On the Influence of the Various Parameters of the Restarting Automaton on its Expressive Capacity and Descriptional Complexity

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Outline:

Otto, F.: RESTARTING AUTOMATA - Automata Inspired by the Linguistic Technique of Analysis by Reduction. Monograph, in preparation.

1. Introduction

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The restarting automaton is not simply another variant of the Turing machine, but it is a new type of automaton that is inspired by the linguistic technique of analysis by reduction (see, e.g., [2]).

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We are given a sentence in a natural language, e.g.:

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This sentence is now simplified step by step until a simple sentence is obtained:

The means are mean.

Example (cont.)

To process a sentence in this way, one needs to understand its meaning. To automate this process, additional information is needed, which can be provided in the form of tags:

Word	Tags
They	<Pronoun> <Plural>
mean	<Verb> <Present Tense> <Plural>
that	<Conjunction>
the	<Article>
means	<Noun> <Plural>
she	<Pronoun> <Singular>
means	<Verb> <Present Tense> <Singular>
are	<Verb> <Present Tense> <Plural>
very	<Adverb>
mean	<Adjective>
.	<End of Sentence>

Each word is processed together with its tags.

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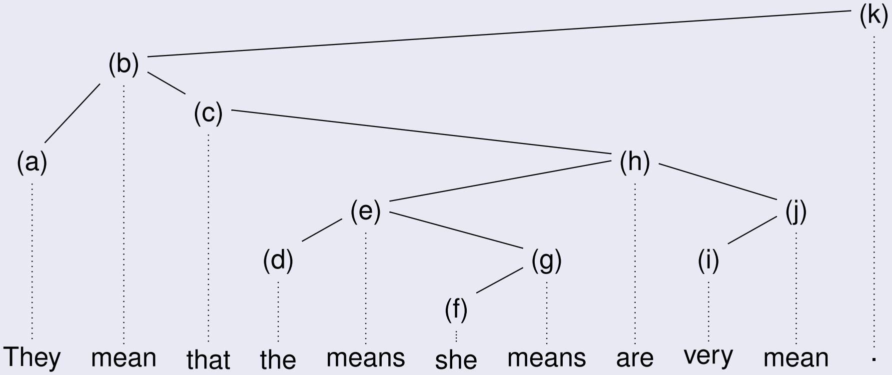
Example (cont.)

In this way dependencies and independencies between words of a sentence can be determined.

This may result in the construction of a dependency tree:



Operations Required for Analysis by Reduction:

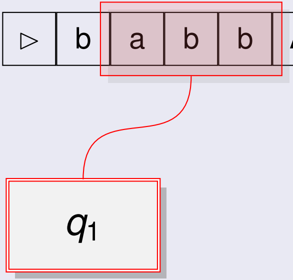
- Scan the current sentence (from left to right).
- Rewrite the current sentence by performing a local transformation.
- Restart in order to process the new sentence.
- Accept (if a correct simple sentence has been obtained).
- Reject (if an error has been detected).

2. The Restarting Automaton and Its Parameters

The Ingredients:

- The input sentence, which is a sequence of morphems, will be represented by a word, i.e., we need a (finite) input alphabet.
- To encode additional information (tags), we need auxiliary letters, i.e., we need a (finite) working alphabet.
- To mark the beginning and end of the current sentence, we need special symbols (sentinels).
- As the sentence analyzed will be transformed, the current word will be stored in a list of cells or a flexible tape.
- To scan the current word, we need a read/write window.
- To process information we need a finite-state control.
- To execute the analysis, we need a program (transition function) that determines the sequence of elementary operations.

The Hardware:



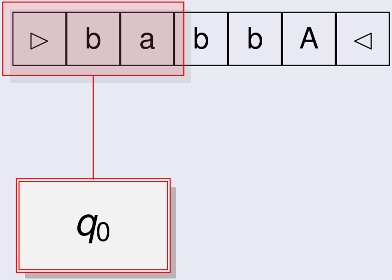
flexible tape
with sentinels

read/write window

finite-state control

The Operations:

Move-Right Step (MVR):



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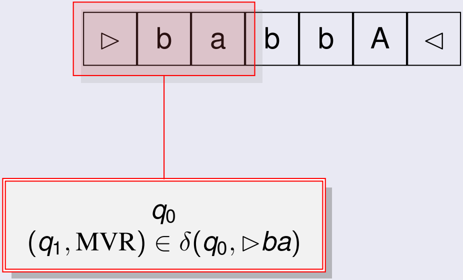

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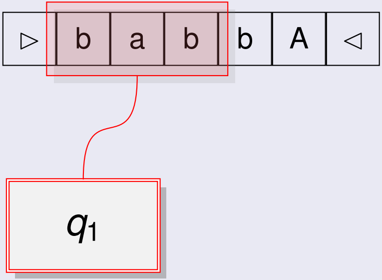
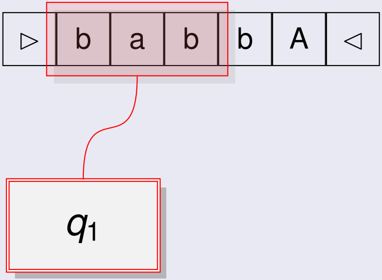


The Operations (cont.):

Move-Left Step (MVL):



The Operations (cont.):

