How to build your own ASP-based system?!

ROLAND KAMINSKI, JAVIER ROMERO, TORSTEN SCHAUB, and PHILIPP WANKO

University of Potsdam, Germany

submitted 1 January 2003; revised 1 January 2003; accepted 1 January 2003

Abstract

Answer Set Programming (ASP) has become a popular and quite sophisticated approach to declarative problem solving. This is arguably due to its attractive modeling-grounding-solving workflow that provides an easy approach to problem solving, even for laypersons outside computer science. Unlike this, the high degree of sophistication of the underlying technology makes it increasingly hard for ASP experts to put ideas into practice.

For addressing this issue, this tutorial aims at enabling users to build their own ASP-based systems. More precisely, we show how the ASP system clingo can be used for extending ASP and for implementing customized special-purpose systems. To this end, we propose two alternatives. We begin with a traditional AI technique and show how meta programming can be used for extending ASP. This is a rather light approach that relies on clingo’s reification feature to use ASP itself for expressing new functionalities. Unlike this, the major part of this tutorial uses traditional programming (in Python) for manipulating clingo via its application programming interface. This approach allows for changing and controlling the entire model-ground-solve workflow of ASP. Central to this is clingo’s new Application class that allows us to draw on clingo’s infrastructure by customizing processes similar to the one in clingo. For instance, we may engage manipulations to programs’ abstract syntax trees, control various forms of multi-shot solving, and set up theory propagators for foreign inferences. Another cross-sectional structure, spanning meta as well as application programming, is clingo’s intermediate format, aspif, that specifies the interface among the underlying grounder and solver. We illustrate the aforementioned concepts and techniques throughout this tutorial by means of examples and several non-trivial case-studies. In particular, we show how clingo can be extended by difference constraints and how guess-and-check programming can be implemented with both meta and application programming.
Contents

1 Introduction
2 Answer set programming
3 Meta programming
  3.1 Reification format
  3.2 Meta encoding
  3.3 Examples
  3.4 Guess-and-check programming
4 About clingo applications
  4.1 Embedded Python code
  4.2 The clingo Python module
  4.3 Implementing a system based on clingo
5 Multi-shot ASP solving
  5.1 A gentle introduction
  5.2 Branch-and-bound-based optimization
  5.3 Incremental ASP solving
6 Theory-enhanced ASP solving
  6.1 Input language
  6.2 Semantic principles
  6.3 Algorithmic aspects
  6.4 Propagator interface
7 Extending ASP with difference constraints
  7.1 Solving flow shop problems
  7.2 Hybrid optimization with difference constraints
8 Guess-and-check programming reloaded
9 Discussion
Appendix A Intermediate language
References
1 Introduction

Answer Set Programming (ASP; Lifschitz 2019) has become an established approach to declarative problem solving, experiencing an increasing popularity in academy as well as industry, and sometimes even beyond Artificial Intelligence and Computer Science. This is arguably due to its pursuit of an integrated modeling-grounding-solving paradigm (Gebser and Schaub 2016; Kaufmann et al. 2016) that enables laypersons to use ASP systems off-the-shelf. However, the underlying technology is highly involved and thus less easily accessible even for ASP experts. This is also reflected by the fact that there are nowadays only two genuine ASP systems, namely dlv (Leone et al. 2006; Alviano et al. 2017) and clingo (Gebser et al. 2019), while other computational approaches mostly rely on extensions to these systems or translations into neighboring solving paradigms. This is not without reason and rather due to the high technical sophistication of full-fledged ASP systems. Hence, it is all the more important to keep this technology open and extensible and so to enable the community to participate in the continuous enhancement of ASP technology. If neglected, we risk a technological gulf that is prone to cut off advances in ASP in the future. Moreover, the extension and integration of ASP technology is indispensable in many real-world applications. Examples include decision support systems for the space shuttle (Nogueira et al. 2001), metabolic network completion (Frioux et al. 2019), and train scheduling (Abels et al. 2019). Hence, empowering the community to master ASP technology also makes it fit for addressing applications at industrial scale.

This empowerment was a guiding motive in the development of the core ASP systems of the Potsdam Answer Set Solving Collection, or POTASSCO for short (Gebser et al. 2011; Gebser et al. 2018), and has meanwhile led to numerous extensions, either being part of POTASSCO at potassco.org or conducted by other scientists worldwide. To further foster such advancements and transfer of ASP technology, we provide in this tutorial an introduction to key techniques enabling the construction of own ASP systems by building upon POTASSCO tools.

No matter whether the envisaged system aims at extending ASP or using it as an implementation platform, the key issue is how to capture the added functionality.

To this end, we propose two alternatives.

We begin with a traditional AI technique and show in Section 3 how meta programming can be used for extending ASP. This is clearly the lightest approach in which ASP itself is used to express new functionalities. This approach draws upon clingo’s reification feature for representing the result of grounding a logic program as a set of facts. The original program is then given as data to a meta program that implements the new functionality. In this way, we use clingo as a black box and implement all examples by consecutive clingo calls. Meta programming is for example used in asprin (Brewka et al. 2015) and plasp (Dimopoulos et al. 2018).

Unlike this, the remainder of this tutorial uses traditional programming for manipulating clingo via its application programming interface (API). This can be seen as treating clingo as a gray box, whose modifications are guided through a well-defined interface. (Before that all functionality had to be done by re-programming. A white box approach that needed quite good programming skills.) For this purpose, we have chosen Python as
our exemplary language, although other choices exist\footnote{For instance, \textit{clingo} also offers APIs to C, C++, Lua, and Rust, as well as third-party APIs for Haskell, Java, and Prolog.}. This approach allows for making changes to the entire model-ground-solve workflow of ASP. We detail capabilities and interfaces supporting the implementation of novel ASP technology such as extending the modeling language of \textit{clingo} by means of grammar-based specifications, manipulating the abstract syntax trees of (non-ground) logic programs, as well as multi-shot and theory solving. While multi-shot solving allows for fine-grained control of ASP reasoning processes, theory solving allows for refining basic ASP solving by incorporating foreign types of constraints. Central to this is \textit{clingo}'s new application class that allows for deriving customized applications from the one of \textit{clingo}. This class constitutes the cornerstone of all recent POTASSCO systems such as \textit{clingcon} \cite{Banbara:2017}, \textit{clingodi} \cite{Jahnunen:2017}, \textit{eclingo} \cite{Cabalar:2020}, and \textit{telingo} \cite{Cabalar:2019}. We discuss its role in Section 4 and use it throughout the remaining sections.

Another cross-sectional structure, spanning meta as well as application programming, is \textit{clingo}'s intermediate format, \textit{aspi}\textsubscript{f}, that specifies the interface among the underlying grounder and solver, namely, \textit{gringo} and \textit{clasp}. This is relevant whenever one deals with ground logic programs, be it as reified rules, in machine-readable format, or via the application interface. The whole input, including rules, customized language expressions, as well as all types of directives, are expressed in their ground form in the \textit{aspi}\textsubscript{f} format. The complete \textit{aspi}\textsubscript{f} specification is given in Appendix A for its machine-readable format. Systems that rely on translating ground logic programs in \textit{aspi}\textsubscript{f} format are \textit{lc2casp} \cite{Cabalar:2016} and \textit{telingo}.

We illustrate the aforementioned techniques throughout this tutorial by means of examples and several non-trivial case-studies. This includes the computation of classical, supported, here-and-there, and diverse models with meta programming, optimization and incremental solving with multi-shot solving in Section 5, hybrid solving and optimization with theory solving in Section 7, and finally guess-and-check programing with both meta and application programming in Section 3.4 and 8, respectively.

In what follows, we refrain from distinguishing features of \textit{gringo} and \textit{clasp} and simply refer to them as \textit{clingo}'s. We deal in this tutorial with \textit{clingo} series 5, in particular, \textit{clingo} version 5.5. The source code of all examples is available at \url{https://cs.uni-potsdam.de/~kaminski/examples.zip}. A complete documentation of \textit{clingo}'s API can be found at \url{https://potassco.org/clingo/python-api/5.5}. We rely on a basic acquaintance with ASP and its underlying concepts. Comprehensive introductions can be found in \cite{Baral:2003,Gebser:2012,Gelfond:2014,Lifschitz:2019}. Otherwise, we presuppose some computer science training that permits a basic understanding of Python. The core of this tutorial is based on material stemming from \cite{Kaminski:2017}.

2 Answer set programming

As usual, a logic program consists of rules of the form
\[
a_1; \ldots; a_n : - a_{m+1}, \ldots, a_n, \text{not } a_{n+1}, \ldots, \text{not } a_o
\]
where each $a_i$ is an atom of form $p(t_1, \ldots, t_k)$ and all $t_i$ are terms, composed of function
symbols and variables. Atoms \( a_1 \) to \( a_m \) are often called head atoms, while \( a_{m+1} \) to \( a_n \) and \( \text{not} \ a_n+1 \) to \( \text{not} \ a_n \) are also referred to as positive and negative body literals, respectively. An expression is said to be ground, if it contains no variables. As usual, \( \text{not} \) denotes (default) negation. A rule is called a fact if \( m = o = 1 \), normal if \( m = 1 \), and an integrity constraint if \( m = 0 \). Semantically, a logic program induces a set of stable models, being distinguished models of the program determined by the stable models semantics; see (Gelfond and Lifschitz 1991) for details.

To ease the use of ASP in practice, several extensions have been developed. First of all, rules with variables are viewed as shorthands for the set of their ground instances. Further language constructs include conditional literals and cardinality constraints (Simmons et al. 2002). The former are of the form \( a:b_1, \ldots, b_m \), the latter can be written as \( s\{d_1; \ldots; d_n\}t \), where \( a \) and \( b_i \) are possibly default-negated (regular) literals and each \( d_j \) is a conditional literal; \( s \) and \( t \) provide optional lower and upper bounds on the number of satisfied literals in the cardinality constraint. We refer to \( b_1, \ldots, b_m \) as a condition. The practical value of both constructs becomes apparent when used with variables. For instance, a conditional literal like \( a(X):b(X) \) in a rule’s antecedent expands to the conjunction of all instances of \( a(X) \) for which the corresponding instance of \( b(X) \) holds. Similarly, \( 2 \{ a(X):b(X) \} \) \( 4 \) is true whenever at least two and at most four instances of \( a(X) \) (subject to \( b(X) \)) are true. Finally, objective functions minimizing the sum of a set of weighted tuples \( (w_i, t_i) \) subject to condition \( c_i \) are expressed as \( \#\text{minimize}\{w_1@l_1, t_1; c_1; \ldots; w_n@l_n, t_n; c_n\} \). Lexicographically ordered objective functions are (optionally) distinguished via levels indicated by \( l_i \). An omitted level defaults to 0.

As an example, consider the rule in Line 6 of Listing 27:

\[
1 \{ \text{move}(D,P,T) : \text{disk}(D), \text{peg}(P) \} 1 : \text{ng}@\text{al}(T-1), T\leq n.
\]

This rule has a single head atom consisting of a cardinality constraint; it comprises all instances of \( \text{move}(D,P,T) \) where \( T \) is fixed by the two body literals and \( D \) and \( P \) vary over all instantiations of predicates \( \text{disk} \) and \( \text{peg} \), respectively. Given 3 pegs and 4 disks as in Listing 28, this results in 12 instances of \( \text{move}(D,P,T) \) for each valid replacement of \( T \), among which exactly one must be chosen according to the above rule.

Full details on the input language of \textbf{clingo} along with various examples can be found in (Gebser et al. 2015); its semantics is given in (Gebser et al. 2015).

3 Meta programming

Meta programming is a technique in which computer programs treat other programs as data. Although this includes traditional compilers and interpreters, it has always played a prominent role in AI languages, such as Lisp or Prolog, since their syntactic proximity of program and data offers an easy way of self-modification. For instance in Prolog, meta programs allow for manipulating the execution of logic programs and constitute an easy way to extend programs with debugging information. Moreover, special-purpose predicates enable the conversion of data into new program parts during run-time.

---

2 More elaborate forms of aggregates can be obtained by explicitly using function (eg. \#\text{count}) and relation symbols (eg. \( \leq \)).

3 Paraphrasing https://en.wikipedia.org/wiki/metaprogramming
Similarly, meta programming can be used in ASP to change the semantics of language constructs and/or implement new languages. Examples include reasoning about action and change [Baral and Gelfond 2000; Son et al. 2006; Dimopoulos et al. 2018], debugging (Gebser et al. 2008), preferences (Gelfond and Son 1997; Delgrande et al. 2003; Eiter et al. 2003) and optimization (Gebser et al. 2011), as well as guess-and-check programming (Eiter and Polleres 2006). The latter is of particular interest to us since we detail its implementation below with meta programming as well as through application interfaces in Section 8.

A common difficulty of such approaches is the conversion of programs into data, or in terms of ASP, the transformation of (non-ground) logic programs into sets of facts. Either a dedicated parser is built or a user is expected to put down a program in terms of a prescribed fact format. As a result, the resulting systems are mostly propositional and only offer a limited set of language constructs. This is because they cannot draw upon the infrastructure of an ASP system for parsing and grounding.

This issue is addressed in clingo, or better its grounder gringo, by means of a fact-based representation of the grounded logic program. This enables quite sophisticated meta programming that may draw on the full-featured non-ground input language of clingo, a highly effective grounding procedure, and ultimately a factual representation reflecting all features of the input language. The remainder of this section provides an introduction to meta programming with clingo. The extension of logic programs during run-time is explained in the subsequent sections.

3.1 Reification format

The process of turning a (ground) logic program into a set of facts is also called reification. clingo’s fact format of reified programs follows its intermediate language aspif, detailed in Appendix A. This is no coincidence since both must capture ground programs in their full generality. In what follows, however, we concentrate on dealing with the actual logic program part and disregard non-logical statements.

A logic program consists of a set of rules, each of which is composed of a head and a body. While heads are formed from atoms, bodies are made of literals.

The fact format is the result of serializing the syntax tree of the ground logic program rule by rule. To this end, heads and bodies are identified via non-negative integers. Also, positive and negative integers are used to represent positive or negative literals, respectively. Hence, 0 is no valid literal.

A rule is represented as a binary fact, using predicate rule/2, whose arguments reflect the head and the body of the rule. In aspif, a head is either a disjunction or a choice, which is indicated by the unary function symbols disjunction/1 and choice/1. Similarly, a body is either a collection of literals or a weight constraint, indicated by normal/1 and sum/1, respectively. All four constituents are treated as tuples, the two former consisting of atoms and the latter either of regular literals or a single weighted literal, respectively. Let us illustrate this with the example program in Listing 1.

4 Reification was originally introduced in gringo 4 for implementing complex optimization via meta programming (Gebser et al. 2011). There, however, it was recreated after the intermediate format of smodels.
How to build your own ASP-based system?! 7

1 \{a\}.
2 b :- a.
3 c :- not a.

Listing 1. A simple logic program (ezy.lp)

a set of facts by means of the command

\texttt{clingo --output=reify ezy.lp}

The result is given in Listing 2. More precisely, the first rule in Listing 1 is represented

\begin{verbatim}
1 rule(choice(0),normal(0)). atom_tuple(0). literal_tuple(0).
2 atom_tuple(0,1).
4 rule(disjunction(1),normal(1)). atom_tuple(1). literal_tuple(1).
5 atom_tuple(1,2). literal_tuple(1,-1).
7 rule(disjunction(2),normal(2)). atom_tuple(2). literal_tuple(2).
8 atom_tuple(2,3). literal_tuple(2,1).
10 output(a,2).
12 output(b,3).
13 literal_tuple(3).
14 literal_tuple(3,3).
15 output(c,4).
16 literal_tuple(4).
16 literal_tuple(4,2).
\end{verbatim}

Listing 2. (Rearranged) facts representing the simple logic program from Listing 1

by the facts in lines 1 and 2 of Listing 2. The choice in the head of the rule is captured by \texttt{choice(0)} and linked via the identifier 0 to a tuple of atoms declared by the fact \texttt{atom_tuple(0)}. Each member of an atom tuple is represented by instances of predicate \texttt{atom_tuple/2} that share the same tuple identifier as their first argument. The atom tuple 0 has one member, as indicated by the single fact \texttt{atom_tuple(0,1)}. While 0 stands for the tuple, 1 is the integer identifying atom \texttt{a} (see below). Analogously, the (empty) body of the choice rule is represented by the tuple of literals that is also identified by 0. This tuple happens to be empty, as reflected by the lack of corresponding instances of \texttt{literal_tuple/2}. Note that this tuple-centered representation treats atom and literal tuples independently and numbers both consecutively.

The second rule in Listing 1 is represented by the facts in lines 7 and 8 of Listing 2. Unlike above, its head is a single atom and is thus represented as a one-element \texttt{disjunction} associated with atom tuple 2. This tuple has a single element, which is accounted for by the fact \texttt{atom_tuple(2,3)}. Hence, \texttt{b} is represented by 3. Similarly, its body, also marked with 2, comprises a single literal captured by \texttt{literal_tuple(2,1)}. Here, 1 stands for

\footnote{Heads and bodies of the same rule may thus be represented by atom and literal tuples having distinct identifiers.}

\footnote{Although \texttt{clingo} grounds rule by rule, the order of the rules is determined internally.}
the positive body literal \(a\). The last rule in Listing 1 is represented analogously in lines 4 and 5 just that its negative body literal is mapped to a negative integer, namely, \(\neg a\) is associated with \(-1\).

The remaining facts in Line 10 to 16 account for implicit output statements. That is, unless any restrictions are formulated, all satisfied atoms are placed at the disposal of being output. This is done by means of the binary predicate `output/2`. For instance, the output of atom \(c\) is linked via literal tuple 4 to integer 2 (cf. lines 15 and 16). The indirection via the tuple representation is due to the fact that clingo's output statements may be conditioned by several literals (cf. Appendix A). Finally, it is interesting to observe that no new literal tuple is generated for `output(a,2)`. Rather the one in Line 7 and 8 is reused. This redundancy-free representation is a general feature of clingo's reification.

**Remark 1**

Although we do not detail this here, it is worth mentioning that the reified output format of clingo accounts for the full spectrum of language constructs supported by clingo's input language (including its generic grammar-based theory language). Its structure is aligned with the intermediate language aspif, which is a machine-oriented format used as the default output of gringo and (preferred) input of clasp. This format is detailed in Appendix A. In addition, clingo offers the options `--reify-sccs` and `--reify-steps` to calculate the strongly connected components of the ground logic program's (positive) dependency graph and to add step numbers to reified output, in case multi-shot solving is used.

### 3.2 Meta encoding

The facts obtained from reifying a logic program can now be used to instantiate meta encodings. Such an encoding allows us to reestablish the original or attribute a different meaning to program constructs.

