# Deriving Conclusions From Non-Monotonic Cause-Effect Relations∗

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## Abstract

We present an extension of Logic Programming (under stable models semantics) that, not only allows concluding whether a true atom is a cause of another atom, but also *deriving new conclusions* from these causal-effect relations. This is expressive enough to capture informal rules like "if some agent's actions $\mathcal{A}$ have been *necessary* to cause an event $E$ then conclude atom $caused(\mathcal{A}, E)$," something that, to the best of our knowledge, had not been formalised in the literature. To this aim, we start from a first attempt that proposed extending the syntax of logic programs with so-called *causal literals*. These causal literals are expressions that can be used in rule bodies and allow inspecting the derivation of some atom $A$ in the program with respect to some query function $\psi$. Depending on how these query functions are defined, we can model different types of causal relations such as sufficient, necessary or contributory causes, for instance. The initial approach was specifically focused on monotonic query functions. This was enough to cover sufficient cause-effect relations but, unfortunately, necessary and contributory are essentially *non-monotonic*. In this work, we define a semantics for non-monotonic causal literals showing that, not only extends the stable model semantics for normal logic programs, but also preserves many of its usual desirable properties for the extended syntax. Using this new semantics, we provide precise definitions of *necessary* and *contributory* causal relations and briefly explain their behaviour on a pair of typical examples from the Knowledge Representation literature.

## 1 Introduction

An important difference between classical models and most Logic Programming (LP) semantics is that, in the latter, true atoms must be founded or justified by a given derivation. Consequently, falsity is understood as absence of proof: for instance, a common informal way of reading for default literal *not A* is "there is no way to derive $A$." Although this idea seems quite intuitive and, in fact, several approaches have studied how to syntactically build these derivations or *justifications* (Specht 1993; Pemmasani et al. 2004; Pontelli et al. 2009; Denecker et al. 2015; Schulz and Toni 2016), it actually resorts to a concept, the *ways to derive A*, outside the scope of the standard LP semantics.

Such information on justifications for atoms can be of great interest for Knowledge Representation (KR), and especially, for dealing with problems related to causality. For instance, in the area of legal reasoning where determining a legal responsibility usually involves finding out which agent or agents have eventually caused a given result, regardless the chain of effects involved in the process. In this sense, an important challenge in causal reasoning is the capability

of not only deriving facts of the form "$A$ has caused $B$," but also being able to represent and reason about them. As an example, take the assertion:

$$\text{"If somebody causes an accident, (s)he would receive a fine"} \tag{1}$$

This law does not specify the possible ways in which a person may cause an accident. Depending on a representation of the domain, the chain of events from the agent's action(s) to the final effect may be simple (a direct effect) or involve a complex set of indirect effects and defaults like inertia. Focussing on representing (1) in an elaboration tolerant manner (McCarthy 1998), we should be able to write a single rule whose body only refers to the $agent$ involved and the $accident$. For instance, consider the following program

$$accident \leftarrow oil \tag{2}$$
$$oil \quad\quad \leftarrow suzy \tag{3}$$
$$suzy \tag{4}$$

representing that $accident$ is an indirect effect of Suzy's actions. We may then represent (1) by the following rule

$$fine(suzy) \leftarrow suzy \ \texttt{necessary for} \ accident \tag{5}$$

that states that Suzy would receive a $fine$ whenever the fact $suzy$ was necessary to cause the atom $accident$.

With this long term goal in mind, (Cabalar et al. 2014a) proposed a multi-valued semantics for LP that extends the stable model semantics (Gelfond and Lifschitz 1988) and where justifications are treated as *algebraic* constructions. In this semantics, *causal stable models* assign, to each atom, one of these algebraic expressions that captures the set of all non-redundant logical proofs for that atom. Recently, this semantics was used in (Fandinno 2015b) to extend the syntax of logic programs with a new kind of literal, called *causal literal*, that allow representing rules like

$$fine(suzy) \leftarrow suzy \ \texttt{sufficient for} \ accident \tag{6}$$

and derive, from a program $P_1$ containing rules (2-4,6), that $fine(suzy)$ holds. However, the major limitation of this semantics is that causal literals must be monotonic and, therefore, rule (5) cannot be represented. It is easy to see that rule (5) is non-monotonic: in a program $P_2$ containing rules (2-5), the fact $suzy$ is necessary for $accident$ is satisfied and, thus, $fine(suzy)$ must hold, but in a program $P_3$ obtained by adding a fact $oil$ to this last program, $suzy$ is not longer necessary and, thus, $fine(suzy)$ should not be a conclusion.

In this paper, we present a semantics for logic programs with causal literals defined in terms of *non-monotonic* query functions. More specifically, we summarise our contributions as follows. In Section 2, we define the syntax of causal literals and a multi-valued semantics for logic programs whose causal values rely on a completely distributive lattice based on causal graphs. Section 3 shows that positive monotonic program has a least model that can be computed by an extension of the direct consequences operator (van Emden and Kowalski 1976). In Section 4, we define semantics for programs with negation and non-monotonic causal literals and show that it is a conservative extension of the standard stable model semantics. Besides, with a running example, we show how causal literals can be used to derive new conclusion from necessary causal relations and, in Section 5, briefly relate this notion with the actual cause literature. In this section, we also formalise the weaker notion of *contributory cause*, also related to the actual cause literature, and show how causal literals may be used to derive new conclusion from them. In Section 6, we show

that our semantics satisfy the usual properties of the stable modles semantics for the new syntax. Finally, Section 7 concluded the paper. Proofs of formal results from the paper can be found in an extended version (Fandinno 2016).

## 2 Causal Programs

We start by reviewing some definitions from (Cabalar et al. 2014a).

