

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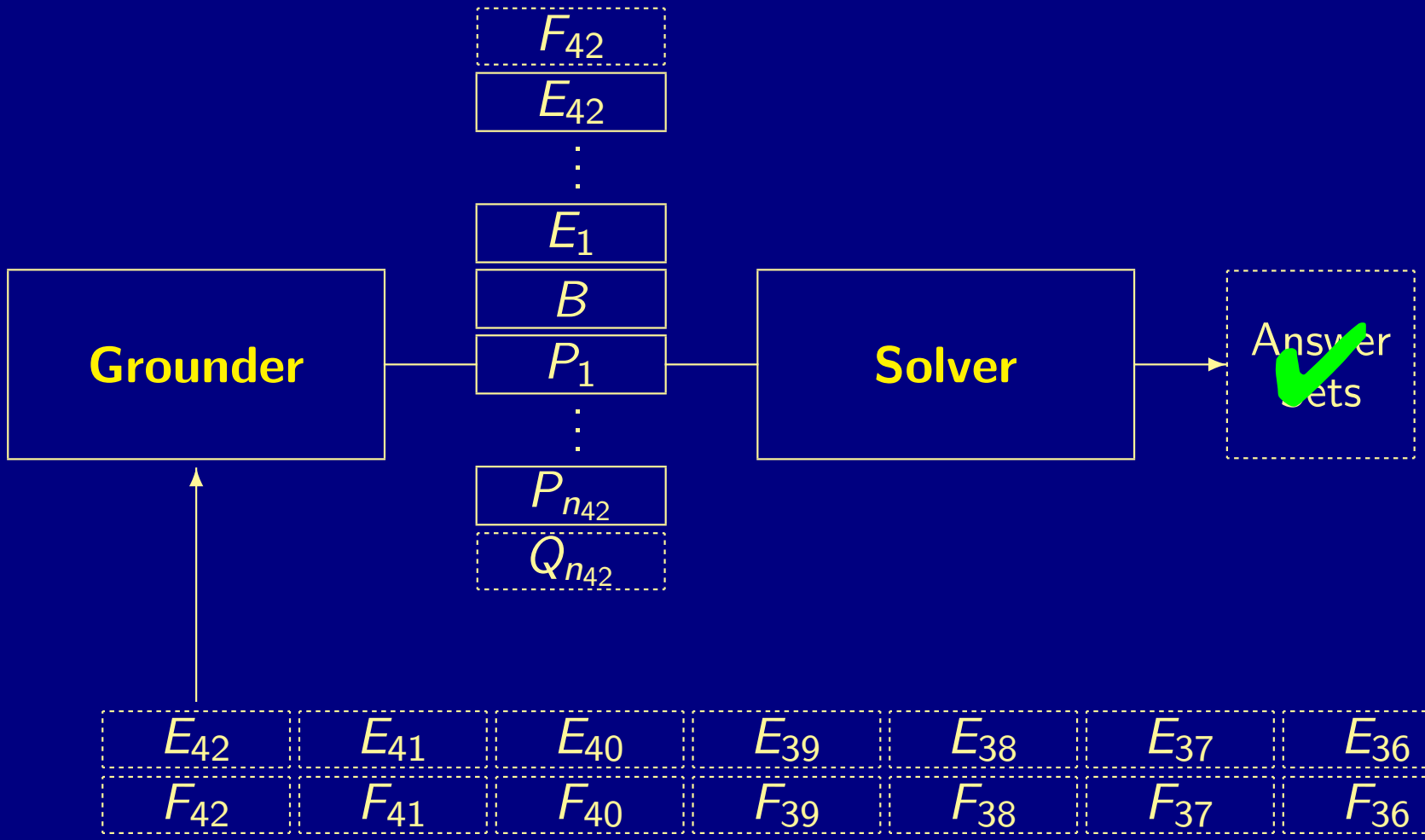