To illustrate this, let us start with the simple meta encoding in Listing 3 which supports all above mentioned language constructs according to their original meaning. Before detailing the functioning of the encoding, let us describe its usage. To compute the stable models of the logic program `ezy.lp` in Listing 1 via meta programming, we proceed in two steps:

```
0 clingo --output=reify ezy.lp | clingo - meta.lp
```

At first, program `ezy.lp` is reified as described above, and then the resulting set of facts along with the meta encoding `meta.lp` are passed to clingo. The possibility of modifying the semantics of language constructs in `meta.lp` is paid by twice as much grounding.

---

7 This mimics the default behavior of clingo, outputting all atoms unless a restrictive `#show` statement is given.
8 This encoding is a subset of the one in https://github.com/potassco/clingo/blob/master/examples/reify/meta.lp which additionally accounts for optimization statements.
9 Note that `|` captures the output of the command before `|`.
10 Alternatively, one may also use an auxiliary file `ezy.rlp` to capture the facts in Listing 2:

```
0 clingo --output=reify ezy.lp > ezy.rlp; clingo ezy.rlp meta.lp
```

11 Adding option `-Wno-atom-undefined` to the second call suppresses warnings due to missing definitions. The same effect can be obtained by using a `#defined` declaration for predicates not occurring among the head atoms of a program.
How to build your own ASP-based system?!

Listing 3. A simple meta program interpreting reified logic programs (meta.lp)

effort. Interestingly, keeping the semantics as done in Listing 3 results in roughly the same solver constraints, no matter whether meta-programming is used or not. Hence, the overall overhead of meta programming is often negligible.

Let us now turn to the actual meta encoding. The logic program in Listing 1 uses the unary predicate hold/1 to express that an atom is true. Such atoms are derived in Lines 10 and 11, provided there is an original choice or disjunctive rule, whose body is satisfied. Both rules use conditional literals to gather all hold atoms belonging to the same atom tuple H, identifying the head of the original rule. In Line 10 this results in a disjunction of atoms, while in Line 11 all such atoms form a set of choosable atoms. In both cases, several, one, or no hold atoms may manifest themselves, depending on the size of the atom tuple. The satisfaction of the body of the original rule, identified by B, is indicated in both lines by the positive body literal body(B). The corresponding atoms are derived by the two rules in lines 5 to 8.

In Line 5 a normal body, composed of regular literals, is satisfied whenever all its literals are found to be true. This is realized by the rule in Line 1 to 3 by using again conditional literals to gather all hold atoms induced by a tuple of literals. Depending on whether the integer representing a literal is positive or negative, the corresponding hold atoms must be satisfied or not. Similarly the rule in Line 6 to 8 implements a weight constraint, just that the hold atoms are expanded within a sum constraint along with their associated weights.

Note that conjunctions of literal tuples may not only stand for rule bodies but also occur in other constructs like conditional output statements. Hence, it makes sense to account for them separately in Line 1 to 3.

3.3 Examples

3.3.1 Classical and supported models

Let us start with some simple modifications to our meta encoding in Listing 3 that change the semantics of logic programs.

For illustration, we consider classical and supported models of logic programs. Take the logic program consisting of the following three rules:
a :- not b. b :- c. c :- b.

It has one stable model, \{a\}, two supported models, \{a\} and \{b, c\}, and three classical models, \{a\}, \{b, c\} and \{a, b, c\}.

This example already illustrates a general relationship between all three semantics: a stable model is also a supported model, which in turn is also a classical model but not vice versa. Intuitively, this difference is the result of how tight each semantics relates the truth of an atom to its derivability (through rules and positive body literals). While no such relation is imposed in the classical setting, supported models require that each of their atoms is supported by a rule having the atom as head and a body satisfied by the model at hand. Stable models reinforce this by stipulating that furthermore all positive body literals of the supporting rule have themselves a supporting rule and that this ends in facts (and thus yields a finite derivation). In our example, only \(a\) warrants this within the only stable model, while \(b\) and \(c\) only satisfy the supportedness criterion in \{\(b, c\)\} but lack a finite derivation. The detachment of truth from derivations (via rules) is exemplified by \(a\) in the classical model \{\(a, b, c\)\}.

For computing classical models in ASP, we have to lift the ban of derivability from atoms. To this end, we extend in Listing 4 our previous meta encoding (via Line 1) with

```lp
#include "meta.lp"

3 atom( A ) :- atom_tuple(_,A).
4 atom(\|L\|) :- literal_tuple(_,L).
5 atom(\|L\|) :- weighted_literal_tuple(_,L).
7 { hold(A) : atom(A) }.
```

Listing 4. Meta encoding computing classic models of logic programs (classic.lp)

the choice rule in Line 7; its subjects are gathered in lines 3 to 5. This choice rule exempts atoms of predicate \texttt{hold/1} from having a derivation by allowing for their inclusion into a stable model at will. Classical models of a logic program can then be computed as above, just by replacing \\texttt{meta.lp} through \texttt{classic.lp}, given in Listing 4.

For computing supported models, we have to make sure that each included \(\texttt{hold}\) atom is supported by some rule. The body of this rule must be satisfied, and its positive literals must themselves have supporting rules, but they do not necessarily have to yield a finite derivation. This can be accomplished by replacing the positive occurrences of \(\texttt{hold}\) literals in Line 2 and 7 in Listing 3 by their double negation. In fact, in ASP, each true positive literal must have a derivation, while its double negated variant is freed from this requirement. The resulting meta encoding is given in Listing 5; cf. Line 2 and 7 in both encodings. As above, supported models are computed by replacing \texttt{meta.lp} by \texttt{supported.lp} in the above system call.

\begin{enumerate}
\item See Remark 2 below.
\item Clearly, more direct meta encodings can be devised, for instance, by turning rules into integrity constraints.
\item See Remark 2.
\end{enumerate}
How to build your own ASP-based system?!

1. conjunction(B) :- literal_tuple(B),
2. not not hold(L) : literal_tuple(B, L), L > 0;
3. not hold(L) : literal_tuple(B,-L), L > 0.

5. body(normal(B)) :- rule(,_normal(B)), conjunction(B).
6. body(sum(B,G)) :- rule(,_sum(B,G)),
7. #sum { W,L : not not hold(L), weighted_literal_tuple(B, L,W), L > 0 ;
8. W,L : not hold(L), weighted_literal_tuple(B,-L,W), L > 0 } >= G.

10. hold(A) : atom_tuple(H,A) :- rule(disjunction(H),B), body(B).
11. { hold(A) : atom_tuple(H,A) } :- rule( choice(H),B), body(B).

Listing 5. Meta encoding computing supported models of logic programs (supported.lp)

Remark 2
Note that the computation of classical and supported models is primarily an academic exercise, since in practice reification is subject to stable-model preserving simplifications that are usually too strong for weaker semantics. In simple cases, like ours, this can be counterbalanced by declaring some atoms as being externally defined. For instance, adding ‘#external b.’ (cf. Section 5.1) to our example program spares b from simplification and produces the above results; otherwise not all models are obtained.

Unfortunately, such techniques become infeasible with programs using integers or function symbols since they may possess infinitely many models in general. For instance, the program consisting of ‘q(f(a)).’ and ‘p(X) :- p(X).’ has a single stable but infinitely many supported and classical models.

3.3.2 Diverse models
Our next exemplary application of meta programming is about computing several diverse stable models of a logic program.\(^{15}\)

To do so within ASP rather than external scripting, we consider several reified stable models within one. To this end, we turn the predicate hold into a binary predicate, whose second argument identifies the respective stable model. These identifiers are generated in Line 1 of Listing 6 where parameter m limits the number of reified stable models. The following lines 3 to 17 constitute an extension of the original meta encoding obtained by adding an additional argument to all non-structural predicates for identifying the associated stable model. This is done throughout with variable M, sometimes bound by model(M). Taking a logic program with n stable models and setting m to 2 makes lines 1 to 17 produce \(n^2\) stable models, each of which comprises two stable models of the original program. To distinguish the comprised models, Line 17 outputs each atom with its associated model identifier.

This initial part of Listing 6 acts as a generator of combinations of \(m\) stable models of the

\(^{15}\) Full-fledged approaches to computing diverse stable models can be found in Eiter et al. 2013 [Romero et al. 2016].
Listing 6. Meta encoding computing several (diverse) stable models (many.lp)

original program. In this spirit, the remainder selects three types of model combinations depending upon the setting of parameter option (and \( k \) in the second case). More precisely, the selected \( m \) reified stable models are

- \( k \)-diverse, if option=1, that is, the Hamming distance between each pair of the \( m \) stable models is greater or equal than \( k \), and

- most-diverse, if option=2, that is, the \( m \) stable models maximize the sum of the Hamming distances between each pair of stable models.

Moreover, the implementation considers only atoms declared to be shown (by using predicate show/2 rather than hold/2 in lines 22 to 25).

The selection of model collections having a pairwise Hamming distance greater or equal than \( k \) is represented by the sum constraint in Line 22 and 23. The condition is embedded into an integrity constraint ruling out all pairs of models \( M \) and \( N \) that differ on less than \( k \) shown atoms. Similarly, the optimization statement in Line 25 maximizes the difference between two models; it attributes one point per difference. Given that such a statement exists for each pair of models the overall sum of differences is maximized. Note that in each case the value of parameter option is tested, so that at most one of them applies.

As an example, consider the logic program in Listing 7.\(^{16}\) The goal in this example is to mark \( m \) cells of an \( n \times n \) grid such that the marked cells are connected to each other (where \( m \) and \( n \) are parameters).

Let us begin by computing three different stable models by relying on the default value of \( k \) in Line 19.\(^{17}\)

\(^{16}\) The symbol #false is the logical constant for falsity.

\(^{17}\) For simplicity, we omit here and in what follows option -Wno-atom-undefined suppressing warnings due to missing definitions.
How to build your own ASP-based system?!  

1 \{ x((1..n,1..n)) \} = m.

3 connect(C) :- x(C), #false: x(C'), C'<C.

4 connect((X',Y')) :- connect((X,Y)), x((X',Y')), |X-X'|+|Y-Y'|=1.

6 :- x(C), not connect(C).

8 #show x/1.

Listing 7. Mark m cells of an n×n grid that must be connected to each other (cells.lp)

```
UNIX> clingo --output=reify cells.lp -c n=3 -c m=3 | \
    clingo - many.lp -c option=1 -c m=3
clingo version 5.5.0
Reading from - ...
Solving...
Answer: 1
(x((2,3)),1) (x((3,2)),1) (x((3,3)),1) \ 
(x((1,3)),2) (x((2,3)),2) (x((3,3)),2) \ 
(x((3,1)),3) (x((3,2)),3) (x((3,3)),3)
SATISFIABLE
```

The obtained three reified stable models can be visualized as follows (by letting (1,1) be the lower left corner).

```
X X X
X X
X
```

Next, consider the result obtained by imposing a Hamming distance of 6.

```
UNIX> clingo --output=reify cells.lp -c n=3 -c m=3 | \
    clingo - many.lp -c option=1 -c m=3 -c k=6
clingo version 5.5.0
Reading from - ...
Solving...
Answer: 1
(x((2,3)),1) (x((3,2)),1) (x((3,3)),1) \ 
(x((1,1)),2) (x((1,2)),2) (x((1,3)),2) \ 
(x((2,1)),3) (x((2,2)),3) (x((3,1)),3)
SATISFIABLE
```

Unlike above, the three solutions do not overlap and possess the imposed pairwise Hamming distance, as illustrated below.

```
X X X
X X X
X
```

Note that no three stable models are obtainable with a pairwise Hamming distance exceeding 6.

A similar result is obtained by maximizing the sum of Hamming distances, as shown next.

```
UNIX> clingo --output=reify cells.lp -c n=3 -c m=3 | \
```

\[18\] We use --quiet=1,2,2 to suppress intermediate models.
The obtained solution has the same quality as the previous one, namely, $18^{19}$ three times a Hamming distance of six. Note that this relation is a particularity of our example.

3.3.3 Here-and-there models

The logical foundations of ASP rest upon the logic of Here-and-There (HT; [Heyting 1930]), or more specifically its non-monotonic extension called Equilibrium Logic (EL; [Pearce 1997]). Informally, interpretations in HT consist of pairs of interpretations \((H, T)\) such that \(H \subseteq T\). The intuition of using two such sets is that atoms in \(H\) are the ones that can be proved, atoms not in \(T\) are those for which no proof exists, and, finally, atoms in \(T \setminus H\) are assumed to hold but have not been proved. A total HT model \((T, T)\) is an equilibrium model if there is no HT model \((H, T)\) with \(H \subset T\). In such a case, \(T\) is also called a stable model. See [Pearce 1997] for a comprehensive account of these logics.

The meta encoding for computing HT models of logic programs is given in Listing 8; it builds upon several constructions already used in the previous meta encodings. The first part in lines 1 to 9 is similar to the computation of classical models in Listing 4 in generating all admissible HT interpretations. As above, we use terms, namely, \(h\) and \(t\), to distinguish the components in HT interpretations such as \((H, T)\). Line 7 generates atoms in \(H\), Line 8 the ones in \(T\), and the integrity constraint in Line 9 makes sure that \(H \subseteq T\). The type of generated HT interpretations is once more determined by parameter \(option\).

A generated pair of interpretations \((H, T)\) is

- an HT interpretation, if \(option=1\),
- an HT interpretation with minimal \(H \subseteq T\), if \(option=2\) (or undefined),
- a total HT interpretation with \(H = T\), if \(option=3\).

The aforementioned description captures the first case. In both remaining cases, the free generation of atoms in \(H\) is dropped (cf. Line 7) and they must rather be derived via program rules. In addition, Option 3 enforces \(T \subseteq H\) via the integrity constraint in Line 11.

The satisfaction of rules and the derivation of hold atoms in lines 13 to 23 is similar to the computation of diverse models in Listing 6 with a few exceptions due to the different semantic setting. First of all, a negative literal only holds in an HT interpretation \((H, T)\)

\(^{19}\) The value of $-18$ is due to the fact that \texttt{maximize} statements are internally converted to \texttt{minimize} statement along with a corresponding conversion of all weights.
How to build your own ASP-based system?

1 atom( A ) :- atom_tuple(_,A).
2 atom([L|]) :- literal_tuple(_,L).
3 atom([L|]) :- weighted_literal_tuple(_,L).

5 model(h). model(t).
7 { hold(A,h) } :- atom(A), option = 1.
8 { hold(A,t) } :- atom(A).
9 :- hold(L,h), not hold(L,t).
11 :- not hold(L,h), hold(L,t), option = 3.
13 conjunction(B,M) :- model(M), literal_tuple(B),
14 hold(L,M) : literal_tuple(B, L, W), L > 0;
15 not hold(L,t) : literal_tuple(B, -L, W), L > 0.
17 body(normal(B),M) :- rule(_,normal(B)), conjunction(B,M).
18 body(sum(B,G),M) :- model(M), rule(_,sum(B,G)),
19 #sum { W,L : hold(L,M), weighted_literal_tuple(B, L, W), L > 0 ;
20 W,L : not hold(L,t), weighted_literal_tuple(B, -L, W), L > 0 } >= G.
22 hold(A,M) : atom_tuple(H,A) :- rule(disjunction(H),B), body(B,M).
23 hold(A,M); not hold(A,t) :- atom_tuple(H,A), rule(choice(H),B), body(B,M).
25 #show.
26 #show ( T,M ) : output(T,B), conjunction(B,M).

Listing 8. A meta encoding for computing here-and-there models

if its atom does not belong to \( T \). Accordingly, the tests for negative body literals in lines 15 and 20 only refer to \( \text{hold} \) atoms associated with \( t \). Second, the choice of an atom \( \{ a \} \) amounts in HT to an instance of the law of the excluded middle \( a \lor \neg a \). This classical treatment of an atom leaves only two possibilities in Line 23; either \( a \) is false, and thus does not belong to \( T \) (and neither to \( H \)), or \( a \) is true in \( H \) or \( T \). Note that whenever \( \text{option}=1 \), this variant of our meta encoding merely checks the satisfaction of rules in both components of an HT interpretation. Unlike this, \( \text{hold} \) atoms associated with \( h \) must be derived via the meta encoding in all other cases.

For illustration, consider the logic programs \( \text{or}.lp \) containing the disjunction \( 'a;b.' \) and \( \text{even}.lp \) with rules \( 'a :- \neg b. b :- \neg a.' \). Both programs are equivalent in the sense that they share the same stable models \( \{ a \} \) and \( \{ b \} \). However, putting each program together with rules \( 'a :- b. b :- a.' \) yields different stable models, indicating that both programs are not interchangeable in an encompassing program; in formal terms, they are not strongly equivalent (Lifschitz et al. 2001). Interestingly, the strong equivalence of logic programs corresponds to their equivalence in HT; their equivalence can be read off the same set of equilibrium models.

Let us verify this by means of our meta encoding in Listing 8. Program \( \text{or}.lp \) has five HT models.

|UNIX| clingo --output=reify or.lp |
|    | clingo -ht.lp 0 -c option=1 |
|    | clingo version 5.5.0 |
|    | Reading from - ... |
|    | Solving... |

20 Obviously, this also implies that the atom does not belong to \( H \) since \( H \subseteq T \).
21 Strictly speaking, Rule 23 could be specialized by adding \( M=h \), since the instance with \( M=t \) is subsumed by Rule 8, which is equivalent to \( \text{hold}(A,t); \text{not hold}(A,t) :- \text{atom}(A) \).
22 For simplicity, we omit once more option \(-\text{who-atom-undefined}\) suppressing warnings due to missing definitions (cf. Footnote 17).
In contrast to ‘a;b.’, Program even.lp has the additional HT model (\emptyset, \{a, b\}), as witnessed by the first of its six models:

```
UNIX > clingo --output=reify even.lp | \
clingo -ht.lp 0 -c option=1
clingo version 5.5.0
Reading from - ...
Solving...
Answer: 1
(a,t) (b,t)
Answer: 2
(a,t) (a,h)
Answer: 3
(a,t) (b,t) (a,h)
Answer: 4
(b,t) (b,h)
Answer: 5
(a,t) (b,t) (b,h) (a,h)
SATISFIABLE
```

Unlike above, both programs have the same equilibrium models, which establishes their equivalence.

```
UNIX > clingo --output=reify or.lp | \
clingo -ht.lp 0 -c option=3
clingo version 5.5.0
Reading from - ...
Solving...
Answer: 1
(b,t) (b,h)
Answer: 2
(a,t) (a,h)
SATISFIABLE
```

```
UNIX > clingo --output=reify even.lp | \
clingo -ht.lp 0 -c option=3
clingo version 5.5.0
Reading from - ...
Solving...
Answer: 1
(a,t) (a,h)
Answer: 2
(b,t) (b,h)
SATISFIABLE
```

### 3.4 Guess-and-check programming

So far, we addressed problems at the first level of the polynomial hierarchy, sharing the same complexity as normal logic programs in ASP [Dantsin et al. 2001]. In fact, ASP
How to build your own ASP-based system?! can also be used for expressing problems at the second level, when disjunctive heads or non-monotonic aggregates are used.