**Definition 1** (Term). *Given a set of labels Lb, a* term *$t$ is recursively defined as one of the following expressions*

$$t \ ::= \ l \ \Big| \ \prod S \ \Big| \ \sum S \ \Big| \ t_1 \cdot t_2$$

*where $l \in Lb$ is a label, $t_1, t_2$ are in their turn terms and $S$ is a (possibly empty and possible infinite) set of terms.* ☐

When $S = \{t_1, \ldots, t_n\}$ is a finite set, we will write $t_1 * \ldots * t_n$ and $t_1 + \ldots + t_n$ instead of $\prod S$ and $\sum S$, respectively. When $S = \emptyset$, we denote $\prod S$ and $\sum S$ by 1 and 0, respectively. We assume that application '·' has higher priority than product '*' and, in its turn, product '*' has higher priority than addition '+'. *Application* '·' represents application of a rule label to a previous justifications. For instance, the justification in program $P_1$ for atom $suzy$ is the fact $suzy$ itself. If rules (2-3) in program $P_1$ are labelled in the following way

$$\begin{aligned} r_1 : & \quad accident \leftarrow oil & (7) \\ r_2 : & \quad oil \quad\quad \leftarrow suzy & (8) \end{aligned}$$

we may represent the justification of $oil$ as $suzy \cdot r_2$, in other words, $oil$ is true because of the the the application of rule $r_2$ to the fact $suzy$. Similarly, we may represent the justification of $accident$ as $suzy \cdot r_2 \cdot r_1$. Addition '+' is used to capture alternative independent causes: each addend is one of those independent causes. For instance, the justification of $oil$, in program $P_3$, may be represented as $suzy \cdot r_2 + oil$ and the justification of $accident$ as $(suzy \cdot r_2 + oil) \cdot r_1$. As we will see below application distributes over addition, so that, the justification of $accident$ can also be written as $suzy \cdot r_2 \cdot r_1 + oil \cdot r_1$, which better illustrates the existence of two alternatives. Product '*' represents conjunction or joint causation. For instance, in a program $P_4$ obtained by adding the fact $billy$ to $P_3$ and replacing rule (8) by

$$r_2 : \quad oil \leftarrow suzy, \, billy \tag{9}$$

the justifications of $oil$ will be $(suzy * billy) \cdot r_2 + oil$. Similarly, the justification of $accident$ will be $(suzy * billy) \cdot r_2 \cdot r_1 + oil \cdot r_1$. Intuitively, terms without addition '+' represent individual causes while terms with addition '+' represent sets of causes. It is worth to mention that these algebraic expressions are in a one-to-one correspondence with non-redundant proofs of an atom (Cabalar et al. 2014a) and that they may also be understood as a formalisation of Lewis' concept of causal chain (Lewis 1973) (see Fandinno 2015b).

**Definition 2** (Value). (Causal) values *are the equivalence classes of terms under axioms for a completely distributive (complete) lattice with meet '*' and join '+' plus the axioms of Figure 1. The set of values is denoted by $\mathbf{V}_{Lb}$. Furthermore, by $\mathbf{C}_{Lb}$ we denote the subset of causal values with some representative term without sums '+'.* ☐

All three operations, '*', '+' and '·' are associative. Product '*' and addition '+' are also

| *Associativity* | | | *Absorption* | | | *Identity* | | | *Annihilator* | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $t \cdot (u{\cdot}w)$ | $=$ | $(t{\cdot}u) \cdot w$ | $t$ | $=$ | $t + u \cdot t \cdot w$ | $t$ | $=$ | $1 \cdot t$ | $0$ | $=$ | $t \cdot 0$ |
| | | | $u \cdot t \cdot w$ | $=$ | $t * u \cdot t \cdot w$ | $t$ | $=$ | $t \cdot 1$ | $0$ | $=$ | $0 \cdot t$ |

| *Indepotence* | *Addition distributivity* | | | *Product distributivity* | | |
|---|---|---|---|---|---|---|
| $l \cdot l = l$ | $t \cdot (u{+}w)$ | $=$ | $(t{\cdot}u) + (t{\cdot}w)$ | $c \cdot d \cdot e$ | $=$ | $(c \cdot d) * (d \cdot e)$ with $d \neq 1$ |
| | $(t + u) \cdot w$ | $=$ | $(t{\cdot}w) + (u{\cdot}w)$ | $c \cdot (d * e)$ | $=$ | $(c \cdot d) * (c \cdot e)$ |
| | | | | $(c * d) \cdot e$ | $=$ | $(c \cdot e) * (d \cdot e)$ |

Fig. 1. Properties of the '$\cdot$'operators: $t, u, w$ are terms, $l$ is a label and $c, d, e$ are terms without '$+$'. Addition and product distributivity are also satisfied over infinite sums and products.

commutative, and they satisfy the usual absorption and distributive laws with respect to infinite sums and products of a completely distributive lattice. The lattice order relation is defined as:

$$t \leq u \qquad \text{iff} \qquad t * u = t \qquad \text{iff} \qquad t + u = u$$

An immediately consequence of this definition is that product, addition, $1$ and $0$ respectively are the greatest lower bound, the least upper bound and the top and the bottom element of the $\leq$-relation. Term $1$ represents a value which holds by default, without an explicit cause, and will be assigned to the empty body. Term $0$ represents the absence of cause or the empty set of causes, and will be assigned to false. Furthermore, applying distributivity (and absorption) of product and application over addition, every term can be represented in *(minimal) disjunctive normal form* in which addition is not in the scope of any other operation and every pair of addends are pairwise $\leq$-incomparable. In the following, we will assume that every term is in disjunctive normal form.

This semantics was used in (Fandinno 2015b), to define the concept of causal query, here *m-query*: a monotonic function $\phi : \mathbf{C}_{Lb} \longrightarrow \{0, 1\}$. Unfortunately, m-queries are not expressive enough to capture necessary causation for two reasons: $(i)$ they are monotonic and $(ii)$ they cannot capture relations between sets of causes. We introduced here the following definition which removes these two limitations.

**Definition 3** (Causal query). *A causal query* $\psi \colon \mathbf{C}_{Lb} \times \mathbf{V}_{Lb} \longrightarrow \{0, 1\}$ *is a function mapping pairs cause-value into* $1$ *(true) and* $0$ *(false) which is anti-monotonic in the second argument, that is,* $\psi(G, t) \leq \psi(G, u)$ *for every* $G \in \mathbf{C}_{Lb}$ *and* $\{t, u\} \subseteq \mathbf{V}_{Lb}$ *such that* $t \geq u$. $\square$

***Syntax.*** We define the semantics of logic programs using its grounding. Therefore, for the remainder of this paper, we restrict our attention to ground logic programs. A *signature* is a triple $\langle At, Lb, \Psi \rangle$ where $At$, $Lb$ and $\Psi$ respectively represent sets of *atoms* (or *propositions*), *labels* and causal queries. We assume the signature of every program contains a causal query $\psi^1 \in \Psi$ s.t. $\psi^1(G, t) \stackrel{\text{def}}{=} 1$ for every $G \in \mathbf{C}_{Lb}$ and value $t \in \mathbf{V}_{Lb}$.