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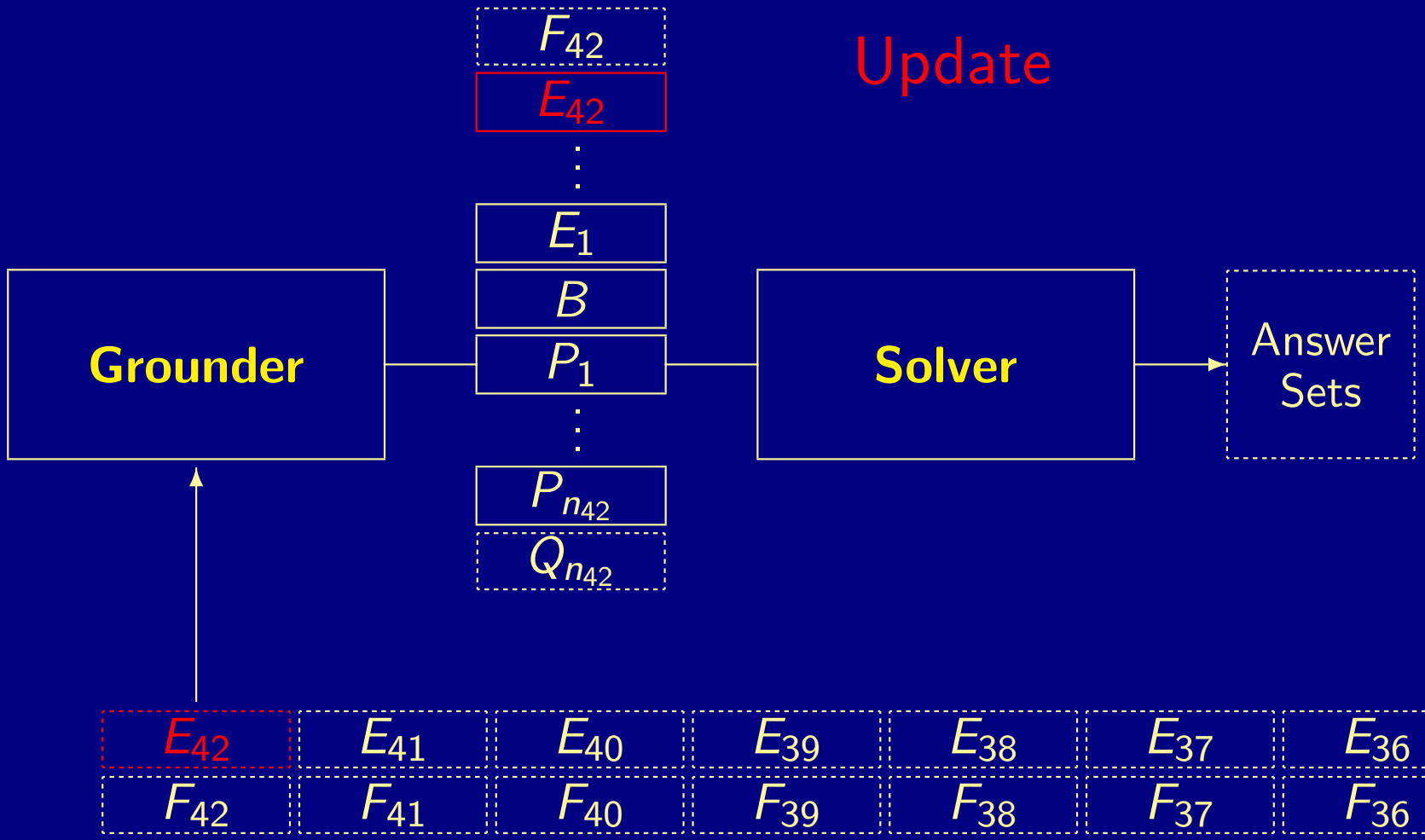

