More on noMoRe

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Abstract. This paper focuses on the efficient computation of answer sets for normal logic programs. It concentrates on a recently proposed rule-based method (implemented in the noMoRe system) for computing answer sets. We show how noMoRe and its underlying method can be improved tremendously by extending the computation of deterministic consequences. With these changes noMoRe is able to deal with problem classes it could not handle so far.

1 Introduction

Answer set programming (ASP) is a programming paradigm, which allows for solving problems in a compact and highly declarative way. The basic idea is to specify a given problem in a declarative language, e.g. normal logic programs[1], such that the different answer sets given by answer sets semantics [8] correspond to the different solutions of the initial problem [10]. As an example, consider the independent set problem, which is to determine if there exists a maximal (wrt set inclusion) independent subset of nodes for a given graph. A subset $S \subseteq V$ of nodes of a graph $G = (V, E)$ is called *independent* if there are no edges between nodes in S. Let

$$P = \left\{ \begin{array}{ll} in(a) \leftarrow not\ in(d), not\ in(b) & in(b) \leftarrow not\ in(a), not\ in(c) \\ in(c) \leftarrow not\ in(b), not\ in(d) & in(d) \leftarrow not\ in(c), not\ in(a) \end{array} \right\} \quad (1)$$

be a logic program and let us call the rules r_a, r_b, r_c and r_d, respectively. Then program (1) encodes the independent set problem for graph $G = (\{a, b, c, d\}, \{(a, b), (b, c), (c, d), (d, a)\})$. Program (1) has two answer sets $X_1 = \{in(a), in(c)\}$ and $X_2 = \{in(b), in(d)\}$ corresponding to the two independent sets of graph G.

Currently there exist a number of interesting applications of answer set programming (e.g. [3, 13, 14]) as well as reasonably efficient implementations (e.g. smodels [15] and dlv [5]). Since computation of answer sets is NP-complete for normal logic programs (and Σ_2^P-complete for disjunctive logic programs), most algorithms contain a non-deterministic part (making choices) and a part computing deterministic consequences (for these choices). In contrast to [7], where different heuristics are investigated in order to make "good" choices, we improve the deterministic consequences of the recently proposed rule-based noMoRe system [1] in this paper. For all ASP systems mentioned so far, non-deterministic choices and deterministic consequences determine the behavior of the resulting algorithm.

[1] The language of normal logic programs is not the only one suitable for ASP. Others are disjunctive logic programs, propositional logic or DATALOG with constraints [4].

We proposed in [11] a graph theoretical approach for computing answer sets for normal logic programs with at most one positive body atom. A special program transformation was introduced in order to be able to deal with multiple positive body atoms. The drawback was a lack of efficiency. One of the contributions of this paper is the extension of the underlying theory and its implementation in order to deal directly with multiple positive body atoms. Furthermore, we redefine propagation of so-called a-colorings as introduced in [11] such that we are able to include *backward propagation*. In addition, we introduce a technique called *jumping* to ensure complete backward propagation and give experimental results showing the impact of the presented concepts.

2 Background

We deal with normal logic programs which contain the symbol not used for *negation as failure*. A *normal logic program* is a set of rules of the form $p \leftarrow q_1, \ldots, q_n, not\ s_1, \ldots, not\ s_k$ where p, q_i $(0 \leq i \leq n)$ and s_j $(0 \leq j \leq k)$ are propositional atoms. A rule is a *fact* if $n = k = 0$, it is called *basic* if $k = 0$ and *quasi-fact* if $n = 0$. For a rule r like above we define $head(r) = p$ and $body(r) = \{q_1, \ldots, q_n, not\ s_1, \ldots, not\ s_k\}$. Furthermore, let $body^+(r) = \{q_1, \ldots, q_n\}$ denote the set of positive body atoms and $body^-(r) = \{s_1, \ldots, s_k\}$ the set of negative body atoms. Definitions of the head, the body, the positive and negative body of a rule are generalized to sets of rules in the usual way. We denote the set of all facts of a program P by $Facts(P)$ and the set of all atoms of P by $Atoms(P)$.

Let r be a rule. r^+ denotes the rule $head(r) \leftarrow body^+(r)$, obtained from r by deleting all negative body atoms in the body of r. For a logic program P let $P^+ = \{r^+ \mid r \in P\}$. A set of atoms X is *closed under* a basic program P iff for any $r \in P$, $head(r) \in X$ whenever $body(r) \subseteq X$. The smallest set of atoms which is closed under a basic program P is denoted by $\mathrm{Cn}(P)$. The *reduct*, P^X, of a program P *relative to* a set X of atoms is defined by $P^X = \{r^+ \mid r \in P \text{ and } body^-(r) \cap X = \emptyset\}$. We say that a set X of atoms is an *answer set* of a program P iff $\mathrm{Cn}(P^X) = X$.