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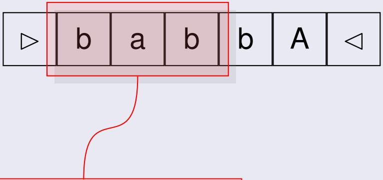
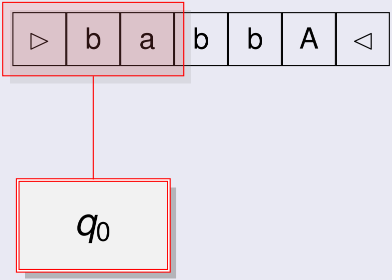


q_1

$(q_0, \mathrm{MVL}) \in \delta(q_1, bab)$

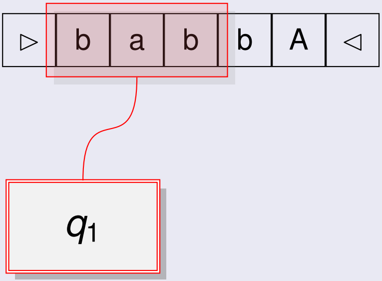
The Operations (cont.):

Move-Left Step (MVL):



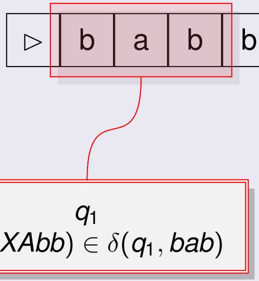
The Operations (cont.):

Rewrite Step:



The Operations (cont.):

Rewrite Step:



The Operations (cont.):

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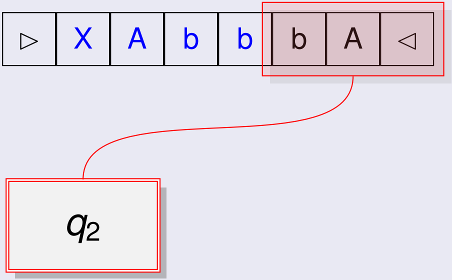
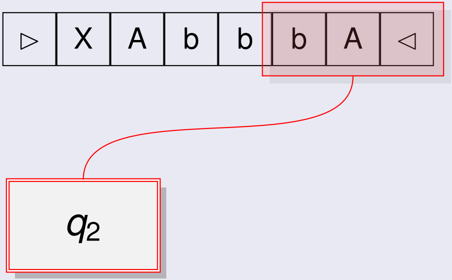


The Operations (cont.):

Restart Step:



The Operations (cont.):

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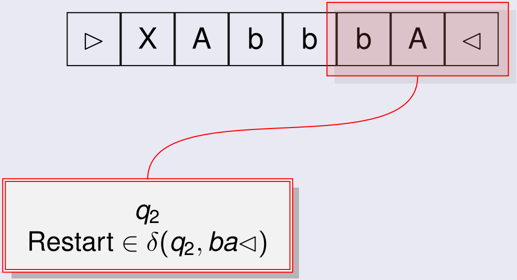
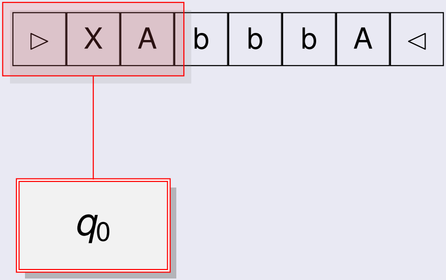


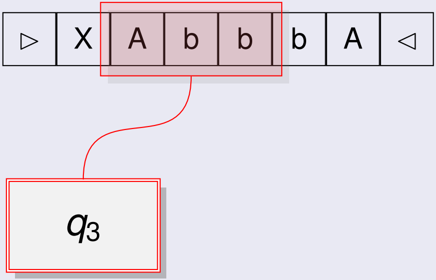
The Operations (cont.):

Restart Step:



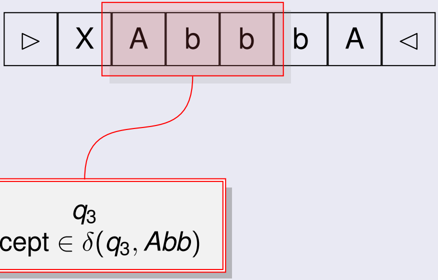
The Operations (cont.):

Accept Step:



The Operations (cont.):

Accept Step:



The Operations (cont.):

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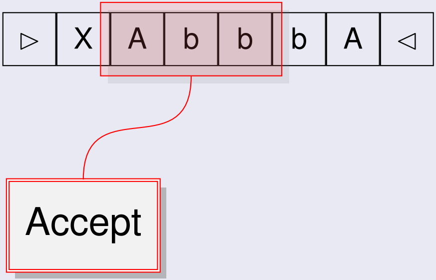
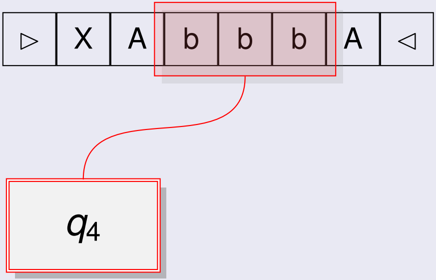


The Operations (cont.):

Reject Step:



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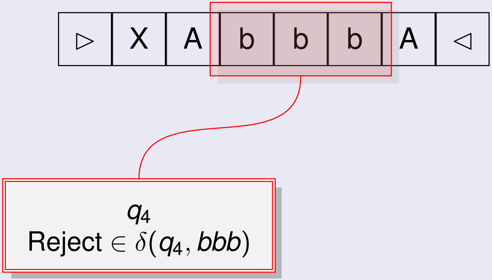

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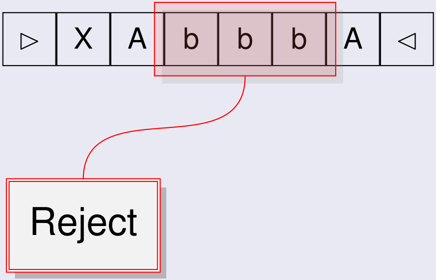