**Definition 4** (Causal literal). *A (causal) literal is an expression* $(\psi :: A)$ *where* $A \in At$ *is an atom and* $\psi \in \Psi$ *is a causal query.* $\square$

A causal atom $(\psi^1 :: A)$ is said to be *regular* and, by abuse of notation, we will use atom $A$ as shorthand for regular causal literals of the form $(\psi^1 :: A)$. We will see below the justification for this notation. A *literal* is either a causal literal $(\psi :: A)$ *(positive literal)*, or a negated causal literal $not(\psi :: A)$ *(negative literal)* or a double negated causal literal $not\,not(\psi :: A)$ *(consistent literal)* with $A \in At$ an atom and $\psi \in \Psi$ a causal query.

**Definition 5** (Causal program). *A (causal) program $P$ is a set of rules of the form:*

$$r_i : \quad A \leftarrow B_1, \ldots, B_m \tag{10}$$

*where $0 \leq m$ is a non-negative integer, $r_i \in Lb$ is a label or $r_i = 1$, $A$ (the head of the rule) is an atom and each $B_i$ with $1 \leq i \leq m$ (the body of the rule) is a literal or a term.* □

A rule $r$ is said to be *positive* iff all literals in its body are positive and it is said to be *regular* if all causal literals in its body are regular. When $m = 0$, we say that the rule is a *fact* and omit the body and sometimes the symbol '$\leftarrow$.' Furthermore, for clarity sake, we also assume that, for every atom $A \in At$, there is an homonymous label $A \in Lb$ and that the label of an unlabelled rule is assumed to be its head. In this sense, a fact $A$ in a program actually stands for the labelled rule $(A : A \leftarrow)$. A program $P$ is *positive* or *regular* when all its rules are positive (i.e. it contains no default negation) or regular, respectively. A *standard program* is a regular program in which the label of every rule is '1 :'.

***Semantics.*** A *(causal) interpretation* is a mapping $I : At \longrightarrow \mathbf{V}_{Lb}$ assigning a value to each atom. For interpretations $I$ and $J$, we write $I \leq J$ when $I(A) \leq J(A)$ for every atom $A \in At$. Hence, there is a $\leq$-bottom interpretation $\mathbf{0}$ (resp. a $\leq$-top interpretation $\mathbf{1}$) that stands for the interpretation mapping every atom $A$ to $0$ (resp. 1). For an interpretation $I$ and atom $A \in At$, by $\max I(A)$ we denote the set

$$\max I(A) \ \stackrel{\text{def}}{=} \ \big\{\, G \in \mathbf{C}_{Lb} \ \big| \ G \leq I(A) \text{ and there is no } G' \in \mathbf{C}_{Lb} \text{ s.t. } G < G' \leq I(A) \,\big\}$$

containing the maximal terms without addition (or individual causes) of $A$ w.r.t. $I$.

**Definition 6** (Causal literal valuation). *The valuation of a causal literal of the form $(\psi :: A)$ with respect to an interpretation $I$, in symbols $I(\psi :: A)$, is given by*

$$I(\psi :: A) \ \stackrel{\text{def}}{=} \ \sum \big\{\, G \in \max I(A) \ \big| \ \psi(G, I(A)) = 1 \,\big\}$$

*We say that $I$ satisfies a causal literal $(\psi :: A)$, in symbols $I \models (\psi :: A)$, iff $I(\psi :: A) \neq 0$.* □

Notice now that $I(\psi^1 :: A) = I(A)$ for any atom $A$ and, thus, writing a standard atom $A$ as a shorthand for causal literal $(\psi^1 :: A)$ does not modify its intended meaning. Causal literals can be used to represent the body of rule (5). For instance, given a set of labels $\mathcal{A} \subseteq Lb$ representing the actions of some agent $\mathcal{A}$, we may define the query function

$$\psi_{\mathcal{A}}^{\text{nec}}(G, t) \ \stackrel{\text{def}}{=} \ \begin{cases} 1 & \text{if } t \leq \sum \mathcal{A} \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

and represent the body of rule (5) by a causal literal of the form $(\psi_{Suzy}^{\text{nec}} :: accident)$ where $Suzy$ is the set of labels $\{suzy\}$. In the sake of clarity, we usually will write $(\mathcal{A} \ \texttt{necessary for} \ A)$ in rule bodies instead $(\psi_{\mathcal{A}}^{\text{nec}} :: A)$.

If we consider an interpretation $I$ which assigns to the atom $accident$ its justification in program $P_2$, that is, $I(accident) = suzy \cdot r_2 \cdot r_1$, then any term without addition $G \in \mathbf{C}_{Lb}$, satisfies

$$\begin{aligned} \psi_{Suzy}^{\text{nec}}(G, I)(accident) = 1 \quad &\text{iff} \quad suzy \cdot r_2 \cdot r_1 \ \leq \ \sum \{suzy\} \\ &\text{iff} \quad suzy \cdot r_2 \cdot r_1 \ \leq \ suzy \\ &\text{iff} \quad suzy \cdot r_2 \cdot r_1 + suzy \ = \ suzy \end{aligned}$$

which holds applying identity and associativity of application and absorption w.r.t. addition

$$suzy{\cdot}r_2{\cdot}r_1 + suzy \;\; = \;\; 1 \cdot suzy \cdot (r_2{\cdot}r_1) + suzy \;\; = \;\; suzy$$

Similarly, in program $P_3$, $\psi^{\mathrm{nec}}_{Suzy}(G, I'(accident)) = 1$ iff $suzy{\cdot}r_2{\cdot}r_1 + oil \leq suzy$ which does not hold. In other words, Suzy's actions has been necessary in program $P_2$ but not in program $P_3$.

The valuation of a causal term $t$ is the class of equivalence of $t$. The valuation of non-positive literals is defined as follows

$$I(not(\psi :: A)) \quad \stackrel{\text{def}}{=} \quad \begin{cases} 1 & \text{iff } I(\psi :: A) \; = \; 0 \\ 0 & \text{otherwise} \end{cases}$$

$$I(not\,not(\psi :: A)) \quad \stackrel{\text{def}}{=} \quad \begin{cases} 1 & \text{iff } I(\psi :: A) \; \neq \; 0 \\ 0 & \text{otherwise} \end{cases}$$

Furthermore, for any literal or term $L$, we write $I \models L$ iff $I(L) \neq 0$.