A set of rules P is *grounded* iff there exists an enumeration $\langle r_i \rangle_{i \in I}$ of P such that for all $i \in I$ we have that $body^+(r_i) \subseteq head(\{r_1, \ldots, r_{i-1}\})$. Observe that there exists a unique maximal (wrt set inclusion) grounded set $P' \subseteq P$ for each program P. For a set of rules P and a set of atoms X we define the set of generating rules of P wrt X as $GR(P, X) = \{r \in P \mid body^+(r) \subseteq X, body^-(r) \cap X = \emptyset\}$. Then X is an answer set of P iff we have $X = \mathrm{Cn}(GR(P, X)^+)$. This characterizes answer sets in terms of generating rules. Observe, that in general $GR(P, X)^+ \neq P^X$ (take $P = \{a \leftarrow, b \leftarrow c\}$ and $X = \{a\}$).

We need some graph theoretical terminology . A *directed graph* (or *digraph*) G is a pair $G = (V, A)$ such that V is a finite, non-empty set (vertices) and $A \subseteq V \times V$ is a set (arcs). For a digraph $G = (V, A)$ and a vertex $v \in V$, we define the set of all *predecessors* of v as $\mathrm{Pred}(v) = \{u \mid (u, v) \in A\}$. Analogously, the set of all *successors* of v is defined as $\mathrm{Succ}(v) = \{u \mid (v, u) \in A\}$. Let $G = (V, A)$ and $G' = (V', A')$ be digraphs. Then G' is a *subgraph* of G if $V' \subseteq V$ and $A' \subseteq A$. G' is an *induced subgraph* of G if G' is a subgraph of G s.t. for each $v, v' \in V'$ we have that $(v, v') \in A'$ iff $(v, v') \in A$.

In order to represent more information in a directed graph, we need a special kind of arc labeling. $G = (V, A^0 \cup A^1)$ is a directed graph whose arcs $A^0 \cup A^1$ are labeled zero (A^0) and one (A^1). We call arcs in A^0 and A^1 *0-arcs* and *1-arcs*, respectively. For G we distinguish 0-predecessors (0-successors) from 1-predecessors (1-successors) denoted by Pred0(v) (Succ0(v)) and Pred1(v) (Succ1(v)) for $v \in V$, respectively.

Block Graphs for Normal Logic Programs Next we summarize the central definitions of block graphs for logic programs and a-colorings of block graphs (cf. [11, 12]).

Definition 1 *Let P be a logic program and let P' be the maximal grounded subset of P. The* block graph $\Gamma_P = (V_P, A_P^0 \cup A_P^1)$ *of P is a directed graph with vertices $V_P = P'$ and two different kinds of arcs defined as follows*

$$A_P^0 = \{(r', r) \mid r', r \in P' \text{ and } head(r') \in body^+(r)\}$$
$$A_P^1 = \{(r', r) \mid r', r \in P' \text{ and } head(r') \in body^-(r)\}.$$

This definition captures the conditions under which a rule r' blocks another rule r (i.e. $(r', r) \in A^1$). We introduce a 1-arc (r', r) in Γ_P if $r' = (q \leftarrow \ldots)$ and $r = (\ldots \leftarrow \ldots, not\ q, \ldots)$. We also gather all groundedness information in Γ_P, because we only introduce a 0-arc (r', r) (between rules $r' = (q \leftarrow \ldots)$ and $r = (\ldots \leftarrow q, \ldots)$) if r and r' are in the maximal grounded subset of P.[2] Figure 1 shows the block graph of program (1). Since the nodes of Γ_P are the rules of logic program P, operations $head(r)$, $body^+(r)$ and $body^-(r)$ (for $r \in P$) are also operations on the nodes of Γ_P.

In order to define so-called *application colorings* or *a-colorings* for block graphs we need the following definition.

Definition 2 *Let P be a logic program and let $\Gamma_P = (V_P, A_P^0 \cup A_P^1)$ be the corresponding block graph. Furthermore, let $r \in P$ and let $G_r = (V_r, A_r)$ be a graph. Then G_r is a* grounded 0-graph for r in Γ_P *iff the following conditions hold:*

1. *G_r is an acyclic subgraph of Γ_P s.t. $A_r \subseteq A_P^0$*
2. *$r \notin V_r$ and $body^+(r) \subseteq head(V_r)$*
3. *for each node $r' \in V_r$ and for each $q' \in body^+(r')$ there exists a node $r'' \in V_r$ s.t. $q' = head(r'')$ and $(r'', r') \in A_r$.*

Observe, that the nodes of a grounded 0-graph are grounded according to definition. Furthermore, the different grounded 0-graphs for rule r in Γ_P correspond to the different classical "proofs" for $head(r)$ in P^+, ignoring the default negations of all rules.

Definition 3 *Let P be a logic program, let $\mathcal{C} : P \to \{\ominus, \oplus\}$ be a total mapping*[3]*. We call r* grounded wrt Γ_P and $\mathcal{C}$ *iff there exist a grounded 0-graph $G_r = (V_r, A_r)$ for r in Γ_P s.t. $\mathcal{C}(V_r) = \oplus$. A rule r is called* blocked wrt Γ_P and $\mathcal{C}$ *if there exists $r' \in Pred1(r)$ s.t. $\mathcal{C}(r') = \oplus$.*

Now we are ready to define a-colorings.