Definition 1

A restarting automaton is defined by an 8-tuple

$$M = (Q, \Sigma, \Gamma, \rhd, \lhd, q_0, k, \delta),$$

where

- *Q is a finite set of states,*
- *Σ is a finite input alphabet,*
- *Γ is a finite working alphabet such that $\Sigma \subseteq \Gamma$,*
- *$\rhd, \lhd \notin \Gamma$ are the markers for the end of the tape (sentinels),*
- *$q_0 \in Q$ is the restart state, which also serves as initial state,*
- *$k \geq 1$ is the size of the read/write window, and*
- *$\delta : Q \times (\Gamma \cup \{\rhd, \lhd\})^k \to 2^{((Q\times\{\mathrm{MVR},\mathrm{MVL},\Gamma^*\})\cup\{\mathrm{Restart},\mathrm{Accept},\mathrm{Reject}\})}$ is the transition relation.*

Example 2

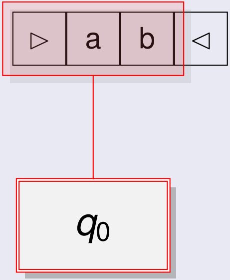
Let $M = (Q, \Sigma, \Gamma, \rhd, \lhd, q_0, 3, \delta)$ *be the following restarting automaton:*

- $Q = \{q_0\}$, $\Sigma = \{a, b\}$, $\Gamma = \Sigma \cup \{a_1, e, e_1, f, f_1, f_2\}$,
- *and* δ *is defined with combined rewrite/restart steps:*

(0)	$\delta(xyz)$	$\ni$	MVR	*for all* $x \in \Gamma \cup \{\rhd\}$ *and* $y, z \in \Gamma$,			
(1)	$\delta(\rhd\lhd)$	$\ni$	Accept,	(13)	$\delta(\rhd a_1 e_1)$	$\ni$	$\rhd e_1$,
(2)	$\delta(ab\lhd)$	$\ni$	$ae\lhd$,	(14)	$\delta(ae_1 e)$	$\ni$	ae,
(3)	$\delta(bb\lhd)$	$\ni$	$be\lhd, bf\lhd$,	(15)	$\delta(\rhd e_1 \lhd)$	$\ni$	$\rhd\lhd$,
(4)	$\delta(bbe)$	$\ni$	bee,	(16)	$\delta(aaf)$	$\ni$	$aa_1 f$,
(5)	$\delta(abe)$	$\ni$	aee,	(17)	$\delta(\rhd af)$	$\ni$	$\rhd a_1 f$,
(6)	$\delta(bbf)$	$\ni$	bff,	(18)	$\delta(a_1 ff)$	$\ni$	$a_1 f_1 f$,
(7)	$\delta(abf)$	$\ni$	aff,	(19)	$\delta(f_1 ff)$	$\ni$	$f_1 f_2 f$,
(8)	$\delta(aae)$	$\ni$	$aa_1 e$,	(20)	$\delta(f_1 f \lhd)$	$\ni$	$f_1 f_2 \lhd$,
(9)	$\delta(\rhd ae)$	$\ni$	$\rhd a_1 e$,	(21)	$\delta(a_1 f_1 f_2)$	$\ni$	$a_1 f_2$,
(10)	$\delta(a_1 ee)$	$\ni$	$a_1 e_1 e$,	(22)	$\delta(aa_1 f_2)$	$\ni$	af_2,
(11)	$\delta(a_1 e \lhd)$	$\ni$	$a_1 e_1 \lhd$,	(23)	$\delta(\rhd a_1 f_2)$	$\ni$	$\rhd f_2$,
(12)	$\delta(aa_1 e_1)$	$\ni$	ae_1,	(24)	$\delta(af_2 f)$	$\ni$	af,
				(25)	$\delta(\rhd f_2 \lhd)$	$\ni$	$\rhd\lhd$.

Example 2 (cont.)

This restarting automaton proceeds as follows:



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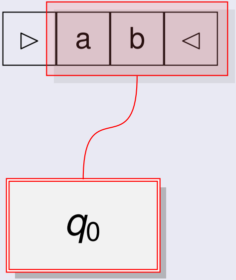

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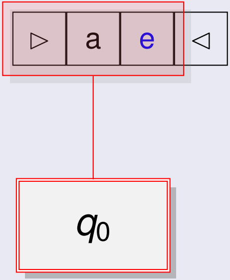

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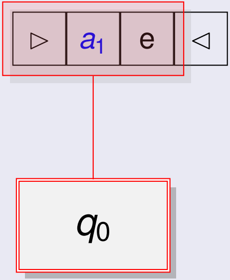

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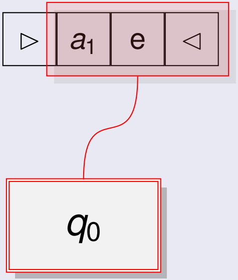

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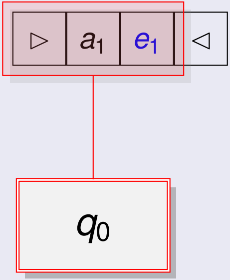

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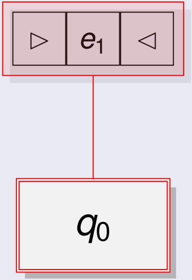

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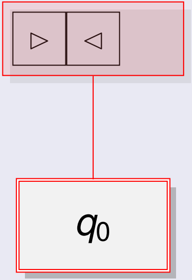

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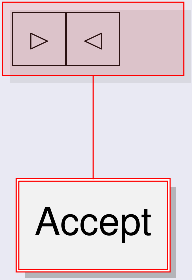

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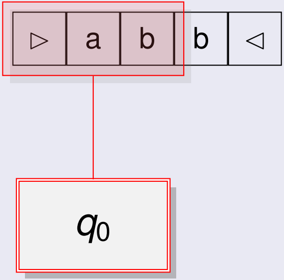