An interesting class of such problems consists of two subproblems (Eiter and Polleres 2006): A guess-and-check logic program is a pair \((P, Q)\) whose solution is a stable model of \(P\) that results in an unsatisfiable program once its atoms are added as facts to \(Q\). This combines a satisfiability problem with an unsatisfiability problem, in the most interesting case, an \(NP\) with a \(coNP\) problem since this lifts the joined problem to the second level of the polynomial hierarchy. An example of this is preference handling, where the first problem defines feasible solutions, while the second one ensures that there are no better solutions. Another example is (bounded) conformant planning under incomplete information, where the first problem gives a plan in some scenario, while the second one makes sure that it is not invalidated in any other scenario. The difficulty then lies in finding a single ASP encoding that combines both problems and returns the solution despite the unsatisfiability of the second subproblem.

For implementing such problems, Eiter and Gottlob invented in (1995) the saturation technique, using the elevated complexity of disjunctive logic programming. In stark contrast to the ease of common ASP modeling, however, this technique is rather involved and hardly usable by ASP laymen. This shortcoming was addressed by means of meta programming by Eiter and Polleres in (2006). The idea is to represent both components of a guess-and-check program as facts and to combine them with a meta encoding to obtain a single joint program after grounding. The meta encoding implements the saturation technique and discharges users from this intricate modeling task. Now, the reification functionalities of clingo allow us to simplify this even further by relieving users from the specification of guess-and-check program as facts.

In what follows, we showcase the computation of solutions to guess-and-check programs by means of meta programming with clingo. To this end, let us start with an outline of a meta encoding implementing saturation and tie it up to guess-and-check programming afterwards. More precisely, we build on an encoding of saturation put forward in (Gebser et al. 2011) and present in Listing 9 a revised version based on the aspif format. A detailed account of the saturation technique goes beyond the scope of this paper and can be found in (Eiter and Gottlob 1995; Eiter and Polleres 2006). From the perspective of plain ASP, a saturation-based meta encoding acts as an ordinary one just that it yields the set of all atoms whenever a program is unsatisfiable. That is, it generates one stable model for each original stable model, whenever the program is satisfiable, and otherwise, it yields a unique stable model containing the set of all atoms of the program. Also, unlike Listing 3, the encoding in Listing 9 takes a reified normal logic program as input and results in a disjunctive logic program after grounding.

As an example, consider the simple logic program \texttt{a.lp}:

\[
1 \{ a(1..2) \}.
\]

Computing its stable models with the meta encoding in Listing 9 (along with the auxiliary \#show statements from Listing 10) yields the three expected models:

---

23 The original meta encoding in (Gebser et al. 2011) relies on the smodels format used for reification in clingo version 4 (cf. Footnote 5).

---

24 The motivation for using option \texttt{-reify-sccs} is explained below.
sum(B, G, T) :- rule(_, sum(B, G)), T = \#\{ W, L : weighted_literature_tuple(B, L, W) \}.  

supp(A, B) :- rule( choice(H), B), atom_tuple(H, A).  
supp(A, B) :- rule(disjunction(H), B), atom_tuple(H, A).  
supp(A) :- supp(A, _).  

atom(L) :- weighted_literature_tuple(_, L, _).  
atom(L) :- literal_tuple(_, L).  
atom(A) :- atom_tuple(_, A).  

fact(A) :- rule(disjunction(H), normal(B)), atom_tuple(H, A), not literal_tuple(B, _).  
true(atom(A)) :- fact(atom(A)).  
fail(atom(A)) :- supp(atom(A), not fact(atom(A))).  

true(normal(B)) :- literal_tuple(B), L > 0; fail(normal(B)) :- literal_tuple(-B, L), L > 0.  
true(normal(B)) :- literal_tuple(B, L), fail(normal(B)), L > 0; fail(normal(B)) :- literal_tuple(-B, -L), true(normal(B)), L > 0.  

true(sum(B, G)) :- sum(B, G, T), \#\{ W, L : true(atom(L)), weighted_literature_tuple(B, L, W), L > 0 ; W, L : fail(atom(L)), weighted_literature_tuple(B, -L, W), L > 0 \} >= G.  
fail(sum(B, G)) :- sum(B, G, T), \#\{ W, L : fail(atom(L)), weighted_literature_tuple(B, L, W), L > 0 ; W, L : true(atom(L)), weighted_literature_tuple(B, -L, W), L > 0 \} >= T-G+1.  

bot :- rule(disjunction(H), B), true(B), fail(atom(A)) : atom_tuple(H, A).  
bot :- true(atom(A)), fail(B) : atom_tuple(A, B).  
internal(C, normal(B)) :- scc(C, A), supp(atom(A), normal(B)), scc(C, A'), literal_tuple(B, A').  
internal(C, sum(B, G)) :- scc(C, A), supp(atom(A), sum(B, G)), scc(C, A'), weighted_literature_tuple(B, A', W).  
external(C, normal(B)) :- scc(C, A), supp(atom(A), normal(B)), not internal(C, normal(B)).  
external(C, sum(B, G)) :- scc(C, A), supp(atom(A), sum(B, G)), not internal(C, sum(B, G)).  

steps(C, Z-1) :- scc(C, _), Z = \{ scc(C, A) : not fact(A) \}.  
wait(C, atom(A), 0) :- scc(C, A), fail(B) : external(C, B), supp(A, B).  
wait(C, normal(B), I) :- internal(C, normal(B)), fail(normal(B)), steps(C, Z), I = 0..Z-1.  
wait(C, normal(B), I) :- internal(C, normal(B)), literal_tuple(B, A), wait(C, atom(A), I), steps(C, Z), I < Z.  
wait(C, sum(B, G), I) :- internal(C, sum(B, G)), steps(C, Z), I = 0..Z-1, sum(B, G, T).  
#\{ W, L : true(atom(L)), weighted_literature_tuple(B, L, W), L > 0, not scc(C, L) ; W, L : fail(atom(L)), weighted_literature_tuple(B, -L, W), L > 0, scc(C, L) ; W, L : true(atom(L)), weighted_literature_tuple(B, -L, W), L > 0 \} >= T-0+1.  
wait(C, atom(A), I) :- wait(C, atom(A), I-1), steps(C, Z), I = 1..Z, wait(C, atom(A), I-1) : supp(A, B), internal(C, B).  
bot :- scc(C, A), true(atom(A)), wait(C, atom(A), Z), steps(C, Z).  
true(atom(A)) :- supp(A), not fact(A), bot.  
fail(atom(A)) :- supp(A), not fact(A), bot.  

Listing 9. A disjunctive meta encoding implementing saturation (metaD.lp)

```
UNIX> clingo --output=reify --reify=sccs a.lp | 
     clingo - metaD.lp show.lp 0
clingo version 5.5.0
Reading from ...  
Solving...  
Answer: 1  
a(2)  
Answer: 2  
a(1)  
Answer: 3  
a(1) a(2)
```
How to build your own ASP-based system?!

1. #show.
2. #show X : output(X,B), literal_tuple(B,A), true(atom(A)).
3. #show X : output(X,B), not literal_tuple(B,_).

Listing 10. Auxiliary #show statements for Listing 9 (show.lp)

SATISFIABLE

Now, the addition of an empty integrity constraint, namely ‘:-.’, makes the program unsatisfiable. This is reflected by a single answer set containing all atoms of the program. This should not to be confused with the third model obtained above.

UNIX> clingo --output=reify --reify=sccs a.lp <( echo ":-." ) \ 
    clingo - metaD.lp show.lp 0
clingo version 5.5.0
Reading from - ...
Solving ...
Answer: 1
a(1) a(2)
SATISFIABLE

In fact, additional show statements would reveal that the actual stable model also contains the artificial atom bot from which all atoms occurring in the original program are derivable (cf. Line 53 and 54 in Listing 9). In other words, this special atom expresses the non-existence of stable models, and by saturating the model with all atoms it can only exist if no true stable models exist. This is because the semantics of disjunctive logic programs is based on subset minimization. Saturation makes sure that bot is derived only if it is inevitable, that is, if it is impossible to construct any other models.

The saturation-based meta encoding in Listing 9 relies on a partition of the atoms of the input program induced by the strongly connect components of its positive dependency graph. Each atom and each loop of the program is contained in some part. The idea is to mimic the consecutive application of the immediate consequence operator to each component of the partition. In a nutshell, the encoding in Listing 9 combines the following parts (cf. Gebser et al. 2011):

1. guessing an interpretation (in lines 14 to 29),
2. deriving the unsatisfiability-indicating atom bot if the interpretation is not a supported model (where each true atom occurs positively in the head of some rule whose body holds; cf. Line 31 and 32),
3. deriving bot if the true atoms of some non-trivial strongly connected component are not acyclically derivable (checked via determining the complement of a fixpoint of the immediate consequence operator; cf. lines 34 to 51), and
4. saturating interpretations that do not correspond to stable models by deriving all truth assignments (for atoms) from bot (in Line 53 and 54).

In fact, without the two saturating rules in lines 53 and 54, Listing 9 would produce a stable model for each interpretation of the original program. The ones without bot represent stable models, while the ones with bot are mere interpretations. By saturation, all these interpretations are mapped to the set of all atoms. Given that the latter is a superset of all conceivable stable models, it can only exist if no stable models exist.

A strongly connected component is a maximal subgraph of the positive dependency graph of a program such that all nodes are pairwisely connected via paths. A strongly connected component is trivial if it does not contain any edge, and non-trivial otherwise.
A comprehensive description of the specific concepts underlying Listing 9 is given in (Gebser et al. 2011).

Now, for implementing guess-and-check programming, we exploit the property of Listing 9 that the non-existence of stable models is indicated by atom \( \text{bot} \). Hence, to make sure that an (augmented) check program is unsatisfiable, we can simply add an integrity constraint that enforces \( \text{bot} \) to hold:

\[
: - \text{not bot}.
\]

Listing 11. Integrity constraint enforcing unsatisfiability (bot.lp)

What remains to be done is to foresee the import of the guessed atoms into the check program and to align the stable models of the guess and the check program. For simplicity, we address both tasks in a rather direct way and show below a more principled alternative. For addressing the first task, we simply add a choice rule over the possible guess atoms to the check program to allow for exchanging all possible interpretations. For this to work, however, the guess atoms must not occur among the heads of the check program. Otherwise a guess atom may be false but become true in the check program. For illustration, consider the simple guess and check programs in Listing 12 and 13 along with the import of guess atoms into the check program in Listing 14.

\[
1 \{ \text{a(1..2)} \}.
\]

\[
\#\text{show a/1}.
\]

Listing 12. Guess program (guess.lp)

\[
{ \text{a(1..2)} }.
\]

Listing 14. Import guess atoms (in.lp)

The second task, synchronizing the respective stable models, is also handled by saturation. Whenever two different models in the guess and in the check part are encountered, the artificial atom \( \text{bot} \) is derived and leads to the saturation of the check part with all atoms, which in turn invalidates the checker’s stable model (since it is obviously a subset of all atoms). In our example, the comparison of the stable models of the guess and check program is done with the two rules in Listing 15.

\[
1 \text{ bot :- output(a(X),B), a(X), fail(normal(B))}. \\
2 \text{ bot :- output(a(X),B), not a(X), true(normal(B))}. \\
\]

Listing 15. Comparing stable models (glue.lp)

The body literals of both rules already hint at the design decision to only reify the check program, while leaving the guess program intact. At last, let us put all this together for our example guess and check program:

```
UNIX> clingo --output=reify --reify -sccs check.lp in.lp | \
        clingo - metaD.lp bot.lp glue.lp guess.lp 0
clingo version 5.5.0
Reading from - ...
Solving...
Answer: 1
a(2)
SATISFIABLE
```

Combining program guess.lp and check.lp from Listing 12 and 13 in a guess-and-check
program eliminates all stable models of \texttt{guess.lp} that contain \texttt{a(1)}. Note that joining both in a regular program eliminates models excluding \texttt{a(1)}.

The first call to \texttt{clingo} merely reifies program \texttt{check.lp}, so that it gets interpreted by the meta encoding \texttt{metaD.lp} in the second call. Unlike this, program \texttt{guess.lp} remains untouched. Hence, the problem handed to the second call of \texttt{clingo} combines both the guess and the check program, whereby the latter is encoded via saturation in order to succeed upon unsatisfiability. And finally, program \texttt{glue.lp} is in charge of aligning the guessed stable models to the checker by saturating the non-aligned ones. Clearly, our illustrative example is super simple and could also be solved directly. In what follows, we present some more substantial use-cases.

\textit{Remark 3}

It is instructive to observe the effect of programs \texttt{bot.lp} and \texttt{glue.lp} on the formation of the joint solving result:

- Without both \texttt{bot.lp} and \texttt{glue.lp}, we get all combinations of stable models of the guess program with stable models of the check program (choice rules included), if the check program is satisfiable, and with the unique saturated set of atoms containing \texttt{bot} otherwise.
- Without \texttt{bot.lp} but with \texttt{glue.lp}, we get all combinations of stables model of the guess program with stables model of the check program (conjoined with all facts stemming from guess atoms), if this program is satisfiable, and with the unique saturated set of atoms containing \texttt{bot} otherwise.
- With both \texttt{bot.lp} and \texttt{glue.lp}, we get all stable models of the previous item that contain \texttt{bot}.

That is, we get all combinations of stable models of the guess program with the unique saturated set of atoms containing \texttt{bot} whenever the check program (conjoined with all facts stemming from guess atoms) is unsatisfiable, and no stable models otherwise.

A more generic approach can be obtained by capturing the guessed atoms by a dedicated predicate, say \texttt{guess/1}. This also eases the restriction of the relayed atoms to a distinguished subset.

In our example, the guess program in Listing 12, viz.

\begin{verbatim}
1  1 { a(1..2) }.
2  #show a/1.
\end{verbatim}

is then augmented with the specification of the actual guess:

\begin{verbatim}
1  guess(a(X)) :- a(X).
\end{verbatim}

Listing 16. Exporting guess atoms (\texttt{out.lp})

Accordingly, the guessed atoms become enveloped in predicate \texttt{guess/1} in the check program \texttt{check.lp}, viz.

\begin{verbatim}
1  :- not guess(a(1)).
\end{verbatim}

Listing 17. Importing guess atoms (\texttt{in.lp})
The above version of Program in.lp replaces the specific choice rule ‘\{ a(1..2) \}.’ in Listing [13] by a generic rule. The instantiation of the first rule is obtained from the symbol table of the guess program, as shown below.

With this approach, both the guess as well as the check programs come in pairs, the former accompanied by the specification of guessed atoms in out.lp in Listing [16] and the latter with choices delineating all possible sets of guessed atoms in in.lp in Listing [17].

While the first fixes all guessable atoms, the second one is generic.

And finally, also the synchronization of stable models can now be expressed in a generic way via the two next rules.

1. \( \text{bot} :- \text{output} (\text{guess}(X), B), \text{guess}(X), \text{fail}(\text{normal}(B)). \)

2. \( \text{bot} :- \text{output} (\text{guess}(X), B), \text{not} \text{guess}(X), \text{true}(\text{normal}(B)). \)

Listing 18. Generic comparison of stable models (superglue.lp)

Note that except for Listing [16], all auxiliary programs are problem independent.

The purpose of the first two lines is to extract all guessable atoms from the symbol table of the guess program. (In passing, this illustrates a pragmatic way of exploiting the symbol table of reified programs.) In our example, this results in the facts \( \text{output}(\text{guess}(a(1)), 5) \) and \( \text{output}(\text{guess}(a(2)), 6) \). These atoms are then used to instantiate the rules in Line 1 of Listing [17] and Line 1 and 2 in Listing [18]. Otherwise, the two remaining calls work just as described above, and obviously yield the same result.

In fact, the effort of encapsulating the guessed atoms along with the resulting generality pays off, as we demonstrate in the following two use-cases.

Preferences. In this example, we use the above setup to compute superset maximal stable models of logic programs. The idea is to guess candidate stable models and to check whether a superset larger stable model of the program is obtainable. If this fails, the candidate is superset maximal.

To illustrate this, reconsider Program guess.lp in Listing [12] along with the specification of guessable atoms in Listing [16]. The check program consists once more of program guess.lp yet extended with the following two rules.

1. \( \text{better} :- a(Y), \text{not guess}(a(Y)), a(X) : \text{guess}(a(X)). \)

2. \( :- \text{not better}. \)

Listing 19. Identifying superset maximal stable models (superset.lp)

The atom better is derived whenever the guessed stable model is a strict superset of the stable model generated by the check program. The candidate model is superset maximal whenever no strictly larger model is obtainable, as checked in Line 2 of Listing [19].
same set of stable models. A guessed stable model is optimal, if no better model can be obtained by the checker. This nicely reflects the checker’s role of generating potential counterexamples.

Proceeding in the same manner as above, we compute the only superset maximal model of program guess.lp as follows.

Proceeding in the same manner as above, we compute the only superset maximal model of program guess.lp as follows.

`UNIX > clingo -- output = reify guess.lp out.lp <(echo "#show guess/1.") | grep "output(guess(.*))" | \ clingo -- output = reify -- reify - sccs - guess.lp superset.lp in.lp | \ clingo - metaD.lp bot.lp superglue.lp guess.lp out.lp 0 clingo version 5.5.0 Reading from - ... Solving... Answer : 1 a(1) a(2) SATISFIABLE`

**Tic-tac-toe.** In this example, we consider a simplified $3 \times 3$ Tic-tac-toe puzzle. Player O has to place her three tokens in a winning configuration such that afterwards Player X cannot place hers in a winning position. Hence, the game is not played in turns and just guaranteed winning configurations of Player O are determined.