Definition 4 *Let P be a logic program, let Γ_P be the corresponding block graph and let $\mathcal{C} : P \to \{\ominus, \oplus\}$ be a total mapping. Then $\mathcal{C}$ is an* a-coloring *of Γ_P iff the following condition holds for each $r \in P$*

[2] Observe, that for program $P = \{p \leftarrow q, q \leftarrow p\}$ the maximal grounded subset of rules is empty and therefore Γ_P contains no 0-arcs.

[3] A mapping $\mathcal{C} : P \to C$ is called *total* iff for each node $r \in P$ there exists some $\mathcal{C}(r) \in C$. Oppositely, mapping $\mathcal{C}$ is called partial if there are some $r \in P$ for which $\mathcal{C}(r)$ is undefined.

AP $\mathcal{C}(r) = \oplus$ *iff* r *is grounded and* r *is not blocked wrt* Γ_P *and* $\mathcal{C}$.

Let $\mathcal{C}$ be an a-coloring of some block graph Γ_P. Rules are then intuitively applied wrt some answer set of P iff they are colored $\oplus$, that is, condition $\mathbf{AP}$ captures the intuition of applying a rule wrt to some answer set. Similarly, the negation of condition $\mathbf{AP}$ (r is **not** grounded **or** r is blocked) captures the intuition when a rule is not applicable.

The main result in [11] states that Program P has an answer set X iff Γ_P has an a-coloring $\mathcal{C}$ s.t. $GR(P, X) = \{r \in P \mid \mathcal{C}(r) = \oplus\}$. This result constitutes a rule-based method to compute answer sets by computing a-colorings. In Figure 1 we have depicted the two a-colorings of the block graph of program (1) left and right from '/', respectively. Observe, that there are programs for which the corresponding block graph

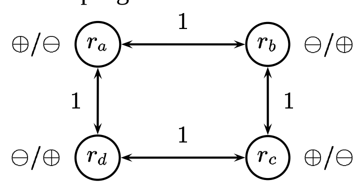

Fig. 1. Block graph and a-colorings of program (1).

has no a-coloring and thus no answer set. Let r_p be rule $p \leftarrow not\ p$. Then the 1-loop (r_p, r_p) is the only arc of the block graph of program $P = \{r_p\}$. By Definition 4 there is no a-coloring of Γ_P. If we color r_p with $\oplus$ we get a direct contradiction to $\mathbf{AP}$, since then r_p is blocked. On the other hand, if we color r_p with $\ominus$ then r_p is trivially grounded and not blocked.

3 Propagation

In the **no**n**mo**notonic **re**asoning system `noMoRe` [1] the approach described in the last section is implemented. Let us assume that each program is grounded. In order to describe the deterministic part of the implementation and its improvements, we need some central properties of nodes. All those properties are defined wrt partial a-colorings. We call a partial mapping $\mathcal{C} : P \rightarrow \{\ominus, \oplus\}$ a *partial a-coloring of* Γ_P if $\mathcal{C}$ is an a-coloring of the induced subgraph of Γ_P with nodes $Dom(\mathcal{C})$[4].

Definition 5 *Let* P *be a logic program and let* $\mathcal{C}$ *be a partial a-coloring of* Γ_P*. For each node* $r \in P$ *we define the following properties wrt* Γ_P *and* $\mathcal{C}$*:*

1. p-grounded(r) *iff* $\forall q \in body^+(r) : \exists r' \in Pred0(r) : q = head(r')$ *and* $\mathcal{C}(r') = \oplus$
2. p-notgrounded(r) *iff* $\exists q \in body^+(r) : \forall r' \in Pred0(r) : q \neq head(r')$ *or* $\mathcal{C}(r') = \ominus$
3. p-blocked(r) *iff* $\exists r' \in Pred1(r) : \mathcal{C}(r') = \oplus$
4. p-notblocked(r) *iff* $\forall r' \in Pred1(r) : \mathcal{C}(r') = \ominus$.

Notice the difference between total and partial a-colorings. For example, if p-notgrounded(r) holds for r wrt some total a-coloring $\mathcal{C}$ then p-grounded(r) is the negation of p-notgrounded(r). This does not hold for partial a-coloring $\mathcal{C}$, since there may be nodes for which $\mathcal{C}$ is undefined. For this reason, we have to define both

[4] $Dom(\mathcal{C})$ denotes the domain of mapping $\mathcal{C}$.

p-grounded (p-blocked) and p-notgrounded (p-notblocked), respectively, because they cannot be defined through each other wrt partial a-colorings. However, we have the following result for total a-colorings:

Theorem 1 *Let P be a logic program and let $\mathcal{C}$ be a total a-coloring of Γ_P. Then a node $r \in P$ is grounded wrt Γ_P and $\mathcal{C}$ iff p-grounded(r) holds wrt Γ_P and $\mathcal{C}$. Furthermore, r is blocked wrt Γ_P and $\mathcal{C}$ iff p-blocked(r) holds wrt Γ_P and $\mathcal{C}$.*

Clearly, to be grounded (wrt Γ_P and $\mathcal{C}$ for node r) is a global concept wrt Γ_P whereas p-grounded(r) is defined locally wrt Γ_P and $\mathcal{C}$. Furthermore, observe the difference between r is blocked and p-blocked(r) wrt Γ_P and $\mathcal{C}$. Even if the definitions of both concepts are the same (cf Definition 3), the former is defined wrt to total a-colorings, whereas the latter one is defined wrt partial a-colorings. In a situation like in Figure 2

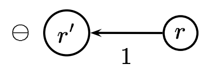

Fig. 2. Some block graph with partial a-coloring.

we do not have p-blocked(r') and we do not have p-notblocked(r') wrt the depicted partial a-coloring. But we always have either that r is blocked or not blocked wrt total a-colorings.

