Merging two Hierarchies of External Contextual Grammars with Subregular Selection

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- Many papers have been published by different authors on subregular families of languages.
- Focus is often on the decrease of descriptional or computational complexity when going from arbitrary regular languages to special ones.
- Here, the generative capacity of contextual grammars with special selection languages is considered.

Contextual Grammars

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– introduced by Solomon Marcus in 1969,
– formal model for generating languages,
– starting with an initial finite set of words,
– wrapping a context around a (sub)word

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Contexts equipped with conditions where they can be applied

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Definitions

Contextual grammar with selection in $\mathcal{F}$ is a construct

$$G = (V, \{(S_1, C_1), (S_2, C_2), \ldots, (S_n, C_n)\}, A)$$

where

- V is an alphabet,
- for $1 \leq i \leq n$, S_i is a language over some $U \subseteq V$ in $\mathcal{F}$ (selection language),
- for $1 \leq i \leq n$, C_i is a finite set of pairs (u, v) (contexts) with $u, v \in V^*$,
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Internal derivation step: $x \underset{\mathrm{in}}{\Longrightarrow} y$ if $x = x_1x_2x_3$ with $x_2 \in S_i$ for some $i \in \{1, \ldots, n\}$ and $y = x_1ux_2vx_3$ for some $(u, v) \in C_i$.

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Languages generated:

$$L_{\mathrm{ex}}(G) = \{z \mid x \underset{\mathrm{ex}}{\overset{*}{\Longrightarrow}} z \text{ for some } x \in A\},$$

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$\mathcal{EC}(\mathcal{F})$ and $\mathcal{IC}(\mathcal{F})$: family of all languages generated externally or internally by contextual grammars with selection in $\mathcal{F}$.

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- *suffix-closed* (or *fully initial* or *multiple-entry* language) if and only if, for any words $x \in V^*$ and $y \in V^*$, the relation $xy \in L$ implies $y \in L$,

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- *ordered* if and only if the language is accepted by some deterministic finite automaton $\mathcal{A} = (V, Z, z_0, F, \delta)$ where $(Z, \preceq)$ is a totally ordered set and, for any $a \in V$, the relation $z \preceq z'$ implies $\delta(z, a) \preceq \delta(z', a)$,

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- *commutative* if and only if it contains with each word also all permutations of this word,
- *circular* if and only if it contains with each word also all circular shifts of this word,
- *non-counting* (or *star-free*) if and only if there is a natural number $k \geq 1$ such that, for any words $x \in V^*$, $y \in V^*$, and $z \in V^*$, it holds $xy^kz \in L$ if and only if $xy^{k+1}z \in L$,

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Let L be a language over an alphabet V.
We say that the language L – with respect to the alphabet V – is

- *power-separating* if and only if, there is a natural number $m \geq 1$ such that for any $x \in V^*$, either $J_x^m \cap L = \emptyset$ or $J_x^m \subseteq L$ where $J_x^m = \{x^n \mid n \geq m\}$,

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- *union-free* if and only if L can be described by a regular expression which is only built by product and star.

Further Subregular Language Families

$$REG_n^Z = \{ L \mid L \in REG \text{ with } State(L) \leq n \},$$

where

$$State(L) = \min \{ State(A) \mid A \text{ is a det. finite automaton accepting } L \},$$

with

$$State(A) = |Z|$$

Further Subregular Language Families

$$REG_n^Z = \{\, L \mid L \in REG \text{ with } State(L) \leq n \,\}\,,$$

$$RL_n^V = \{\, L \mid L \in REG \text{ with } Var_{RL}(L) \leq n \,\}\,,$$

where

$$State(L) = \min\{\, State(A) \mid A \text{ is a det. finite automaton accepting } L \,\}\,,$$

$$Var_{RL}(L) = \min\{\, Var(G) \mid G \text{ is a right-linear grammar generating } L \,\}\,,$$

with

$$State(A) = |Z|,\ Var(G) = |N|$$

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$$RL_n^P = \{ L \mid L \in REG \text{ with } Prod_{RL}(L) \leq n \},$$

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$$Var_{RL}(L) = \min \{ Var(G) \mid G \text{ is a right-linear grammar generating } L \},$$
$$Prod_{RL}(L) = \min \{ Prod(G) \mid G \text{ is a right-linear grammar generating } L \}.$$

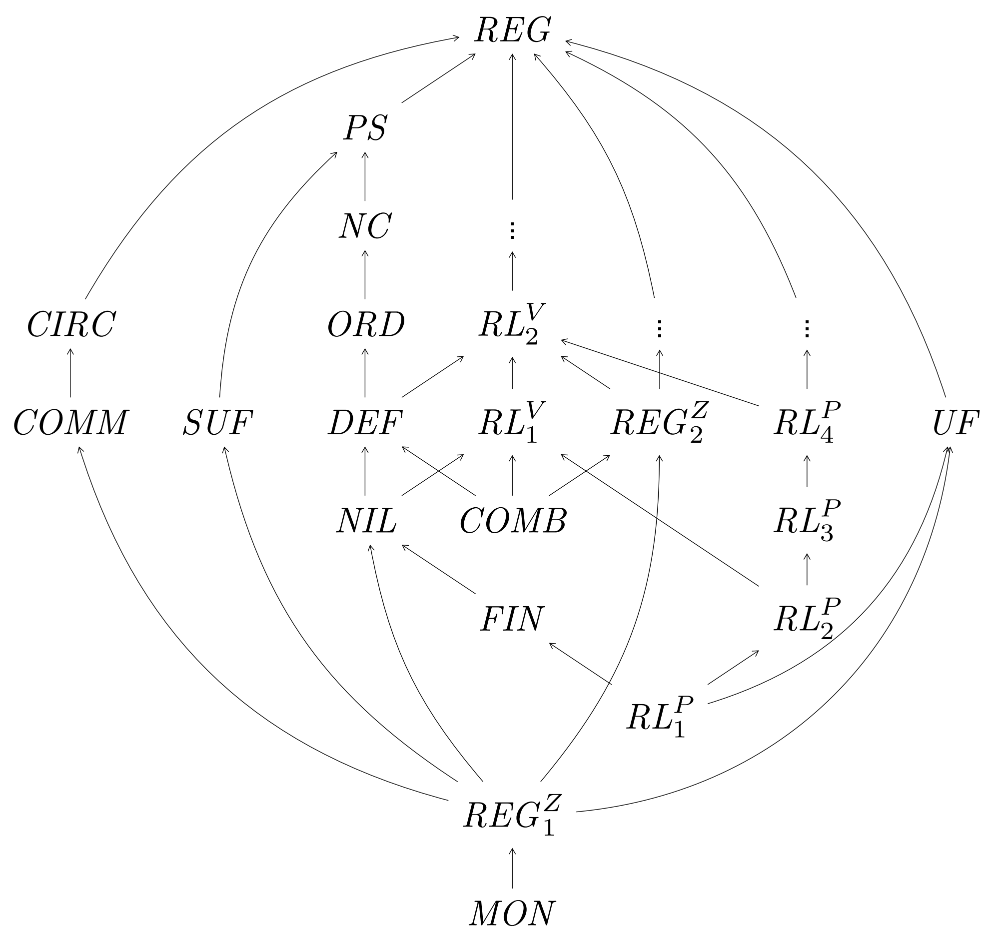
with

$$State(A) = |Z|, Var(G) = |N|, Prod(G) = |P|.$$

Families Under Consideration

$$\mathcal{F} \in \{FIN, MON, NIL, COMB, DEF, SUF, ORD\}$$
$$\cup \{COMM, CIRC, NC, PS, UF, REG\}$$
$$\cup \{\, REG_n^Z \mid n \geq 1 \,\} \cup \{\, RL_n^V \mid n \geq 1 \,\} \cup \{\, RL_n^P \mid n \geq 1 \,\}$$

Hierarchy of Subregular Language Families



EC Grammars with Subregular Selection – Previous Work

Jürgen Dassow: *FI* 2005.

$\mathcal{F} \in \{FIN, MON, COMB, NIL, DEF, COMM, SUF\}$
(finite, monoidal, combinational, nilpotent, definite, commutative, suffix-closed)

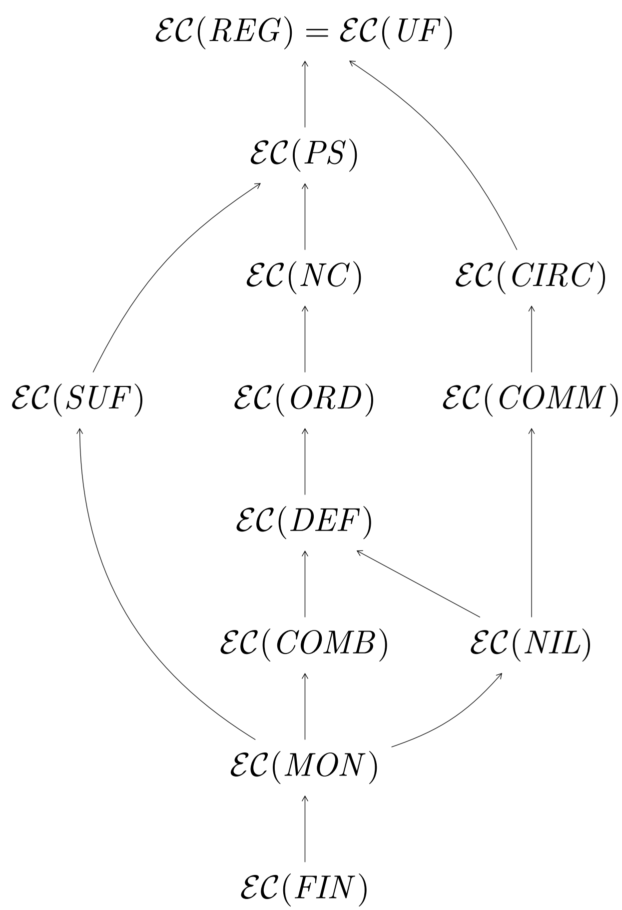
Jürgen Dassow, Florin Manea, Bianca Truthe: *TCS* 2012 (DCFS 2011)

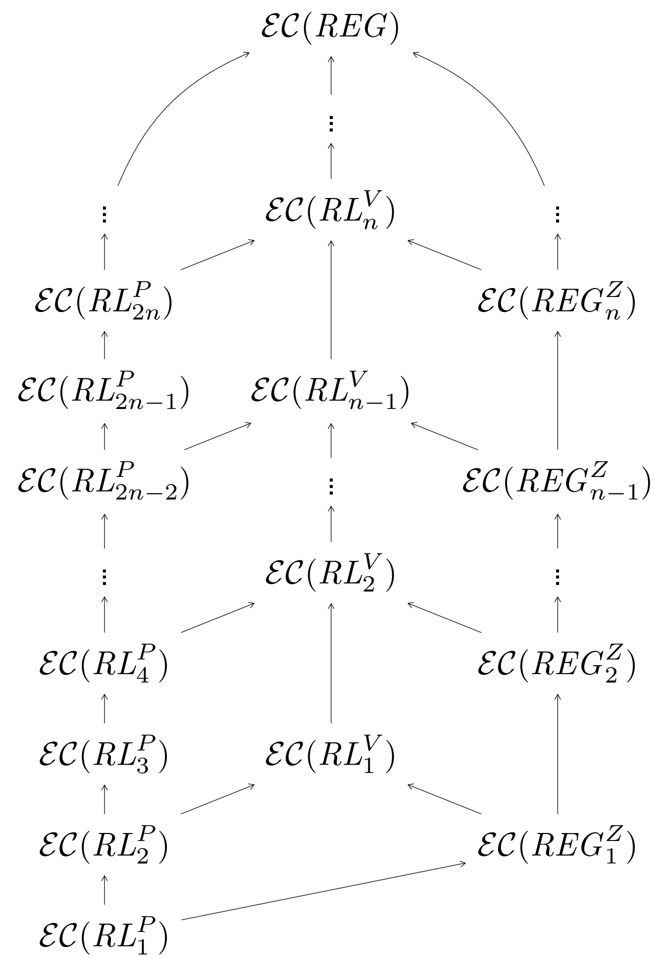
$\mathcal{F} \in \{CIRC, ORD, UF\} \cup \{ REG_n^Z, RL_n^V, RL_n^P \mid n \geq 1 \}$
(circular, ordered, union-free)

Bianca Truthe: *FI* 2021 (NCMA 2017)

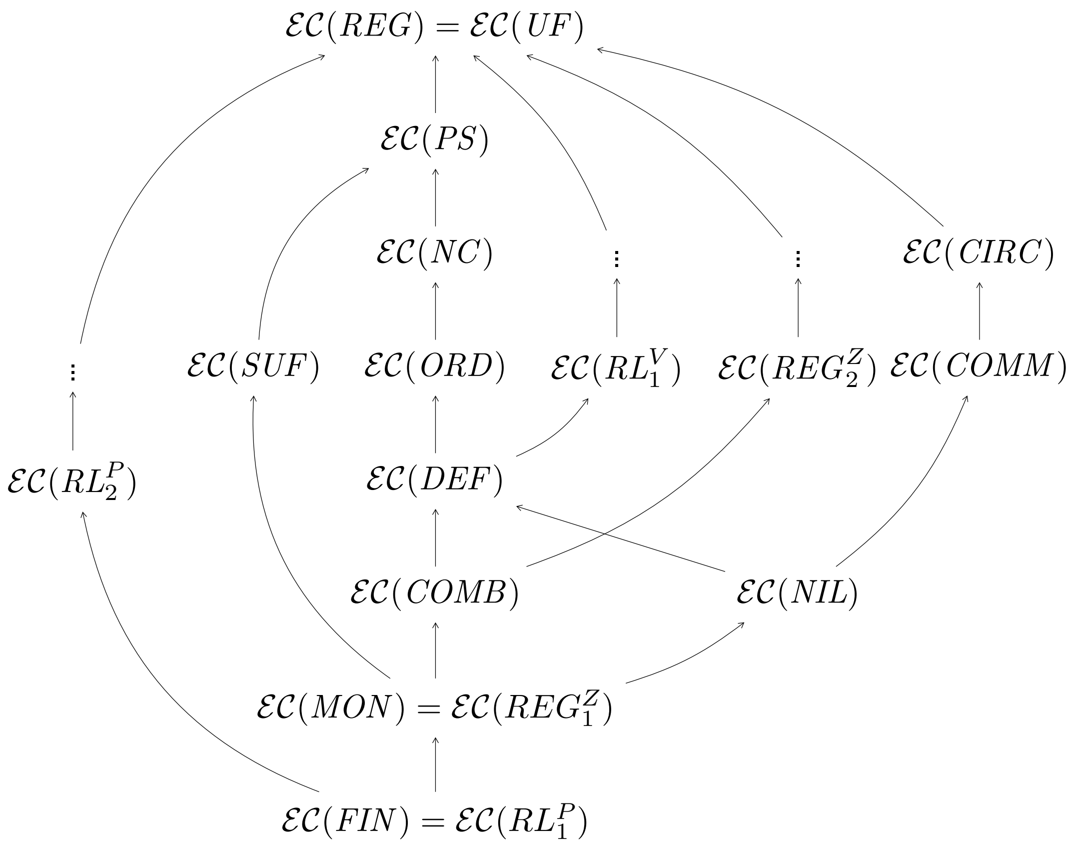
$\mathcal{F} \in \{ REG_n^Z, RL_n^V, RL_n^P \mid n \geq 1 \}$, Survey