The encodings of Player O and X are given in Listing 20 and 21 below. The former acts as a guess program and the latter as a checker program; only the positions of Player O are passed to Player X, as fixed in Listing 22. In this way, the positions occupied by O become blocked for Player X in Line 9 of Listing 21.

```
1 #const n=3.
2 n { o(1..n,1..n) } n.
3 :- not win.
4 win :- I=1..n, o(I,J) : J=1..n.
5 win :- J=1..n, o(I,J) : I=1..n.
6 win :- o(I,I) : I=1..n.
7 win :- o(I,n+1-I) : I=1..n.
9 #show o/2.

Listing 20. Player O (playero.lp)
```

```
1 #const n=3.
2 n { x(1..n,1..n) } n.
3 :- not win.
4 win :- I=1..n, x(I,J) : J=1..n.
5 win :- J=1..n, x(I,J) : I=1..n.
6 win :- x(I,I) : I=1..n.
7 win :- x(I,n+1-I) : I=1..n.
9 :- guess(o(I,J)), x(I,J).

Listing 21. Player X (playerx.lp)
```

```
1 guess(o(I,J)) :- o(I,J).

Listing 22. Exporting guess atoms (out.lp)
```

This example involves two similar yet different programs. Unlike above, a guess involves only a subset of the generated stable models. Relying on the above setup, we can then compute the two undefeatable diagonal configurations in Tic-tac-toe:

```
UNIX > clingo -- output = reify playero.lp out.lp \ <(echo "#show guess/1.") | grep "output(guess(.*))" | \ clingo -- output = reify -- reify - sccs - playero.lp in.lp | \ clingo - metaD.lp bot.lp superglue.lp playero.lp out.lp 0 clingo version 5.5.0 Reading from - ... Solving... Answer : 1 o(1,1) o(2,2) o(3,3) Answer : 2 o(1,3) o(2,2) o(3,1) SATISFIABLE
```
4 About clingo applications

We have seen in the last section how the functionality of ASP systems can be changed by using meta programming. In particular, the reification of logic programs allows us to control ASP by means of ASP. The remainder of this tutorial parallels this by showcasing several ways of how ASP can be managed with other programming languages. As mentioned in the introduction, we have chosen Python as our exemplar, although other choices exist.

This section focuses on the overall setup before the following ones delve into particular functionalities and case studies.

clingo offers three alternative ways of combining ASP with other programming languages, either via an embedded script, module import, or its application class. Although all three options allow us to change the behavior of clingo by overwriting its main function, they aim at rather different use cases. We discuss the three choices below and show how they treat the common example in Listing 23. The idea is to outsource the computation of divisors to Python. The term \texttt{@divisors(N)} in Line 3 assumes the definition of a corresponding method in Python that takes the instantiation of N as argument; its results are added as a term pool (cf. \cite{Gebser et al. 2015}) so that one or several values can be accommodated. In our example, the head of the third rule thus results in the atoms \texttt{div(3,(1;3))} and \texttt{div(6,(1;2;3;6))}.

4.1 Embedded Python code

The simplest way to extend an ASP encoding with a Python method is to add it as an embedded script.

In our example, this can be done by supplying clingo with the embedded Python script in Listing 24. It shows that foreign language scripts are enclosed in \texttt{#script} and \texttt{#end}, and supplied with an argument indicating the used language. The code in this block is arbitrary and executed before grounding. Functions defined in it, such as the \texttt{divisors} function, can be called from ASP by prepending an \texttt{@} symbol. Finally, such scripts are meant to be part of the input of clingo just as the encoding in Listing \ref{ex:example} as shown below.

```
UNIX> clingo example.lp embedded.lp
clingo version 5.5.0
Reading from example.lp ...
Solving ...
Answer: 1
num(3) num(6) div(3,1) div(3,3) div(6,1) div(6,2) div(6,3) div(6,6)
SATISFIABLE
```

Such external term evaluation (during grounding) is a predestined use case for embedded scripts. The clingo object invoked on the command line is in charge of loading, grounding, and solving, unless these tasks are taken from it (cf. \cite{Gebser et al. 2019}). Hence, the usage of embedded scripts is generally most suitable for small amendments to the logic
# How to build your own ASP-based system?

1. #script (python)

import clingo

def divisors(a):
    a = a.number
    for i in range(1, a+1):
        if a % i == 0:
            yield clingo.Number(i)

#end.

Listing 24. Embedded Python code (embedded.lp)

program, anything on the term level during grounding. Often they are used to perform calculations that are hard or inconvenient to express in ASP.

4.2 The clingo Python module

The second alternative is to write a Python script using clingo's Python module. The module provides high level functions to interact with the grounder and solver including input and output processing as well as fine-grained control over the grounding and solving process. This provides a convenient way to use clingo as part of a larger project. The surrounding application is in charge of the control flow and ASP is used to perform specific computations. Even for simple computations, this avoids error prone string processing like transforming data into ASP facts or parsing the solver's output.

Listing 25 implements the previous example by using the Python module. The main difference is that we have to construct a Control object and take care of the control flow ourselves. Such a Control object encapsulates an instance of clingo. First, the program is added to the Control object using its add function. Then, the base part is grounded. To be able to call the divisors function, we pass the self argument as context to the ground function. Finally, note that we use Python's print function as on_model callback. An on_model callback is a function passed to solve that is called for each model. It allows for inspecting (and printing) the current model. This construction is necessary because, unlike with the clingo system, there is no output foreseen when using the clingo module in Python. This is nicely reflected by the plain output produced by Python’s print function when solving our example with Listing 25.

```
UNIX> python module.py example.lp
num(3) num(6) div(3,1) div(3,3) div(6,1) div(6,2) div(6,3) div(6,6)
```

This is different from the ASP-specific output produced by clingo in the previous section. While there ASP is extended with Python, it is here the other way around. This renders the use of ASP completely opaque.

27 Without any declarations, rules belong to a logic program referred to as base (cf. Section 5.1).
import clingo

class ExampleApp:
    @staticmethod
    def divisors(a):
        a = a.number
        for i in range(1, a+1):
            if a % i == 0:
                yield clingo.Number(i)

def run(self):
    ctl = clingo.Control()
    ctl.load("example.lp")
    ctl.ground([("base", [])], context=self)
    ctl.solve(on_model=print)

if __name__ == "__main__":
    ExampleApp().run()

Listing 25. Example with external function (module.py)

4.3 Implementing a system based on clingo

Finally, we present a third way that aims at building custom systems based on clingo. This is similar to embedded Python code but gives more control to customize the system. For example, parts of the text output can be modified, additional options can be registered, or the way input files are treated can be changed completely.

Unlike the previous example, we now derive our ExampleApp class from clingo’s application class. The resulting class can then be used with the clingo_main function, which starts a process similar to the one in clingo but possibly with some overwritten functions. First, the program name and version are defined. These values are then used in the status, help, and version output of clingo. Furthermore, each application class must implement a main function, which is called right after option parsing and is in charge of the grounding and solving process. The function receives a Control object and a list of paths to the files passed on the command line. The subsequent code processes the files just as clingo would. Similar to the previous example, we pass in Listing 25 the ExampleApp object to the ground function to be able to call its divisors method during grounding. Note that we do not need to use a print function to output models. Rather the one of clingo is used in a seamless way.

Again, this is reflected by the output of solving our example with Listing 26.

```bash
UNIX> python app.py example.lp
example version 1.0
Reading from example.lp
Solving...
Answer: 1
num(3) num(6) div(3,1) div(3,3) div(6,1) div(6,2) div(6,3) div(6,6)
SATISFIABLE
```

Furthermore, this output already hints at the advantage of using clingo’s Application.
import sys
import clingo

class ExampleApp(clingo.Application):
    program_name = "example"
    version = "1.0"

    @staticmethod
def divisors(a):
        a = a.number
        for i in range(1, a+1):
            if a % i == 0:
                yield clingo.Number(i)

def main(self, ctl, files):
    for path in files: ctl.load(path)
    if not files:
        ctl.load("-")
    ctl.ground([("base", []), context=self])
    ctl.solve()

if __name__ == "__main__":
    clingo.clingo_main(ExampleApp(), sys.argv[1:])

Listing 26. Example application (app.py)

While control is exercised from Python, as in the last section, it allows us to draw on clingo’s infrastructure, similar to using embedded scripting.

The utility of this class becomes apparent in the rest of this paper, since it is used throughout as the basic building block of all clingo based systems.

5 Multi-shot ASP solving

Multi-shot solving allows for solving continuously changing logic programs in an operative way. This can be controlled via APIs implementing reactive procedures that loop on grounding and solving while reacting, for instance, to outside changes or previous solving results. Such reactions may entail the addition or retraction of rules that clingo’s operative approach can accommodate while leaving unaffected program parts intact within the solver. This avoids re-grounding and benefits from heuristic scores and constraints learned over time.

We begin with an informal overview of the central features and language constructs of clingo’s multi-shot solving capabilities. We illustrate them in Section 5.2 and 5.3 by showcasing two exemplary reasoning modes, namely branch-and-bound-based optimization
and incremental ASP solving. A comprehensive introduction to multi-shot solving with clingo can be found in (Gebser et al. 2019).

5.1 A gentle introduction

Clingo allows us to structure (non-ground) rules into subprograms. To this end, a program can be partitioned into several subprograms by means of the directive `#program`; it comes with a name and an optional list of parameters. Once given in the input, the directive gathers all rules up to the next such directive (or the end of file) within a subprogram identified by the supplied name and parameter list. As an example, we specify two subprograms `base` and `acid(k)` in file `chemistry.lp` as follows:

```plaintext
1  a(1).
2  #program acid(k).
3  b(k).
4  c(X,k) :- a(X).
5  #program base.
6  a(2).
```

Note that `base` is a special subprogram (with an empty parameter list). In addition to the rules in its scope, it gathers all rules not preceded by any `#program` directive. Hence, in the above example, the `base` subprogram includes the facts `a(1)` and `a(2)`, although, only the latter is in the actual scope of the directive in Line 5. Without further control instructions (see below), clingo grounds and solves the `base` subprogram only, essentially, yielding the standard behavior of ASP systems. The processing of other subprograms such as `acid(k)` is subject to external governance.

We first have a look at customizing grounding and solving by creating a Control object, as put forward in Section 4.2. For illustration, let us consider two Python code snippets:

```python
1  import clingo
2  ctl = clingo.Control()
3  ctl.load("chemistry.lp")
4  ctl.ground([("base", [])])
5  ctl.solve(on_model=print)
```

While the above control program matches the default behavior of clingo, the one below ignores all rules in the `base` program but rather contains a ground instruction for `acid(k)` in Line 4, where the parameter `k` is to be instantiated with the term 42.

```python
1  import clingo
2  ctl = clingo.Control()
3  ctl.load("chemistry.lp")
4  ctl.ground([("acid", [42])])
5  ctl.solve(on_model=print)
```

The treatment of parameter `k` is similar to that of a constant, defined with `#const`, yet

---

28 A full specification of clingo’s Python API can be found at [https://potassco.org/clingo/python-api/current/clingo.html](https://potassco.org/clingo/python-api/current/clingo.html)

29 The ground routine takes a list of pairs as argument. Each such pair consists of a subprogram name (e.g. `base` or `acid`) and a list of actual parameters (e.g. `[ ]` or `[42]`).
restricted to the rules in the scope of the respective subprogram. Accordingly, the schematic fact $b(k)$ is turned into $b(42)$. No ground rule is obtained from $c(X,k) :- a(X)$ due to lacking instances of $a(X)$. Hence, the `solve` call in Line 9 yields a stable model consisting of $b(42)$ only. Note that `ground` instructions apply to the subprograms given as arguments, while `solve` triggers reasoning with respect to all accumulated ground rules.

For more elaborate reasoning processes, it is indispensable to activate and/or deactivate ground rules on demand. For instance, former initial or goal state conditions may need to be relaxed or completely replaced when modifying a planning problem, e.g., by extending its horizon. To expire transient rules, `clingo` provides the `#external` directive. This directive goes back to `lpars` (Syriänen 2001), where it was used to exempt (input) atoms from simplifications during grounding. Its functionality is generalized in `clingo` to provide a flexible handling of yet undefined atoms in the course of grounding and solving.

For continuously assembling ground rules evolving at different stages of a reasoning process, `#external` directives declare atoms that may still be defined by rules added later on. In terms of module theory (Oikarinen and Janhunen 2006) such atoms correspond to inputs, which (unlike undefined output atoms) must not be simplified. For declaring such input atoms, `clingo` offers schematic `#external` directives that are instantiated along with the rules of their respective subprograms.

For instance, the directive in the second line below

```latex
8  #program acid(k).
9  #external d(X,k) : c(X,k).
10  e(X,k) :- d(X,k).
```

is treated similar to the rule ‘$d(X,k) :- c(X,k)$’ during grounding, just that only the head atoms of the resulting ground instances are collected as inputs. Hence, adding the above lines to program `chemistry.lp` and grounding subprograms `base` and `acid(42)` yields the external atoms $d(1,42)$ and $d(2,42)$. Thus, we furthermore obtain the ground rules ‘$e(1,42) :- d(1,42)$’ and ‘$e(2,42) :- d(2,42)$’.

Once grounded, the truth value of external atoms can be changed via `clingo`’s API (until the atoms become defined by corresponding rules or are released). By default, the initial truth value of external atoms is set to false. Then, for example, with `clingo`’s Python API, the call `ctl.assign_external(d(2,42),True)` can be used to set the truth value of the external atom $d(2,42)$ to true. This can be used to activate and deactivate rules in logic programs. For instance, the rule ‘$e(1,42) :- d(1,42)$’ is ineffective because $d(1,42)$ is false by default. Hence, a subsequent `solve` call yields the stable model consisting of atoms $a(1), a(2), c(1,42), c(2,42), d(2,42)$, and $e(d,42)$. One further interesting use case is to release an external atom. The call `ctl.release_external(d(1,42))` removes the external atom and the rule ‘$e(1,42) :- d(1,42)$’ from the `Control` object.

**Remark 4**
Module theory (Oikarinen and Janhunen 2006) is used to characterize the composition of ground subprograms during multi-shot solving. For this, each ground subprogram is

---

30 The planning horizon is the maximum number of steps a planner takes into account when searching for a plan.

31 In order to construct atoms, symbolic terms, or function terms, respectively, the `clingo` API function `Function` has to be used. Hence, the expression $d(2,42)$ actually stands for `Function("d", [2,42])`. 

associated with a module. Accordingly, the restrictions of module composition apply: First, no two subprograms may define the same atom. Second, loops cannot spread across subprograms. See (Gebser et al. 2019) for details.

The first condition can be enforced by equipping rule heads with parameters, as done above.

5.2 Branch-and-bound-based optimization

We illustrate clingo’s multi-shot solving machinery in the next two sections via a simple Towers of Hanoi puzzle. Our example consists of three pegs and four disks of different size; it is shown in Figure 1. The goal is to move all disks from the left peg to the right one. Only the topmost disk of a peg can be moved at a time. Furthermore, a disk cannot be moved to a peg already containing a disk of smaller size. Although there is an efficient algorithm to solve our simple puzzle, we do not exploit it and below merely specify conditions for sequences of moves being solutions. The Towers of Hanoi puzzle constitutes a typical planning problem, aiming at finding a plan, that is, a sequence of actions, that leads from an initial state to a state satisfying a goal.

To illustrate how multi-shot solving can be used for realizing branch-and-bound-based optimization, we consider the problem of finding the shortest plan solving our puzzle within a given horizon. To this end, we adapt the Towers of Hanoi encoding from (Gebser et al. 2012) in Listing 27. Here, the length of the horizon is given by parameter \( n \). The problem instance in Listing 28 together with Line 1 in Listing 27 gives the initial configuration of disks in Figure 1. Similarly, the goal is checked in lines 3-4 of Listing 27 (by drawing on the problem instance in Listing 28). Because the overall objective is to solve the problem in the minimum number of steps within a given bound, it is successively tested in Line 3. Once the goal is established, it persists in the following steps. This allows us to read off whether the goal was reached at the planning horizon (in Line 4). The state transition function along with state constraints are described in lines 6-16. Since the encoding of the Towers of Hanoi problem is fairly standard, we refer the interested reader for details to (Gebser et al. 2012) and devote ourselves in the sequel to implementing branch-and-bound-based minimization. In view of this, note that Line 6 ensures that moves are only permitted if the goal is not yet achieved in the previous state. This ensures that the following states do not change anymore and allows us to express the optimization function in Line 20 as minimizing the number of states in which the goal is not reached.

Fig. 1. Towers of Hanoi: initial and goal situation

\[\begin{align*}
\text{a} & \quad \text{b} & \quad \text{c} \\
4 & \quad 3 & \quad 4 \\
3 & \quad 2 & \quad 1 \\
2 & \quad 1 & \quad 1 \\
1 & \quad 1 & \quad 1
\end{align*}\]

\[\begin{align*}
\text{a} & \quad \text{b} & \quad \text{c} \\
4 & \quad 3 & \quad 4 \\
3 & \quad 2 & \quad 1 \\
2 & \quad 1 & \quad 1 \\
1 & \quad 1 & \quad 1
\end{align*}\]

The complete source code of this example is available at https://github.com/potassco/clingo/tree/master/examples/clingo/opt
**How to build your own ASP-based system?!**

1. `on(D,P,0) :- init_on(D,P).`

3. `ngoal(T) :- on(D,P,T), not goal_on(D,P).`
4. `:- ngoal(n).`

6. `{ move(D,P,T) : disk(D), peg(P) } 1 :- ngoal(T-1), T<=n.`

8. `move(D,T) :- move(D,P,T).`
9. `on(D,P,T) :- move(D,P,T).`
10. `on(D,P,T) :- on(D,P,T-1), not move(D,T), T<=n.`
11. `blocked(D-1,P,T) :- on(D,P,T-1).`
12. `blocked(D-1,P,T) :- blocked(D,P,T), disk(D).`

14. `:- move(D,P,T), blocked(D-1,P,T).`
15. `:- move(D,T), on(D,P,T-1), blocked(D,P,T).`
16. `:- disk(D), not 1 { on(D,P,T) } 1, T=1..n.`

18. `#show move/3.`

20. `_minimize(1,T) :- ngoal(T).`

Listing 27. Bounded towers of hanoi encoding (`tohB.lp`)

1. `peg(a;b;c).`
2. `disk(1..4).`
3. `init_on(1..4,a).`
4. `goal_on(1..4,c).`

Listing 28. Towers of hanoi instance (`tohI.lp`)

The idea of branch-and-bound-based optimization is to compute an optimal solution by producing a series of increasingly better solutions until no better solution is found. The solution obtained last is then an optimal one. Listing 29 implements the corresponding optimization algorithm via `clingo’s Application` class from Section 4. It starts by over-riding `clingo’s main` function in Line 19 and begins with reading the input either from files provided on the command line or from standard input in Line 20 and 23.

The basic building block of our algorithm consist of a weight constraint embedded in the following subprogram.