Definition 6 *Let $\mathcal{C}$ be a partial a-coloring of Γ_P and let U be the set of uncolored nodes wrt $\mathcal{C}$. Then each node $r \in U$ can be* colored $\oplus$ by propagation of $\mathcal{C}$ *iff we have p-grounded(r) and p-notblocked(r) wrt Γ_P and $\mathcal{C}$. Node r can be* colored $\ominus$ by propagation of $\mathcal{C}$ *iff we have p-notgrounded(r) or p-blocked(r) wrt Γ_P and $\mathcal{C}$.*

Notice, that propagation of partial a-colorings to uncolored nodes is global wrt Γ_P, since in order to propagate $\mathcal{C}$ as much as possible we have to check all nodes in U which in general are distributed over Γ_P. According to Definitions 5 and 6 nodes colored by propagation always have colored predecessors. Therefore we obtain a more procedural way to propagate partial a-colorings by *localized* propagation conditions.

Definition 7 *Let P be a logic program, let Γ_P be the corresponding block graph and let $\mathcal{C}$ be a partial a-coloring of Γ_P. We define an extended mapping $\mathcal{C}^e$ of $\mathcal{C}$ s.t. for each $r \in Dom(\mathcal{C})$ we have $\mathcal{C}^e(r) = \mathcal{C}(r)$ and for each $r, r' \in P$ the following conditions hold wrt Γ_P and $\mathcal{C}^e$:*

(A) *if $r \in Succ1(r')$ and $\mathcal{C}^e(r') = \oplus$ then $\mathcal{C}^e(r) = \ominus$*
(B) *if $r \in Succ1(r')$ and $\mathcal{C}^e(r') = \ominus$ and p-notblocked(r) and p-grounded(r) then $\mathcal{C}^e(r) = \oplus$*
(C) *if $r \in Succ0(r')$ and $\mathcal{C}^e(r') = \oplus$ and p-notblocked(r) and p-grounded(r) then $\mathcal{C}^e(r) = \oplus$*
(D) *if $r \in Succ0(r')$ and $\mathcal{C}^e(r') = \ominus$ and p-notgrounded(r) then $\mathcal{C}^e(r) = \ominus$.*

Let $\mathcal{C} : P \to \{\ominus, \oplus\}$ be a partial mapping. $\mathcal{C}$ is represented by a pair of (disjoint) sets $(\mathcal{C}_\ominus, \mathcal{C}_\oplus)$ s.t. $\mathcal{C}_\ominus = \{r \in P \mid \mathcal{C}(r) = \ominus\}$ and $\mathcal{C}_\oplus = \{r \in P \mid \mathcal{C}(r) = \oplus\}$. Since $\mathcal{C}$ is not total we do not necessarily have $P = \mathcal{C}_\ominus \cup \mathcal{C}_\oplus$. We refer to a partial mapping $\mathcal{C}$ with the pair $(\mathcal{C}_\ominus, \mathcal{C}_\oplus)$ and vice versa. We have the following result:

Theorem 2 *Let P be a logic program and let $\mathcal{C}$ and $\mathcal{C}^e : P \rightarrow \{\ominus, \oplus\}$ be partial mappings. Then we have if $\mathcal{C}$ is a partial a-coloring of Γ_P and $\mathcal{C}^e$ is an extension of $\mathcal{C}$ as in Definition 7 then $\mathcal{C}_\ominus \subseteq \mathcal{C}^e_\ominus$, $\mathcal{C}_\oplus \subseteq \mathcal{C}^e_\oplus$ and $\mathcal{C}^e$ is a partial a-coloring of Γ_P.*

This theorem gives the conditions for four different propagation cases in arc direction: if a node r is colored $\ominus$ or $\oplus$ then this color can be propagated over 1- and over 0-arcs to the successors of r, according to localized propagation conditions (A), (B), (C) and (D).

Now let P be some logic program and let $\mathcal{C}$ be a partial a-coloring. Assume that Γ_P is a global parameter of each of the presented procedures. Let U and N be sets of nodes s.t. U contains the currently uncolored nodes $U = P \setminus (\mathcal{C}_\ominus \cup \mathcal{C}_\oplus)$ and N contains colored nodes whose color has to be propagated. Figure 3 shows the main procedure of `noMoRe` in pseudo code. Notice that procedures **color**$_P$ and **propagate**$_P$ return

```
function   colorP(U, N : list; C : partial mapping)
var  r : node
  if  propagateP(N, C)  then
    U := U \ (C⊖ ∪ C⊕)
    if  chooseP(U, C, r)  then
      U := U \ {r}
      if  colorP(U, {r}, (C⊖, C⊕ ∪ {r}))  then
          return  true
      else
          return  colorP(U, {r}, (C⊖ ∪ {r}, C⊕))
    else
        return  propagateP(U, (C⊖ ∪ U, C⊕))
  else
      return  false
```

