Intelligent Data Analysis

Tutorial 7

Kernel Ridge Regression

Paul Prasse
Ahmed Abdelwahab
Dr. Niels Landwehr
Prof. Tobias Scheffer

Date: 04-06-2015

Goals

In this tutorial we will implement the *Kernel Ridge Regression* algorithm. The goal is a deeper understanding of this learning process, the functioning of kernel models and their advantages and disadvantages.

Problem setting

We'll use the Boston Housing Dataset (as in the tutorial before). This data set contains information collected by the U.S Census Service concerning housing in the city of Boston in the state of Massachusetts in 1978. Our goal is to predict the median value of the houses in a particular town in the city of Boston given its attributes. Check the file 'housing.names' for more information on the attributes.

Task 1

Write the following MATLAB-Function

```
function Z = rbf(X,Y,sigma)
```

which computes the RBF-Matrix $\mathbf{Z}$. The matrix $\mathbf{X}$ is a $m \times n$-matrix with m n-dimensional row vectors. The matrix $\mathbf{Y}$ is a $k \times n$-matrix with k n-dimensional row vectors. Both matrices can have a different number of rows; only the number of columns must be the same (number of attributes). The entry (i, j) of the RBF-matrix is defined as $\mathbf{Z}_{ij} = exp(-\frac{1}{2\sigma^2}\|\mathbf{x}_i - \mathbf{y}_j\|^2)$, where $\mathbf{x}_i$ is the i's row vector of the matrix $\mathbf{X}$, and $\mathbf{y}_j$ it the j's row vector of the matrix $\mathbf{Y}$. Try to avoid using `for`-loops by implementing this function.
Hint: The instructions `repmat` and/or `norm` could be helpful.

Task 2

Write the following MATLAB-Function

```
function alpha = learnKernelReg(X,y,lambda,f_krnl,krnl_p)
```

which computes the parameter `alpha` for a kernel ridge regression model for a given $m \times n$-matrix $\mathbf{X}$ (m n-dimensional row vectors) and numerical label $\mathbf{y} \in \mathbb{R}^n$. The trade-off-parameter between the empirical loss and the regularizer is $\lambda > 0$. The kernel function is given by the function handle `f_krnl` with the kernel parameter `krnl_p`. Use the analytic solution for the kernel ridge regression from the slides (slide 28).

Task 3

Write the following MATLAB-Function

```
function y = classKernelReg(alpha,Xtr,f_krnl,krnl_p,x)
```

which returns the numeric label $y \in \mathbb{R}$ for the data point `x` using the previously learned parameter `alpha` and the kernel function `f_krnl` and kernel parameter `krnl_p`.

Task 4

Use the data set `housing.mat` and divide this data set in a training and test set using the function `split_train_test` (`split`= 0.7, `seed`= 3, ...). Train a model using the instruction `alpha = learnKernelReg(train,y_train,0.1,@rbf,2)`. Classify the test data and compute the mean absolute difference between the label and the predicted label. Try to find optimal parameter for the kernel parameter `sigma` and the regularizer `lambda`.