1. `#program bound(b).`
2. `:- #sum { V,I: _minimize(V,I) } >= b.`

This program ensures that the next stable model yields a better bound than the one of the solution at hand. More technically, it expects a bound `b` as parameter and adds the integrity constraint in Line 2 to enforce a better solution. A new instance of this program is added for each consecutive stable model. The addition of the (non-ground) constraint in Line 2 as part of program `bound(b)` is accomplished in Line 24 and 25.
import sys
import clingo

class OptExampleApp(clingo.Application):
    program_name = "opt-example"
    version = "1.0"

    def __init__(self):
        self._bound = None

    def _on_model(self, model):
        self._bound = 0
        for atom in model.symbols(atoms=True):
            if (atom.match("_minimize", 2)
                and atom.arguments[0].type is clingo.SymbolType.Number):
                self._bound += atom.arguments[0].number

    def main(self, ctl, files):
        if not files:
            files = ['-']
        for f in files:
            ctl.load(f)
        ctl.add("bound", ["b"],
            ":- #sum { V,I : _minimize(V,I) } >= b."
        )
        ctl.ground(["base", []])
        while ctl.solve(on_model=self._on_model).satisfiable:
            print("Found new bound: {}".format(self._bound))
        ctl.ground(["bound", [self._bound]])

        if self._bound is not None:
            print("Optimum found")

    clingo.clingo_main(OptExampleApp(), sys.argv[1:])

Listing 29. Branch-and-bound optimization (opt.py)

The actual minimization algorithm starts by grounding the base program in Line 27 before it enters the loop in lines 28–30. This loop implements the branch-and-bound-based search for the minimum by searching for stable models while updating the bound until the problem is unsatisfiable. Note that we pass a callback to the solve function in Line 28. With it, the _on_model function in lines 11–17 is called for every model found. If there is a stable model, lines 13–17 iterate over the atoms of the stable model while summing up the current bound by extracting the weight of atoms over predicates _minimize/2. We
check that the first argument of the atom is an integer and ignore atoms where this is not the case; just like the \texttt{#sum} aggregate in Line 25. When a model was found, the body of the loop in lines 28–30 is processed. First, the algorithm prints the bound in Line 29. Then, it adds an integrity constraint in Line 30 making sure that the next stable model is strictly better than the current one. Finally, if the program becomes unsatisfiable, the branch-and-bound loop in lines 28–30 is exhausted. At last, control continues in lines 32–33 by printing that the previously found stable model (if any) is the optimal solution.

When running the augmented logic program in Listing 27, 28, and 29 with a horizon of 17, the solver finds plans of length 17, 16, and 15 and shows that no plan of length 14 exists. This is reflected by \texttt{clingo}'s output indicating four solver calls and three found stable models:

```
UNIX> python opt.py tohB.lp tohI.lp -c n=17
opt-example version 1.0
Reading from tohB.lp ...
Solving ...
Answer: 1
move(4,b,1) move(3,c,2) move(4,a,3) move(4,c,4) move(2,b,5) \move(4,a,6) move(3,b,7) move(4,c,8) move(4,b,9) move(1,c,10) \move(4,c,11) move(3,a,12) move(4,a,13) move(2,c,14) move(4,b,15)move(3,c,16) move(4,c,17)
Found new bound: 17
Solving ...
Answer: 1
move(4,b,1) move(3,c,2) move(4,c,3) move(4,b,4) move(4,a,5) \move(3,b,6) move(4,c,7) move(4,b,8) move(1,c,9) move(4,c,10) \move(3,a,11) move(4,a,12) move(2,c,13) move(4,b,14) move(3,c,15)move(4,c,16)
Found new bound: 16
Solving ...
Answer: 1
move(4,b,1) move(3,c,2) move(4,c,3) move(2,b,4) move(4,a,5) \move(3,b,6) move(4,b,7) move(1,c,8) move(4,c,9) move(3,a,10) \move(4,a,11) move(2,c,12) move(4,b,13) move(3,c,14) move(4,c,15)move(3,c,16)
Found new bound: 15
Solving ...
Optimum found
UNSATISFIABLE
```

Note that at the end the solver comprises the ground program obtained from Listing 27 and 28 grounded with parameter \texttt{n=17} along with the three integrity constraints obtained from subprograms \texttt{bound(17)}, \texttt{bound(16)}, and \texttt{bound(15)}.

Last but not least, note that the functionality implemented above is equivalent to using \texttt{clingo}'s inbuilt optimization mode by replacing Line 20 in Listing 27 with

\begin{verbatim}
23 #minimize { 1,T : ngoal(T) }.
\end{verbatim}

### 5.3 Incremental ASP solving

Incremental ASP solving offers a step-oriented approach to ASP that avoids redundancies by gradually processing the extensions to a problem rather than repeatedly re-processing the entire growing problem. To this end, a program is partitioned into a base part, describing static knowledge independent of the step parameter \( t \), a cumulative part, capturing knowledge accumulating with increasing \( t \), and a volatile part specific for each value of \( t \). In \texttt{clingo}, all three parts are captured by \texttt{#program} declarations along with
#program base.
on(D,P,0) :- init_on(D,P).

#program check(t).
goal_on(D,P), not on(D,P,t), query(t).

#program step(t).
1 { move(D,P,t) : disk(D), peg(P) } 1.

move(D,t) :- move(D,P,t).
on(D,P,t) :- move(D,P,t).
on(D,P,t) :- on(D,P,t-1), not move(D,t).
blocked(D-1,P,t) :- on(D,P,t-1).
blocked(D-1,P,t) :- blocked(D,P,t), disk(D).

:- move(D,P,t), blocked(D-1,P,t).
:- move(D,t), on(D,P,t-1), blocked(D,P,t).
:- disk(D), not 1 { on(D,P,t) } 1.

#show move/3.

Listing 30. Towers of hanoi incremental encoding (tohE.lp)

#external atoms for handling volatile rules, namely, subprograms named base, step, and check along with external atoms of form query(t).

We illustrate this by adapting the Towers of Hanoi encoding from Listing 27 to an incremental version in Listing 30. To this end, we arrange the original encoding in program parts base, check(t), and step(t), use t instead of T as time parameter, and simplify checking the goal. Checking the goal is easier here because the incremental approach guarantees a shortest plan and, hence, does not require additional minimization.

At first, we observe that the problem instance in Listing 28 as well as Line 2 in Listing 30 constitute static knowledge and thus belong to the base program. More interestingly, the query is expressed in Line 5 of Listing 30. Its volatility is realized by making it subject to the truth assignment to the external atom query(t). For convenience, this atom is predefined in Line 41 in Listing 31 as part of the check program. Hence, subprogram check consists of a user- and a pre-defined part. Finally, the transition function along with state constraints are described in the subprogram step in lines 7–18.

The idea is now to control the successive grounding and solving of the program parts in Listing 28 and 30 by the Python script in Listing 31. To this end, we use five variables governing the loop in lines 47–62. Variables imin and imax prescribe a least and largest number of iterations, respectively; istop gives a termination criterion, e.g., "SAT" or

These names have no general, predefined meaning; their meaning emerges from their usage in the associated script (see below).

For brevity, we have stripped class IncConfig in Line 7 from the parsing methods parse_int and parse_stop, which are given at.

This follows the original implementation of incremental ASP solving in iclingo (Gebser et al. 2008)
import sys, clingo

class IncConfig:
    def __init__(self):
        self.imin, self.imax, self.istop = 1, None, "SAT"

class IncExampleApp(clingo.Application):
    program_name = "inc-example"
    version = "1.0"

    def __init__(self):
        self._conf = IncConfig()

    def register_options(self, options):
        group = "Inc-Example Options"
        options.add(group, "imin",
                    "Minimum number of steps [{}].".format(self._conf.imin),
                    parse_int(self._conf, "imin", min_value=0),
                    argument="<n>")

        options.add(group, "imax",
                    "Maximum number of steps [{}].".format(self._conf.imax),
                    parse_int(self._conf, "imax", min_value=0, optional=True),
                    argument="<n>")

        options.add(group, "istop",
                    "Stop criterion [{}].".format(self._conf.istop),
                    parse_stop(self._conf, "istop"))

    def main(self, ctl, files):
        if not files:
            files = ["-"]
        for f in files:
            ctl.load(f)
        ctl.add("check", ["t"], "#external query(t).")

        conf = self._conf
        imin, imax, istop = conf.imin, conf.imax, conf.istop

        step, ret = 0, None
        while ((imax is None or step < imax) and
                (step == 0 or step < imin or (istop == "SAT" and not ret.satisfiable)) or
                (istop == "UNSAT" and not ret.unsatisfiable)):
            parts = []
            parts.append(("check", [step]))
            if step > 0:
                ctl.release_external(clingo.Function("query", [step - 1]))
            parts.append(("step", [step]))
            ctl.cleanup()
        else:
            parts.append(("base", []))
            ctl.ground(parts)
            ctl.assign_external(clingo.Function("query", [step]), True)
            ret, step = ctl.solve(), step + 1

clingo.clingo_main(IncExampleApp(), sys.argv[1:])

Listing 31. Python script implementing incremental ASP solving in clingo (inc.py)
"UNSAT". The value of \texttt{step} is used to instantiate the parametrized subprograms and \texttt{ret} comprises the solving result. While the initial values of \texttt{step} and \texttt{ret} are set in Line \ref{line:initial-values}, the first three variables are user-defined. We showcase how such user-defined variables are integrated into \texttt{clingo}'s option handling below, but first describe the actual implementation of incremental ASP solving.

The subprograms grounded in each iteration are accumulated in the list \texttt{parts} (cf. Line \ref{line:parts}). Each of its entries is a pair consisting of a subprogram name along with its list of actual parameters. In the very first iteration, the subprograms \texttt{base} and \texttt{check(0)} are grounded. Note that this involves the declaration of the external atom \texttt{query(0)} and the assignment of its default value false. The latter is changed in Line \ref{line:query-value} to true in order to activate the actual query. The \texttt{solve} call in Line \ref{line:solve} then amounts to checking whether the goal situation is already satisfied in the initial state. As well, the value of \texttt{step} is incremented to 1.

As long as the termination condition remains unfulfilled, each following iteration takes the respective value of variable \texttt{step} to replace the parameter in subprograms \texttt{step} and \texttt{check} during grounding. In addition, the current external atom \texttt{query(t)} is set to true, while the previous one is permanently set to false. This disables the corresponding instance of the integrity constraint in Line \ref{line:integrity-constraint} of Listing \ref{listing:inc} before it is replaced in the next iteration. In this way, the query condition only applies to the current horizon.

An interesting feature is given in Line \ref{line:cleanup}. As its name suggests, this function cleans up domains used during grounding. That is, whenever the truth value of an atom is ultimately determined by the solver, it is communicated to the grounder where it can be used for further simplifications.

In our example, the \texttt{solver} is called 16 times before a plan of length 15 is found:

| UNIX> python inc.py tohE.lp tohI.lp |
| inc-example version 1.0 |
| Reading from tohE.lp ... |
| Solving... |
| [...] |
| Solving... |
| Answer: 1 |
| move(4,b,1) move(3,c,2) move(4,c,3) move(2,b,4) \ |
| move(4,a,5) move(3,b,6) move(4,b,7) move(1,c,8) \ |
| move(4,c,9) move(3,a,10) move(4,a,11) move(2,c,12) \ |
| move(4,b,13) move(3,c,14) move(4,c,15) |
| SATISFIABLE |
| Models : 1+ |
| Calls : 16 |

Last but not least, let us explain how option processing can be added by sketching how the three variables \texttt{imin}, \texttt{imax}, and \texttt{istop} can be set from the command line. For this purpose, \texttt{clingo}'s API offers the two methods \texttt{register_options} and \texttt{validate_options}. In our simple example, we only use the former since the latter is meant to handle situations with conflicting options\footnote{We also omit describing the underlying option parsers; cf. Footnote \ref{note:option-parsers}.}. In fact, \texttt{register_options} receives an \texttt{ApplicationOptions} object as parameter to register additional options. For example, option \texttt{--imin} is added in Lines \ref{line:imin-option}--\ref{line:imin-option-end} by calling \texttt{ApplicationOptions.add}. Its first parameter is the name of an option group, used as the section heading when printing the application’s help when called.
with --help. In our example, all options are grouped in the “Inc-Example Options” section. More concretely, we get the following help output (omitting descriptions for clingo’s options).

```
UNIX> python inc.py --help
[...]
Inc-Example Options:
   --imin=<n> : Minimum number of steps [1]
   --imax=<n> : Maximum number of steps [None]
   --istop=<arg> : Stop criterion [SAT]
[...]
```

The second parameter is the name of the option on the command line. Here, we pass "imin" to add option --imin. This is followed by the option description and the (omitted) option parser. Registration and parsing of options happens once before the IncExampleApp.main method is called. Hence, at that time the attributes of the applications IncExampleApp._conf object either have their default values or were overwritten by options passed on the command line. The last keyword parameter configures the placeholder, which is printed in the help output. Since we are adding a numeric option, it is good practice to call it <n>. By default it is called <arg>.

6 Theory-enhanced ASP solving

This section provides the fundamental concepts for extending clingo with theory-specific reasoning. We begin by showing how its input language can be customized with theory-specific constructs. We then sketch clingo’s algorithmic approach to ASP solving with theory propagation to put the description of clingo’s theory reasoning interface on firm grounds.

6.1 Input language

We begin by introducing the theory-related features of clingo’s input language. They are situated in the underlying grounder gringo and can thus also be used independently of clingo. We start with a detailed description of the generic means for defining theories and complement this in Appendix A with an overview of the corresponding intermediate language.

The generic approach to theory specification rests upon two languages: the one defining theory languages and the theory language itself. Both borrow elements from the underlying ASP language, foremost an aggregate-like syntax for formulating variable length expressions. To illustrate this, consider Listing \[2\] where a logic program is extended by constructs for handling difference and linear constraints. While the former are binary constraints of the form \[x_1 - x_2 \leq k\], the latter have a variable size and are of form \[a_1 x_1 + \cdots + a_n x_n \circ k\], where \(x_i\) are integer variables, \(a_i\) and \(k\) are integers, and \(\circ \in \{\leq, \geq, <, >, =\}\) for \(1 \leq i \leq n\). Note that solving difference constraints is polynomial, while solving linear equations (over integers) is NP-complete. The theory language for expressing both types of constraints is defined in lines 11–15 and preceded by the directive.

\[37\] For simplicity, we consider normalized difference constraints rather than general ones of form \(x_1 - x_2 \circ k\).
#theory lc {
  constant { - : 0, unary };  
  diff_term { - : 0, binary, left };  
  linear_term { + : 2, unary; - : 2, unary;  
    * : 1, binary, left;  
    + : 0, binary, left; - : 0, binary, left };  
  domain_term { .. : 1, binary, left };  
  show_term { / : 1, binary, left };  
}

#theory

&dom/0 : domain_term, {=}, linear_term, any;
&sum/0 : linear_term, { <=,=, >=,<,>, !=} , linear_term, any;
&diff/0 : diff_term, {<=}, constant, any;
&show/0 : show_term, directive
}

#const n=2. #const m=1000.
task (1..n).
duration (T,200*T) :- task (T).

&dom { 1..m } = start (T) :- task (T).
&dom { 1..m } = end (T) :- task (T).
&diff { end (T)-start (T) } <= D :- duration (T,D).
&sum { end (T) : task (T); -start (T) : task (T) } <= m.
&show { start /1; end /1 }.

Listing 32. Logic program enhanced with difference and linear constraints (lc.lp)

#theory. The elements of the resulting theory language are preceded
by & and used as regular atoms in the logic program in lines [17][27].

To be more precise, a theory definition has the form

#theory T {D_1;...;D_n}.

where T is the theory name and each D_i is a definition for a theory term
or a theory atom for 1 ≤ i ≤ n. The language induced by a theory definition
is the set of all theory atoms constructible from its theory atom definitions.

A theory atom definition has form

&p/k : t,o or &p/k : t, {o_1,...,o_m}, t',o

where p is a predicate symbol and k its arity, t,t' are names of theory term definitions,
each o_i is a theory operator for m ≥ 1, and o ∈ {head, body, any, directive} determines
where theory atoms may occur in a rule. Examples of theory atom definitions are given
in lines [1][14] of Listing 32. The language of a theory atom definition as above contains
all theory atoms of form

&a {C_1:L_1;...;C_n:L_n} or &a {C_1:L_1;...;C_n:L_n} ◦ c

where a is an atom over predicate p of arity k, each C_i is a tuple of theory terms
in the language for t, c is a theory term in the language for t', ◦ is a theory operator
among {o_1,...,o_m}, and each L_i is a regular condition (i.e., a tuple of regular literals) for
1 ≤ i ≤ n. Whether the last part '◦ c' is included depends on the form of a theory atom.
definition. Further, observe that theory atoms with occurrence type any can be used both in the head and body of a rule; with occurrence types head and body, their usage can be restricted to rule heads and bodies only. Occurrence type directive is similar to type head but additionally requires that the rule body must be completely evaluated during grounding. Five occurrences of theory atoms can be found in lines 22–27 of Listing 32.

Remark 5

Having conditions, such as \( L_i \) above, is useful to address variable length constraints as in Listing 32. However, atoms like \texttt{task(T)} are given as facts and therefore not subject to solving. In general, however, conditions may involve atoms that are not decided during grounding. If this is the case, they have to be handled with care since there is no predefined behavior. For instance, a rule \( a : - \ \&\text{sum}( x : a ) \gg 0 \) may be unsatisfiable under certain semantic principles (cf. (Gelfond and Zhang 2019)). We explore the formal foundations and implementation techniques of conditional theory atoms in (Cabalar et al. 2020a; Cabalar et al. 2020b).

A theory term definition has form
\[
 t \{ D_1; \ldots; D_n \}
\]
where \( t \) is a name for the defined terms and each \( D_i \) is a theory operator definition for \( 1 \leq i \leq n \). A respective definition specifies the language of all theory terms that can be constructed via its operators. Examples of theory term definitions are given in lines 3–9 of Listing 32. Each resulting theory term is one of the following:

- a constant term: \( c \)
- a variable term: \( v \)
- a binary theory term: \( t_1 \diamond t_2 \)
- a unary theory term: \( \diamond t_1 \)
- a function theory term: \( f(t_1, \ldots, t_k) \)
- a tuple theory term: \( (t_1, \ldots, t_i) \)
- a set theory term: \( \{t_1, \ldots, t_l\} \)
- a list theory term: \( [t_1, \ldots, t_l] \)

where each \( t_i \) is a theory term, \( \diamond \) is a theory operator defined by some \( D_i \), \( c \) and \( f \) are symbolic constants, \( v \) is a first-order variable, \( k \geq 1 \), and \( l \geq 0 \). (The trailing comma in tuple theory terms is optional if \( l \neq 1 \).) Parentheses can be used to specify operator precedence.