Previous Work – Two Hierarchies





Results - One Hierarchy



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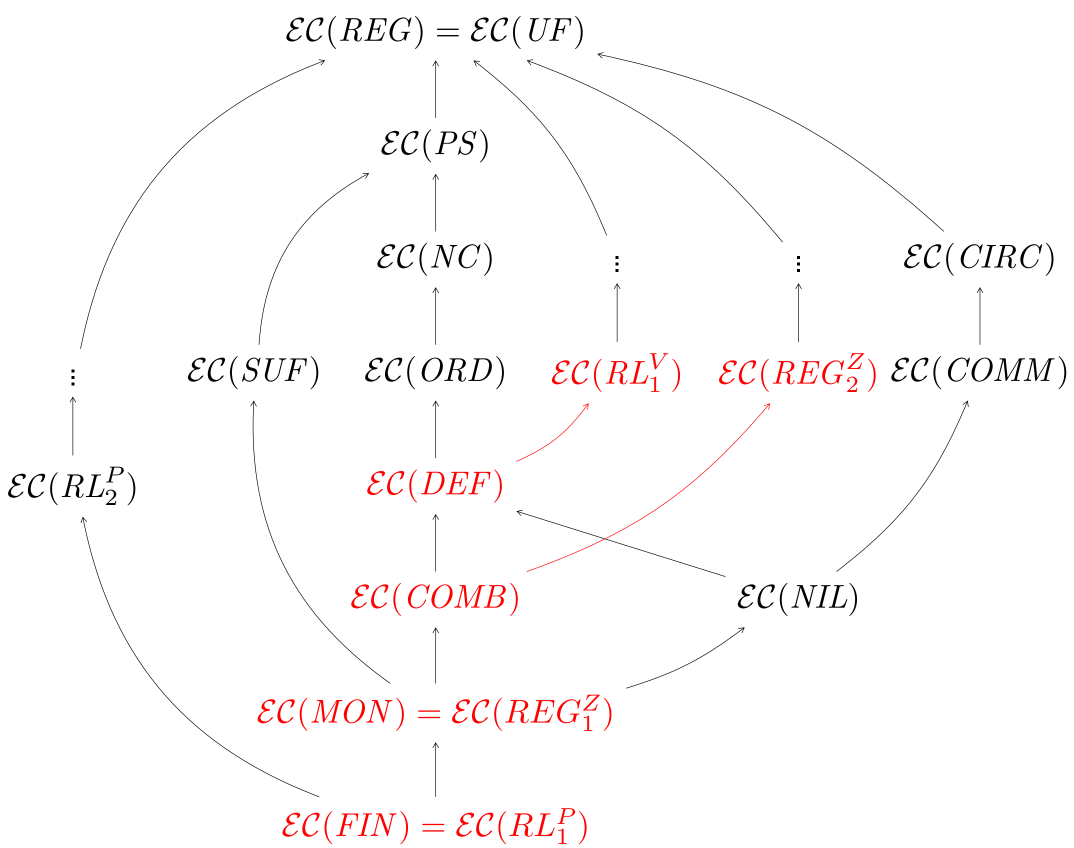

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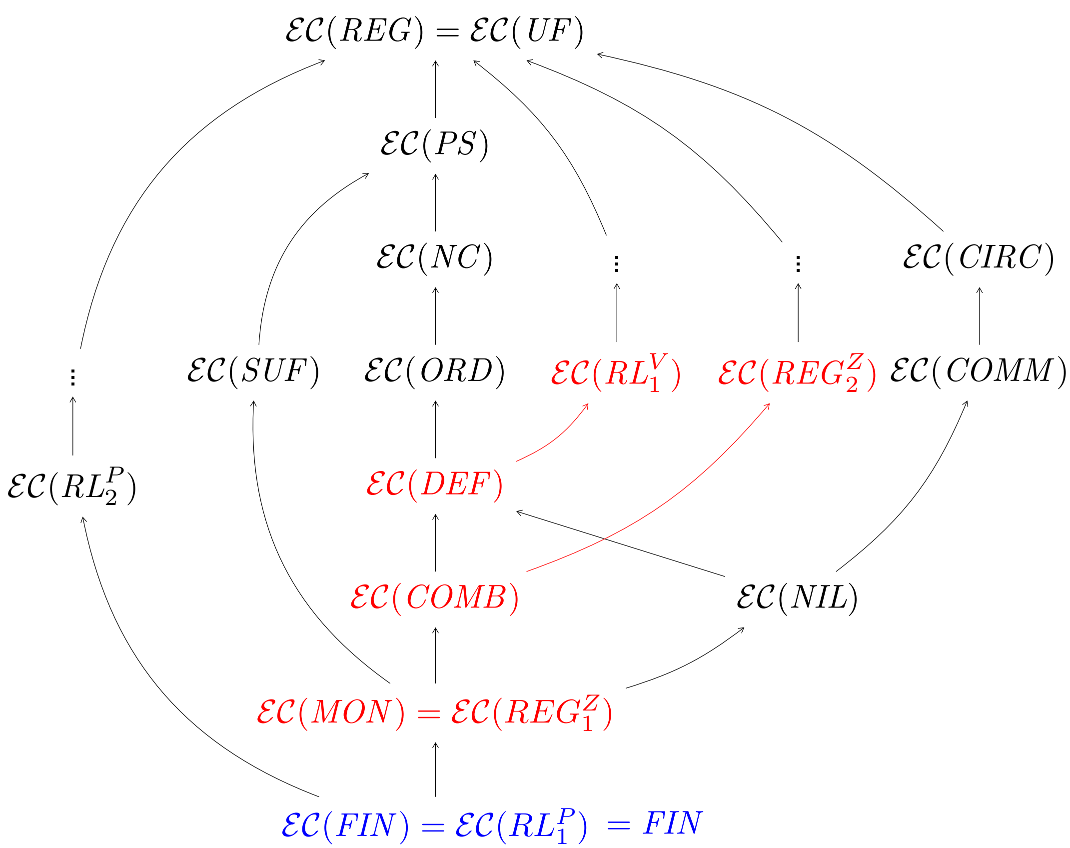

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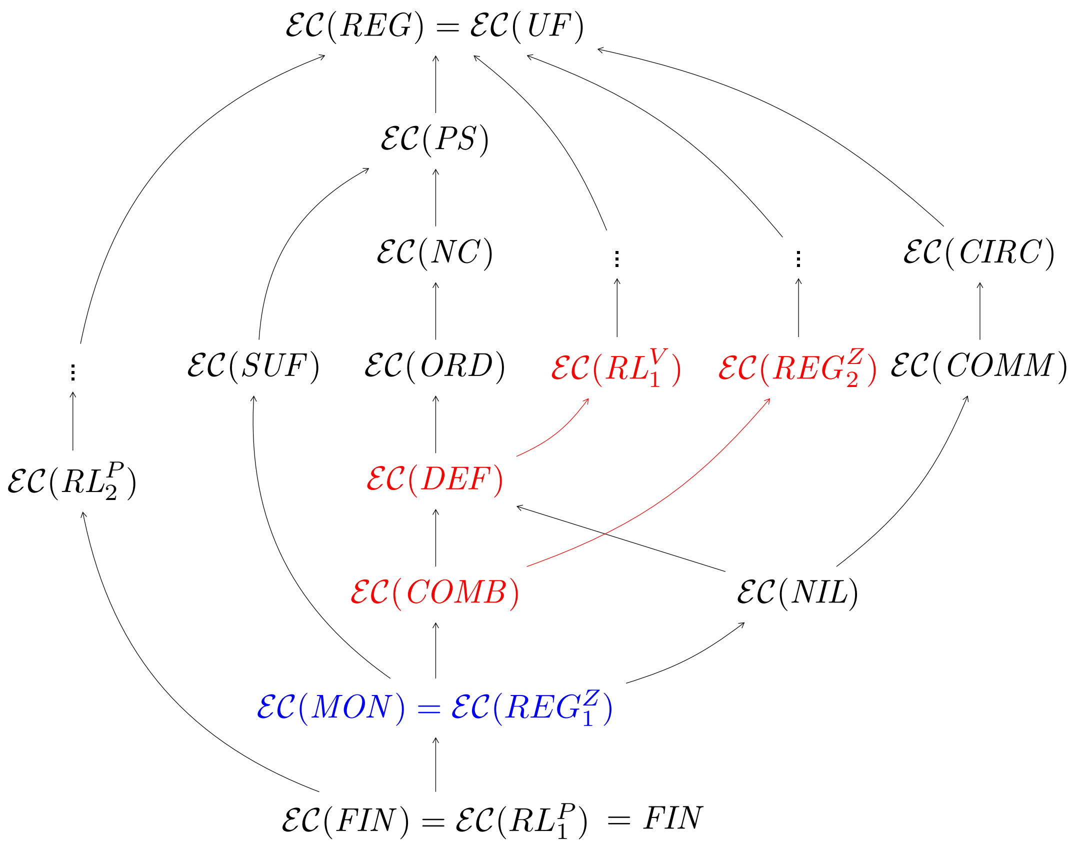

$$REG_1^Z = MON \cup \{\emptyset\}$$

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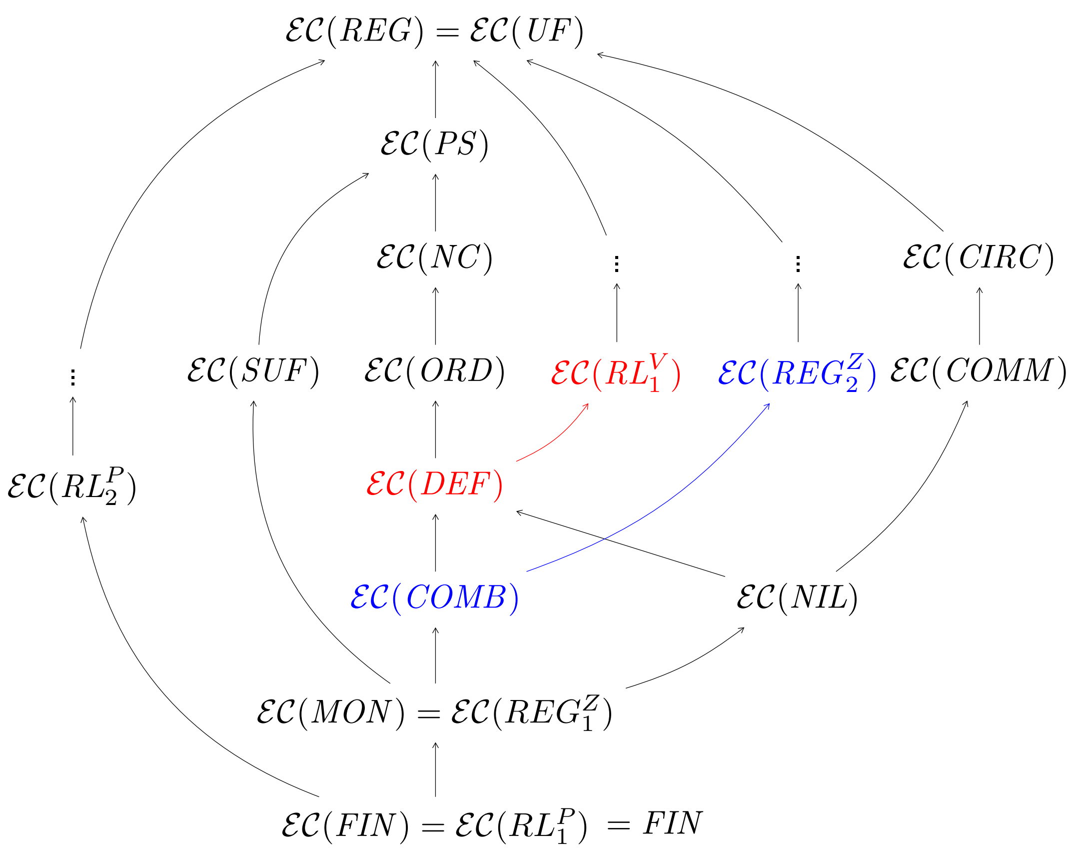

Comb. Lang.: V^*A with $A \subseteq V$

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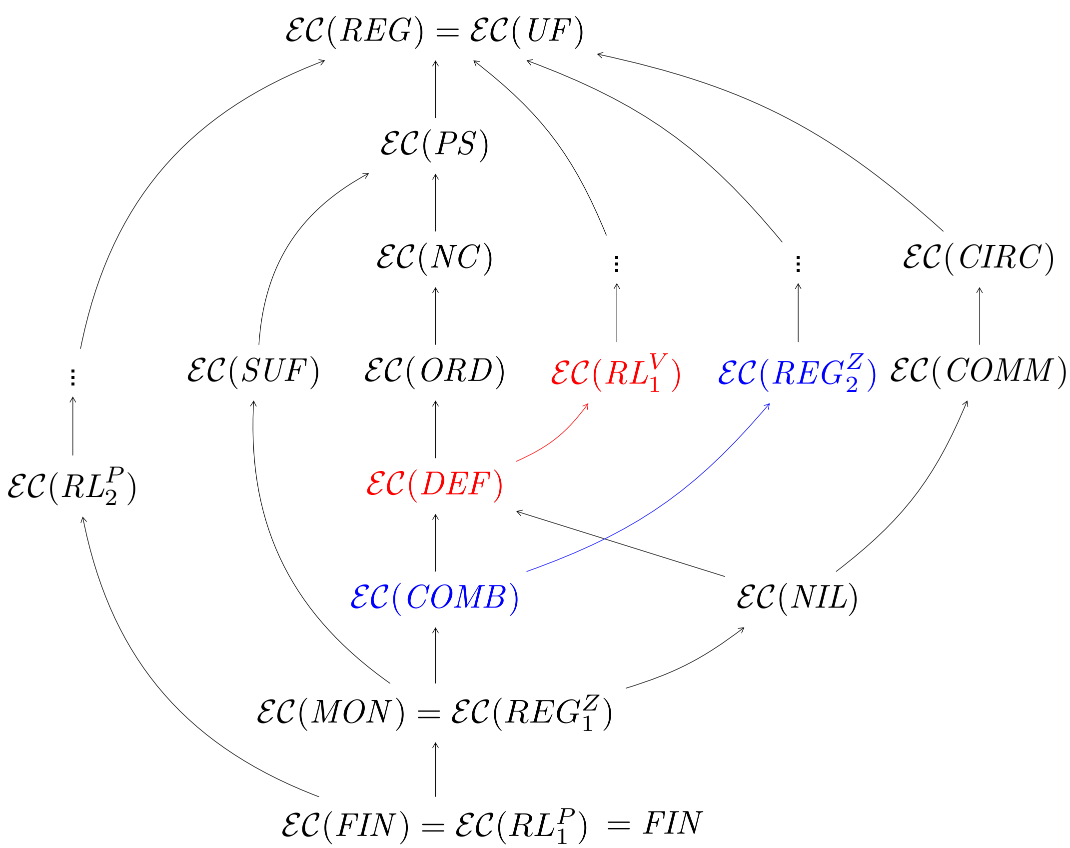