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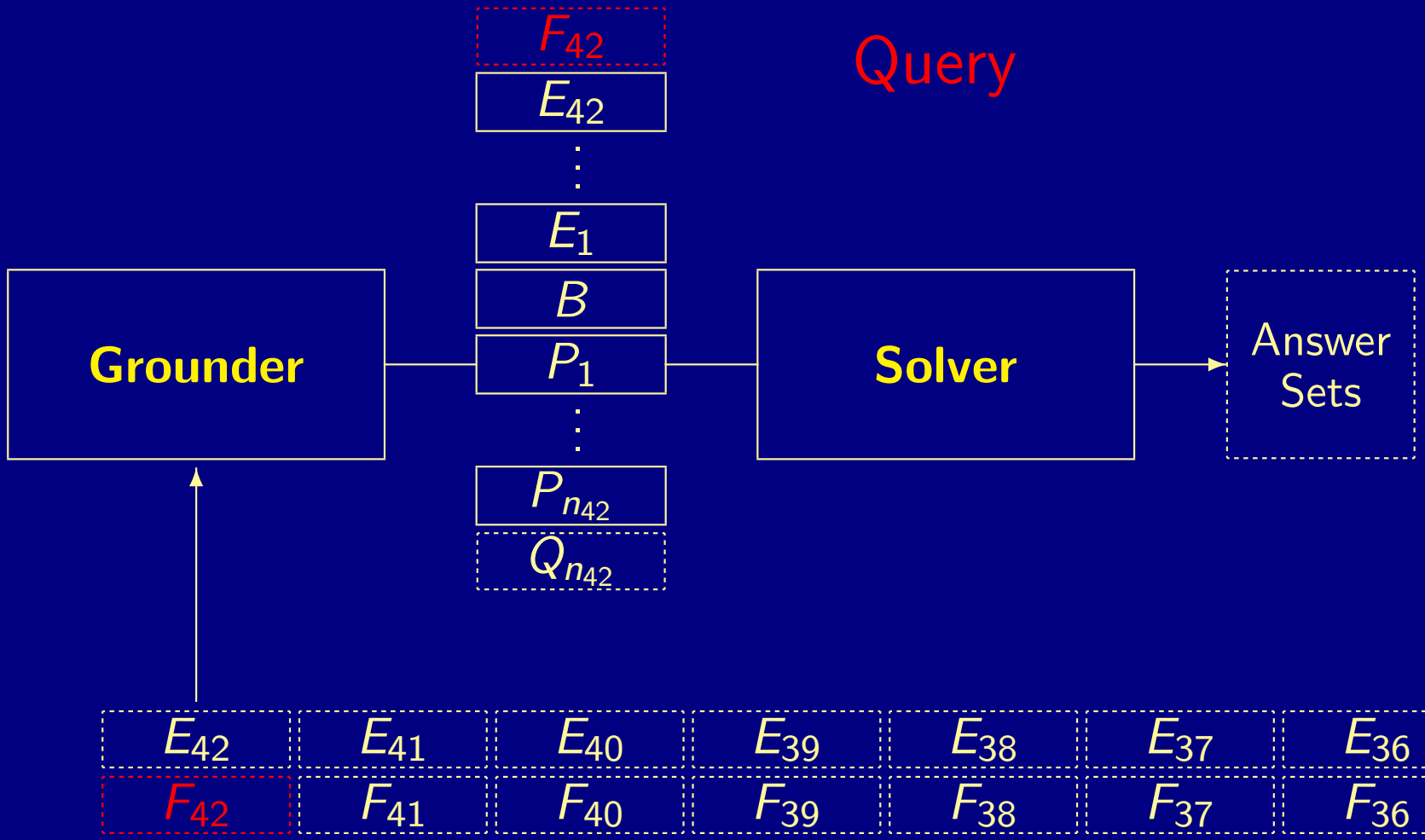