A theory operator definition has form
\[
 \diamond : \mathit{p,\text{unary}} \quad \text{or} \quad \diamond : \mathit{p,\text{binary},a}
\]
where \( \diamond \) is a unary or binary theory operator with precedence \( p \geq 0 \) (determining implicit parentheses). Binary theory operators are additionally characterized by an associativity \( a \in \{\text{right, left}\} \). As an example, consider lines 5–6 of Listing 32, where the left associative binary operators + and * are defined with precedence 2 and 1. Hence, parentheses in terms like \( \langle X+(2*Y)\rangle+Z \) can be omitted. In total, lines 3–9 of Listing 32 include nine theory operator definitions. Specific theory operators can be assembled (written consecutively without spaces) from the symbols ‘!’, ‘<’, ‘=’, ‘>’, ‘+’, ‘-’, ‘/’, ‘\', ‘?’; ‘&’, ‘|’, ‘;’, ‘::’, ‘:::’, ‘::=’, and ‘\’\’\’\’\’. For instance, in Line 8 of Listing 32 the operator ‘..’ is defined as the concatenation of two periods. The tokens ‘\’\’\’\’\’, ‘::’, ‘:::’, and ‘::=’ must be combined with other symbols due to their dedicated usage. Instead, one may write ‘\’\’\’\’\’\’, ‘:::’, ‘:::’, ‘:::‘, etc.

While theory terms are formed similar to regular ones, theory atoms rely upon an aggregate-like construction for forming variable-length theory expressions. In this way,
standard grounding techniques can be used for gathering theory terms. (However, the actual atom \&a within a theory atom comprises regular terms only.) The treatment of theory terms still differs from their regular counterparts in that the grounder skips simplifications like, e.g., arithmetic evaluation. This can be nicely seen on the different results in Listing 33 of grounding terms formed with the regular and theory-specific variants of operator ‘..’. Observe that the fact task(1..n) in Line 19 of Listing 32 results in n ground facts, viz. task(1) and task(2) because of n=2. Unlike this, the theory expression 1..m stays structurally intact and is only transformed into 1..1000 in view of m=1000. That is, the grounder does not evaluate the theory term 1..1000 and leaves its interpretation to a downstream theory solver. A similar situation is encountered when comparing the treatment of the regular term ‘200*T’ in Line 20 of Listing 32 to the theory term ‘end(T)-start(T)’ in Line 24. While each instance of ‘200*T’ is evaluated during grounding, instances of the theory term ‘end(T)-start(T)’ are left intact in lines 11 and 12 of Listing 33. In fact, if ‘200*T’ had been a theory term as well, it would have resulted in the unevaluated instances ‘200*1’ and ‘200*2’.

6.2 Semantic principles

Given the hands-on nature of this work, we only give an informal idea of the semantic principles underlying theory solving in ASP. A full logical account in terms of denotational semantics can be found in (Cabalar et al. 2016; Cabalar et al. 2020b).

A logic program induces a set of stable models. To extend this concept to logic programs with theory expressions, we follow the approach of lazy theory solving (Barrett et al. 2009). We abstract from the specific semantics of a theory by considering the theory atoms representing the underlying theory constraints. The idea is that a regular stable model of a program over regular and theory atoms is only valid with respect to a theory, if the constraints induced by the truth assignment to the theory atoms are satisfiable in the theory.

In the above example, this amounts to finding a numeric assignment to all theory
variables satisfying all difference and linear constraints associated with theory atoms. The ground program in 33 has a single stable model consisting of all regular and theory atoms in lines 1-16. Here, we easily find assignments satisfying the induced constraints, e.g. \( \text{start}(1) \rightarrow 1, \text{end}(1) \rightarrow 2, \text{start}(2) \rightarrow 2, \) and \( \text{end}(1) \rightarrow 3. \)

In fact, there are alternative semantic options for capturing theory atoms, as discussed in (Gebser et al. 2016). First of all, we may distinguish whether imposed constraints are only determined outside or additionally inside a logic program. This leads to the distinction between \textit{defined} and \textit{external} theory atoms. While external theory atoms must only be satisfied by the respective theory, defined ones must additionally be derivable through rules in the program. A second distinction concerns the interplay of ASP with theories. More precisely, it is about the logical correspondence between theory atoms and theory constraints. This leads us to the distinction between \textit{strict} and \textit{non-strict} theory atoms. The strict correspondence requires a constraint to be satisfied \textit{iff} the associated theory atom is true. A weaker since only implicative condition is imposed in the non-strict case. Here, a constraint must hold \textit{only if} the associated theory atom is true. In other words, only non-strict theory atoms assigned true impose requirements, while constraints associated with falsified non-strict theory atoms are free to hold or not. However, by contraposition, a violated constraint leads to a false non-strict theory atom.

6.3 Algorithmic aspects

The algorithmic approach to ASP solving modulo theories of \textit{clingo}, or more precisely that of its underlying ASP solver clasp, follows the lazy approach to solving in Satisfiability Modulo Theories (SMT; (Barrett et al. 2009)). We give below an abstract overview that serves as light algorithmic underpinning for the description of \textit{clingo}'s implementation given in the next section.

As detailed in (Gebser et al. 2012), a ground program \( P \) induces completion and loop nogoods, called \( \Delta_P \) or \( \Lambda_P \), respectively, that can be used for computing stable models of \( P \). Nogoods represent invalid partial assignments and can be thought of as negative Boolean constraints. We represent (partial) assignments as consistent sets of literals. An assignment is total if it contains either the positive or negative literal of each atom. We say that a nogood is violated by an assignment if the former is contained in the latter; a nogood is unit if all but one of its literals are in the assignment. Each total assignment not violating any nogood in \( \Delta_P \cup \Lambda_P \) yields a regular stable model of \( P \), and such an assignment is called a solution (for \( \Delta_P \cup \Lambda_P \)). To accommodate theories, we identify a theory \( T \) with a set \( \Delta_T \) of \textit{theory nogoods} and extend the concept of a solution in the straightforward way.

The nogoods in \( \Delta_P \cup \Lambda_P \cup \Delta_T \) provide the logical fundament for the Conflict-Driven Constraint Learning (CDCL) procedure (cf. (Marques-Silva et al. 2009; Gebser et al. 2012)) outlined in Figure 2. While the completion nogoods in \( \Delta_P \) are usually made explicit and subject to unit propagation, the loop nogoods in \( \Lambda_P \) as well as theory

---

38 This distinction is analogous to that between head and input atoms, defined via rules or \#external directives (Gebser et al. 2019), respectively.

39 See (Gebser et al. 2016) for different ways of associating theories with nogoods.

40 Unit propagation extends an assignment with literals complementary to the ones missing in unit nogoods.
Fig. 2. Basic algorithm for Conflict-Driven Constraint Learning (CDCL) modulo theories

nogoods in $\Delta_T$ are typically handled by dedicated propagators and particular members are selectively recorded.

While a dedicated propagator for loop nogoods is built-in in systems like clingo, those for theories are provided via the interface Propagator in Figure 3. To utilize custom propagators, the algorithm in Figure 2 includes an initialization step in line (I). In addition to the “registration” of a propagator for a theory as an extension of the basic CDCL procedure, common tasks performed in this step include setting up internal data structures and so-called watches for (a subset of) the theory atoms, so that the propagator is invoked (only) when some watched literal gets assigned.

As usual, the main CDCL loop starts with unit propagation on completion and loop nogoods, the latter handled by the respective built-in propagator, as well as any nogoods already recorded. If this results in a non-total assignment without conflict, theory propagators for which some of their watched literals have been assigned are invoked in line (P). A propagator for a theory $T$ can then inspect the current assignment, update its data structures accordingly, and most importantly, perform theory propagation determining theory nogoods $\delta \in \Delta_T$ to record. Usually, any such nogood $\delta$ is unit in order to trigger a conflict or unit propagation, although this is not a necessary condition. The interplay of unit and theory propagation continues until a conflict or total assignment arises, or no (further) watched literals of theory propagators get assigned by unit propagation. In the latter case, some non-deterministic decision is made to extend the partial assignment at hand and then to proceed with unit and theory propagation.

If no conflict arises and an assignment is total, in line (C), theory propagators are called, one by one, for a final check. The idea is that, e.g., a “lazy” propagator for a theory $T$ that does not exhaustively test violations of its theory nogoods by partial assignments can make sure that the assignment is indeed a solution for $\Delta_T$, or record some violated nogood(s) from $\Delta_T$ otherwise. Even in case theory propagation on partial assignments is exhaustive and a final check is not needed to detect conflicts, the information that search led to a total assignment can be useful in practice, e.g., to store values for integer variables like start(1), start(2), end(1), and end(2) in Listing 33 that witness the existence of a solution for $T$. 
Finally, in case of a conflict, i.e., some completion or recorded nogood is violated by the current assignment, provided that some non-deterministic decision is involved in the conflict, a new conflict constraint is recorded and utilized to guide backjumping in line (U), as usual with CDCL. In a similar fashion as the assignment of watched literals serves as trigger for theory propagation, theory propagators are informed when they become unassigned upon backjumping. This allows the propagators to undo earlier operations, e.g., internal data structures can be reset to return to a state taken prior to the assignment of watches.

In summary, the basic CDCL procedure is extended in four places to account for custom propagators: initialization, propagation of (partial) assignments, final check of total assignments, and undo steps upon backjumping.

### 6.4 Propagator interface

We now turn to the implementation of theory propagation in clingo and detail the structure of its interface depicted in Figure 3. The interface Propagator has to be implemented by each custom propagator. After registering such a propagator with clingo, its functions are called during initialization and search as indicated in Figure 2. Function Propagator.init is called once before solving (line [41] in Figure 2) to allow for initializing data structures used during theory propagation. It is invoked with a PropagatorInit object providing access to symbolic (SymbolicAtom) as well as theory (TheoryAtom) atoms.

Both kinds of atoms are associated with program literals,[42] which are in turn associated

---

41 For brevity, we below drop the qualification Propagator and use its function names unqualified.
42 Program literals are also used in the aspif format (see Appendix A).
with solver literals. Program as well as solver literals are identified by non-zero integers, where positive and negative numbers represent positive or negative literals, respectively. In order to get notified about assignment changes, a propagator can set up watches on solver literals during initialization.

During search, function `propagate` is called with a `PropagateControl` object and a (non-empty) list of watched literals that got assigned in the recent round of unit propagation (line (P) in Figure 2). The `PropagateControl` object can be used to inspect the current assignment, record nogoods, and trigger unit propagation. Furthermore, to support multi-threaded solving, its `thread_id` property identifies the currently active thread, each of which can be viewed as an independent instance of the CDCL algorithm in Figure 2. Function `undo` is the counterpart of `propagate` and called whenever the solver retracts assignments to watched literals (line (U) in Figure 2). In addition to the list of watched literals that have been retracted (in chronological order), it receives the identifier and the assignment of the active thread. Finally, function `check` is similar to `propagate`, yet invoked without a list of changes. Instead, it is (only) called on total assignments (line (C) in Figure 2), independently of watches. Overriding the empty default implementations of propagator methods is optional.

7 Extending ASP with difference constraints

In this section, we develop a case-study featuring the extension of ASP with difference constraints by augmenting `clingo` with a corresponding propagator. To this end, we extend the language of Section 2 with difference constraint atoms of form

\[ \&\text{diff}\{u-v\} \leq d \]

where \( u \) and \( v \) are (regular) terms and \( d \) is an integer constant. Such atoms may either occur in the head or the body of a rule. Hence, stable models may now also include theory atoms of form `\&\text{diff}\{u-v\} \leq d`. More precisely, for a stable model \( X \), let \( C_X \) be the set of difference constraints \( u - v \leq d \) associated with theory atoms `\&\text{diff}\{u-v\} \leq d` in \( X \) and \( V_X \) be the set of all (integer) variables occurring in the difference constraints in \( C_X \). In our case, a stable model \( X \) is then DC-stable, if there is a mapping from \( V_X \) to the integers, first, satisfying all constraints in \( C_X \), and second, falsifying all constraints not in \( C_X \) that are associated with a strict difference constraint atom (cf. Janhunen et al. 2017).

Next, let us discuss the semantic principles guiding our implementation. Recall from Section 6.2 that theory atoms may have different semantic properties, either defined or external depending upon their occurrence, or their logical connection to the constraint they represent, either strict or non-strict. In Janhunen et al. 2017, we discussed the four different resulting settings and concluded that the combinations of strict and external as well as non-strict and defined are the most intuitive combinations for difference constraints (although this may differ in other contexts).

43 Note that `clasp`’s preprocessor might associate a positive or even negative solver literal with multiple atoms.

44 Depending on the configuration of `clasp`, threads can communicate with each other. For example, some of the recorded nogoods can be shared. This is transparent from the perspective of theory propagators.

45 In SMT, the underlying formal system is also referred to as quantifier free integer difference logic.
To illustrate this, consider the following example:

1. \&diff \{ 0-x \} \leq -2.
2. a :- \&diff \{ 0-x \} \leq -1.

This program states that \( x \) is greater or equal 2 and that \( a \) is derived if \( x \) is greater or equal than 1. An intuitive result is to assign \( x \) a value greater or equal than 2 and to derive \( a \). However, in case atom \&diff \{ 0-x \} \leq -1' is non-strict, we also obtain answer sets without \( a \). This is because the falsity of \&diff \{ 0-x \} \leq -1' does not imply that \( 0-x \leq -1 \) is false as well. This is enforced by interpreting the relation between \&diff \{ 0-x \} \leq -1' and \( 0-x \leq -1 \) as strict, and then \( a \) is obtained. Intuitively, the combination of external and strict can be seen as interpreting theory atoms relative to an external oracle, according to which all possibilities have to be considered as the logic program is oblivious to the meaning of the theory atom.

For a complement, consider the example:

1. \&diff \{ 0-x \} \leq -2.
2. \&diff \{ 0-x \} \leq -1 :- a.

Again, the program states that \( x \) is greater equal 2 but now atom \( a \) derives that \( x \) is even greater or equal than 1. As we do not have a definition of \( a \), we expect answer sets not containing \( a \) and assigning \( x \) a value greater equal 2. However, once we interpret the atom \&diff \{ 0-x \} \leq -1' as strict, the program becomes unsatisfiable. In this case the falsity of \&diff \{ 0-x \} \leq -1' implies that \( 0-x \leq -1 \) is false as well. A non-strict interpretation avoids this and yields the expected result. The combination of non-strict and defined lets the logic program decide which theory atoms hold. Specifically, the absence of an atom in an answer set does not imply that the constraint is false but rather that it is not enforced. As a result, we handle occurrences of difference constraint atoms in the head as defined and non-strict and atoms occurrences in the body as external and strict.

Let us now turn to the actual extension of clingo. The overall implementation is divided in two, on the one hand, the actual application class \texttt{DLApp} addressing grounding and solving in Listing 36, and on the other hand, five classes dealing with various aspects of difference constraints. Among them, we concentrate on the \texttt{HeadBodyTransformer} class in Listing 35 illustrating the manipulation of a logic program’s abstract syntax tree (AST), as well as the \texttt{DLPropagator} class in Listing 37 showcasing a propagator adding foreign inferences to ASP. To support this, we also describe the interface of the \texttt{Graph} class but refrain from presenting its implementation. For expressing difference constraints, we define theory language \texttt{dl} in Listing 31 a subset of the theory language \texttt{lc} presented in Listing 32 above. Note that to accommodate the additional argument indicating the location of the difference constraint atom, we have replaced \texttt{&diff/0} by \texttt{&diff/1}.

To achieve the above distinction between head and body occurrences of theory atoms without changing the input language, we use clingo’s functionalities to modify the AST of non-ground programs for tagging theory atoms with their respective occurrence. Although

\footnote{Note that this amounts to treating occurrences of the same constraint atom in different ways. This is not unusual since the same constraint may be represented by syntactically different constraint atoms.}

\footnote{The complete source code is available at https://github.com/potassco/clingo/tree/master/examples/clingo/dl}
THEORY = ""
#theory dl{
  diff_term {
    - : 3, unary;
    ** : 2, binary, right;
    * : 1, binary, left;
    / : 1, binary, left;
    \ : 1, binary, left;
    + : 0, binary, left;
    - : 0, binary, left
  };
  &diff/1 : diff_term, {<=}, diff_term, any
};
""

Listing 34. Theory language dl for difference constraints (dl.py)

class HeadBodyTransformer(Transformer):
    def visit_Rule(self, rule):
        head = rule.head
        body = rule.body
        head = self.visit(head, loc="head")
        body = self.visit(body, loc="body")
        return ast.Rule(rule.location, head, body)

    def visit_TheoryAtom(self, atom, loc="body"):
        term = atom.term
        if term.name == "diff" and not term.arguments:
            atom.term = ast.Function(
                term.location,
                term.name,
                [ast.Function(term.location, loc, [], False)], False)
        return atom

Listing 35. HeadBodyTransformer class for tagging occurrence of theory atoms (dl.py)

pragmatic, the annotation of theory atoms has turned out to be very useful in several implementations. Moreover, it serves us as a first example of how non-ground programs can be modified through clingo’s API.

Remark 6
The user still writes difference constraints over &diff/0. The AST modification occurs on the non-ground level during parsing. Once grounded, it is checked whether all theory atoms are valid with regards to a theory language. At this point, difference constraint are constructed over &diff/1, which is opaque to the user.

As mentioned, this is accomplished by the HeadBodyTransformer class in Listing 35. This class nicely illustrates how the visitor design pattern is used by clingo to manipulate the AST of (non-ground) logic programs. Function visit_Rule in lines 143-148 ensures...
import sys, clingo, dl

class DLApp(clingo.Application):
    program_name = "clingo-dl"
    version = "1.0"

    def __init__(self):
        self._propagator = dl.DLPropagator()

    def on_model(self, model):
        self._propagator.on_model(model)

    def _read(self, path):
        if path == "-":
            return sys.stdin.read()
        with open(path) as file_
            return file_.read()

    def main(self, prg, files):
        prg.register_propagator(self._propagator)
        prg.add("base", [], dl.THEORY)

        if not files:
            files = ["-"]

        with prg.builder() as b:
            t = dl.HeadBodyTransformer()
            for path in files:
                clingo.parse_program(self._read(path),
                    lambda stm: b.add(t.visit(stm)))

        prg.ground([("base", [])])
        prg.solve(on_model=self.on_model)

if __name__ == "__main__":
    sys.exit(int(clingo.clingo_main(DLApp(), sys.argv[1:])))

Listing 36. Application class DLApp with main loop for difference constraints (dl-app.py)

that whenever a rule is visited (during parsing), the body and head atoms are aware of
their respective locations. Similarly, function visit_TheoryAtom in lines [150][157] is called
whenever a theory atom is visited and returns the theory atom with the location (either
head or body) as an argument. For instance, treating the example above using this class
results in the following program:

1 &diff(head) { 0-x } <= -2.
2 a :- &diff(body) { 0-x } <= -1.