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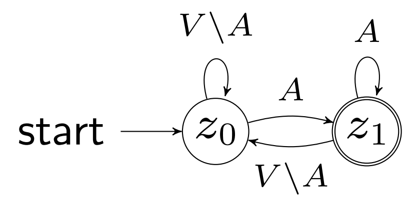

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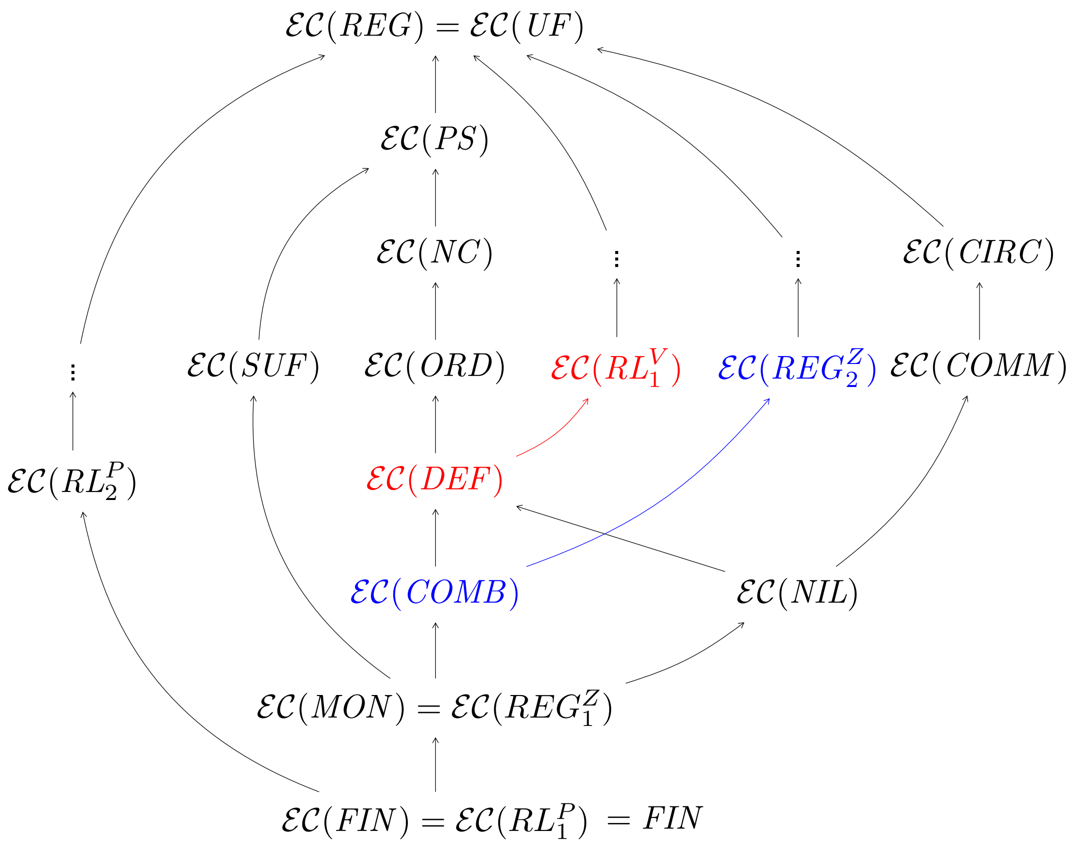

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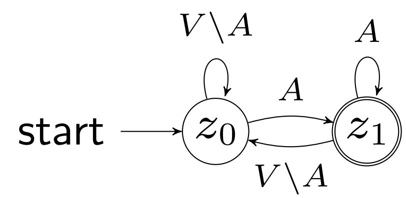

$\rightsquigarrow \mathcal{EC}(COMB) \subseteq \mathcal{EC}(REG_2^Z)$

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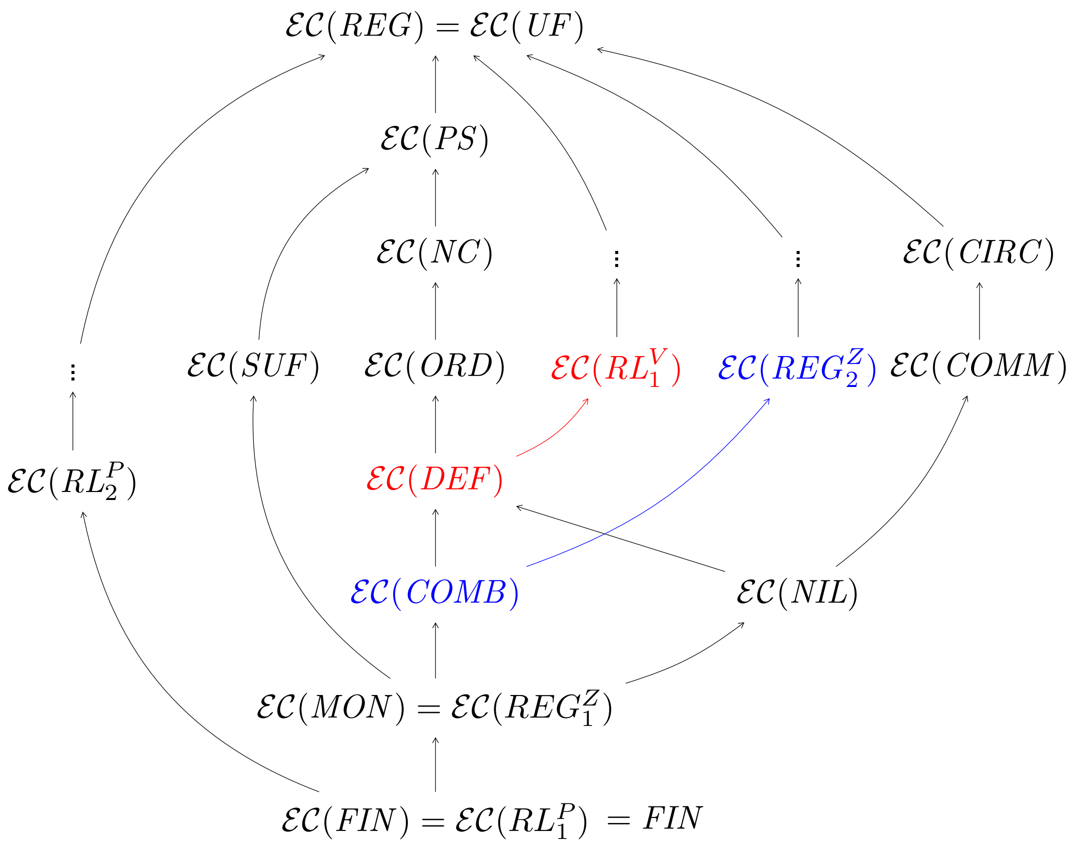

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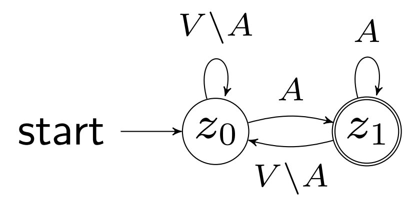

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$$L = \{a\}^+ \cup \{a\}^*\{b\}\{a,b\}^* \cup \{c\}\{a\}^*\{b\}\{a,b\}^*\{c\} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(COMB)$$

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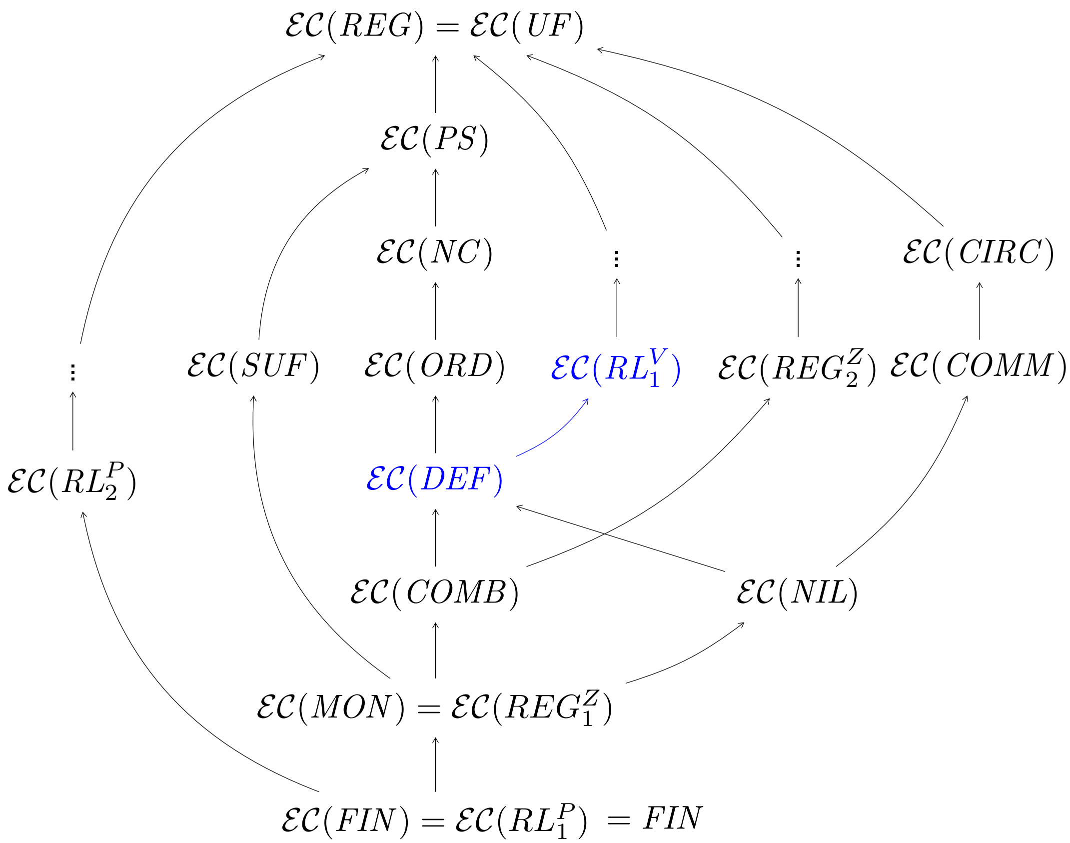

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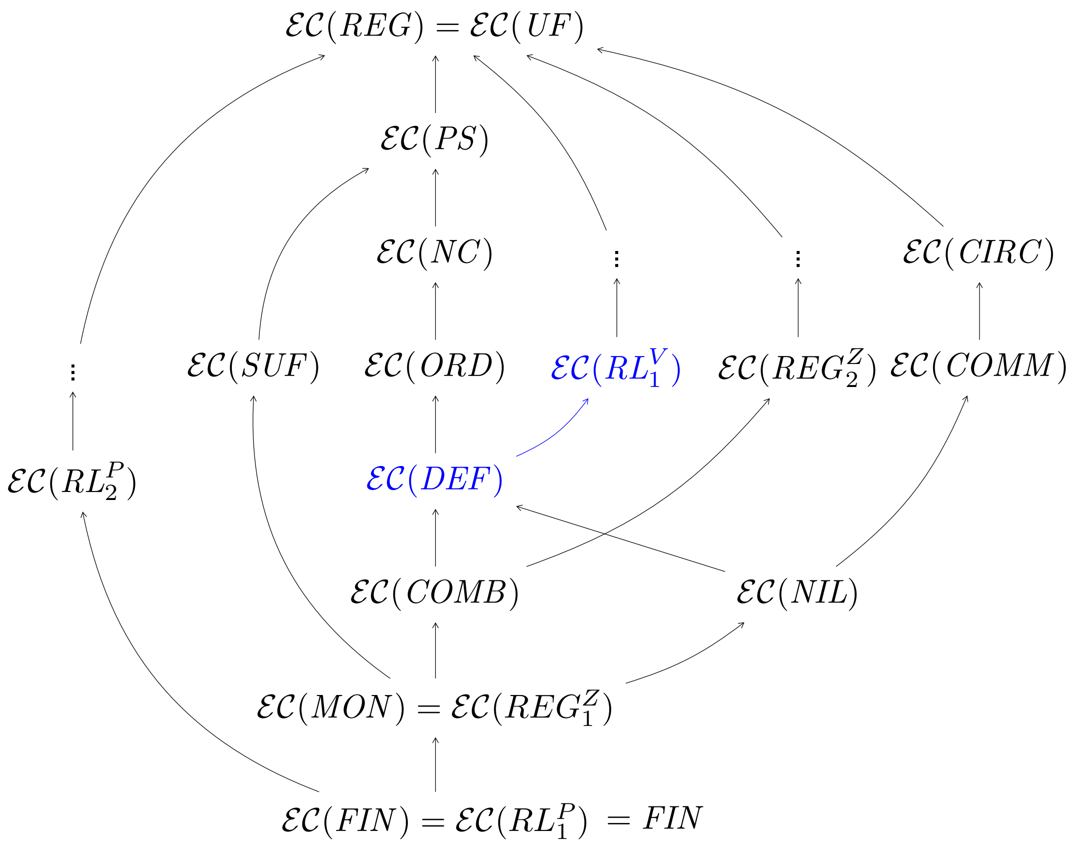

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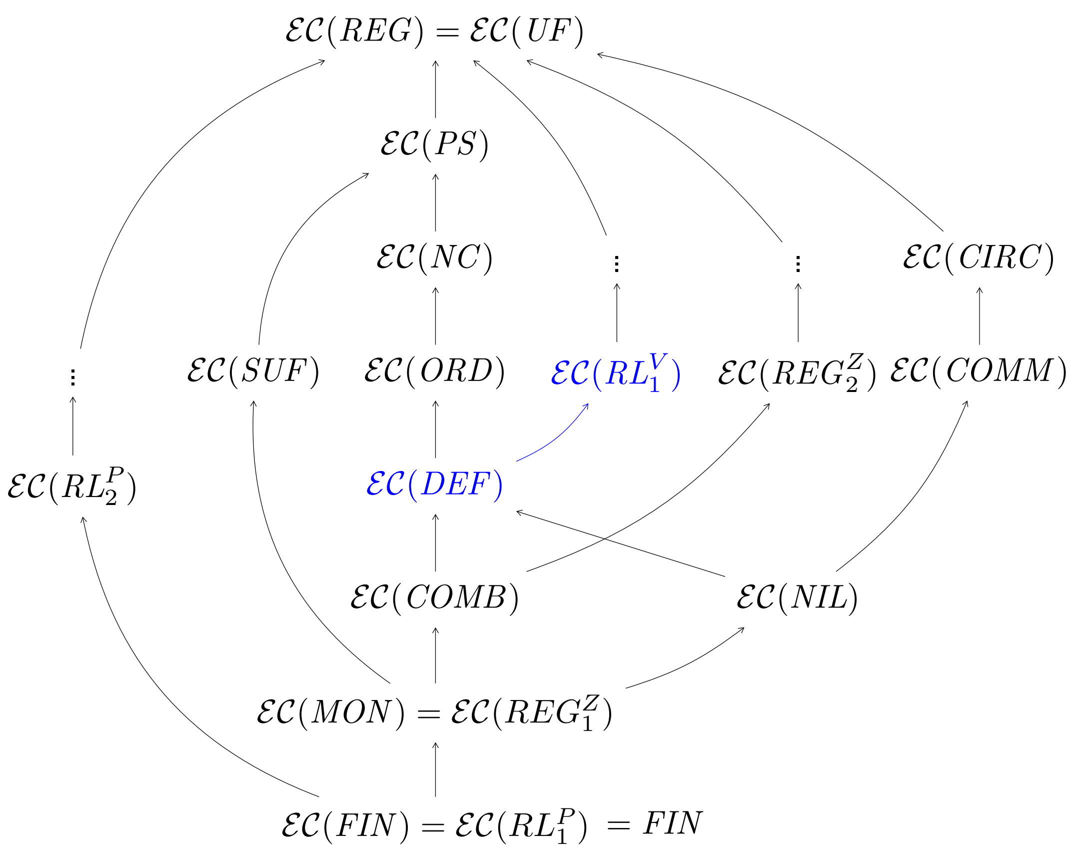