**Definition 7** (Causal model)**.** *Given a rule $r$ of the form* (10)*, we say that an interpretation $I$ satisfies $r$, in symbols $I \models r$, if and only if the following condition holds:*

$$\big( I(B_1) * \ldots * I(B_m) \big) \cdot r_i \;\; \leq \;\; I(A) \tag{12}$$

*An interpretation $I$ is a* causal model *of $P$, in symbols $I \models P$, iff $I$ satisfies all rules in $P$.*  □

Let $P_5$ be the program containing rules (7) and (8) plus the labelled fact $(suzy : suzy \leftarrow)$ and $P_6$ be the program containing rules (7) and (9) plus the labelled facts $(suzy : suzy \leftarrow)$ and $(billy : billy \leftarrow)$. Then, it can be checked that these programs respectively have unique $\leq$-minimal models $I_5$ and $I_6$ which satisfy

$$I_5(accident) \;\; = \;\; suzy{\cdot}r_2{\cdot}r_1 \qquad\qquad I_6(accident) \;\; = \;\; (suzy * billy){\cdot}r_2{\cdot}r_1 + oil$$

Let now $P_7$ and $P_8$ be the labelled programs respectively obtained by adding the following rule

$$r_3 : \quad fine(suzy) \leftarrow suzy \;\texttt{necessary for}\; accident \tag{13}$$

(resulting of labelling rule (5) with $r_3$) to programs $P_5$ and $P_6$. Then it can be checked that these programs also have unique $\leq$-minimal models $I_7$ and $I_8$ which respectively agree with $I_5$ and $I_6$ in all atoms but in $fine(suzy)$ and, as we have seen above,

$$I_7(\psi^{\mathrm{nec}}_{Suzy} :: accident) = I_7(accident) = suzy{\cdot}r_2{\cdot}r_1 \qquad I_8(\psi^{\mathrm{nec}}_{Suzy} :: accident) = 0$$

Furthermore, by definition, it holds that $I_j(fine(suzy)) = I_j(\psi^{\mathrm{nec}}_{Suzy} :: accident){\cdot}r_3$ for $j \in \{7, 8\}$ which implies that

$$I_7(fine(suzy))) \;\; = \;\; suzy{\cdot}r_2{\cdot}r_3$$
$$I_8(fine(suzy))) \;\; = \;\; 0{\cdot}r_3 = 0$$

That is, Suzy would receive a fine for causing the accident, $I_7 \models fine(suzy)$, w.r.t $P_7$, but not w.r.t. program $P_8$ because $I_8 \not\models fine(suzy)$.

It is worth to note that positive programs may contain non-monotonic causal literals that, somehow, play the role of negation and, hence, they may have several $\leq$-minimal causal models. Consider, for instance, the following positive program $P_9$

$$r_1 : \quad p \qquad\qquad\qquad r_2 : \quad q \leftarrow \mathcal{A}_1 \;\texttt{necessary for}\; p$$

where $\mathcal{A}_1 \stackrel{\text{def}}{=} \{r_1\}$. Program $P_9$ has two $\leq$-minimal causal models. The first one which satisfies $I_9(p) = r_1$ and $I_9(q) = r_1 \cdot r_2$; and a second unintended one which satisfies $I'_9(p) = r_1 + r_2$ and $I'_9(q) = 0$. In the following section, we introduce the notion of *monotonic programs* which have a least model and a well-behaved direct consequences operator (when they are positive). In Section 4, we will see that, in fact, only $I_9$ is a causal stable model of program $P_9$.

## 3 Positive monotonic Programs

A causal query $\psi$ is said to be *monotonic* iff $\psi(G, u) \leq \psi(G', w)$ for any values $\{G, G'\} \subseteq \mathbf{C}_{Lb}$ and $\{u, w\} \subseteq \mathbf{V}_{Lb}$ such that $G \leq G'$. A causal literal $(\psi :: A)$ is *monotonic* if $\psi$ is monotonic. A program $P$ is *monotonic* iff all causal literals occurring in $P$ are monotonic. We show next that every monotonic program can be reduced to the syntax and semantics of (Fandinno 2015b). For space reasons, we omit here the details of (Fandinno 2015b), which can be found in the extended verison (Fandinno 2016).

**Definition 8.** *Given a query $\psi$ (resp. m-query $\phi$), its* corresponding m-query (resp. query) *is given by $\phi_\psi(G) \stackrel{\text{def}}{=} \psi(G, 1)$ (resp. $\psi_\phi(G, t) \stackrel{\text{def}}{=} \phi(G)$). Similarly, for any program $P$ (resp. m-program $Q$) its* corresponding m-program $Q$ (resp. program $P$) *is obtained by replacing every query $\psi$ in $P$ (resp. m-query $\phi$ in $Q$) by its corresponding m-query $\phi_\psi$ (resp.query $\psi_\phi$).* □

**Theorem 1.** *If $P$ is the corresponding program of some positive m-program $Q$ with the syntax of Definition 5 or $Q$ is the corresponding m-program of some positive monotonic program $P$, then an interpretation $I$ is a model of $P$ iff $I$ is a model of $Q$.* □

An immediate consequence of Theorem 1, plus Theorem 3.8 in (Fandinno 2015b), is that positive monotonic programs have a least model that can be computed by iteration of the following extension of the direct consequences operator of van Emden and Kowalski (1976).

**Definition 9** (Direct consequences). *Given a causal program $P$, the operator of* direct consequences *is a function $T_P$ from interpretations to interpretations such that*

$$T_P(I)(A) \stackrel{\text{def}}{=} \sum \left\{ \left( I(B_1) * \ldots * I(B_m) \right) \cdot r_1 \ \middle| \ (r_i : A \leftarrow B_1, \ldots, B_m) \in P \right\}$$

*for any interpretation $I$ and any atom $A \in At$. The iterative procedure is defined as usual*

$$
\begin{aligned}
T_P^{\uparrow\alpha}(\mathbf{0}) &\stackrel{\text{def}}{=} T_P(T_P^{\uparrow\alpha-1}(\mathbf{0})) && \textit{if $\alpha$ is a successor ordinal} \\
T_P^{\uparrow\alpha}(\mathbf{0}) &\stackrel{\text{def}}{=} \sum_{\beta < \alpha} T_P^{\uparrow\beta}(\mathbf{0}) && \textit{if $\alpha$ is a limit ordinal}
\end{aligned}
$$

*As usual $0$ and $\omega$ respectively denote the first limit ordinal and the first limit ordinal that is greater than all integers. Thus, $T_P^{\uparrow 0}(\mathbf{0}) = \mathbf{0}$.* □