Fig. 3. Basic `noMoRe` procedure.

true (plus some extended partial mapping through parameter $\mathcal{C}$) or return false. If in **color**$_P$ the procedure **choose**$_P$ returns true, it also returns some uncolored node r s.t. we have p-grounded(r) wrt the current partial a-coloring. Clearly, **choose**$_P$ implements the non-deterministic part of **color**$_P$. Oppositely, **propagate**$_P$ implements the deterministic consequences of `noMoRe`. Let $L_1(P) = \{r \in P \mid r \in \text{Pred1}(r)\}$ denote the set of all 1-loops in Γ_P. When calling **color**$_P$ the first time, we start with $\mathcal{C} = (L_1(P), Facts(P))$, $U = P \setminus (Facts(P) \cup L_1(P))$ and $N = Facts(P) \cup L_1(P)$. That is, we start with all facts colored $\oplus$ and all 1-loops colored $\ominus$. Basically, **color**$_P$ takes both a partial mapping $\mathcal{C}$ and a set of uncolored nodes U and aims at coloring these nodes. That is, **color**$_P$ computes an extended partial mapping or if this is impossible returns false. This is done by choosing some uncolored node r ($r \in U$) with **choose**$_P$ and by trying to color it $\oplus$ first. If this does not give a solution **color**$_P$ tries to color node r with $\ominus$. If both possibilities fail **color**$_P$ returns false. Therefore, we say that node r is used as a *choice point*. To be a choice point is not a property of a node, because choice points are dynamic wrt each solution. Observe, that all different a-colorings are obtained via backtracking over choice points in **color**$_P$.

Notice, that procedure **propagate**$_P$ works locally according to conditions (A) to (D) of Definition 7 and colors are propagated only in arc direction (if possible). The color of a node is propagated immediately after getting colored, because the test whether the node was colored correctly is done during propagation. That is, **color**$_P$ fails only during propagation.

4 Backward Propagation

Partial a-colorings can also be propagated in opposite arc direction which improves our algorithm. Clearly, as for propagation in arc direction, we have four backward propagation cases. However, there is a problem with defining localized conditions for backward propagation (as in Definition 7). Assume that Figure 4 depicts a part of some block

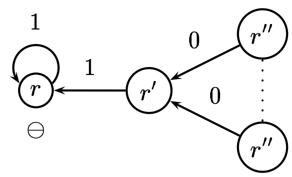

Fig. 4. Part of some block graph with partial a-coloring.

graph together with some partial a-coloring. On the one hand, we know that r' has to be colored $\oplus$ (provided that there are no other predecessors of r), because this is the only way to block r. If r is not blocked the depicted partial a-coloring cannot be extended to a total one. On the other hand, we cannot color r' with $\oplus$, because we do not have p-grounded(r') (see Definition 6). Therefore we need so-called *transitory a-colorings* .

Definition 8 *Let P be some logic program. We call a partial mapping $\mathcal{C} : P \to \{\ominus, \oplus, +\}$ a* transitory a-coloring of Γ_P *iff $\mathcal{C}$ is an a-coloring of the induced subgraph of Γ_P with nodes $\mathcal{C}_\ominus \cup \mathcal{C}_\oplus$.*

A transitory a-coloring is a partial a-coloring where some nodes may be uncolored or colored with $+$. Color $+$ is used instead of $\oplus$ to color node r' in situations like in Figure 4, where p-grounded(r') does not hold, yet. In order to transform some transitory a-coloring (during the execution of **color**$_P$) to a total a-coloring, color $+$ is replaced by color $\oplus$, if possible. This is achieved by propagation. Whenever a node is colored, this color is propagated to all its neighbors immediately, no matter whether these already have been colored or not. In case a node already colored $+$ ($\oplus$) has to be colored $\ominus$ via propagation, propagation fails due to contradiction. When a node already colored $+$ has to be colored $\oplus$ via propagation, color $+$ is simply replaced by $\oplus$. That is, either every color $+$ will become $\oplus$, or **color**$_P$ fails. We need the following properties wrt transitory a-colorings:

Definition 9 *Let P be a logic program, let $\mathcal{C}$ be a transitory a-coloring of Γ_P and let $r \in P$ be some node. Then* groundable(r) holds wrt Γ_P and $\mathcal{C}$ *iff $\forall q \in body^+(r) : \exists r' \in Pred0(r)$ with $q = head(r')$ s.t. $\mathcal{C}(r') = \oplus$ or r' is uncolored.*

Here groundable(r) means that either r is grounded or that there is some uncolored 0-predecessor, which can possibly be colored $\oplus$ when $\mathcal{C}$ is further extended. For each $r \in P$ and $q \in body^+(r)$ we define $S_q \subseteq \text{Pred0}(r)$ as $S_q = \{r' \mid r' \in \text{Pred0}(r) \text{ and } q = head(r')\}$. Furthermore, for a set of rules $S \subseteq P$ we define p-grounded(S) wrt Γ_P and transitory a-coloring $\mathcal{C}$ iff there is some $r \in S$ s.t. $\mathcal{C}(r) = \oplus$. Now we are ready to define the four localized backward propagation cases.