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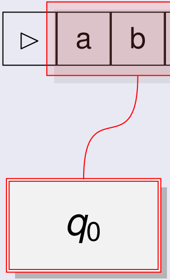

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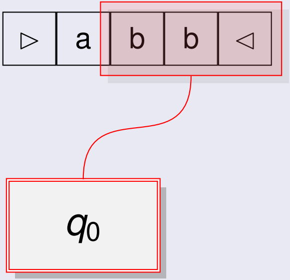

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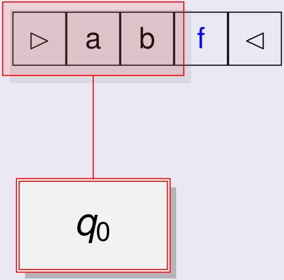

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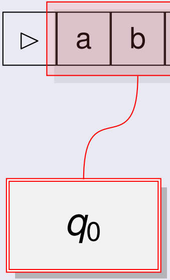

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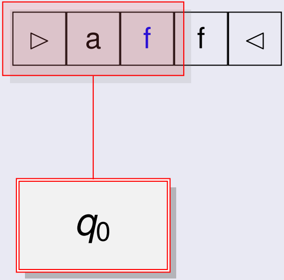

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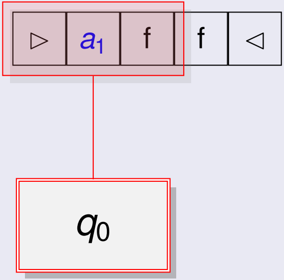

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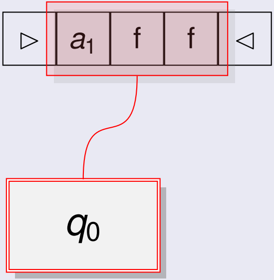

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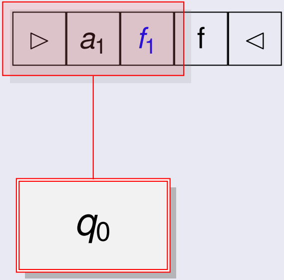

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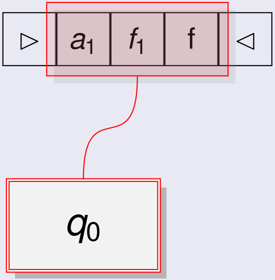

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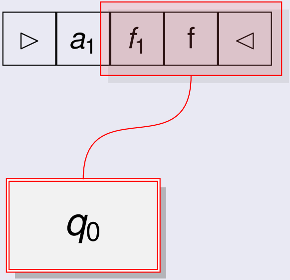

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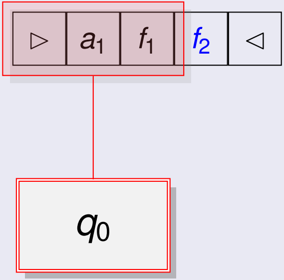

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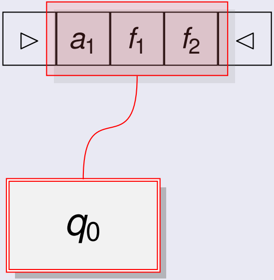

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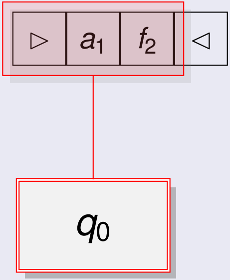

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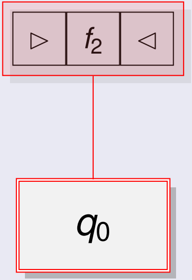

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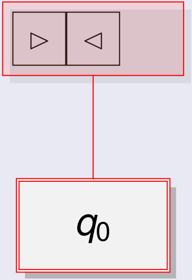

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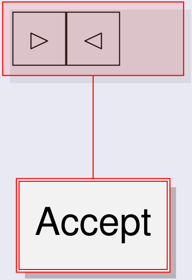

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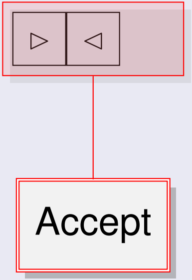

It accepts the language $L = \{ a^n b^n, a^n b^{2n} \mid n \geq 0 \} \in \mathrm{CFL} \smallsetminus \mathrm{DCFL}$.

Definition 1 (cont.)

Let $M = (Q, \Sigma, \Gamma, \rhd, \lhd, q_0, k, \delta)$ be a restarting automaton.

(a) A configuration of M is a word of the form $\rhd uqvw\lhd$, where $q \in Q$ and $u, v, w \in \Gamma^*$ with $|v| = k$.

(b) A restart configuration is of the form $q_0 \rhd vw\lhd$, where $v, w \in \Gamma^*$ with $|v| = k - 1$. If $v, w \in \Sigma^*$, then $q_0 \rhd vw\lhd$ is an initial configuration.

(c) A sequence of steps leading from a restart configuration $q_0 \rhd vw\lhd$ to the next restart configuration $q_0 \rhd xy\lhd$ is called a cycle. It is written as $q_0 \rhd vw \lhd \vdash_M^c q_0 \rhd xy\lhd$.
Condition: Each cycle of each computation of M must contain at least one rewrite step!

(d) A sequence of steps leading from a restart configuration to a final configuration (accepting or rejecting) without going through another restart configuration is called a tail.

Remark 3

A terminating computation of M consists of a finite sequence of cycles that is followed by a tail.

Definition 1 (cont.)

(e) $L_C(M) = \{\, w \in \Gamma^* \mid q_0 \rhd w \lhd \vdash_M^* \text{Accept} \,\}$
is the basic (or characteristic) language of M.