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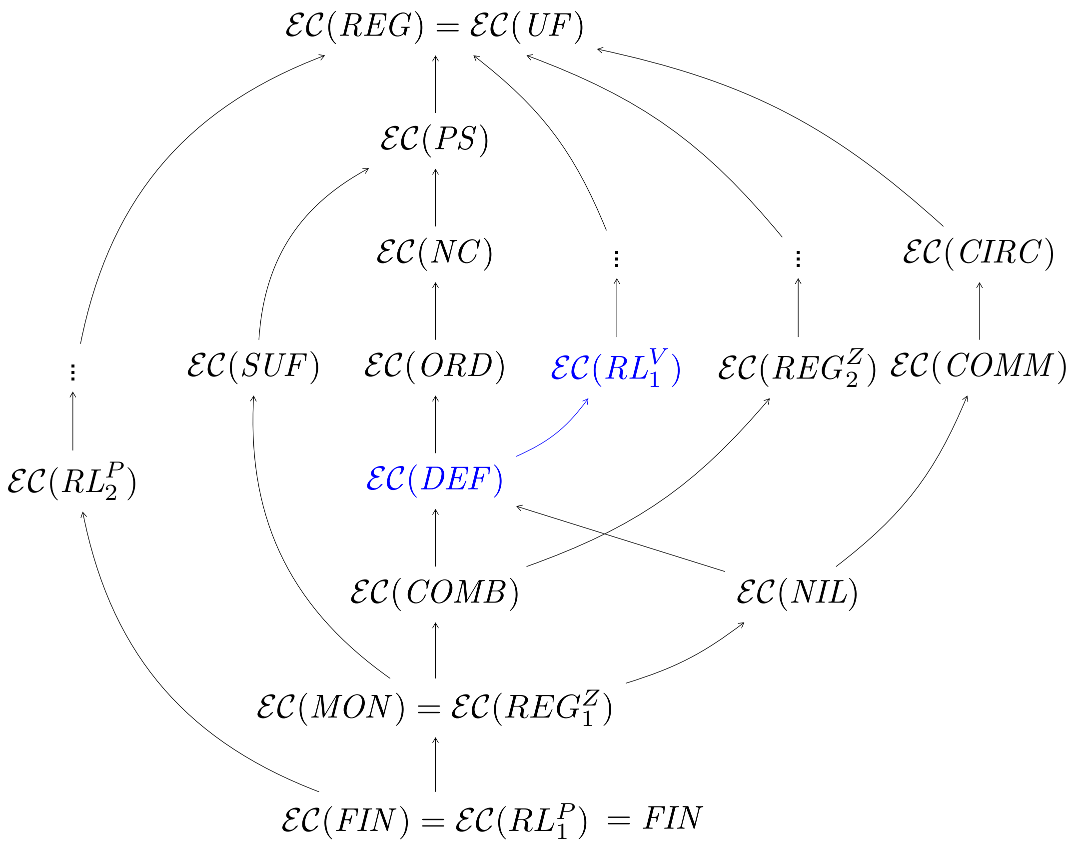

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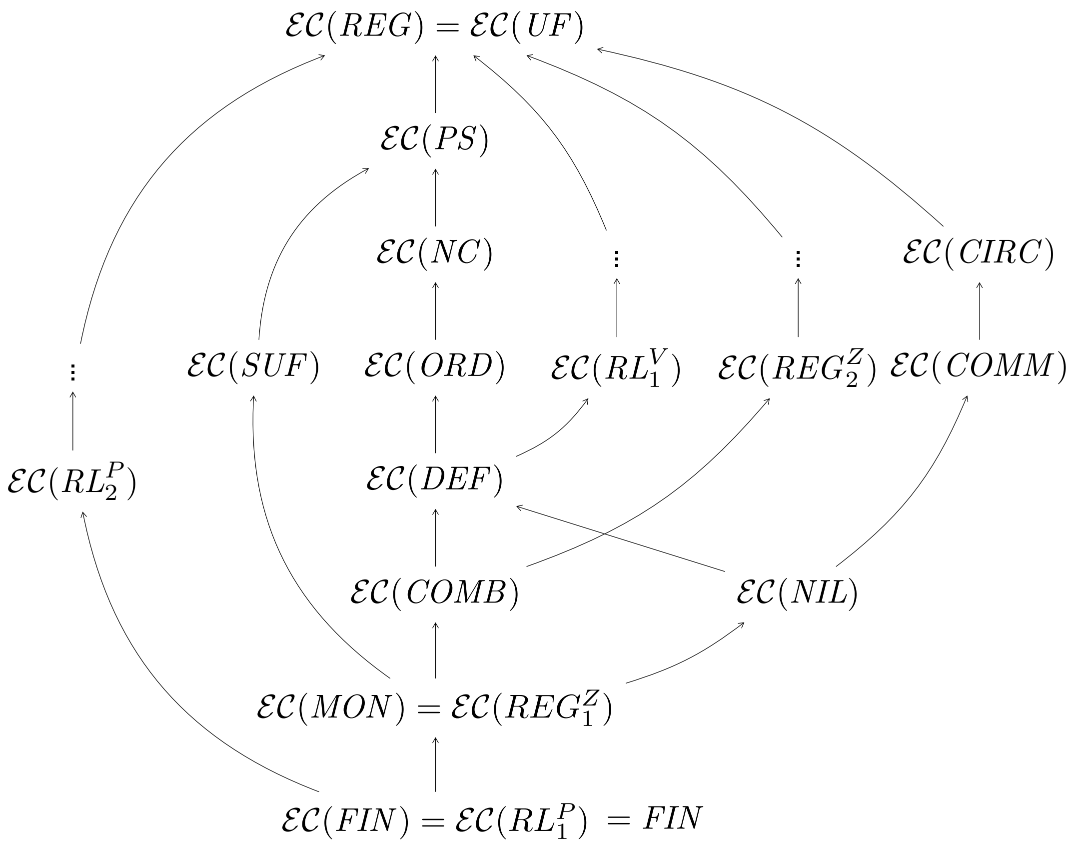
$$L = \{ a^n \mid n \geq 1 \} \cup \{ ba^{2n}b \mid n \geq 1 \} \in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(PS)$$

Results - One Hierarchy: Incomparabilities



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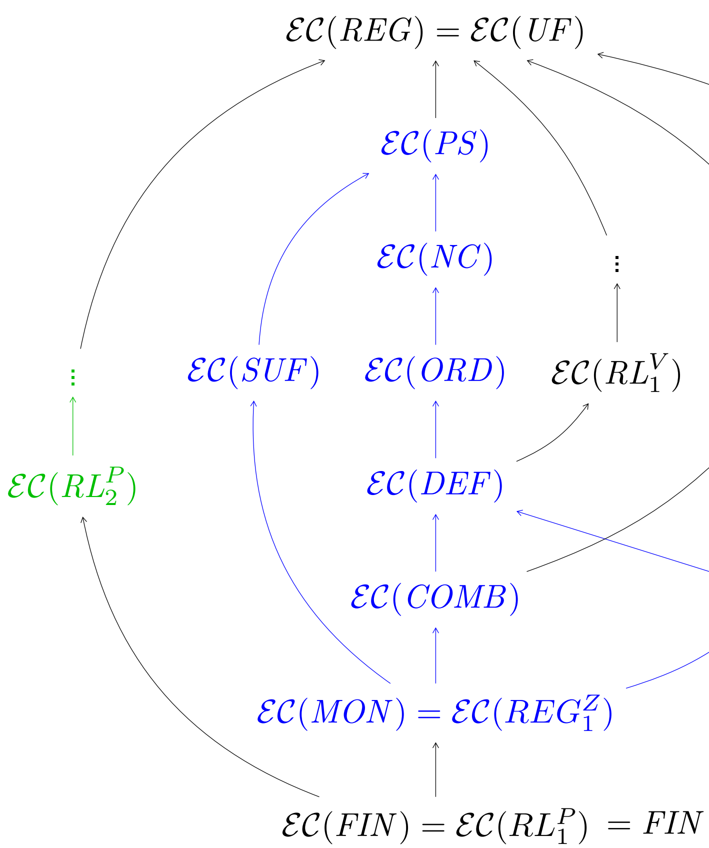

$\mathcal{EC}(RL_n^P)$ with $n \geq 2$ to

$\mathcal{EC}(MON) \ldots \mathcal{EC}(PS)$

$\mathcal{EC}(MON) \ldots \mathcal{EC}(CIRC)$

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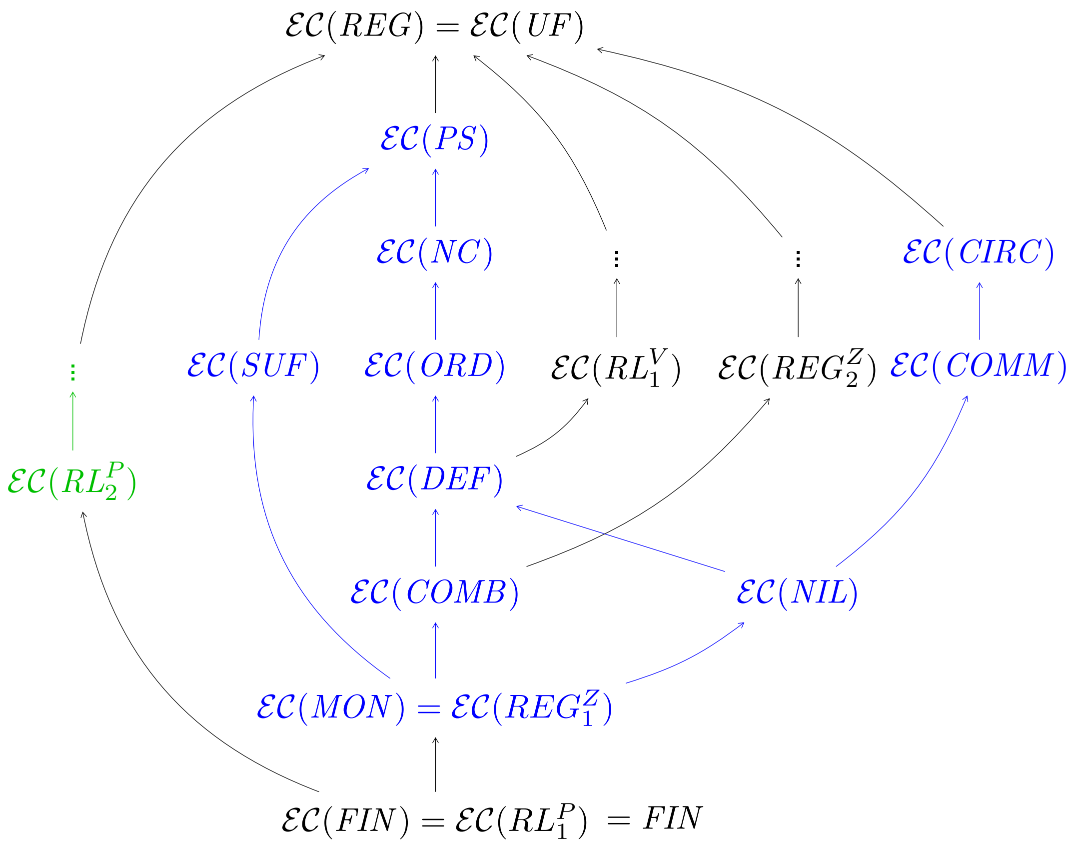

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$$L_{RL_2^P,\neg PS} = \{\, a^n \mid n \geq 1 \,\} \cup \{\, ba^{2n}b \mid n \geq 1 \,\} \in \mathcal{EC}(RL_2^P) \setminus \mathcal{EC}(PS)$$

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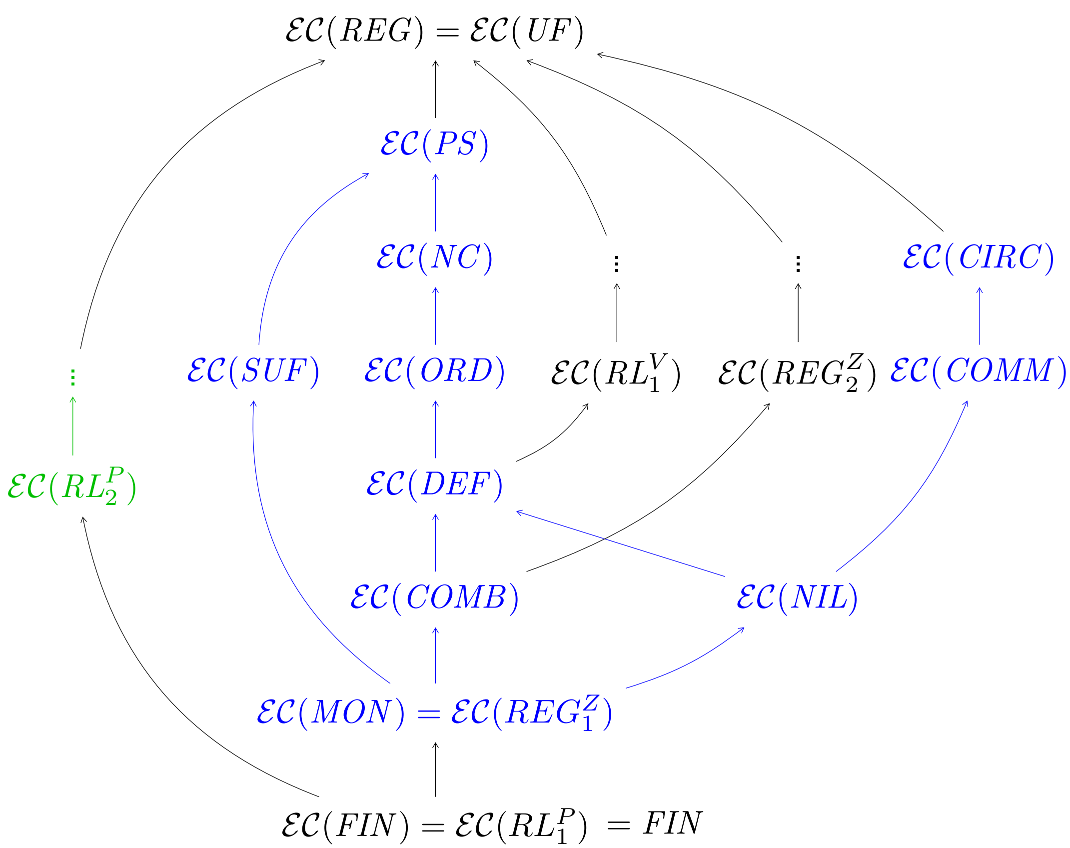