Definition 10 *Let P be a logic program, let Γ_P be the corresponding block graph and let $\mathcal{C}$ be a transitory a-coloring of Γ_P. We define an extended mapping $\mathcal{C}^e : P \to \{\ominus, \oplus, +\}$ of $\mathcal{C}$ s.t. for each $r \in Dom(\mathcal{C})$ we have $\mathcal{C}^e(r) = \mathcal{C}(r)$ and conditions (A) to (D) of Definition 7 as well as the following conditions hold for all $r, r' \in P$ wrt Γ_P and $\mathcal{C}^e$:*

(bA) *if $\mathcal{C}^e(r') = \oplus$ and $r \in Pred1(r')$ then $\mathcal{C}^e(r) = \ominus$*

(bB) *if $\mathcal{C}^e(r') = \ominus$ and p-grounded(r') and $r \in Pred1(r')$ s.t. $\forall r'' \in Pred1(r')$: $(\mathcal{C}^e(r'') = \ominus$ iff $r'' \neq r)$ then $\mathcal{C}^e(r) = +$*

(bC) *if $\mathcal{C}^e(r') = \oplus$ and there is some $q \in body^+(r')$ s.t. $q = head(r)$ for some $r \in S_q$ and groundable(r) and for each $r'' \in S_q$: $(\mathcal{C}^e(r'') = \ominus$ iff $r'' \neq r)$ then $\mathcal{C}^e(r) = +$*

(bD) *if $\mathcal{C}^e(r') = \ominus$ and p-notblocked(r') and there is some $q \in body^+(r')$ with $q = head(r)$ for some $r \in S_q$ s.t. for each $q' \in body^+(r')$: (p-grounded$(S_{q'})$ iff $S_{q'} \neq S_q$) then $\mathcal{C}^e(r) = \ominus$.*

Intuitively, these cases ensure that an already $\oplus$-colored node is grounded (bC) and not blocked (bA) while an already $\ominus$-colored node is blocked (bB) or not grounded (bD). So in a sense, the purpose of these cases is to justify the color of a node. Observe, that cases (bB) and (bC) use color $+$ instead of $\oplus$ (see Definition 8). We have the following result corresponding to Theorem 2:

Theorem 3 *Let P be a logic program and let $\mathcal{C}$ and $\mathcal{C}^e : P \to \{\ominus, \oplus, +\}$ be a partial mappings. Then we have: If $\mathcal{C}$ is a transitory a-coloring of Γ_P and $\mathcal{C}^e$ is an extension of $\mathcal{C}$ as in Definition 10 then $\mathcal{C}_\ominus \subseteq \mathcal{C}^e_\ominus$, $\mathcal{C}_\oplus \subseteq \mathcal{C}^e_\oplus$ and $\mathcal{C}^e$is a transitory a-coloring of Γ_P.*

Let us show how **color**$_P$ computes the a-colorings of the block graph of program (1) (see Figure 1). At the beginning we cannot propagate anything, because there is no fact and no 1-loop. We take r_a as a choice. First, we try to color r_a with $\oplus$ by calling **color**$_P(U, N, \mathcal{C})$ with $U = P \setminus \{r_a\}$, $N = \{r_a\}$ and $\mathcal{C} = (\emptyset, \{r_a\})$. Now, **propagate**$_P(N, \mathcal{C})$ is executed. By propagating $\mathcal{C}(r_a) = \oplus$ with case (A) we get $\mathcal{C}(r_b) = \ominus$ and $\mathcal{C}(r_d) = \ominus$. Recursively, through case (B) $\mathcal{C}(r_c) = \oplus$ is propagated. This gives $\mathcal{C} = (\{r_b, r_d\}, \{r_a, r_c\})$. Since U becomes the empty set, **choose**$_P$ fails and $\mathcal{C}$ is the first output. So far we did not need backward propagation.

Now, we color r_a with $\ominus$ through calling **color**$_P(U, N, \mathcal{C})$ with U and N as above and $\mathcal{C} = (\emptyset, \{r_a\})$. Since no (backward) propagation is possible we have to compute the next choice. For **choose**$_P$ all three uncolored nodes are possible choices. Assume $\mathcal{C}(r_b) = \oplus$ as next choice. Through propagation case (A) we get $\mathcal{C}(r_c) = \ominus$. This color of r_c has to be propagated by executing **propagate**$_P(\{r_c\}, (\{r_a, r_c\}, \{r_c\}))$. By using propagation case (B) we obtain $\mathcal{C}(r_d) = \oplus$. Recursively, propagation of $\oplus$ for r_d gives no contradiction and $\mathcal{C} = (\{r_a, r_c\}, \{r_b, r_d\})$ is the second a-coloring. By assuming $\mathcal{C}(r_b) = \ominus$, that is, $\mathcal{C} = (\{r_a, r_b\}, \emptyset)$, r_c is colored with $\oplus$ through backward propagation case (bB). By propagation of this color with case (A) node r_d is colored with $\ominus$. By applying case (B) to the color of r_d we obtain that r_a has to be colored with $\oplus$, because it is not blocked, but this is a contradiction to $\mathcal{C}(p_a) = \ominus$. Thus, there is no further solution and we have found the two solutions with two choices. Observe that the usage of (bB) saves one additional choice, since without backward propagation the partial coloring $\mathcal{C} = (\{r_a, r_b\}, \emptyset)$ could not have been extended any more and another choice would have been necessary. Thus backward propagation reduces the number of necessary choices.