(f) $L(M) = \{\, w \in \Sigma^* \mid q_0 \rhd w \lhd \vdash_M^* \text{Accept} \,\}$
is the input language of M.

Remark 4

$L(M) = L_C(M) \cap \Sigma^*$.

Parameters of the Restarting Automaton

The type of a restarting automaton depends on many parameters:

- the number $|Q| \geq 1$ of states
- the number $|\Gamma \smallsetminus \Sigma| \geq 0$ of auxiliary letters
- the size $k \geq 1$ of the read/write window
- the type of move steps allowed
- the type of rewrite steps allowed
- the number $j \geq 1$ of allowed rewrite steps per cycle
- the positions at which rewrite steps are executed

The automaton from Example 2 satisfies $|Q| = 1$, $|\Gamma \smallsetminus \Sigma| = 6$, $k = 3$, only MVR-steps, rewrite steps replace or delete only the symbol in the middle of the window, and only one rewrite step per cycle that is immediately followed by a restart.

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Remark 5

(a) *Each recursively enumerable language is accepted by a deterministic restarting automaton with window size 1 and MVR- and MVL-steps that executes a single rewrite step per cycle which replaces a single letter or inserts a single letter.*

(b) *Each context-sensitive language is accepted by a (nondeterministic) restarting automaton with window size 1 and MVR- and MVL-steps that executes a single rewrite step per cycle which replaces a single letter.*

Proof.

The given type of restarting automaton can simulate a single-tape Turing machine (linear-bounded automaton), where each step of the Turing machine (LBA) is simulated by several cycles. □

Definition 6

Let $M = (Q, \Sigma, \Gamma, \triangleright, \triangleleft, q_0, k, \delta)$ be a restarting automaton.

(a) *M is called* length-reducing *if $|v| < |u|$ holds for each rewrite step $(q', v) \in \delta(q, u)$ of M.*

(b) *M is called* weight-reducing *(or* shrinking*) if there exists a weight function $\omega : \Gamma \to \mathbb{N}_+$ such that $\omega(v) < \omega(u)$ holds for each rewrite step $(q', v) \in \delta(q, u)$ of M.*

Fact 7

If M is a weight-reducing restarting automaton, then each computation that begins with a tape contents of length n consists of at most $c \cdot n$ cycles and a tail, where $c \geq 1$ is a constant that depends on the underlying weight function.

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Corollary 8

If M is a weight-reducing restarting automaton, then $L(M) \in \mathrm{NTIMESPACE}(n^2, n)$.

Proof.

A computation of M can be simulated by a nondeterministic 2-tape Turing machine that runs in quadratic time and that needs linear space. □

From now on we only consider restarting automata that are weight-reducing (or even length-reducing)!

The restarting automaton from Example 2 is weight-reducing: Choose the weight function ω in such a way that

$$\omega(a), \omega(b) > \omega(a_1), \omega(e), \omega(f) > \omega(e_1), \omega(f_1), \omega(f_2)$$

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3. Some Important Results on Formal Languages

3.1 Parameter 1: The Type of Operations

In [1] and other papers by Petr Jančar et al., only length-reducing restarting automata were studied that execute exactly one rewrite step in each cycle. The resulting types of restarting automata are the following:

	with auxiliary symbols (-WW)	no auxiliary symbols (-W)	deletions only (-ε)
MVL-steps (RL-)	RLWW	RLW	RL
no MVL-steps (RR-)	RRWW	RRW	RR
no MVL-steps, rewrite followed by restart (R-)	RWW	RW	R

Definition 9

Let M be an RLWW*-automaton.*

(a) *If a cycle C of M contains the rewrite transition*

$$\triangleright uqvw\triangleleft \vdash_M \triangleright uxq'w\triangleleft,$$

then $D_r(C) = |vw| + 1$ is the right distance of C.

(b) *M is monotone if, in each computation of M, the right distance does not increase from one cycle to the next.*

(c) *M is weakly monotone if there exists a constant $c \geq 0$ such that, in each computation of M, the right distance increases by at most c from one cycle to the next.*

Prefixes are used to denote subtypes of restarting automata:

deterministic automata	det-
monotone automata	mon-
weakly monotone automata	wmon-
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Theorem 10 (Plátek 2001 [3])

$$\mathcal{L}((\text{s})\text{RL}(\text{W})(\text{W})) = \mathcal{L}((\text{s})\text{RR}(\text{W})(\text{W})).$$
$$\mathcal{L}((\text{w})\text{mon-}(\text{s})\text{RL}(\text{W})(\text{W})) = \mathcal{L}((\text{w})\text{mon-}(\text{s})\text{RR}(\text{W})(\text{W})).$$

Theorem 11 (Jurdziński, Otto 2007 [4])

$\mathcal{L}(\text{sRWW}) = \mathcal{L}(\text{sRRWW}) = \mathcal{L}(\text{sRLWW}) = \mathcal{L}(\text{FCA})$.

Theorem 12 (Jančar et al. 1999 [5])

(a) $\mathcal{L}(\text{mon-RWW}) = \mathcal{L}(\text{mon-RRWW}) = \text{CFL}$.
(b) $\mathcal{L}(\text{det-mon-R}) = \mathcal{L}(\text{det-mon-RRWW}) = \text{DCFL}$.

Theorem 13 (Jurdziński et al. 2004 [6])

$\mathcal{L}(\text{wmon-RWW}) = \mathcal{L}(\text{wmon-RRWW}) = \text{GCSL}$.

Theorem 14 (Niemann, Otto 2000 [7])

$\mathcal{L}(\text{det-RWW}) = \mathcal{L}(\text{det-RRWW}) = \text{CRL}$.