**Corollary 1.** *Any (possibly infinite) positive monotonic program $P$ has a least causal model $I$ which (i) coincides with the least fixpoint $\mathrm{lfp}(T_P)$ of the direct consequences operator $T_P$ and (ii) can be iteratively computed from the bottom interpretation $I = \mathrm{lfp}(T_P) = T_P^{\uparrow\omega}(\mathbf{0})$.* □

Corollary 1 guarantees that the least fixpoint of $T_P$ is well-behaved and corresponds to the least model of the program $P$. In fact, we can check now that the least model $I_6$ of program $P_6$ satisfies $I_6(accident) = (suzy * billy) \cdot r_2 \cdot r_1 + oil \cdot r_1$. First note, that program $P_6$ contains facts $suzy$, $billy$ and $oil$ whose label is the same as the name atom and, thus, $T_{P_6}^{\uparrow 1}(\mathbf{0})(A) = A$ for each atom $A \in \{suzy, \ billy, \ oil\}$. Then, since $T_{P_6}^{\uparrow 1}(\mathbf{0})(suzy) = suzy$, $T_{P_6}^{\uparrow 1}(\mathbf{0})(billy) = billy$ and

rule (8) and fact *oil* belong to program $P_6$, it follows that $T_{P_6}^{\uparrow 2}(\mathbf{0})(oil) = (suzy * billy) \cdot r_2 + oil$. Similarly, we can check that

$$T_{P_6}^{\uparrow 3}(\mathbf{0})(accident) \;\;=\;\; (\,(suzy * billy) \cdot r_2 + oil\,) \cdot r_1 \;\;=\;\; (suzy * billy){\cdot}r_2{\cdot}r_1 + oil{\cdot}r_1$$

and, thus, $I_6 = T_{P_6}^{\uparrow 3}(\mathbf{0})$ is the least fixpoint of $T_{P_6}$. Checking that $T_{P_5}^{\uparrow 3}(\mathbf{0}) = I_5$, that $T_{P_7}^{\uparrow 4}(\mathbf{0}) = I_7$ and that $T_{P_8}^{\uparrow 4}(\mathbf{0}) = I_8$ are the least fixpoint and the least models respectively of programs $P_5$, $P_7$ and $P_8$ is analogous.

It is easy to see that every true atom, according to the standard least model semantics, has a non-zero causal value associated in the causal least model of the program, that is, some associated cause. An interpretation $I$ is *two-valued* when it maps each atom into the set $\{0, 1\}$. By $I^{cl}$, we denote the two-valued (or "classic") interpretation corresponding to some interpretation $I$ s.t.

$$I^{cl}(A) \;\; \stackrel{\text{def}}{=} \;\; \begin{cases} 1 & \text{iff } I(A) > 0 \\ 0 & \text{iff } I(A) = 0 \end{cases}$$

**Corollary 2.** *Let $P$ be a regular, positive monotonic program and $Q$ its standard unlabelled version obtained by removing all labels from the rules in $P$. Let $I$ and $J$ be the least models of $P$ and $Q$, respectively. Then, $I^{cl} = J$.* $\qquad\square$

## 4 Non-monotonic causal queries and negation

We introduce now the semantics for programs with non-monotonic causal queries and negation by extending the concept of reduct (Gelfond and Lifschitz 1988) to causal queries.

**Definition 10** (Reduct). *For any term $t$, by $\psi^t$ we denote a query such that*

$$\psi^t(G, u) \;\; \stackrel{\text{def}}{=} \;\; \begin{cases} 1 & \text{iff exists some } G' \leq G \text{ s.t. } G' \in \max t \text{ and } \psi(G', t) = 1 \\ 0 & \text{otherwise} \end{cases}$$

*The reduct of a causal literal $(\psi :: A)$ w.r.t some interpretation $I$ is itself if $\psi$ is monotonic and $(\psi^{I(A)} :: A)$ if $\psi$ is non-monotonic. The reduct of a program $P$ w.r.t. an interpretation $I$, in symbols $P^I$, is the result of (i) removing all rules whose body contains a non satisfied negative or consistent literal, (ii) removing all the negative and consistent literals for the remaining rules and (iii) replacing the remaining causal literals $(\psi :: A)$ by their reducts $(\psi :: A)^I$.* $\qquad\square$

It is easy to see that the reduct $P^I$ of any program $P$ is a positive monotonic program and, therefore, it has a least causal model.

**Definition 11** (Causal stable model). *We say that an interpretation $I$ is a causal stable model of a program $P$ iff $I$ is the least model of the positive program $P^I$.* $\qquad\square$

We can check now that interpretation $I_9$ is, in fact, the unique causal stable model of program $P_9$. Let $Q = P_9^{I_9}$ be the reduct of program $P_9$ w.r.t. $I_9$ consisting in the following rules

$$r_1: \quad p \qquad\qquad r_2: \quad q \leftarrow (\psi :: p)$$

where $\psi(G, t) = 1$ iff there exists some $G' \leq G$ s.t. $G' \in \max I_9(p) = r_1$ and $\psi_{\mathcal{A}_1}^{\text{nec}}(G', I_9(p))$ iff $r_1 \leq G$ and $r_1 \leq \sum \mathcal{A}_1 = r_1$ iff $r_1 \leq G$. First note that $T_Q^{\uparrow \alpha}(\mathbf{0})(p) = r_1 = I_9(p)$ for any ordinal $\alpha \geq 1$ because $r_1$ is the only rule with the atom $p$ in the head. Then, note that

$T_Q^{\uparrow\alpha}(\mathbf{0})(\psi :: p) = T_Q^{\uparrow\alpha}(\mathbf{0})(p)$ because $r_1 \leq G$ for every $G \in \max T_Q^{\uparrow\alpha}(\mathbf{0})(p) = r_1$ (there is only one such $G = r_1$) and, thus,

$$T_Q^{\uparrow\beta}(\mathbf{0})(q) \;=\; T_Q^{\uparrow\alpha}(\mathbf{0})(\psi :: p) \cdot r_2 \;=\; T_Q^{\uparrow\alpha}(\mathbf{0})(p) \cdot r_2 \;=\; r_1 \cdot r_2 \;=\; I_9(q)$$

for any ordinal $\beta \geq 2$. Hence, $I_9$ is a causal stable model of $P_9$. On the other hand, we can check that $I_9'$ is not a causal stable model of $P_9$. Let $Q' = P_9^{I_9'}$ be the reduct of program $P_9$ w.r.t. $I_9'$ consisting in the same rules than program $Q$, but replacing $\psi$ by $\psi'$ where $\psi'(G, t) = 1$ iff there exists some $G' \leq G$ s.t. $G' \in \max I_9'(p) = r_1 + r_2$ and $\psi_{\mathcal{A}_1}^{\mathrm{nec}}(G', I_9'(p))$. As above, $T_{Q'}^{\uparrow\alpha}(\mathbf{0})(p) = r_1 \neq I_9'(p) = r_1 + r_2$ for any ordinal $\alpha \geq 1$ and, therefore, $I_9$ is not a causal stable model of program $P_9$.