Listing 36 shows the application that addresses grounding and solving. Lines [19][34]
implement a customized main function. The difference to clingo’s regular one is that a
propagator for difference constraints is registered, the string variable dl.THEORY containing
the above theory language is added as a program, and the input programs are parsed to
add the locations of the difference constraint atoms; grounding and solving then follow as usual. Note that the `solve` function in Line 34 takes a model callback as argument. Whenever a dc-stable model $X$ is found, this callback adds symbols to the answer set representing a mapping satisfying the corresponding difference constraints $C_X$. The model $X$ (excluding theory atoms) is printed as part of clingo’s default output. The callback function `on_model` in Line 10 calls in turn the `on_model` function of the propagator (Line 10 in Listing 37) that adds symbols of the form $dl(x, v)$ to the model, where $x$ is the name of an integer variable and $v$ the assigned value in $X$.

Our exemplary propagator for difference constraints in Listing 37 implements the algorithm presented in (Cotton and Maler 2006). The idea is that deciding whether a set of difference constraints is satisfiable can be mapped to a graph problem. Given a set of difference constraints, let $(V, E)$ be the weighted directed graph such that $V$ is the set of variables occurring in the constraints and $E$ the set of weighted edges $(u, v, d)$ for each constraint $u - v \leq d$. The set of difference constraints is satisfiable if the corresponding graph does not contain a negative cycle.\footnote{A cycle is negative, if the sum of its edges is negative.} The Graph class is in charge of cycle detection;

\begin{figure}[h]
\centering
\begin{tabular}{|c|}
\hline
\textbf{dl} \\
\hline
\textbf{Graph} \\
\hline
+ add_edge(level, edge) \\
+ backtrack(level) \\
+ get_assignment() \\
\hline
\end{tabular}
\caption{Class diagram for the Graph class}
\end{figure}

its interface is given in Figure 4. We refrain from giving the code of the Graph class and rather concentrate on describing its interface:

- Function `add_edge` adds an edge of form $(u, v, d)$ to the graph. If adding an edge to the graph leads to a negative cycle, the function returns the cycle in form of a list of edges; otherwise, it returns `None`. Furthermore, each edge added to the graph is associated with a decision level.\footnote{The ASP solver’s assignment comprises the decision level; it is incremented for each decision made and decremented for each decision undone while backjumping; initially, the decision level is zero.} This additional information is used to backtrack to a previous state of the graph, whenever the solver has to backtrack to recover from a conflict.
- Function `backtrack` takes a decision level as argument. It removes all edges added on that level from the graph. For this to work, decision levels have to be backtracked in chronological order. Note that the CDCL algorithm in Figure 2 calling our propagator also backtracks decision levels in chronological order.
- As a side effect, the Graph class internally maintains an assignment of integers to nodes. This assignment can be turned into an assignment to the variables such that the difference constraints corresponding to the edges of the graph are satisfied. Function `get_assignment` returns this assignment in form of a list of pairs of variables and integers.
How to build your own ASP-based system?!

class DLPropagator(clingo.Propagator):
    def __init__(self):
        self.__l2e = {}  # {literal: [(node, node, weight)]}
        self.__e2l = {}  # {(node, node, weight): [literal]}
        self.__states = []  # [Graph]

    def _add_edge(self, init, lit, u, v, w):
        edge = (u, v, w)
        self.__l2e.setdefault(lit, []).append(edge)
        self.__e2l.setdefault(edge, []).append(lit)
        init.add_watch(lit)

    def init(self, init):
        for atom in init.theory_atoms:
            term = atom.term
            if term.name == "diff" and len(term.arguments) == 1:
                u = _evaluate(atom.elements[0].terms[0].arguments[0])
                v = _evaluate(atom.elements[0].terms[0].arguments[1])
                w = _evaluate(atom.guard[1]).number
                lit = init.solver_literal(atom.literal)
                self._add_edge(init, lit, u, v, w)
                if term.arguments[0].name == "body":
                    self._add_edge(init, -lit, v, u, -w - 1)

    def propagate(self, control, changes):
        state = self.__state(control.thread_id)
        level = control.assignment.decision_level
        for lit in changes:
            for edge in self.__l2e[lit]:
                cycle = state.add_edge(level, edge)
                if cycle is not None:
                    c = [self.__literal(control, e) for e in cycle]
                    control.add_nogood(c) and control.propagate()
        return

    def undo(self, thread_id, assign, changes):
        self.__state(thread_id).backtrack(assign.decision_level)

    def on_model(self, model):
        assignment = self.__state(model.thread_id).get_assignment()
        model.extend(
            clingo.Function("dl", [var, value]) for var, value in assignment)

Listing 37. DLPropagator class for difference constraints (dl.py)

The difference logic propagator implements the Propagator interface (except for check) in Figure 3 in lines 236-260; it features aspects like incremental propagation and backtracking, while supporting solving with multiple threads, and multi-shot solving. Whenever the set of edges associated with the current partial assignment of a solver induces a negative cycle and, hence, the corresponding difference constraints are unsatisfiable, it adds a nogood forbidding the negative cycle. To this end, it maintains data structures for
detecting whether there is a conflict upon the addition of new edges. More precisely, the
propagator has three data members:

1. The self.__l2e dictionary in Line 226 maps solver literals for difference constraint
theory atoms to their corresponding edges [50]
2. the self.__e2l dictionary in Line 227 maps edges back to solver literals [51] and
3. the self.__state list in Line 228 stores for each solver thread its current graph
with the edges assigned so far.

Function init in lines 236–246 sets up watches as well as the dictionaries in self.__l2e
and self.__e2l. To this end, it traverses the theory atoms over diff/1 in lines 237–246.
Note that the loop simply ignores other theory atoms treated by other propagators. In
lines 240–241, we extract the edge from the theory atom. Each such atom is associated
with a solver literal, obtained in Line 243. The mappings between solver literals and
corresponding edges are then stored in the self.__l2e and self.__e2l dictionaries in
lines 232 and 233. In Line 234 of the loop, a watch is added for each solver literal at
hand, so that the solver calls propagate whenever the edge has to be added to the graph.
Up to here, we accommodated the non-strict semantics as we only consider the constraint
occurring in the program and not its negation. If the difference constraint atom occurs in
the body, we impose the strict semantics, meaning that, in case that the assigned literal
is false, we make sure that the negation of the difference constraint holds. We check if the
atom occurs in the body in Line 245, and if this is the case, we add an edge representing
the negation of the difference constraint associated with the negated literal and watch
the negated literal as well.

Function propagate, given in lines 248–257, accesses control.thread_id in Line 249
to obtain the graph associated with the active thread. The loops in lines 251–257 then
iterate over the list of changes and associated edges. In Line 253 each such edge is
added to the graph. If adding the edge produced a negative cycle, a nogood is added
in Line 256. Because an edge can be associated with multiple solver literals, we use
function __literal retrieving the first solver literal associated with an edge that is true,
to construct the nogood forbidding the cycle. Given that the solver has to resolve the
conflict and backjump, the call to add_nogood always yields false, so that propagation is
stopped without processing the remaining changes any further [54].

Given that each edge added to the graph in Line 253 is associated with the current
decision level, the implementation of function undo is quite simple. It calls function
backtrack on the solver’s graph to remove all edges added on the current decision level.

50 A solver literal might be associated with multiple edges, see Footnote 43.
51 In one solving step, the clingo API guarantees that a (grounded) theory atom is associated with exactly
one solver literal. Theory grounded in later solving steps can be associated with fresh solver literals
though.
52 Here we assume that the user supplied a valid theory atom. A propagator for production should check
validity and provide proper error messages.
53 Python’s setdefault function is used to update the mappings. Depending on whether the given key
already appears in the dictionary, the function either retrieves the associated value or inserts and
returns the second argument.
54 The optional arguments tag and lock of add_nogood can be used to control the scope and lifetime
of recorded nogoods. Furthermore, if a propagator adds nogoods that are not necessarily violated,
function control.propagate can be invoked to trigger unit propagation.
Remark 7

Here, we used a simplified Python version of the difference constraints propagator as a showcase. In practice, performance might fall short compared to solutions implemented in C++ or C. Our project Theory Loading for clingo (tefoli) found at https://github.com/potassco/tefoli provides a framework for loading shared C libraries implementing propagators thus combining comfortable scripting with performance.

7.1 Solving flow shop problems

To see our propagator in action, we consider the flow shop problem, dealing with a set of tasks $T$ that have to be consecutively executed on $m$ machines. Each task has to be processed on each machine from 1 to $m$. Different parts of one task are completed on each machine resulting in the completion of the task after execution on all machines is finished. Before a task can be processed on machine $i$, it has to be finished on machine $i-1$. The duration of different tasks on the same machine may vary. A task can only be executed on one machine at a time and a machine must not be occupied by more than one task at a time. An (optimal) solution to the problem is a permutation of tasks so that all tasks are finished as early as possible.

<table>
<thead>
<tr>
<th>task</th>
<th>duration on machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5. Flow shop: instance with three tasks and two machines

Figure 5 depicts a possible instance for the flow shop problem. The three tasks a, b, and c have to be scheduled on two machines. The colored boxes indicate how long a task has to run on a machine. Lighter shades of the same color are for the first and darker ones for the second machine. For example, task a needs to be processed for 3 time units on the first and 4 time units on the second machine.

Listing 38. Flow shop instance (fsI.lp)

```
1 machine(1). machine(2).
2 task(a). duration(a,1,3). duration(a,2,4).
3 task(b). duration(b,1,1). duration(b,2,6).
4 task(c). duration(c,1,5). duration(c,2,5).
```

Next, we encode this problem using difference constraints. We give in Listing 38 a straightforward encoding of the instance in Figure 5. Listing 39 provides the encoding of the flow shop problem. Following the generate, define, and test methodology of ASP, we first generate in lines 1–10 all possible permutations of tasks, where atoms of form permutation(T,U) encode that task $T$ has to be executed before task $U$. Then, in the following lines 12–16 we use difference constraints to calculate the duration of the generated permutation. The difference constraint in Line 15 guarantees that the tasks
are executed in the right order. For example, \((a,1) - (a,2) \leq -d\) ensures that task \(a\) can only be executed on machine 2 if it has finished on machine 1. Hence, variable \((a,2)\) has to be assigned so that it is greater or equal to \((a,2) - d\) where \(d\) is the duration of task \(a\) on machine 1. Similarly, \((a,1) - (b,1) \leq -d\) makes sure that task \(b\) can only be executed on machine 1 if task \(a\) has finished on machine 1. While the first constraint is a fact (see Line 12), the latter is subject to the generated permutation of tasks (see Line 13). The difference constraint in Line 16 ensures that all time points at which a task is started are greater than zero. Note that this constraint is in principle redundant but since sets of difference constraints always have infinitely many solutions it is good practice to encode relative to a starting point. Furthermore, note that 0 is actually a variable. In fact, the Graph class takes care of subtracting the value of variable 0 from all other variables when returning an assignment to get easier interpretable solutions.

Running encoding and instance with the dl propagator results in the following six solutions corresponding to the solutions in Figure 6.

```
UNIX> python dl-app.py fsE.lp fsI.lp 0
clingo-dl version 1.0
Reading from fsE.lp ...
Solving...
Answer: 1
permutation(b,a) permutation(a,c)
dl((a,1),1) dl((a,2),7) dl((b,1),0)
dl((b,2),1) dl((c,1),4) dl((c,2),11)
```

\(^{55}\) Note that in each solution all tasks are executed as early as possible. This is no coincidence and actually guaranteed by the algorithm implemented in the Graph class.
How to build your own ASP-based system?!  

<table>
<thead>
<tr>
<th>machine</th>
<th>solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>a &lt; b &lt; c</td>
<td>18</td>
</tr>
<tr>
<td>a &lt; c &lt; b</td>
<td>19</td>
</tr>
<tr>
<td>b &lt; a &lt; c</td>
<td>16</td>
</tr>
<tr>
<td>b &lt; c &lt; a</td>
<td>16</td>
</tr>
<tr>
<td>c &lt; a &lt; b</td>
<td>20</td>
</tr>
<tr>
<td>c &lt; b &lt; a</td>
<td>20</td>
</tr>
</tbody>
</table>

Fig. 6. Flow shop: solutions for all possible permutations with the total execution length in the top right corner and optimal solutions with a blue background

7.2 Hybrid optimization with difference constraints

Finally, to find optimal solutions, we combine the algorithms in Listing 29 and Listing 36 to minimize the total execution time of the tasks. The adapted algorithm is given in Listing 40. As with the algorithm in Listing 36, a propagator and theory language is registered before solving and the program is parsed to accommodate a uniform semantic treatment. And the control flow is similar to the branch-and-bound-based optimization algorithm in Listing 29 except that we now minimize the variable bound; or better the difference between variable 0 and bound by adding the difference constraint 0 – bound ≤ b to the program in Line 29 where b is the best known execution time of the tasks as obtained from the assignment in Line 17 minus 1. To bound maximum execution time of the task, we have to add one more line to the encoding in Listing 39.
import sys, clingo, dl

class DLOptApp(clingo.Application):
    program_name = "clingo-dl-opt"
    version = "1.0"

    def __init__(self):
        self._bound = None
        self._propagator = dl.DLPropagator()

    def _on_model(self, model):
        self._propagator.on_model(model)
        for symbol in model.symbols(theory=True):
            if symbol.match("dl", 2):
                n, v = symbol.arguments
                if n.match("bound", 0):
                    self._bound = v.number
                    break

    def _read(self, path):
        if path == "-":
            return sys.stdin.read()
        with open(path) as f:
            return f.read()

    def main(self, prg, files):
        prg.register_propagator(self._propagator)
        prg.add("base", [], dl.THEORY)
        prg.add("bound", ["b"], "@diff(head) { bound -0 } <= b.")

        if not files:
            files = ["-"

        with prg.builder() as b:
            t = dl.HeadBodyTransformer()
            for path in files:
                clingo.parse_program(
                    self._read(path),
                    lambda stm: b.add(t.visit(stm)))

            prg.ground([{"base", []}])
            while prg.solve(on_model=self._on_model).satisfiable:
                print("Found new bound: {}".format(self._bound))
                prg.ground([{"bound", [self._bound - 1]}])

        if self._bound is not None:
            print("Optimum found")

        if __name__ == "__main__":
            sys.exit(int(clingo.clingo_main(DLOptApp(), sys.argv[1:1])))

Listing 40. Application class DLOptApp for difference constraints with optimization (dl0-app.py)
How to build your own ASP-based system?!

22  \&diff \{ (T,M)-bound \} <= -D :- duration(T,M,D).

This makes sure that each task ends within the given bound. Running encoding and instance with the dl propagator results in the optimum bound 16 where the obtained solution corresponds to the left of the two optimal solutions indicated by a light blue background in Figure 6:

```
UNIX > python dlO-app.py fsE.lp fsI.lp
clingo-dl-opt version 1.0
Reading from fsE.lp ...  
Solving...  
Answer: 1  
permutation(b,a) permutation(a,c) dl(bound,16)  
dl((a,1),1) dl((a,2),7) dl((b,1),0)  
dl((b,2),1) dl((c,1),4) dl((c,2),11)  
Found new bound: 16  
Solving...  
Optimum found  
UNSATISFIABLE
```

8 Guess-and-check programming reloaded

Finally, let us present an implementation of guess-and-check programming that relies on a combination of two clingo solvers. In contrast to the approach taken in Section 3.4 where the logic programs comprising the guess and check parts are combined in a single disjunctive program and thus solved by a single solver, the idea is now to deal with both programs separately by means of two interacting solvers. This last case-study nicely contrasts the efforts involved in meta-programming and the usage of solver APIs. Also, it further illustrates features of clingo’s API, namely, the manipulation of a program’s abstract syntax tree (AST), the interaction of (multi-threaded) clingo instances via the propagator interface, the usage of assumptions during solving, and the addition of constraints to a program during runtime.

Unlike Section 3.4, we use #program directives to declare rules belonging to the guess and checker program. As above, guess atoms must not occur among the head atoms of the check program. For example, the simple guess and check programs from Listing 12 and 13 can now be rolled into one, as shown in Listing 41.

```
1  #program guess.
2  1 { a(1..2) }.

4  #program check.
5  :- not a(1).
```

Listing 41. Guess-and-check program (guess-check.lp)

Passing this to our guess-and-check application app.py yields the analogous output as with meta-programming.

```
UNIX > python app.py guess-check.lp 0  
guess-and-check version 1.0  
Reading from guess-check.lp  
Solving...  
Answer: 1  
a(2)  
SATISFIABLE
```

In what follows, we detail the inner working of the approach. The idea is to have
one solver guessing solution candidates, and another checking their compliance. Their interaction is realized through *clingo*’s propagator interface and restricted to testing total candidates, rather than partial ones as done in Section 7. In this way, the checking solver acts as a propagator within the guessing one.

The overall design is partitioned in four classes. Our description concentrates on these classes by following the overall workflow, although the line numbers reflect positions in the source code.

As before, we derive from *clingo*’s Application class to tie things together. To this end, we implement *clingo*’s main function, as shown in Listing 42. As mentioned, the primary

```python
class GACApp(clingo.Application):
    def __init__(self):
        self.program_name = "guess-and-check"
        self.version = "1.0"

    def main(self, ctl, files):
        if not files:
            files = ["-"

        check = []
        with ctl.builder() as builder:
            trans = Transformer(builder, check)
            for path in files:
                if path == ":-":
                    program = sys.stdin.read()
                else:
                    with open(path) as handle:
                        program = handle.read()
                        clingo.parse_program(program, trans.add)
            ctl.register_propagator(GACPropagator(check))
            ctl.ground([("base", [])])
            ctl.solve()
```

Listing 42. The GACApp class for guess-and-check programming (app.py)

solver object `ctl` acts as the guesser, while the checker is encapsulated as its propagator. The main function starts by parsing the input programs, registers the propagator, grounds, and solves. The task of the Transformer in Line 111 is to add rules from the guess part to the program in the primary solver `ctl` via its builder and to collect rules from the check part in the list initialized in Line 109. This is done during parsing in Line 118 by means of the add function defined in the Transformer class.