Theorem 10 (Plátek 2001 [3])

$$\mathcal{L}((s)\text{RL}(W)(W)) = \mathcal{L}((s)\text{RR}(W)(W)).$$
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Theorem 15 (Otto 2009 [8])

$\mathcal{L}(\text{det-mon-RL}) = \mathcal{L}(\text{det-mon-RLWW}) = \text{LRR}.$

Theorems 12, 13, 14, and 15 extend to the corresponding weight-reducing types of restarting automata.

Hierarchy of Language Classes Accepted by Various Subtypes of sRLWW-Automata:

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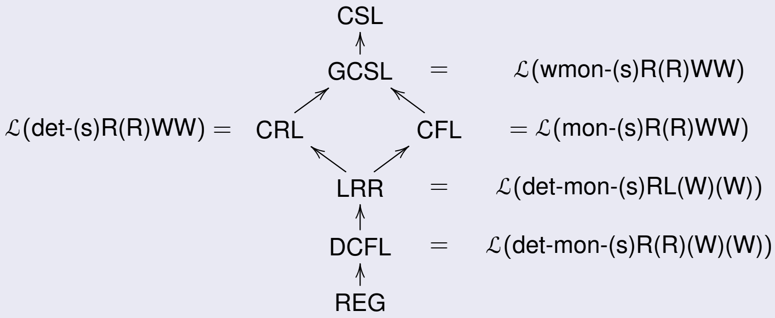

Problem 1

Is the inclusion $\mathcal{L}(\text{sRRWW}) \subseteq \text{CSL}$ *proper?*
Find a context-sensitive language that is not accepted by an sRRWW-automaton!

Problem 2

Is the inclusion $\mathcal{L}(\text{RRWW}) \subseteq \mathcal{L}(\text{sRRWW})$ *proper?*
Find a language that is accepted by a weight-reducing, but not by any length-reducing RRWW-automaton!

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Theorem 16 (Holzer, Kutrib, Reimann 2007 [9])

Let $\mathsf{X}, \mathsf{Y} \in \{\mathsf{R}, \mathsf{RR}, \mathsf{RW}, \mathsf{RRW}, \mathsf{RWW}, \mathsf{RRWW}\}$. *Then the trade-off for the conversion of a restarting automaton of type* X *or* det-X *into an equivalent monotone restarting automaton of type* Y *is non-recursive.*

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3.2 Parameter 2: The Size of the Read/Write Window

For each type X of restarting automaton, let X(k) denote the restarting automata of type X that have a read/write window of size k.

Theorem 18 (Mráz 2001 [11])

For all $k \geq 1$ *and all* $\mathrm{X} \in \{\mathrm{R}, \mathrm{RR}, \mathrm{RW}, \mathrm{RRW}\}$,

$$\mathcal{L}((\text{det-})(\text{mon-})\mathrm{X}(k)) \subsetneq \mathcal{L}((\text{det-})(\text{mon-})\mathrm{X}(k+1)).$$

Theorem 19 (Kutrib, Otto 2012 [12])

For all $\mathrm{X} \in \{\text{det-R(R)W}, \mathrm{R(R)W}\}$ *and all* $k \geq 3$, *the trade-off from* $\mathrm{X}(k)$*-automata to* $\mathrm{X}(k-1)$*-automata is non-recursive.*

Theorem 20 (Kutrib, Otto 2013 [13])

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For all $\mathrm{X} \in \{\text{det-R(R)W}, \text{R(R)W}\}$ and all $k \geq 3$, the trade-off from $\mathrm{X}(k)$-automata to $\mathrm{X}(k-1)$-automata is non-recursive.

Theorem 20 (Kutrib, Otto 2013 [13])

For all $\mathrm{X} \in \{\text{det-R}, \text{det-RR}, \mathrm{R}, \mathrm{RR}\}$ and all $k \geq 5$, the trade-off from $\mathrm{X}(k)$-automata to $\mathrm{X}(k-1)$-automata is non-recursive.

Theorem 21 (Mráz 2001 [11], Kutrib, Reimann 2008 [14])

(a) $\mathcal{L}((\text{det-})(\text{mon-})\text{R}(1)) = \text{REG}.$

(b) $\mathcal{L}(\text{det-}(\text{mon-})\text{RR}(1)) = \text{REG} \subsetneq \mathcal{L}(\text{mon-RR}(1)).$

Theorem 22 (Mráz, Otto 2019 [15])

(a) $\mathcal{L}(\text{det-sRWW}(1)) = \mathcal{L}(\text{mon-sRWW}(1)) = \text{REG}.$

(b) $\mathcal{L}(\text{det-sRRWW}(1)) = \text{REG}.$

(c) $\mathcal{L}(\text{mon-RR}(1)) \subsetneq \mathcal{L}(\text{mon-sRRWW}(1)).$

Problem 4

Is the inclusion $\mathcal{L}(\text{mon-sRRWW}(1)) \subseteq \text{CFL}$ *proper?*
Find a context-free language that is not accepted by any monotone sRRWW-automaton with window size one!
Is $L_{\text{pal}} = \{ ww^R \mid w \in \{a,b\}^* \}$ *such a language?*

Theorem 21 (Mráz 2001 [11], Kutrib, Reimann 2008 [14])

(a) $\mathcal{L}(\text{(det-)(mon-)R(1)}) = \text{REG}.$

(b) $\mathcal{L}(\text{det-(mon-)RR(1)}) = \text{REG} \subsetneq \mathcal{L}(\text{mon-RR(1)}).$

Theorem 22 (Mráz, Otto 2019 [15])

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(c) $\mathcal{L}(\text{mon-RR(1)}) \subsetneq \mathcal{L}(\text{mon-sRRWW(1)}).$

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Is the inclusion $\mathcal{L}(\text{mon-sRRWW(1)}) \subseteq \text{CFL}$ proper?
Find a context-free language that is not accepted by any monotone sRRWW-automaton with window size one!
Is $L_{\text{pal}} = \{ ww^R \mid w \in \{a,b\}^* \}$ such a language?