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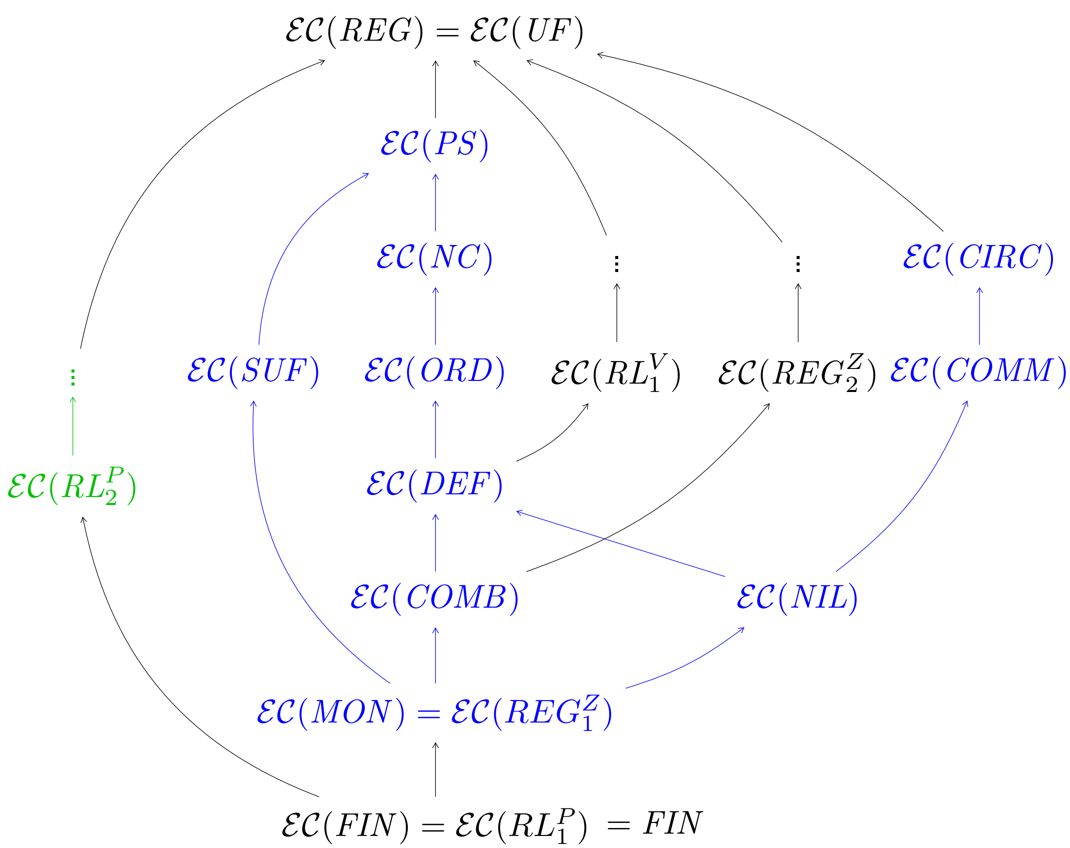

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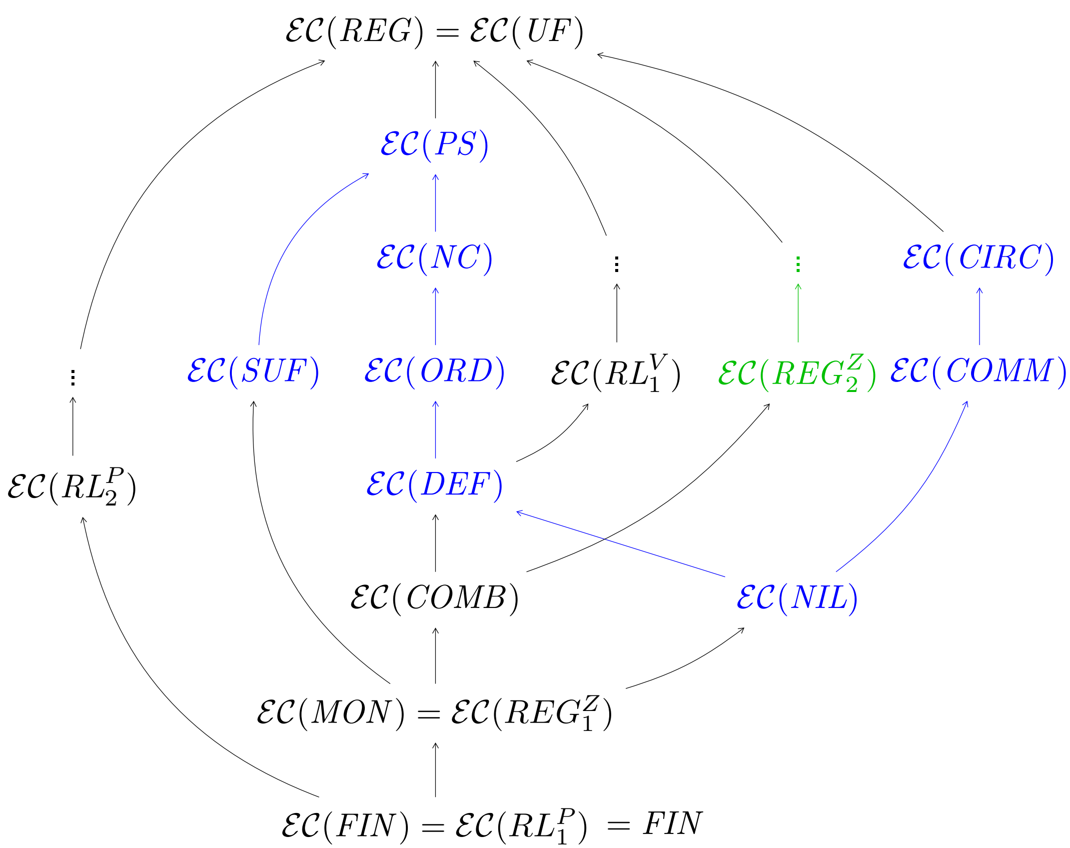
$$L_{MON,\neg RL_n^P} = \{a_1, \ldots, a_n\}^* \in \mathcal{EC}(MON) \setminus \mathcal{EC}(RL_n^P)$$

Results - One Hierarchy: Incomparabilities



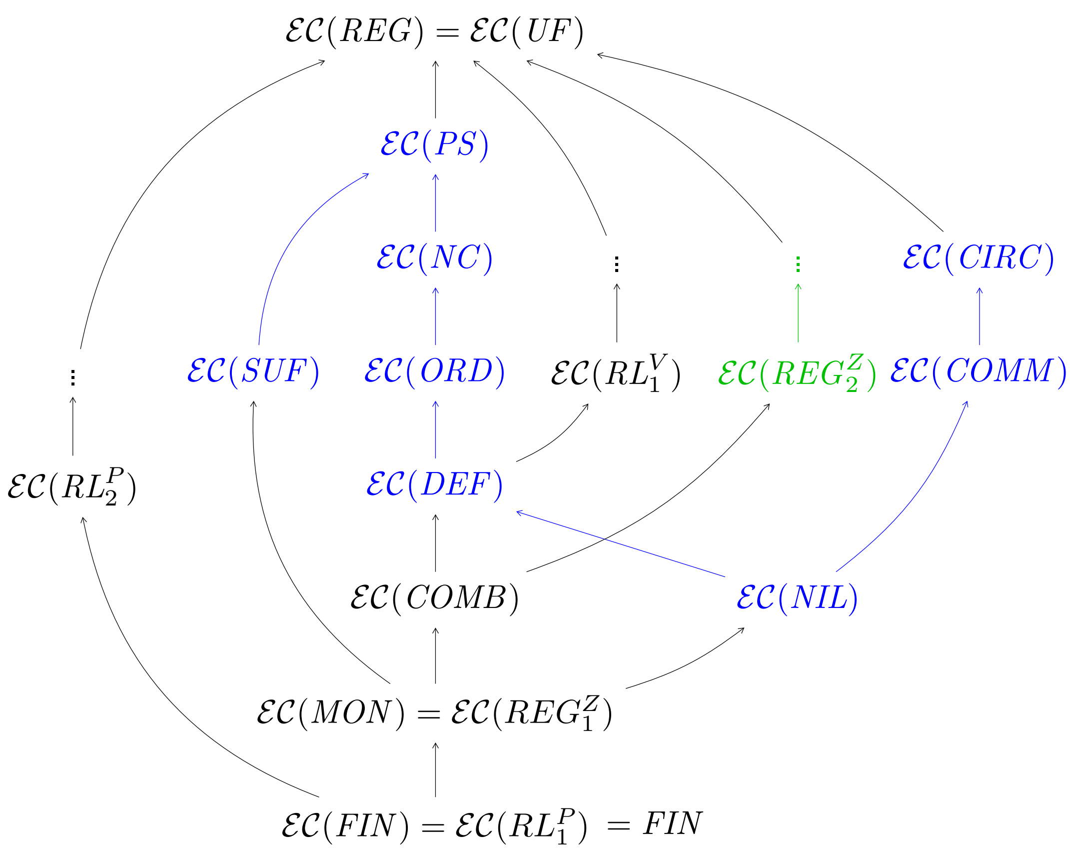
$\mathcal{EC}(REG_n^Z)$ with $n \geq 2$ to

$\mathcal{EC}(NIL) \ldots \mathcal{EC}(PS)$

$\mathcal{EC}(NIL) \ldots \mathcal{EC}(CIRC)$

$\mathcal{EC}(SUF)$

Results - One Hierarchy: Incomparabilities



$\mathcal{EC}(REG_n^Z)$ with $n \geq 2$ to

$\mathcal{EC}(NIL) \ldots \mathcal{EC}(PS)$

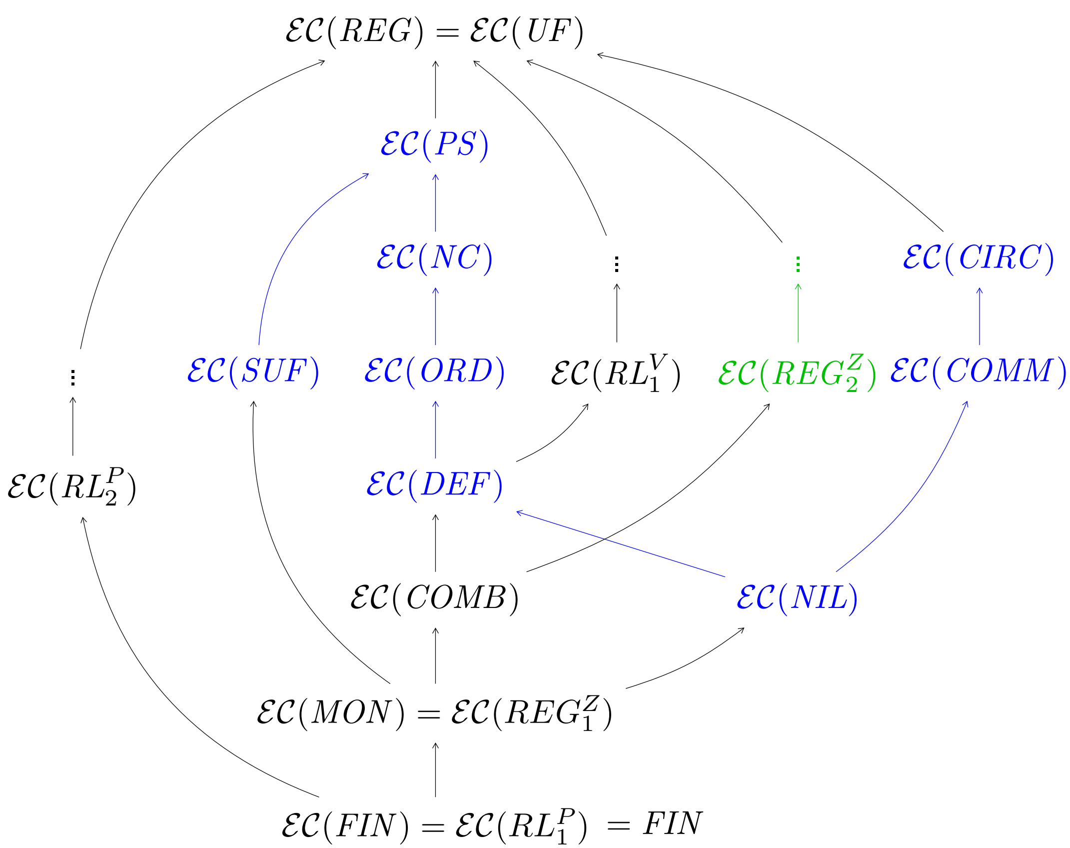
$\mathcal{EC}(NIL) \ldots \mathcal{EC}(CIRC)$

$\mathcal{EC}(SUF)$

$L_{REG_2^Z,\neg PS}$

$\in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(PS)$

Results - One Hierarchy: Incomparabilities



$\mathcal{EC}(REG_n^Z)$ with $n \geq 2$ to

$\mathcal{EC}(NIL) \ldots \mathcal{EC}(PS)$

$\mathcal{EC}(NIL) \ldots \mathcal{EC}(CIRC)$

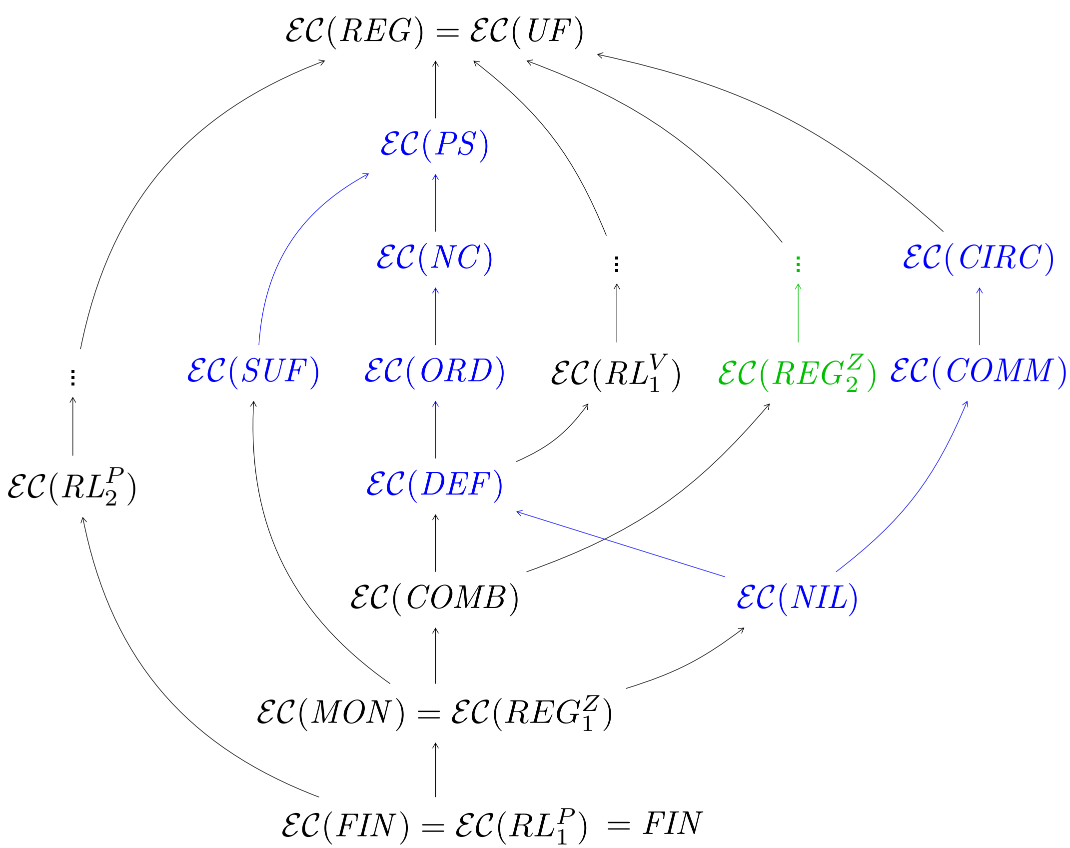
$\mathcal{EC}(SUF)$

$L_{REG_2^Z,\neg PS}$

$\in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(PS)$

$$L_{REG_2^Z,\neg PS} = \{ a^n \mid n \geq 1 \} \cup \{ ba^{2n}b \mid n \geq 1 \}$$