The add function is given in lines 12 to 25 of Listing 43 as the salient part of the Transformer class. It relies on variable _state to distinguish whether a rule is read in the context of a guess (or base) or check program. This variable is “toggled” in lines 16 and 17 whenever a #program directive is encountered. Accordingly, a rule’s AST is either added to the program builder of the guessing solver in Line 23 or appended to the check list (in Line 25) that was passed down from the main function to gather the check program.
How to build your own ASP-based system?! 57

class Transformer:
    def __init__(self, builder, check):
        self._builder = builder
        self._state = "guess"
        self._check = check

    def add(self, stm):
        if stm.type == ast.ASTType.Program:
            if stm.name == "check" and not stm.parameters:
                self._state = "check"
            elif stm.name in ("base", "guess") and not stm.parameters:
                self._state = "guess"
            else:
                raise RuntimeError("unexpected program part")
        else:
            if self._state == "guess":
                self._builder.add(stm)
            else:
                self._check.append(stm)

Listing 43. The Transformer class for classifying rules into the guess and check part (app.py)

Once parsing is finished, the filled list is used to initialize the propagator in Line 120. The corresponding GACPropagator class is given in Listing 44. It administers one or several solver objects, which are encapsulated by the Checker class in Listing 45. Given that no partial checks are performed, the propagation class only implements function init and check of clingo’s Propagator interface from Figure 3.

Let us first detail the initialization of the checkers in Line 64 to 87. In fact, the init function may create several instances of the Checker class, depending on the number of threads of the primary solver. Each such checker (cf. Line 67) is initialized by looping over the atoms of the primary solver that provide the respective guess. While atoms are dropped in Line 74 that have been found to be false after grounding (and pre-processing) the guess program, either a fact or a choice rule is added to the checker in Line 81 and 84 depending on whether the atom was found to be true or unknown, respectively. Clearly, unknown atoms of the guesser are most relevant to the checking solver, since their truth value is still subject to change. To this end, each checker comprises a dictionary mapping (unknown) guess literals to check literals; it is filled in Line 85. Once all facts and choice rules are added to the checker, the check program gathered during parsing is grounded in Line 87 and added as well. The corresponding add and ground functions are defined in the Checker class and implemented in a straightforward way in the lines following lines 35 and 38, respectively.

Just the same way as during initialization, both GACPropagator and Checker work hand in hand in their respective check functions. The propagator’s check function is called once the guessing solver has found a stable model; it immediately calls the check function.

56 In Section 3, this was done via predicate guess/1.
class GACPropagator(clingo.Propagator):
    def __init__(self, check):
        self._check = check
        self._checkers = []

    def init(self, init):
        for _ in range(init.number_of_threads):
            checker = Checker()
            self._checkers.append(checker)
            with checker.backend() as backend:
                for atom in init.symbolic_atoms:
                    guess_lit = init.solver_literal(atom.literal)
                    guess_truth = init.assignment.value(guess_lit)
                    if guess_truth is False:
                        continue
                    check_lit = backend.add_atom(atom.symbol)
                    if guess_truth is True:
                        backend.add_rule([check_lit], [])
                    else:
                        backend.add_rule([check_lit], [], True)
            checker.add(guess_lit, check_lit)
        checker.ground(self._check)

    def check(self, control):
        assignment = control.assignment
        checker = self._checkers[control.thread_id]
        if not checker.check(control):
            conflict = []
            for level in range(1, assignment.decision_level+1):
                conflict.append(-assignment.decision(level))
            control.add_clause(conflict)

Listing 44. The GACPropagator class interfacing guessing and checking solver (app.py)

of the associated checker in Line 93. In doing so, it passes along the control object
providing (limited) access to the underlying solver. This includes access to the assignment
of the solver which of course corresponds to a model. The latter is at once extracted upon
entering the checker’s check function in Line 46 and analyzed afterwards. To this end,
the function loops over the dictionary associating (originally unknown) guess and check
atoms to transfer the guessed literals into a list of checker literals that are then used as
assumptions in the subsequent call of the checker in Line 53. Technically, assumptions
are added to the solver’s assignment and amount semantically to the addition of integrity
constraints (unlike externals; cf. Section 5.1) In this way, the checker is forced to
search for stable models comprising all guessed literals. If this fails, the checker’s check
succeeds, as does the propagator’s check. Otherwise, the propagator extracts from the

57 Also, assumptions only affect the current solve call. Opposed to this, assignments to externals persist
over solve call (as long as the externals are not released, reassigned, or defined).
How to build your own ASP-based system?

Listing 45. The `Checker` class wrapping the checking solver (`app.py`)

guesser’s stable model all underlying decision literals and adds them as an integrity constraint, thus eliminating the combination of literals from the search space.

9 Discussion

This tutorial aims at enabling ASP users to become ASP system developers.

Our role model has been the landmark paper by Eén and N. Sörensson in (2004) that aimed at “give[ing] sufficient details about implementation to enable the reader to construct his or her own solver in a very short time. This will allow users of SAT-solvers to make domain specific extensions or adaptions of current state-of-the-art SAT-techniques, to meet the needs of a particular application area.” Their presentation of the C++ source code of the SAT solver `minisat` significantly boosted research in SAT by “bridge[ing] the gap between existing descriptions of SAT-techniques and their actual implementation” (Eén and Sörensson 2004).

We hope to achieve a similar effect with the tutorial at hand. However, unlike following suit in easing a white box approach to ASP solving, we rather advocate the gray or black box approach put forward in the introduction. This is motivated by the much more elaborate model-ground-solve workflow of ASP systems that must often be addressed in its entirety to provide a certain functionality.

To this end, we describe several essential techniques for extending the ASP system `clingo` or implementing customized special-purpose systems. Although we start with the
lighter approach of meta programming, the greater part deals with application interface programming in Python (although several alternatives are available). Central to this is the new Application class of clingo that permits to draw on clingo’s infrastructure by starting processes similar to the one in clingo. This allows us to tie things together by overriding clingo’s main function, as illustrated by various examples throughout the tutorial. In particular, we have seen how derivatives of the Application class can be used to engage manipulations to programs’ abstract syntax trees, control various forms of multi-shot solving, and set up theory propagators for foreign inferences. Multi-shot solving provides us with fine-grained control of ASP reasoning processes, while theory solving allows us to refine basic ASP solving by incorporating foreign types of constraints. Because of ASP’s model-ground-solve methodology both techniques pervade its whole workflow, starting with extensions to the input language, over means for incremental and theory-enhanced grounding, to stateful and theory-enhanced solving. Multi-shot solving even adds a fourth dimension to control ASP reasoning processes.

Although meta programming has been around in ASP since its beginning (cf. Section 3 for a brief discussion), we hope that the reification feature of clingo makes it more attractive as a light-weight alternative to extend ASP systems. The idea of implementing ASP systems by chaining was first advocated by Tomi Janhunen and used in his normalization toolbox [Janhunen and Niemelä 2011] [Bomanson et al. 2014] [Bomanson et al. 2016]. In both cases, an intermediate ASP format is used to pass data from one solver to the next. While we used a fact-based representation of aspif, the normalization tools rely on the machine-oriented smodels format. Interestingly, lc2casp [Cabalar et al. 2016] implements a system for non-monotonic constraint solving by translating one aspif specification into another. That is, it takes the output of gringo, compiles out non-monotonicity, and feeds the result into clingcon, an extension of clingo with monotonic linear constraints over integers.

As mentioned in the introduction, dlv and clingo constitute nowadays the only genuine ASP systems in use. Accordingly, they are the only possible providers of native APIs for ASP. As detailed by Alviano et al. in (2017), the latest versions of dlv combine the idlv grounder [Calimeri et al. 2017] with the ASP solver wasp [Alviano et al. 2015]. As with gringo, the input language of idlv covers the second ASP language standard [Calimeri et al. 2019]. Furthermore, it offers the integration of computable function, similar to the mechanism (using terms preceded by ‘@’) sketched at the beginning of Section 4. Unlike this, a full-fledged Python API is offered by the ASP solver wasp [Dodaro and Ricca 2020]. Interesting applications of it to heuristic control and propagators for integrity constraints are described in [Dodaro et al. 2016] and [Cuteri et al. 2020], respectively. In fact, former versions of dlv offer powerful Java integration [Febbraro et al. 2012], compliant with the Object-Relational Mapping standard (ORM), and implemented by wrapping dlv at its core. A Python library providing an ORM interface to clingo is available at https://github.com/potassco/clorm. A framework for developing applications embedding ASP on mobile devices is proposed in [Fuscà et al. 2016].

Prior to the availability of APIs various systems extending ASP have been built. For example, dlhex [Redl 2016] [Eiter et al. 2018] provides higher-order logic programs, whose higher-order atoms are implemented externally in C++ or Python; it is build upon clingo’s infrastructure. At the time, such a white box approach was only feasible thanks to a close collaboration between both groups at Vienna and Potsdam, not to
How to build your own ASP-based system?!

mention that this fostered the development of clingo’s API quite a bit. Interestingly, also clingcon [Banbara et al. 2017], an extension of clingo with linear constraints over integers, started out as a white box approach and has just recently been transformed into a gray box approach, since otherwise its maintenance had been infeasible. Similarly yet much earlier, adsolver [Mellarkod et al. 2008] extended smodels [Niemelä and Simons 1997] with linear constraints over integers. Another category of ASP systems, such as ezsmt [Lierler and Susman 2016], dingo [Janhunen et al. 2011], and aspmt [Bartholomew and Lee 2014] translate ASP with constraints to SAT Modulo Theories (SMT; [Nieuwenhuis et al. 2006]) and use appropriate back-ends. Similarly, mingo [Liu et al. 2012] translates to Mixed Integer Linear Programming (MILP).

Both meta and application interface programming greatly facilitate the development of ASP-based systems and therefore ease the transposition of ideas into practise. The lighter approach of meta programming is well suited for rapid prototyping and moreover enjoys elaboration tolerance. However, once more control is needed, application programming is indispensable. Although, it lacks full elaboration tolerance, it has nonetheless the great advantage to offer a high level of abstraction. This makes any project much easier to handle and to maintain than modifying the source code of an ASP systems.

Last but not least, ASP has come a long way to turn into a mature and quite so- phisticated approach to declarative problem solving. However, this sophistication should not become an obstacle to further technological advances. We hope that this tutorial contributes to coping with this challenge. After all, we, the ASP community, have the hard job of making our users’ lives easy.

Appendix A Intermediate language

To accommodate the rich input language, a general grounder-solver interface is needed. Although this could be left internal to clingo, it is good practice in ASP and neighboring fields to explicate such interfaces via an intermediate language. This also allows for using alternative downstream solvers or transformations.

Unlike the block-oriented smodels format, the aspif format is line-based. Notably, it abolishes the need of using symbol tables in smodels’ format for passing along meta-expressions and rather allows gringo to output information as soon as it is grounded. An aspif file starts with a header, beginning with the keyword asp along with version information and optional tags:

```
asp tv_m,v_n,v_r,t_1,...,t_k
```

where \( v_m, v_n, v_r \) are non-negative integers representing the version in terms of major, minor, and revision numbers, and each \( t_i \) is a tag for \( k \geq 0 \). Currently, the only tag is incremental, meant to set up the underlying solver for multi-shot solving. An example header is given in the first lines of Listing 46 and 47 below. The rest of the file is comprised of one or more logic programs. Each logic program is a sequence of lines of aspif statements followed by a 0, one statement or 0 per line, respectively. Positive and negative integers

58 ASP Intermediate Format
59 http://www.tcs.hut.fi/Software/smodels
are used to represent positive or negative literals, respectively. Hence, 0 is not a valid literal.

Let us now briefly describe the format of aspif statements and illustrate them with the simple logic program in Listing 1 as well as the result of grounding a subset of Listing 32 only pertaining to difference constraints in Listing 47.

Listing 46. Representing the logic program from Listing 1 in aspif format

```
1  asp 1 0 0
2  1 1 1 0 0
3  1 0 1 2 0 1
4  1 0 1 3 0 1 -1
5  4 1 a 1 1
6  4 1 b 1 2
7  4 1 c 1 3
8  0
```

Rule statements have form

\[ 1 \_ H \_ B \]

in which head \( H \) has form

\[ h \_ m \_ a_1 \ldots a_m \]

where \( h \in \{0, 1\} \) determines whether the head is a disjunction or choice, \( m \geq 0 \) is the number of head elements, and each \( a_i \) is a positive literal.

Body \( B \) has one of two forms:
- Normal bodies have form
  \[ 0 \_ n \_ l_1 \ldots l_n \]
  where \( n \geq 0 \) is the length of the rule body, and each \( l_i \) is a literal.
- Weight bodies have form
  \[ l \_ n \_ l_1 \_ w_1 \ldots l_n \_ w_n \]
  where \( l \) is a positive integer to denote the lower bound, \( n \geq 0 \) is the number of literals in the rule body, and each \( l_i \) and \( w_i \) are a literal and a positive integer.

All types of ASP rules are included in the above rule format. Heads are disjunctions or choices, including the special case of singular disjunctions for representing normal rules. As in the smodels format, aggregates are restricted to a singular body, just that in aspif cardinality constraints are taken as special weight constraints. Otherwise, a body is simply a conjunction of literals.

The three rules in Listing 1 are represented by the statements in lines 2–4 of Listing 46. For instance, the four occurrences of 1 in Line 2 capture a rule with a choice in the head, having one element, identified by 1. The two remaining zeros capture a normal body with no element. For another example, lines 2–7 of Listing 47 represent 6 of the facts in Listing 33, the four regular atoms in lines 1–4 along two comprising theory atoms in lines 11 and 12.
Minimize statements have form
\[ 2. p.n.l_1.w_1 \ldots l_n.w_n \]
where \( p \) is an integer priority, \( n \geq 0 \) is the number of weighted literals, each \( l_i \) is a literal, and each \( w_i \) is an integer weight. Each of the above expressions gathers weighted literals sharing the same priority \( p \) from all \#minimize directives and weak constraints in a logic program. As before, maximize statements are translated into minimize statements.

Projection statements result from \#project directives and have form
\[ 3. n.a_1 \ldots a_n \]
where \( n \geq 0 \) is the number of atoms, and each \( a_i \) is a positive literal.

Output statements result from \#show directives and have form
\[ 4. m.s.n.l_1 \ldots l_n \]
where \( n \geq 0 \) is the length of the condition, each \( l_i \) is a literal, and \( m \geq 0 \) is an integer indicating the length in bytes of string \( s \) (where \( s \) excludes byte ‘\0’ and newline). The output statements in lines [1][1] of Listing [46] print the symbolic representation of atom \( a \), \( b \), or \( c \), whenever the corresponding atom is true. For instance, the string ‘a’ is printed if atom ‘1’ holds. Unlike this, the statements in lines [8][11] of Listing [47] unconditionally print the symbolic representation of the atoms stemming from the four facts in lines [1][4] of Listing [33].

External statements result from \#external directives and have form
\[ 5. a.v \]
where \( a \) is a positive literal, and \( v \in \{0, 1, 2, 3\} \) indicates free, true, false, and release.

Assumption statements have form
\[ 6. n.l_1 \ldots l_n \]
where \( n \geq 0 \) is the number of literals, and each \( l_i \) is a literal. Assumptions instruct a solver to compute stable models such that \( l_1, \ldots, l_n \) hold. They are only valid for a single solver call.

Heuristic statements result from \#heuristic directives and have form
\[ 7. m.a.k.p.n.l_1 \ldots l_n \]
where \( m \in \{0, \ldots, 5\} \) stands for the \((m+1)\)th heuristic modifier among level, sign, factor, init, true, and false, \( a \) is a positive literal, \( k \) is an integer, \( p \) is a non-negative integer priority, \( n \geq 0 \) is the number of literals in the condition, and the literals \( l_i \) are the condition under which the heuristic modification should be applied.

Edge statements result from \#edge directives and have form
\[ 8. u.v.n.l_1 \ldots l_n \]
where \( u \) and \( v \) are integers representing an edge from node \( u \) to node \( v \), \( n \geq 0 \) is the length of the condition, and the literals \( l_i \) are the condition for the edge to be present.

Let us now turn to the theory-specific part of aspif. Once a theory expression is grounded, gringo outputs a serial representation of its syntax tree. To illustrate this,
we give in Listing 47 the (sorted) result of grounding all lines of Listing 32 related to difference constraints, viz. lines 1–20 and Line 24.

Theory terms are represented using the following statements:

\begin{align}
9_0 &. w \\
9_1 &. n &. s \\
9_2 &. t &. n &. u_1 &.& u_2 &.& u_3 &.& \ldots &. u_n
\end{align}

where $n \geq 0$ is a length, index $u$ is a non-negative integer, integer $w$ represents a numeric term, string $s$ of length $n$ represents a symbolic term (including functions) or an operator, integer $t$ is either -1, -2, or -3 for tuple terms in parentheses, braces, or brackets, respectively, or an index of a symbolic term or operator, and each $u_i$ is an integer for a theory term.
Statements (A1), (A2), and (A3) capture numeric terms, symbolic terms, as well as compound terms (tuples, sets, lists, and terms over theory operators).

Fifteen theory terms are given in lines 12–26 of Listing 47. Each of them is identified by a unique index in the third spot of each statement. While lines 12–20 stand for primitive entities of type (A1) or (A2), the ones beginning with '9/uni24232' represent compound terms. For instance, line 21 and 22 represent `end(1)` or `start(1)`, respectively, and line 23 corresponds to `end(1)-start(1)`.

Theory atoms are represented using the following statements:

\[
\begin{align*}
9.4. & v. _n. u_1. \ldots. u_m. l_1. \ldots. l_m \\
9.5. & a. p. _n. v_1. \ldots. v_n \\
9.6. & a. p. _n. v_1. \ldots. v_n. g. u_1
\end{align*}
\]  

(A4) \hspace{1cm} (A5) \hspace{1cm} (A6)

where \( n \geq 0 \) and \( m \geq 0 \) are lengths, index \( v \) is a non-negative integer, \( a \) is a positive literal or 0 for directives, each \( u_i \) is an integer for a theory term, each \( l_i \) is an integer for a literal, integer \( p \) refers to a symbolic term, each \( v_i \) is an integer for a theory atom element, and integer \( g \) refers to a theory operator. Statement (A4) captures elements of theory atoms and directives, and statements (A5) and (A6) refer to the latter.

For instance, line 27 captures the (single) theory element in '{ `end(1)`-`start(1)` }', and line 29 represents the theory atom `&diff { `end(1)`-`start(1)` } <= 200`.

Comments have form

10. s

where \( s \) is a string not containing a newline.

The aspif format constitutes the default output of gringo 5. With clasp 3.2, ground logic programs can be read in both smodels and aspif format.

References


How to build your own ASP-based system?!