Theorem 21 (Mráz 2001 [11], Kutrib, Reimann 2008 [14])

(a) $\mathcal{L}$((det-)(mon-)R(1)) $=$ REG.
(b) $\mathcal{L}$(det-(mon-)RR(1)) $=$ REG $\subsetneq$ $\mathcal{L}$(mon-RR(1)).

Theorem 22 (Mráz, Otto 2019 [15])

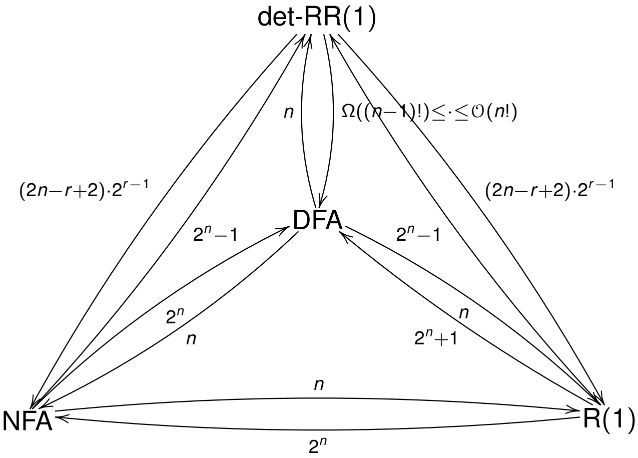
(a) $\mathcal{L}$(det-sRWW(1)) $=$ $\mathcal{L}$(mon-sRWW(1)) $=$ REG.
(b) $\mathcal{L}$(det-sRRWW(1)) $=$ REG.
(c) $\mathcal{L}$(mon-RR(1)) $\subsetneq$ $\mathcal{L}$(mon-sRRWW(1)).

Problem 4

Is the inclusion $\mathcal{L}$(mon-sRRWW(1)) $\subseteq$ CFL proper?
Find a context-free language that is not accepted by any monotone sRRWW-automaton with window size one!
Is $L_{\mathrm{pal}} = \{ ww^R \mid w \in \{a, b\}^* \}$ such a language?

Theorem 23 (Kutrib, Reimann 2008 [16])

The trade-offs between DFAs, NFAs, R(1)- , and det-RR(1)-automata are as depicted below.



Theorem 24 (Schluter 2015 [17])

For all $k \geq 2$,

(a) $\mathcal{L}(\text{mon-RWW}(k)) = \mathcal{L}(\text{mon-RRWW}(k))$.

(b) $\mathcal{L}(\text{det-mon-RWW}(k)) = \mathcal{L}(\text{det-mon-RRWW}(k))$.

(c) $\mathcal{L}(\text{mon-RRWW}(k+1)) = \mathcal{L}(\text{mon-RRWW}(k))$.

(d) $\mathcal{L}(\text{RRWW}(k+1)) = \mathcal{L}(\text{RRWW}(k))$.

Corollary 25 (Schluter 2015 [17])

$\mathcal{L}(\text{mon-RWW}(2)) = \text{CFL} = \mathcal{L}(\text{mon-sRRWW})$.

Theorem 24 (Schluter 2015 [17])

For all $k \geq 2$,

$$\begin{array}{lll} \text{(a)} & \mathcal{L}(\text{mon-RWW}(k)) & = & \mathcal{L}(\text{mon-RRWW}(k)). \\ \text{(b)} & \mathcal{L}(\text{det-mon-RWW}(k)) & = & \mathcal{L}(\text{det-mon-RRWW}(k)). \\ \text{(c)} & \mathcal{L}(\text{mon-RRWW}(k+1)) & = & \mathcal{L}(\text{mon-RRWW}(k)). \\ \text{(d)} & \mathcal{L}(\text{RRWW}(k+1)) & = & \mathcal{L}(\text{RRWW}(k)). \end{array}$$

Corollary 25 (Schluter 2015 [17])

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Theorem 26 (Mráz, Otto 2019 [18])

$\mathcal{L}$(det-(mon-)RWW(2)) $=$ DCFL $=$ $\mathcal{L}$(det-mon-sRRWW).

As $L_{\text{expo}} = \{ a^{2^n} \mid n \geq 0 \} \in \mathcal{L}$(det-RWW(3)), we see that $\mathcal{L}$(det-RWW(2)) $\subsetneq \mathcal{L}$(det-RWW(3)).

Problem 5

Is the inclusion $\mathcal{L}$(det-RWW(k)) $\subseteq \mathcal{L}$(det-RWW($k+1$)) proper for all $k \geq 3$, or is there an integer $\hat{k} \geq 3$ such that $\mathcal{L}$(det-RWW($\hat{k}$)) $=$ CRL?

Theorem 27 (Mráz, Otto 2019 [18])

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3.3 Parameter 3: The Number of States

Definition 28

A restarting automaton $M = (Q, \Sigma, \Gamma, \rhd, \lhd, q_0, k, \delta)$ *is called stateless if* $Q = \{q_0\}$. *We use the prefix stl- to denote stateless types of restarting automata.*

Theorem 29 (Kutrib, Messerschmidt, Otto 2010 [19])

(a)	$\mathcal{L}$(stl-det-mon-R)	$\supsetneq$	REG
(b)	$\mathcal{L}$(stl-det-mon-RWW)	$=$	DCFL
(c)	$\mathcal{L}$(stl-mon-RWW)	$=$	CFL
(d)	$\mathcal{L}$(stl-det-RWW)	$=$	CRL
(e)	$\mathcal{L}$(stl-wmon-RWW)	$=$	GCSL

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Definition 30

An ordered RWW-automaton (ORWW) is an sRWW(3)-automaton in which each rewrite step replaces the letter in the middle of the window by a letter with less weight.