5 Jumping

A further improvement of backward propagation is the so-called *jumping*. Backward propagation according to (bB), (bC) and (bD) requires certain conditions to be fulfilled, which may not be known when a node is colored. For (bA) this is not the case, because in (bA) there is no further condition. Take the following program P and its corresponding block graph Γ_P (see Figure 5):

$$P = \left\{ \begin{array}{ll} a \leftarrow not\ a, not\ b, not\ d & \\ b \leftarrow not\ c & d \leftarrow not\ e \\ c \leftarrow not\ b & e \leftarrow not\ d \end{array} \right\} \qquad (2)$$

We know that $\mathcal{C}(\{r_a\}) = \ominus$, otherwise there would not be an answer set at all. Since

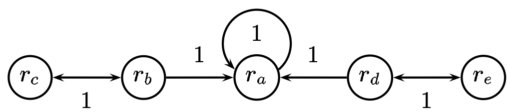

Fig. 5. Block graph of program (2).

r_a is trivially grounded, it has to be blocked. This can be achieved by coloring one of two rules, r_b and r_d with $\oplus$, though one is sufficient. But we do not know yet, which one should be colored $\oplus$. Later on, when e.g. r_b has been used as a choice and is colored $\ominus$, this is achieved via jumping. That is, case (bB) is used again for node r_a and thus r_d is colored $\oplus$. Finally, with (A) node r_e is colored $\ominus$. In this way jumping helps to avoid unnecessary choices, because without jumping we would need another choice to color nodes r_d and r_e. Therefore backward propagation would not be complete without jumping. In general, whenever a $\ominus$ is propagated along a 1-arc to an already $\ominus$-colored node, we check backward propagation case (bB) for this node again. Similarly, we check (bC) and (bD) again for $\oplus$-colored and $\ominus$-colored 0-successors of already colored and propagated nodes, respectively.

As an example, Figure 6 shows the implementation of procedure **jumpB**$_P$, which jumps to an already colored node in order to check backward propagation (bB) again. Procedures **jumpC**$_P$ and **jumpD**$_P$ are implemented analogously. By using procedure **propagate**$_P$ (in Figure 3) as defined in Figure 6 we obtain an algorithm for computing a-colorings including backward propagation and jumping. The four procedures **backpropA**$_P$, **backpropB**$_P$, **backpropC**$_P$, and **backpropD**$_P$ are the implementations of conditions (bA), (bB), (bC) and (bD), respectively. Procedures **propA**$_P$, **propB**$_P$, **propC**$_P$, and **propD**$_P$ are the implementations of conditions (A), (B), (C) and (D), respectively.

```
procedure  propagate_P(N : list, C : part. mapping)
  var  r' : node;
     while  N ≠ ∅  do
       select  r'  from  N;
       if  (r' ∈ C_⊕)  then
(A)        if  propA_P(r', C)  fails  then  fail;
(C)        if  propC_P(r', C)  fails  then  fail;
(bA)       if  backpropA_P(r', C)  fails  then  fail;
(bC)       if  backpropC_P(r', C)  fails  then  fail;
           if  jumpC_P(r', C)  fails  then  fail;
       else
(B)        if  propB_P(r', C)  fails  then  fail;
(D)        if  propD_P(r', C)  fails  then  fail;
(bB)       if  backpropB_P(r', C)  fails  then  fail;
(bD)       if  backpropD_P(r', C)  fails  then  fail;
           if  jumpB_P(r', C)  fails  then  fail;
           if  jumpD_P(r', C)  fails  then  fail;

procedure  jumpB_P(r' : node; C : partial mapping,)
  var  S : set of nodes;
    S  :=  Succ1(r')
     while  S ≠ ∅  do
       select  r'  from  S;
       if  C(r') = ⊖  then  backpropB_P(r', C);
```

Fig. 6. Extended propagation procedures including backward propagation and jumping.

6 Results

For a partial mapping $\mathcal{C} : P \rightarrow \{\ominus, \oplus, +\}$ we define the set of *induced answer sets* $A_{\mathcal{C}}$ as $A_{\mathcal{C}} = \{X \mid X$ is answer set of $P, \mathcal{C}_{\oplus} \subseteq GR(P,X)$ and $\mathcal{C}_{\ominus} \cap GR(P,X) = \emptyset\}$. If $\mathcal{C}$ is undefined for all nodes then $A_{\mathcal{C}}$ contains all answer sets of P. If $\mathcal{C}$ is a total mapping s.t. no node is colored with $+$ then $A_{\mathcal{C}}$ contains exactly one answer set of P (if $\mathcal{C}$ is an a-coloring). With this notation we formulate the following result:

Theorem 4 *Let P be a logic program, let $\mathcal{C}$ and $\mathcal{C}' : P \rightarrow \{\ominus, \oplus, +\}$ be partial mappings. Then for each $r \in (\mathcal{C}_{\ominus} \cup \mathcal{C}_{\oplus} \cup \mathcal{C}_{+})$ we have if* **propagate**$_P(\{r\}, \mathcal{C})$ *succeeds and $\mathcal{C}'$ is the partial mapping after its execution then $A_{\mathcal{C}} = A_{\mathcal{C}'}$.*

This theorem states that **propagate**$_P$ neither discards nor introduces answer sets induced by a partial mapping $\mathcal{C}$. Hence, only nodes used as choices lead to different answer sets. Finally, let C_P be the set of all solutions of **color**$_P$. We obtain correctness and completeness of **color**$_P$.

Theorem 5 *Let P be a logic program, let Γ_P be its block graph, let $\mathcal{C} : P \rightarrow \{\ominus, \oplus\}$ be a mapping and let C_P the set of all solutions of* **color**$_P$ *for program P. Then $\mathcal{C}$ is an a-coloring of Γ_P iff $\mathcal{C} \in C_P$.*

7 Experiments

As benchmarks, we used some instances of NP-complete problems proposed in [2], namely, the independent set problem for circle graphs[5], the problem of finding Hamiltonian cycles in complete graphs and the problem of finding classical graph colorings. Furthermore we have tested some planning problems taken from [6] and the n-queens problem. In Table 1 we have counted the number of choices instead of measuring time, since the number of choices (theoretically) indicates how good an algorithm deals with a non-deterministic problem[6]. That is, for very large examples an algorithm using less choices will be faster than one using more choices (provided that the implementation of the deterministic parts of both algorithms need polynomial time). Therefore counting choices gives rather theoretical than practical results on the behavior of `noMoRe`. For `smodels` results with and without lookahead (results in parentheses) are shown[7]. The

	`noMoRe`			`smodels`	
backprop	no	yes	yes	with	(without)
jumping	no	no	yes	lookahead	
ham_k_7	14335	14335	2945	4814	(34077)
ham_k_8	82200	82200	24240	688595	(86364)
ind_cir_20	539	317	276*	276	(276)
ind_cir_30	9266	5264	4609*	4609	(4609)
p1_step4	-	464	176	7	(69)
p2_step6	-	13654	3779	75	(3700)
col4x4	27680	27680	7811	7811	(102226)
col5x5	-	-	580985	580985	(2.3 Mil)
queens4	84	84	5	1	(11)
queens5	326	326	13	9	(34)

Table 1. Number of choices (all solutions) for different problems.
* minimal number of choices, since at least $(n-1)$ choices are needed for n solutions

influence of backward propagation and jumping on the number of choices is plain to see. There are also some problems where we did not obtain a solution after more than 12 hours without backward propagation. Table 1 impressively shows that `noMoRe` with backward propagation and jumping is now (theoretically) comparable with `smodels` on several problem classes; especially if we disable the lookahead of `smodels`[8]. The difference between `smodels` and `noMoRe` for planning examples and the n-queens problems seems to come from different heuristics for making choices. We have just started to investigate the influence of more elaborated heuristics. The `noMoRe` system including test cases is available at http://www.cs.uni-potsdam.de/~linke/nomore.

[5] A so-called circle graph Cir_n has n nodes $\{v_1, \cdots, v_n\}$ and arcs $A = \{(v_i, v_{i+1}) \mid 1 \leq i < n\} \cup \{(v_n, v_1)\}$.

[6] You can find time measurements for these examples in [1].

[7] Whereas `smodels` and `noMoRe` make exactly one choice at each choice point in the search space, `dlv` makes several choices at the same point of the search space. Therefore there are no results for `dlv` in Table 1.

[8] Observe, that currently `noMoRe` has no lookahead.

8 Conclusion

We have generalized and advanced an approach to compute answer sets for normal logic programs which was introduced in [11]. Now we are able to deal directly with logic programs with multiple positive body literals. We have shown that by introducing backward propagation together with jumping the rule-based algorithm implemented in `noMoRe` can be greatly improved. A related method of backward propagation wrt answer set semantics for normal logic programs was proposed in [9]. However, a lot of the obtained improvement is due to the concept of a third color $+$. There seems to be a close relation between `noMoRe`'s color $+$ and `dlv`'s must-be-true truth value [6], though this has to be studied more thoroughly, because `noMoRe` is rule-based and `dlv` (and `smodels`) is literal-based. Through the conducted experiments the impact of the improvements is shown. `NoMoRe` is now comparable to `smodels` on many different problem classes measured by the number of choices. This improvement was obtained by improving the deterministic consequences of `noMoRe`. However, there are still some interesting open questions. The main one is whether rule-based computation of answer sets is different from atom-based (literal-based) or not. During our experiments we have detected programs (with a high rule per atom ratio) for which atom-based computations are more suitable and other programs (with a low rule per atom ratio) for which rule-based computation performs better. Currently, we have no general answer to this question and a general comparison between atom-based and rules-based methods for logic programs will be necessary.

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