It is worth to mention that, as happened with positive programs, we can stablish a correspondence between the causal stable models of regular programs and the standard stable models of their standard version.

**Definition 12** (Two-valued equivalence). *Two programs $P$ and $Q$ are said to be two-valued equivalent iff for every causal stable model $I$ of $P$ there is an unique causal stable model $J$ of $Q$ such that $I^{cl} = J^{cl}$, and vice-versa.* $\quad\square$

**Theorem 2.** *Let $P$ be a regular program and $Q$ be its corresponding standard program obtained by removing all labels in $P$. Then $P$ and $Q$ are two-valued equivalent.* $\quad\square$

Theorem 2 asserts that, labelling a standard program does not change which atoms are true or false in its stable models, in other words, the causal stable semantics presented here is a conservative extension of the standard stable model semantic.

## 5 Contributory cause and its relation with actual causation

Until now we have considered that an agent is a cause of an event when its actions have been necessary to cause that event. This understanding is similar to the definition of the *modified Halpern-Pearl definition of causality* given by Halpern (2015). However, in some scenarios it makes sense to consider a weaker definition in which those agents whose actions have *contributed* to that event are also considered causes, even if their actions have not been necessary (Pearl 2000). Consider, for instance, the following example from (Hopkins and Pearl 2003).

**Example 1.** *For a firing squad consisting of shooters Billy and Suzy, it is John's job to load Suzy's gun. Billy loads and fires his own gun. On a given day, John loads Suzy's gun. When the time comes, Suzy and Billy shoot the prisoner. The agents who caused the prisoner death would be punished with imprisonment.* $\quad\square$

In this example, although the actions of any of the agents are not necessary for the prisoner's death, commonsense tells that all three should be considered responsible of it. If we represent Example 1 by the following program $P_{10}$

$$
\begin{array}{llll}
r_1 : & dead & \leftarrow shoot(suzy), loaded & shoot(suzy) \\
r_2 : & dead & \leftarrow shoot(billy) & shoot(billy) \\
r_3 : & loaded & \leftarrow load(john) & load(john) \\
r_{\mathcal{A}} : & long\_prison(\mathcal{A}) & \leftarrow \mathcal{A} \texttt{ necessary for } dead &
\end{array}
$$

for $\mathcal{A} \in \{suzy,\ billy,\ john\}$, it can be shown that its unique causal stable model $I_{10}$ satisfies

$$I_{10}(dead) \;=\; \big(load(john){\cdot}r_3 * shoot(suzy)\big) \cdot r_1 \;+\; shoot(billy){\cdot}r_2$$

Recall that, we assume that every fact has a label with the same name. According to $I_{10}$, the actions of the three agents appear in the causes of the atom $dead$, but there is no agent whose actions occur in all causes. Then, the causal literal ($\mathcal{A}$ `necessary for` $dead$) is not satisfied for any agent $\mathcal{A}$ and, therefore, for every agent $\mathcal{A} \in \{suzy,\ billy,\ john\}$, it holds that $I_{10}(long\_prison(\mathcal{A})) = 0$. That is, no agent is punished with imprisonment for the prisoner's death. On the other hand, if $P_{11}$ is a program obtained by replacing rules $r_{\mathcal{A}}$ by rules

$$c_{\mathcal{A}} : \; short\_prison(\mathcal{A}, dead) \leftarrow \mathcal{A} \; \texttt{contributed to} \; dead$$

in program $P_{10}$, we may expect that $short\_prison(\mathcal{A})$ holds, in its unique causal stable model $I_{11}$, for any $\mathcal{A} \in \{suzy,\ billy,\ john\}$. We formalise this by defining the following query

$$\psi_{\mathcal{A}}^{\text{cont}}(G, t) \;\overset{\text{def}}{=}\; \begin{cases} 1 & \text{if } G \leq \sum \mathcal{A} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

In the sake of clarity, we will write ($\mathcal{A}$ `contributed to` $dead$) instead of ($\psi_{\mathcal{A}}^{\text{cont}} :: dead$). It can be checked that $\big(load(john){\cdot}r_3 * shoot(suzy)\big) \cdot r_1 \;\leq\; load(john)$ and, therefore,

$$I_{11}(john \; \texttt{contributed to} \; dead) \;=\; \big(load(john){\cdot}r_3 * shoot(suzy)\big) \cdot r_1$$

Consequently, $I_{11}(short\_prison(john)) \;=\; \big(load(john){\cdot}r_3 * shoot(suzy)\big) \cdot r_1 \cdot c_{john}$. Similarly, it can be shown that

$$I_{11}(short\_prison(suzy)) \;=\; \big(load(john){\cdot}r_3 * shoot(suzy)\big) \cdot r_1{\cdot}c_{suzy}$$
$$I_{11}(short\_prison(billy)) \;=\; shoot(billy){\cdot}r_2 \cdot c_{billy}$$

It is worth to note that contributory causes are non-monotonic when defaults are taken into account. Consider now the following variation of Example 1.

**Example 2.** *Now Suzy also loads her gun as Billy does. However, Suzy's gun was broken and John repaired it.* □

As in Example 1, John's repairing action is necessary in order for Suzy to be able to fire her gun. However, in this case, it seems too severe to consider that John has contributed to the prisoner's death. This consideration has been widely attributed to the fact that we consider that, by default, things are not broken and that causes must be events that deviate from the norm (Maudlin 2004; Hall 2007; Halpern 2008; Hitchcock and Knobe 2009). If we represent this variation by a program $P_{12}$ containing the following rules[1]

| | | | |
|---|---|---|---|
| $r_1$ : | $dead$ | $\leftarrow shoot(suzy), un\_broken$ | $shoot(suzy)$ |
| $r_2$ : | $dead$ | $\leftarrow shoot(billy)$ | $shoot(billy)$ |
| $r_3$ : | $un\_broken$ | $\leftarrow repair(john)$ | $repair(john)$ |
| $c_{\mathcal{A}}$ : | $short\_prison(\mathcal{A})$ | $\leftarrow \mathcal{A} \; \texttt{contributed to} \; dead$ | |