Theorem 31 (Kwee, Otto 2018 [20, 21])

(a) $\mathcal{L}(\text{stl-det-ORWW}) = \mathcal{L}(\text{det-ORWW}) = \text{REG}$

(b) $\mathcal{L}(\text{stl-ORWW}) = \text{REG} \subsetneq \mathcal{L}(\text{ORWW})$

(c) $\mathcal{L}(\text{ORWW})$ *is incomparable to* DLIN *and* GCSL.

Proof of (c):

$\{ a^n b^n \mid n \geq 1 \} \in \text{DLIN} \smallsetminus \mathcal{L}(\text{ORWW})$, while

$$L'_{\text{copy}} = \{ wcu \mid w, u \in \{a, b\}^*, |w|, |u| \geq 2, u \text{ is a scattered factor of } w \}$$

belongs to $\mathcal{L}(\text{ORWW}) \smallsetminus \text{GCSL}$. □

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Remark

For a stl-det-ORWW-automaton $M = (Q, \Sigma, \Gamma, \rhd, \lhd, q_0, k, \delta)$, we use $|\Gamma|$ to measure the size of M.

Theorem 32 (Kwee, Otto 2018 [21])

From a stl-ORWW-automaton M with a tape alphabet of cardinality n, an NFA $A = (Q, \Sigma, S, \delta_A, F)$ can be constructed such that $|Q| \leq 2^{13 \cdot n}$ and $L(A) = L(M)$.

Corollary 33 (Kwee, Otto 2018 [21])

For converting a stl-ORWW-automaton or a stl-det-ORWW-automaton with a tape alphabet of cardinality n into an equivalent DFA (NFA), $2^{2^{\mathcal{O}(n)}}$ ($2^{\mathcal{O}(n)}$) states are sufficient, and there are cases in which these many states are also necessary.

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Definition 34

An ordered restart-delete-automaton (ORD) is an sRWW(3)-automaton in which each rewrite step replaces the letter in the middle of the window by a letter with less weight or by the empty word.

Theorem 35 (Otto 2018 [22, 23])

(a) $\mathrm{DCFL} \subsetneq \mathcal{L}(\text{stl-det-ORD}) = \mathcal{L}(\text{det-ORD}) \subsetneq \mathrm{CRL} \cap \mathrm{CFL}$

(b) $\mathrm{CFL} \subseteq \mathcal{L}(\text{stl-ORD})$

Problem 6

Is the inclusion $\mathrm{CFL} \subseteq \mathcal{L}(\text{stl-ORD})$ *proper?*
Find a non-context-free language that accepted by a stl-ORD-automaton!

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A stl-ORD-automaton $M = (Q, \Sigma, \Gamma, \rhd, \lhd, q_0, k, \delta)$ is called swift, if it can always perform move-right steps unless its window is already at the right end of the tape.

Theorem 37 (Otto 2020 [23, 24])

$\mathcal{L}$(swift-ORD) = CFL

Theorem 38 (Otto 2019 [23, 24])

The trade-off for the conversion of a swift-ORD-automaton into an equivalent stl-ORWW-automaton is non-recursive.

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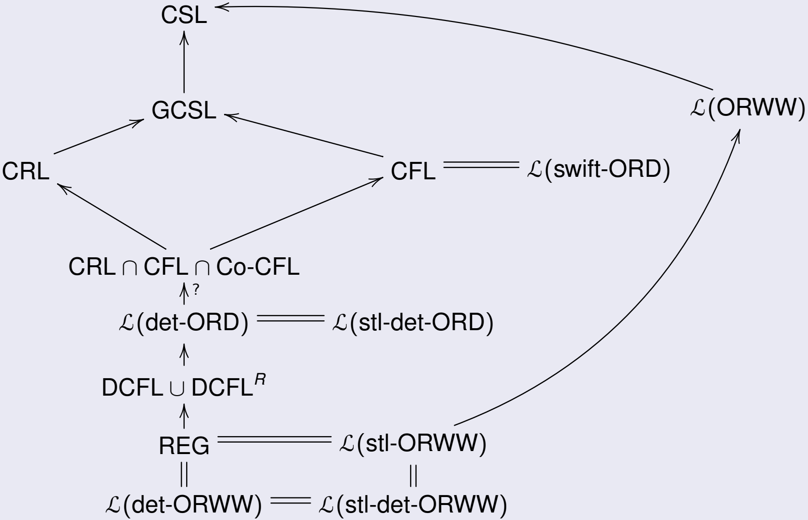
The Taxonomy of Ordered Restarting Automata



4. Conclusion

Further Parameters of Restarting Automata:

1. number of auxiliary letters (Jurdziński, Otto 2006 [25])
2. utilization of auxiliary letters:
   - lexicalized restarting automata (Mráz, Otto, Plátek 2009 [26])
   - h-lexicalized restarting automata (Plátek, Otto 2017 [27])
3. number of rewrites per cycle (Mráz, Otto, Plátek 2014 [28])

Further Variants of Restarting Automata

1. nonforgetting restarting automata [29]
2. cooperating distributed systems of restarting automata [30]
3. parallel communicating systems of restarting automata [31]
4. restarting transducer [32]
5. weighted restarting automata [33]
6. restarting automata with structured output [34]
7. restarting automata for picture languages [35]
8. restarting tree automata [36]

Another Topic:

- Inductive inference of restricted types of restarting automata (See, e.g., Mráz, Plátek, Otto 2006 [37], and Hoffmann 2013 [38])

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Thank you for your attention!

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