Results - One Hierarchy: Incomparabilities



$\mathcal{EC}(REG_n^Z)$ with $n \geq 2$ to

$\mathcal{EC}(NIL) \ldots \mathcal{EC}(PS)$

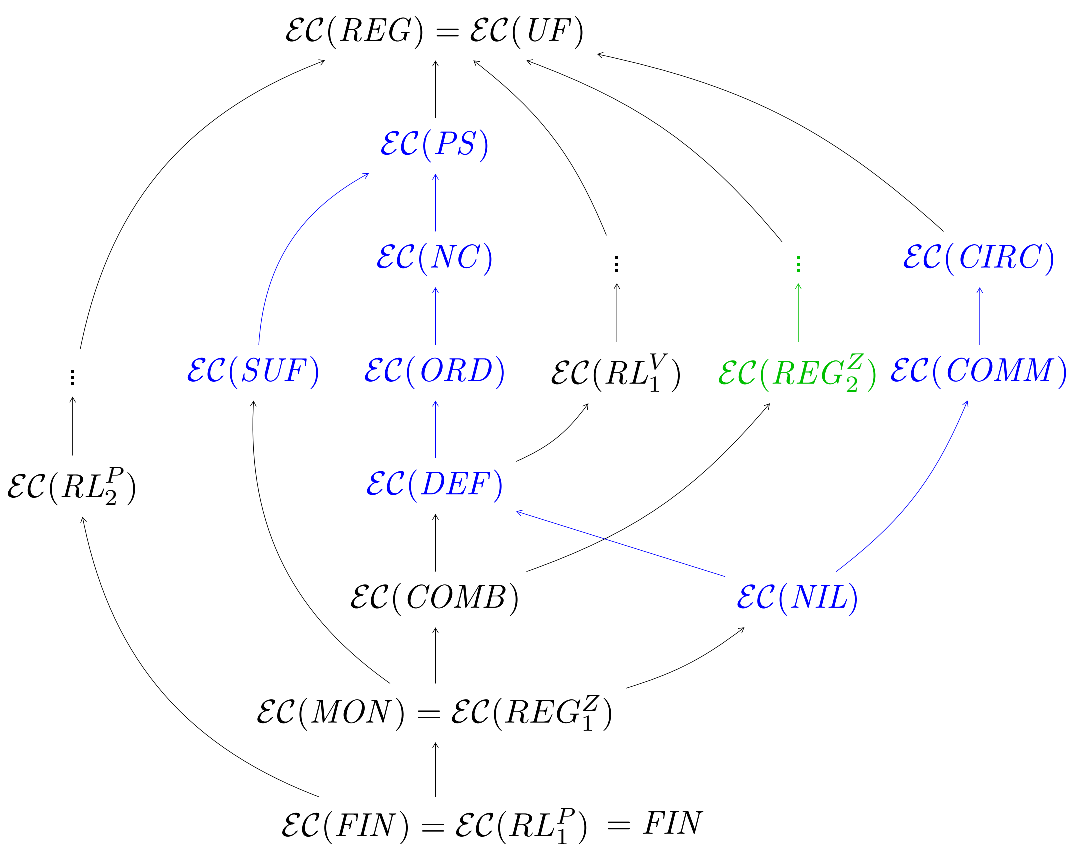
$\mathcal{EC}(NIL) \ldots \mathcal{EC}(CIRC)$

$\mathcal{EC}(SUF)$

$L_{REG_2^Z,\neg PS} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(PS)$

$L_{REG_2^Z,\neg CIRC} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(CIRC)$

Results - One Hierarchy: Incomparabilities



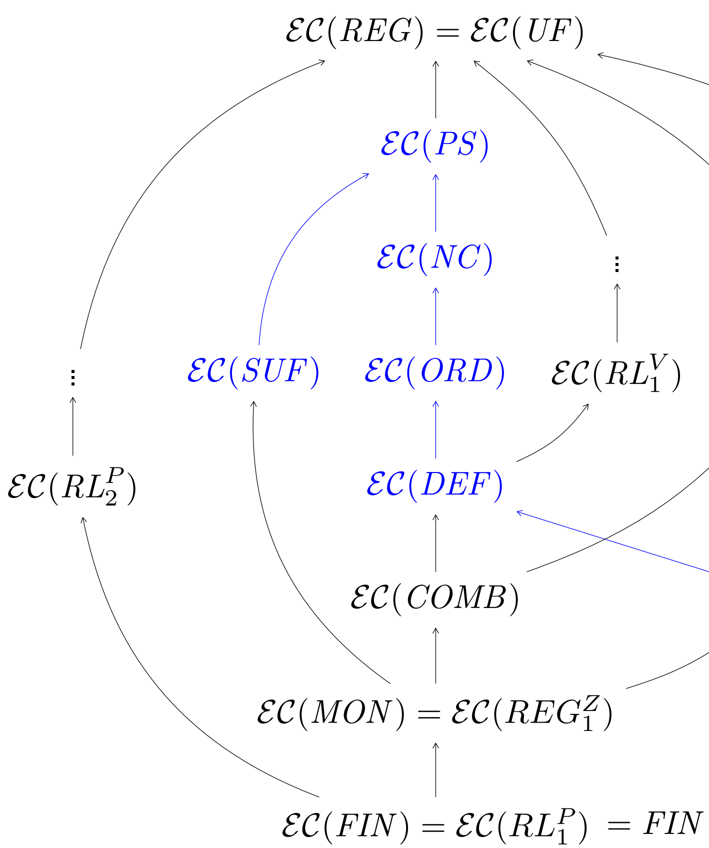
$\mathcal{EC}(REG_n^Z)$ with $n \geq 2$ to
- $\mathcal{EC}(NIL) \ldots \mathcal{EC}(PS)$
- $\mathcal{EC}(NIL) \ldots \mathcal{EC}(CIRC)$
- $\mathcal{EC}(SUF)$

$L_{REG_2^Z,\neg PS} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(PS)$

$L_{REG_2^Z,\neg CIRC} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(CIRC)$

$$L_{REG_2^Z,\neg CIRC} = \{ab\}^+ \cup \{ba\}^+ \cup \{\, c(ba)^n c \mid n \geq 1 \,\}$$

Results - One Hierarchy: Incomparabilities



$\mathcal{EC}(REG_n^Z)$ with $n \geq 2$ to

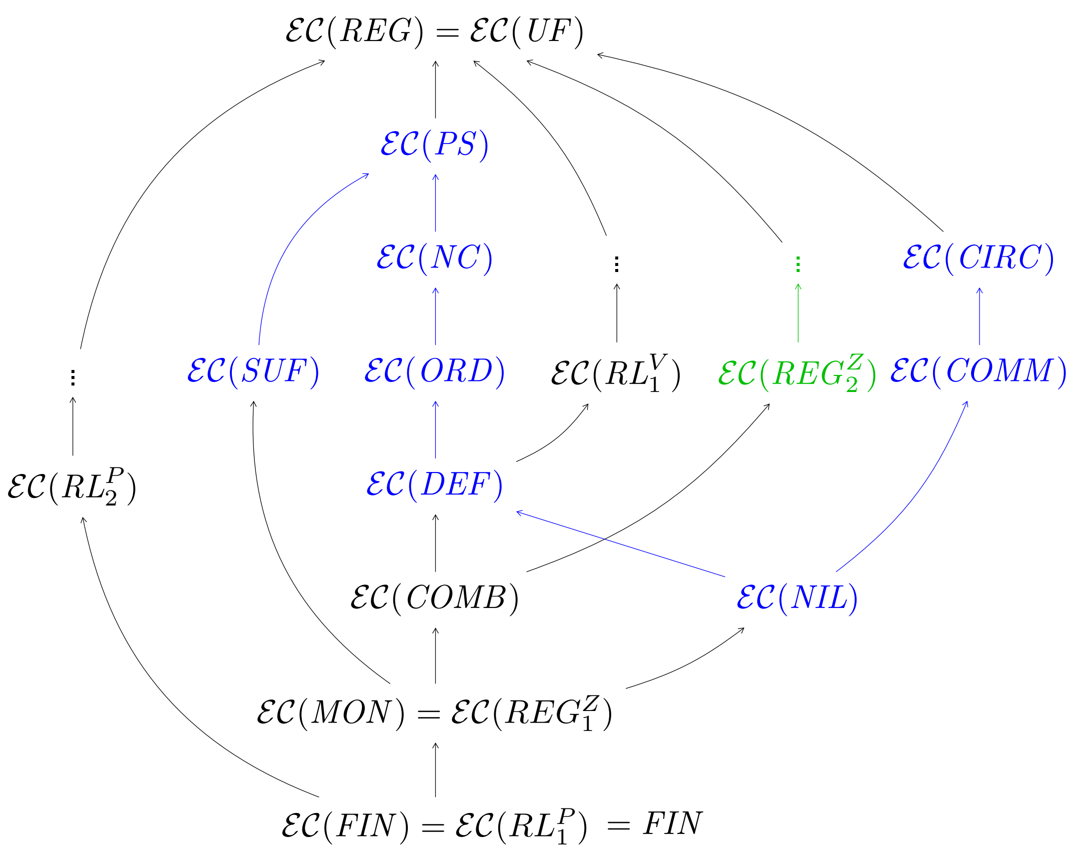
- $\mathcal{EC}(NIL) \ldots \mathcal{EC}(PS)$
- $\mathcal{EC}(NIL) \ldots \mathcal{EC}(CIRC)$
- $\mathcal{EC}(SUF)$

$L_{REG_2^Z,\neg PS} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(PS)$

$L_{REG_2^Z,\neg CIRC} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(CIRC)$

$L_{NIL,\neg REG_n^Z} \in \mathcal{EC}(NIL) \setminus \mathcal{EC}(REG_n^Z)$

Results - One Hierarchy: Incomparabilities



$\mathcal{EC}(REG_n^Z)$ with $n \geq 2$ to

- $\mathcal{EC}(NIL) \ldots \mathcal{EC}(PS)$
- $\mathcal{EC}(NIL) \ldots \mathcal{EC}(CIRC)$
- $\mathcal{EC}(SUF)$

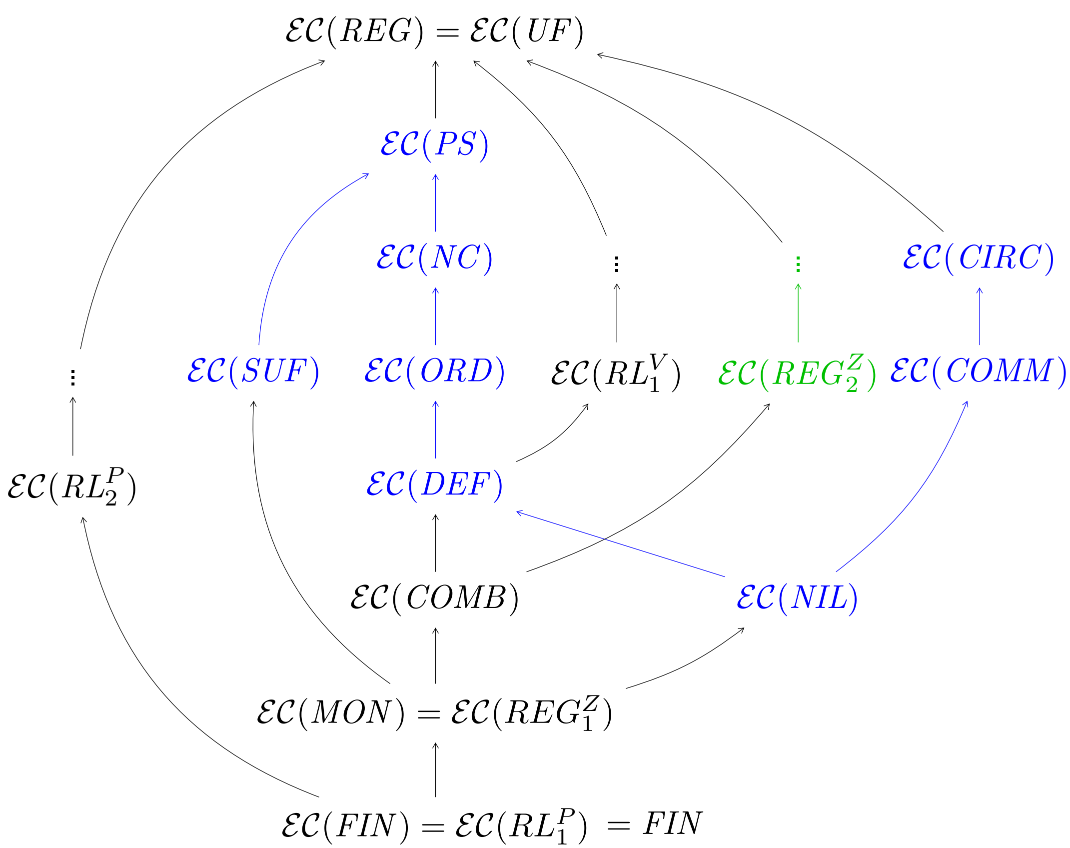
$L_{REG_2^Z,\neg PS} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(PS)$

$L_{REG_2^Z,\neg CIRC} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(CIRC)$

$L_{NIL,\neg REG_n^Z} \in \mathcal{EC}(NIL) \setminus \mathcal{EC}(REG_n^Z)$

$$L_{NIL,\neg REG_n^Z} = \{\, a^i \mid 1 \leq i \leq n \,\} \cup \{\, a^{k(n+1)} \mid k \geq 2 \,\}$$