---

[1] We have chosen this representation in order to illustrate the non-monotonicity of contributory cause. However, solving the Frame and Qualification Problems (McCarthy and Hayes 1969; McCarthy 1987) would require the introduction of time and the inertia laws, plus the replacement of rule $r_1$ by the pair of rules ($r_1$ : $dead \leftarrow shoot(suzy)$, *not ab*) and ($ab \leftarrow broken$). For a detailed discussion of how causality and the inertia laws can combined we refer to (Fandinno 2015a).

for $\mathcal{A} \in \{suzy,\ billy,\ john\}$, then it is easy to see that

$$I_{12}(dead) \;=\; \big(repair(john){\cdot}r_3 * shoot(suzy)\big) \cdot r_1 \;+\; shoot(billy){\cdot}r_2$$

where $I_{12}$ is the least model of program $P_{12}$ and, thus, $responsible(john, dead)$ will be a conclusion of it. Just note that program $P_{12}$ is the result of replacing atoms $loaded$ and $load(john)$ in program $P_{11}$ by $un\_broken$ and $repair(john)$, respectively. Note also that nothing in program $P_{12}$ reflects the fact that by default guns are $un\_broken$. We state that guns are $un\_broken$ by default adding the following rule

$$1: \quad un\_broken \leftarrow not\,broken \tag{15}$$

If $P_{13}$ is the result of adding rule (15) to program $P_{12}$ and $I_{13}$ is the least model of $P_{13}$, then

$$I_{13}(un\_broken) \;=\; I_{12}(un\_broken) \;+\; 1 \;=\; 1$$

and, consequently,

$$\begin{aligned}
I_{12}(dead) \;&=\; \big(1{\cdot}r_3 * shoot(suzy)\big) \cdot r_1 \;+\; shoot(billy){\cdot}r_2 \\
&=\; \big(r_3 * shoot(suzy)\big) \cdot r_1 \;+\; shoot(billy){\cdot}r_2
\end{aligned}$$

which shows that John is not considered to have contributed to the prisoner's death. Hence, $short\_prison(john)$ is not a conclusion of program $P_{13}$. It is worth to mention that besides the two syntactic differences between causal queries and m-queries already mentioned, there is a, perhaps, less noticeable difference in the evaluation of causal literals. Note that,

$$\big(repair(john){\cdot}r_3 * shoot(suzy)\big) \cdot r_1 \;\leq\; \big(r_3 * shoot(suzy)\big) \cdot r_1$$

and, thus, if we replaced $G \in \max I(A)$ by $G \leq I(A)$ in Definition 6 (as done in Fandinno 2015b), it would follows that atom $short\_prison(john)$ would be an unintended conclusion of program $P_{13}$. It is also worth to mention that, besides (Pearl 2000) approach, the notion of contributory cause is also behind the definitions of actual cause given in (Halpern and Pearl 2005; Hall 2007).

## 6 Properties of causal logic programs

Theorem 2 established a correspondence for regular programs, but they say nothing about programs with causal queries. For instance, positive program with non-monotonic causal literals may have more than one causal stable model. Consider the following positive program $P_{14}$

$$
\begin{array}{ll}
r_1: \quad p & \qquad\qquad r_2: \quad q \leftarrow \mathcal{A}_1 \ \texttt{necessary for}\ p \\
r_3: \quad q & \qquad\qquad r_4: \quad p \leftarrow \mathcal{A}_2 \ \texttt{necessary for}\ q
\end{array}
$$

obtained by adding rules $r_3$ and $r_4$ to program $P_9$ and where $\mathcal{A}_2 \stackrel{\text{def}}{=} \{r_3\}$. Program $P_{14}$ has two causal stable causal models. The first that satisfies $I_{14}(p) = r_1 + r_3{\cdot}r_4$ and $I_{14}(q) = r_3$. The second $I'_{14}(p) = r_1$ and $I'_{14}(q) = r_3 + r_1{\cdot}r_2$. Let now $Q = P_{14}^{I_{14}}$ be the reduct of program $P_{14}$ w.r.t. $I_{14}$, which consists in the following rules

$$
\begin{array}{ll}
r_1: \quad p & \qquad\qquad r_2: \quad q \leftarrow (\psi_1 :: p) \\
r_3: \quad q & \qquad\qquad r_4: \quad p \leftarrow (\psi_2 :: q)
\end{array}
$$

where $\psi_1(G, t) = 1$ iff there exists some $G' \leq G$ such that $G' \in \max I_{14}(p) = r_1 + r_3{\cdot}r_4$ and $\psi_{\mathcal{A}_1}^{\text{nec}}(G', I_{14}(p))$ and $\psi_2(G, t) = 1$ iff there exists $G' \leq G$ such that $G' \in \max I_{14}(q) = r_3$

and $\psi_{\mathcal{A}_2}^{\text{nec}}(G', I_{14}(p))$. First, note that $\psi_{\mathcal{A}_1}^{\text{nec}}(G', I_{14}(p))$ iff $I_{14}(p) = r_1 + r_3 \cdot r_4 \leq \sum \mathcal{A}_1 = r_1$ which does not hold. Thus, $\psi_1(G, t) = 0$ for every $G \in \mathbf{C}_{Lb}$ and $t \in \mathbf{V}_{Lb}$. Then, it is clear that the body of rule $r_2$ is never satisfied and, therefore, $T_Q^{\uparrow\alpha}(\mathbf{0})(q) = r_3$ for any ordinal $\alpha \geq 1$. It can also be checked that $\psi_2(r_3, T_Q^{\uparrow\alpha}(\mathbf{0})(q)) = 1$ because there exists $G' = r_3$ such that $G' \in \max I'_{14}(q) = r_3$ and $\psi_{\mathcal{A}_2}^{\text{nec}}(G', I_{14}(q)) = \psi_{\mathcal{A}_2}^{\text{nec}}(r_3, r_3) = 1$ since $r_3 \leq \sum \mathcal{A}_2 = r_3$. Hence, since $r_3 \in \max T_Q^{\uparrow\alpha}(\mathbf{0})(q)$ and $\psi_2(r_3, T_Q^{\uparrow\alpha}(\mathbf{0})(q)) = 1$, it follows that $T_Q^{\uparrow\alpha}(\mathbf{0})(\psi_2 :: q) = r_3$ and $T_Q^{\uparrow\beta}(\mathbf{0})(p) = r_1 + T_Q^{\uparrow\alpha}(\mathbf{0})(q) \cdot r_4 = r_1 + r_3 \cdot r_4 = I_9(p)$ for any ordinal $\beta \geq 2$. Hence, $I_{14}$ is the least model of $P_{14}^{I_{14}}$ and a causal stable model of program $P_{14}$. Showing that $I'_{14}$ is also a causal stable model of $P_{14}$ is symmetric.