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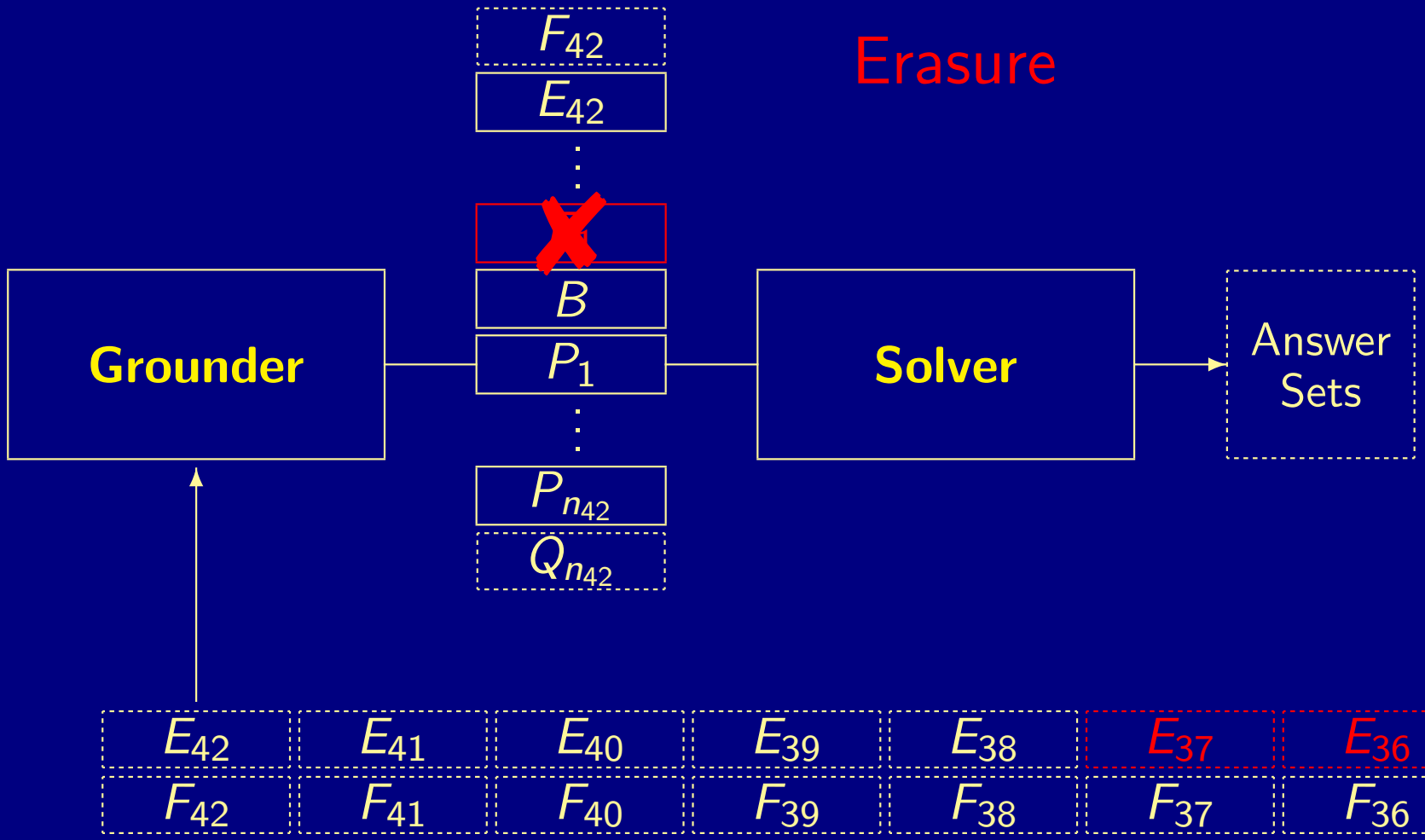

Elevator Control

```
#base.

floor(1..3).
atFloor(1,0).

#cumulative t.
#external request(F,t) :  floor(F).

1 { atFloor(F-1;F+1,t) } 1 :- atFloor(F,t-1), floor(F).
:- atFloor(F,t), not floor(F).
requested(F,t) :- request(F,t),     floor(F), not atFloor(F,t).
requested(F,t) :- requested(F,t-1), floor(F), not atFloor(F,t).
goal(t) :- not requested(F,t) : floor(F).

#volatile t.

:- not goal(t).
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```

Pushing a button

- oClingo acts as a server listening on a port waiting for client requests
- To issue such requests, a separate controller program sends online progressions using network sockets
- For instance,

```
#step 1.
request(3,1).
#endstep.
```

- This process terminates when the client sends

```
#stop.
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Systems: Overview

31 Potassco

32 gringo

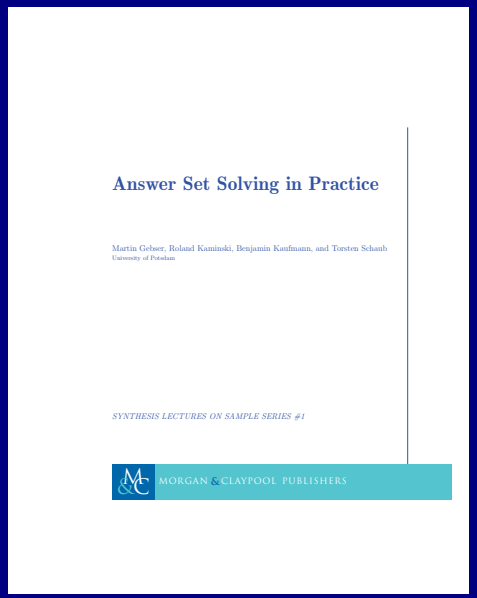
33 clasp

34 Siblings
- claspfolio
- clingcon
- iclingo
- oclingo

35 Book

The (forthcoming) Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



`http://potassco.sourceforge.net/teaching.html`

Summary

- ASP is emerging as a viable tool for Knowledge Representation and Reasoning
- ASP offers efficient and versatile off-the-shelf solving technology
  - `http://potassco.sourceforge.net`
  - ASP'07/09/11, CASC'11, MISC'11, PB'09/11, and SAT'09/11
- ASP offers an expanding functionality and ease of use
  - Rapid application development tool
- ASP has a growing range of applications

ASP = KR+DB+SAT+LP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