Results - One Hierarchy: Incomparabilities



$\mathcal{EC}(REG_n^Z)$ with $n \geq 2$ to
- $\mathcal{EC}(NIL) \ldots \mathcal{EC}(PS)$
- $\mathcal{EC}(NIL) \ldots \mathcal{EC}(CIRC)$
- $\mathcal{EC}(SUF)$

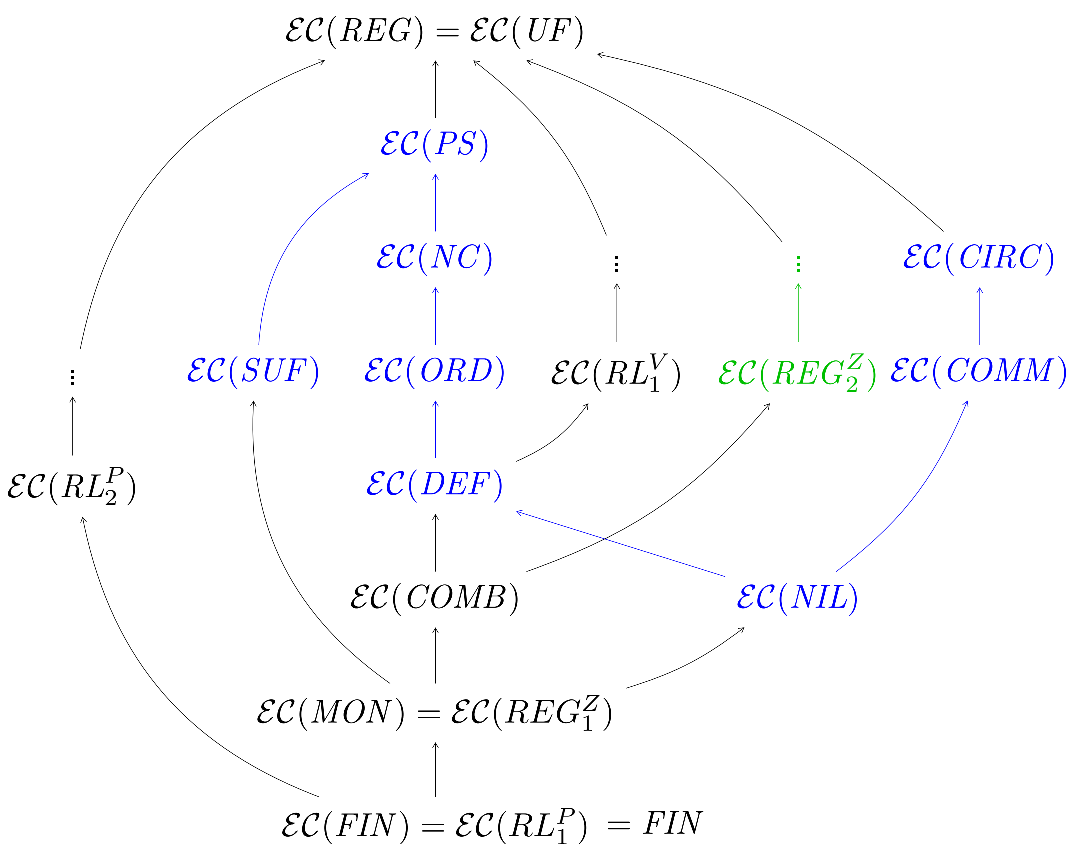
$L_{REG_2^Z,\neg PS} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(PS)$

$L_{REG_2^Z,\neg CIRC} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(CIRC)$

$L_{NIL,\neg REG_n^Z} \in \mathcal{EC}(NIL) \setminus \mathcal{EC}(REG_n^Z)$

$L_{SUF,\neg REG_n^Z} \in \mathcal{EC}(SUF) \setminus \mathcal{EC}(REG_n^Z)$

Results - One Hierarchy: Incomparabilities



$\mathcal{EC}(REG_n^Z)$ with $n \geq 2$ to

$\mathcal{EC}(NIL) \ldots \mathcal{EC}(PS)$
$\mathcal{EC}(NIL) \ldots \mathcal{EC}(CIRC)$
$\mathcal{EC}(SUF)$

$L_{REG_2^Z,\neg PS} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(PS)$

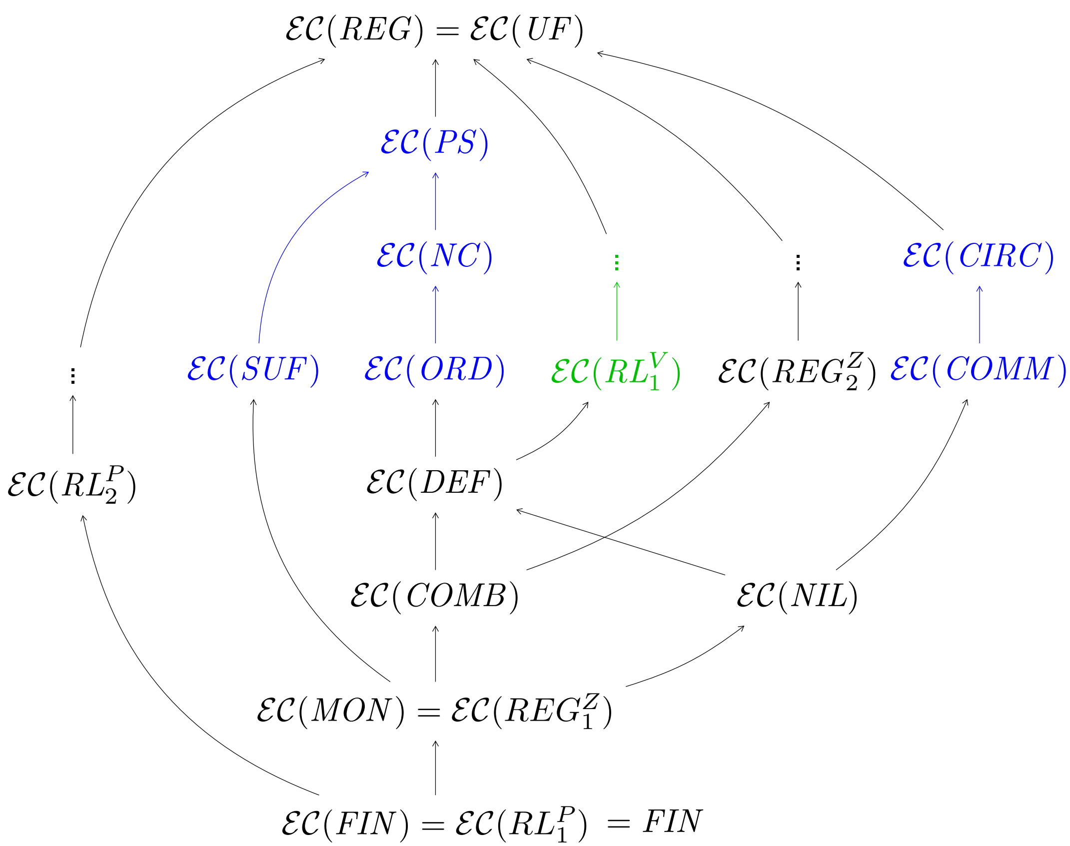
$L_{REG_2^Z,\neg CIRC} \in \mathcal{EC}(REG_2^Z) \setminus \mathcal{EC}(CIRC)$

$L_{NIL,\neg REG_n^Z} \in \mathcal{EC}(NIL) \setminus \mathcal{EC}(REG_n^Z)$

$L_{SUF,\neg REG_n^Z} \in \mathcal{EC}(SUF) \setminus \mathcal{EC}(REG_n^Z)$

$$L_{SUF,\neg REG_n^Z} = \{\, ba^i \mid 1 \leq i \leq n \,\} \cup \{\, ba^{k(n+1)} \mid k \geq 2 \,\}$$

Results - One Hierarchy: Incomparabilities



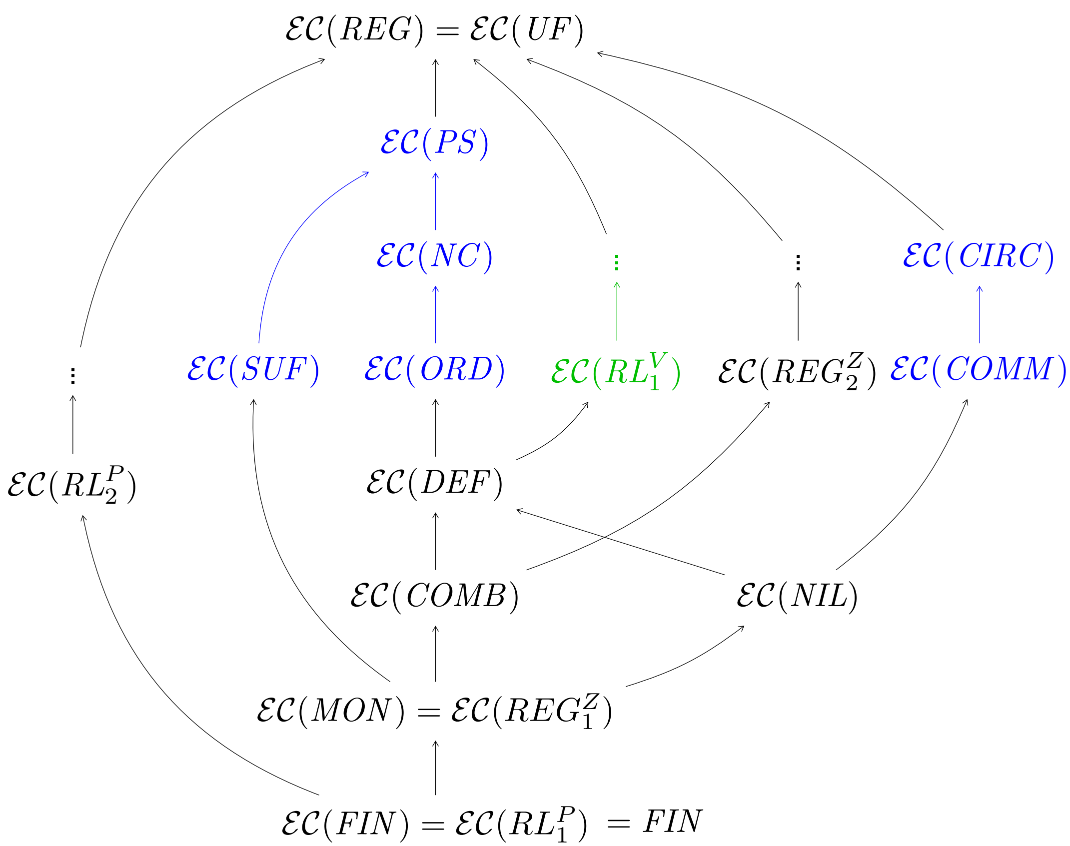
$\mathcal{EC}(RL_n^V)$ with $n \geq 1$ to

$\mathcal{EC}(ORD) \ldots \mathcal{EC}(PS)$

$\mathcal{EC}(COMM) \ldots \mathcal{EC}(CIRC)$

$\mathcal{EC}(SUF)$

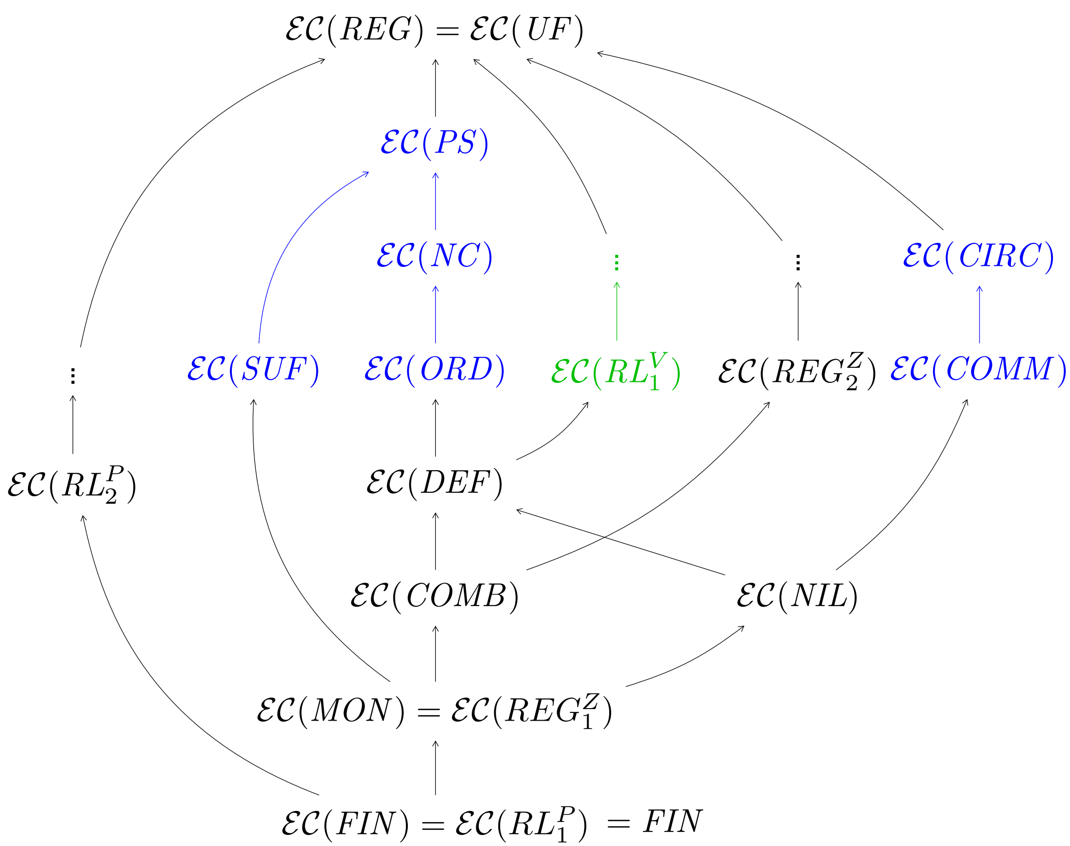
Results - One Hierarchy: Incomparabilities



$\mathcal{EC}(RL_n^V)$ with $n \geq 1$ to
- $\mathcal{EC}(ORD) \dots \mathcal{EC}(PS)$
- $\mathcal{EC}(COMM) \dots \mathcal{EC}(CIRC)$
- $\mathcal{EC}(SUF)$

$L_{RL_1^V, \neg PS}$
$\in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(PS)$

Results - One Hierarchy: Incomparabilities

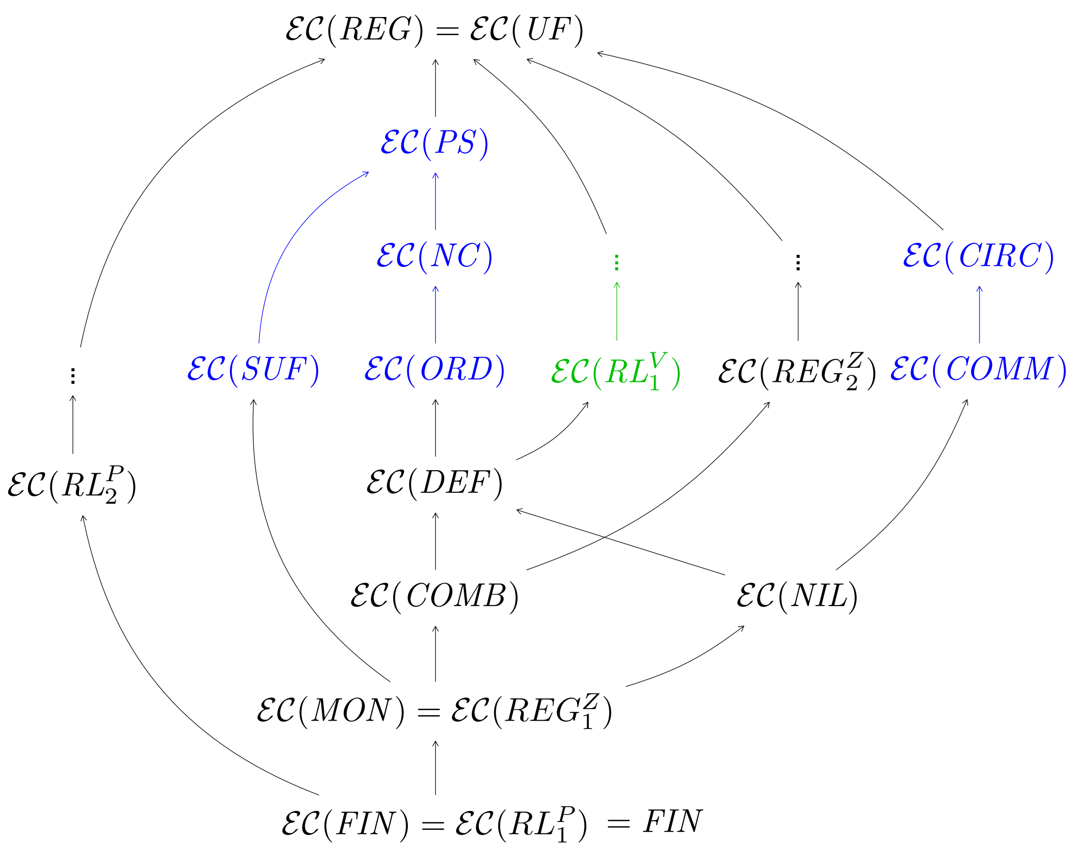


$\mathcal{EC}(RL_n^V)$ with $n \geq 1$ to
- $\mathcal{EC}(ORD) \ldots \mathcal{EC}(PS)$
- $\mathcal{EC}(COMM) \ldots \mathcal{EC}(CIRC)$
- $\mathcal{EC}(SUF)$

$L_{RL_1^V,\neg PS} \in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(PS)$

$$L_{RL_1^V,\neg PS} = \{\, a^n \mid n \geq 1 \,\} \cup \{\, ba^{2n}b \mid n \geq 1 \,\}$$

Results - One Hierarchy: Incomparabilities



$\mathcal{EC}(RL_n^V)$ with $n \geq 1$ to

$\mathcal{EC}(ORD) \ldots \mathcal{EC}(PS)$

$\mathcal{EC}(COMM) \ldots \mathcal{EC}(CIRC)$

$\mathcal{EC}(SUF)$

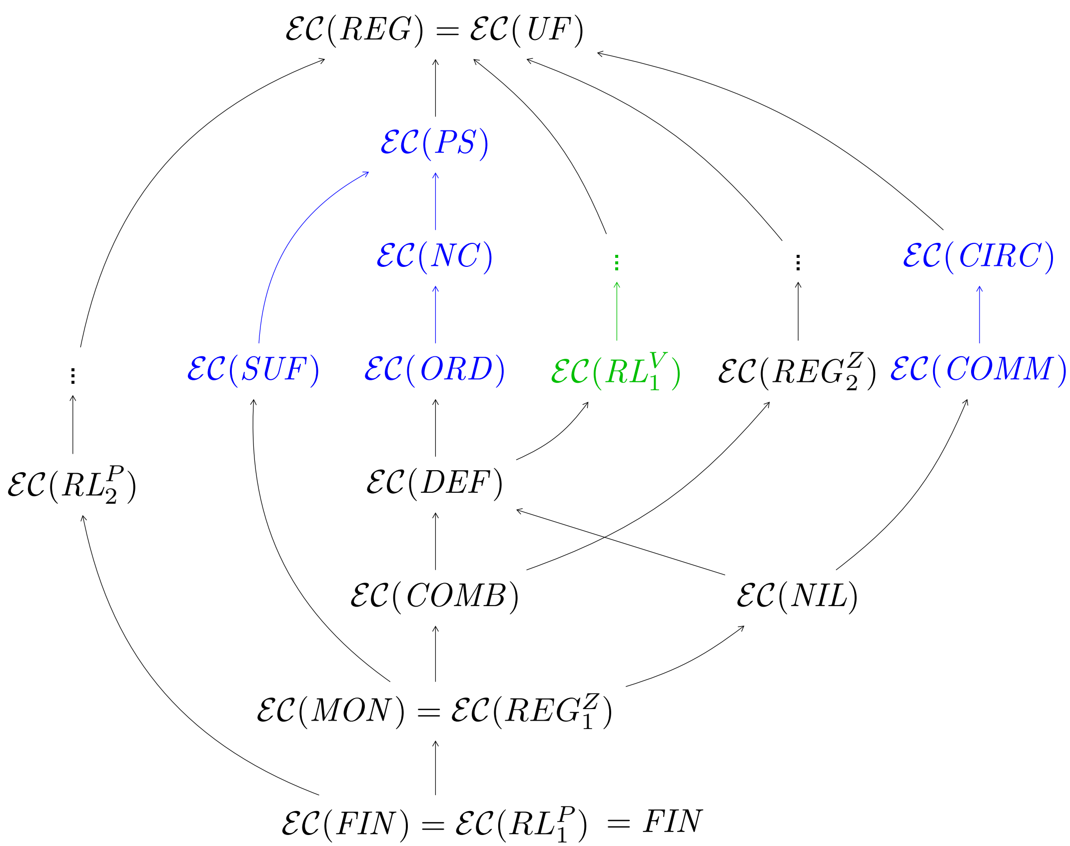
$L_{RL_1^V, \neg PS}$

$\in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(PS)$

$L_{RL_1^V, \neg CIRC}$

$\in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(CIRC)$

Results - One Hierarchy: Incomparabilities



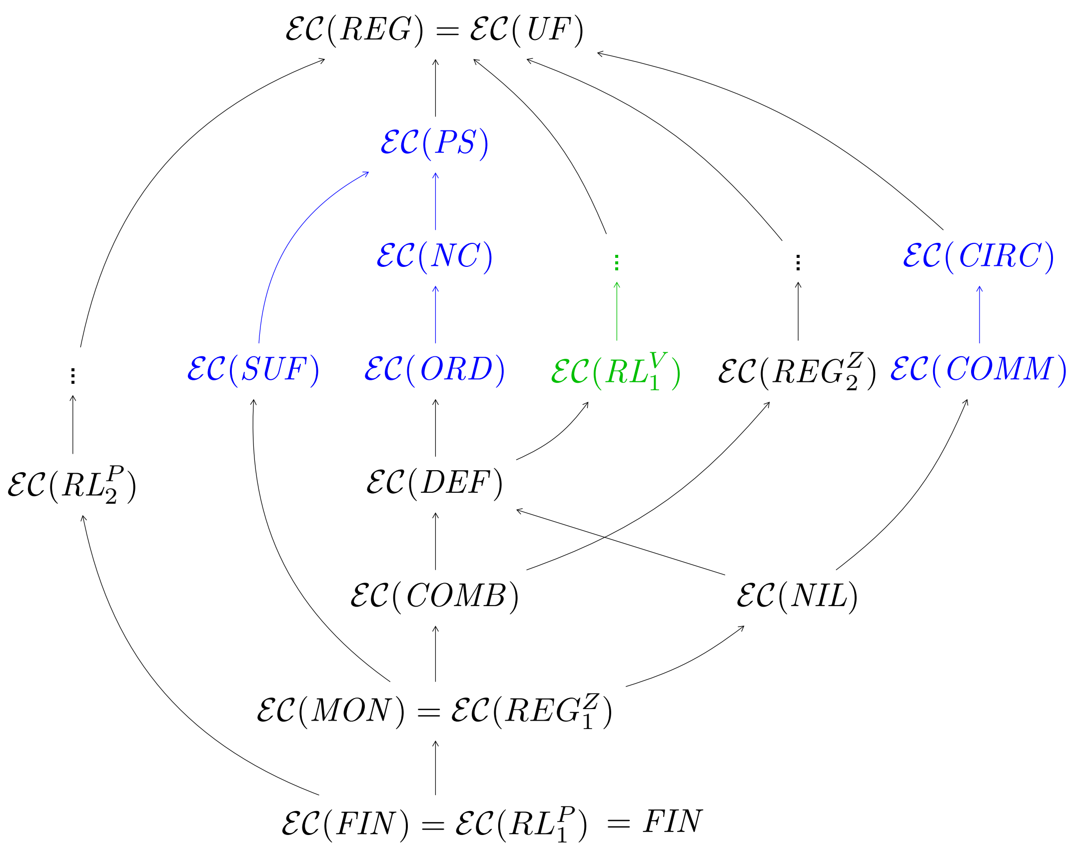
$\mathcal{EC}(RL_n^V)$ with $n \geq 1$ to
- $\mathcal{EC}(ORD) \ldots \mathcal{EC}(PS)$
- $\mathcal{EC}(COMM) \ldots \mathcal{EC}(CIRC)$
- $\mathcal{EC}(SUF)$

$L_{RL_1^V,\neg PS}$
$\in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(PS)$

$L_{RL_1^V,\neg CIRC}$
$\in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(CIRC)$

$$L_{RL_1^V,\neg CIRC} = \{ab\}^+ \cup \{ba\}^+ \cup \{\, c(ba)^n c \mid n \geq 1 \,\}$$

Results - One Hierarchy: Incomparabilities



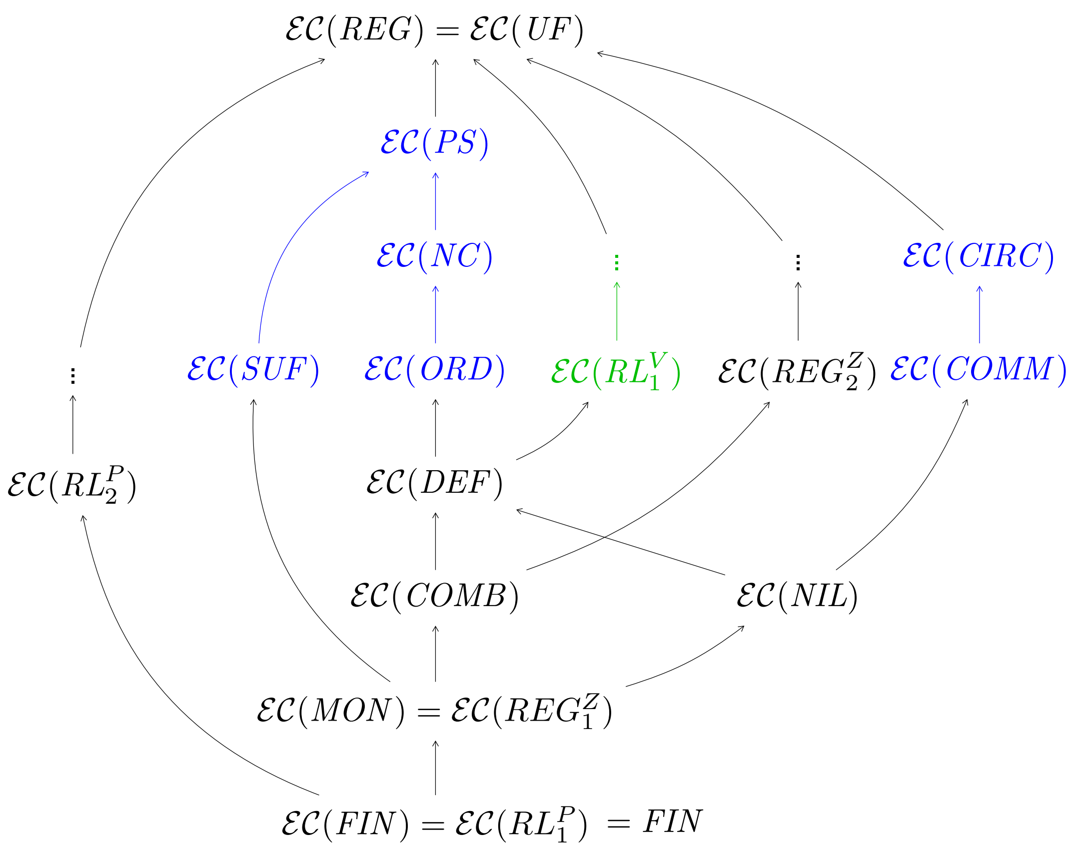
$\mathcal{EC}(RL_n^V)$ with $n \geq 1$ to
$\mathcal{EC}(ORD) \ldots \mathcal{EC}(PS)$
$\mathcal{EC}(COMM) \ldots \mathcal{EC}(CIRC)$
$\mathcal{EC}(SUF)$

$L_{RL_1^V,\neg PS}$
$\in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(PS)$

$L_{RL_1^V,\neg CIRC}$
$\in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(CIRC)$

$L_{ORD,COMM,SUF,\neg RL_n^V}$
$\in (\mathcal{EC}(ORD) \cup \mathcal{EC}(COMM) \cup \mathcal{EC}(SUF)) \setminus \mathcal{EC}(RL_n^V)$

Results - One Hierarchy: Incomparabilities



$\mathcal{EC}(RL_n^V)$ with $n \geq 1$ to
$\mathcal{EC}(ORD) \ldots \mathcal{EC}(PS)$
$\mathcal{EC}(COMM) \ldots \mathcal{EC}(CIRC)$
$\mathcal{EC}(SUF)$

$L_{RL_1^V,\neg PS} \in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(PS)$

$L_{RL_1^V,\neg CIRC} \in \mathcal{EC}(RL_1^V) \setminus \mathcal{EC}(CIRC)$

$L_{ORD,COMM,SUF,\neg RL_n^V} \in (\mathcal{EC}(ORD) \cup \mathcal{EC}(COMM) \cup \mathcal{EC}(SUF)) \setminus \mathcal{EC}(RL_n^V)$

$$L_{ORD,COMM,SUF,\neg RL_n^V} = \{a,b\}^+ \cup \{c\}\{\, w \mid w \in \{a,b\}^*, |w|_a \leq n \,\}\{c\}$$

Future Work

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- Inclusion relations and incomparabilities between classes based on strictly locally testable languages and those classes here

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- Other subregular families for the selection languages

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- Inclusion relations and incomparabilities between classes based on strictly locally testable languages and those classes here
- Other subregular families for the selection languages
- Further properties of the generated language families
- Merging the two hierarchies of internal contextual grammars (submitted)