In the following we revise some desired general properties for a LP semantics. First, causal stable models should also be supported models. Note that the concept of supported model bellow is analogous to the usual concept used in standard LP, but it is stronger in the sense that, not only requires that true atoms are supported, but also all their causes must be supported by a rule and a cause of its body.

**Definition 13.** *A interpretation $I$ is a* (causally) supported model *of a program $P$ iff $I$ is a model of $P$ and for every true atom $A$ and cause $G \in \mathbf{C}_{Lb}$ such that $G \leq I(A)$ there is a rule $r$ in $P$ of the form of* (10) *such that $G \leq (\ I(B_1) * \ldots * I(B_m)\ ) \cdot r_i$.*  □

**Proposition 1.** *Any causal stable model $I$ of a program $P$ is a also supported model of $P$.*  □

Furthermore, as happen with programs with nested negation under the standard stable models semantics (where stable models may not be minimal models of the program), causal stable models may not be minimal models either. In fact, this may happen even when the nested negation is replaced by a non-monotonic causal literal. Consider, for instance, the following program $P_{15}$

$$r_1: \quad p \qquad\qquad\qquad r_2: \quad p \ \leftarrow \ \textit{not}\,(\mathcal{A}_1 \ \texttt{necessary for}\ p)$$

where $\mathcal{A}_1 \overset{\text{def}}{=} \{r_1\}$. Program $P_{15}$ has two causal models. One which satisfies $I_{15}(p) = r_1$. The other which satisfies $I'_{15}(p) = r_1 + r_2$. We define now the notion of *normal program* whose causal stable models are also $\leq$-minimal models. A program $P$ is *normal* iff no body rule in $P$ contains a consistent literal (double negated literal) nor a negated non-monotonic causal literal. In other words, a program is normal iff it does not contains nested negation nor non-monotonic causal literals in the scope of negation.

**Proposition 2.** *Any causal stable model $I$ of normal program $P$ is also a $\leq$-minimal model.*  □

***Splitting programs.*** The intuitive meaning of the causal rule (13) in programs $P_7$ and $P_8$ is to cause the atom $fine(suzy)$ whenever the causal query expressed by its body is true with respect to a programs $P_5$ and $P_6$, respectively. This intuitive understanding can be formalised as a splitting theorem in (Lifschitz and Turner 1994).

**Theorem 3** (Splitting). *Let $\langle P_b, P_t \rangle$ a partition of a program $P$ such that no atom occurring in the head of a rule in $P_t$ occurs in $P_b$. An interpretation $I$ is a causal stable model of $P$ iff there is some causal stable model $J$ of $P_b$ such that $I$ is a causal stable model of $(J \cup P_t)$.*  □

In our running example, the bottom part are $P_{7,b} = P_5$ and $P_{8,b} = P_6$ while their top part $P_{7,t} = P_{8,t}$ is the program containing the rule (13). This result can be generalised to infinite splitting sequences as follows.

**Definition 14.** *A* splitting sequence *of a program $P$ is a family $(P_\alpha)_{\alpha<\mu}$ of pairwise disjoint sets such that $P = \bigcup_{\alpha<\mu} P_\alpha$ and no atom occurring in the head of a rule in some $P_\alpha$ occurs in the body of a rule in $\bigcup_{\beta<\alpha} P_\beta$. A* solution *of a splitting $(P_\alpha)_{\alpha<\mu}$ is a family $(I_\alpha)_{\alpha<\mu}$ such that*

*(1.) $I_0$ is a stable model of $P_0$,*

*(2.) $I_\alpha$ is a stable model of $(J_\alpha \cup P_\alpha)$ for any ordinal $0 < \alpha < \mu$ where $J_\alpha = \sum_{\beta<\alpha} I_\beta$.*

*A splitting sequence is said to be* strict *in $\alpha$ if, in addition, no atom occurring in the head of a rule in $P_\alpha$ occurs (in the head of a rule) in $\bigcup_{\beta<\alpha} P_\beta$ and it is said to be* strict *if it is strict in $\alpha$ for every $\alpha < \mu$.* $\qquad\square$

**Theorem 4** (Splitting sequences)**.** *Let $(P_\alpha)_{\alpha<\mu}$ a splitting sequence of some program $P$. An interpretation $I$ is a causal stable model of $P$ iff there is some solution $(I_\alpha)_{\alpha<\mu}$ of $(P_\alpha)_{\alpha<\mu}$ such that $I = \sum_{\alpha<\mu} I_\alpha$. Furthermore, if such solution is strict in $\alpha$, then $I_\alpha = I_{|S_\alpha}$ where $S_\alpha$ is the set of all atoms not occurring in the head of any rule in $\bigcup_{\alpha<\beta<\mu} P_\beta$ and $I_{|S_\alpha}$ denotes the restriction if $I$ to $S_\alpha$.* $\qquad\square$

A program $P$ is said to be *stratified* if there is a some ordinal $\mu$ and mapping $\lambda$ from the set of atoms $At$ into the set of ordinals $\{\alpha < \mu\}$ such that, for every rule of the form (10) and atom $B$ occurring in its body, it satisfies $\lambda(A) \geq \lambda(B)$ if $B$ does not occur in the scope of negation nor in a non-monotonic causal literal, and $\lambda(A) > \lambda(B)$ if $B$ does occur under the scope of negation or in a non-monotonic causal literal.

**Proposition 3.** *Every stratified causal program $P$ has a unique causal stable model.* $\qquad\square$

## 7  Conclusions, related work and open issues

The main contribution of this work is the introduction of a semantics for non-monotonic *causal literals* that allow deriving new conclusions by inspecting the causal justifications of atoms in an *elaboration tolerant* manner. In particular, we have used causal literals to define *necessary* and *contributory causal relations* which are intuitively related to some of the most established definitions of actual causation in the literature (Pearl 2000; Halpern and Pearl 2005; Hall 2007; Halpern 2015). Besides, by some running examples we have shown that causal literals allow, not only to derive whether some event is the cause or not of another event, but also to derive new conclusions from this fact. From a technical point of view, we have shown that our semantics is a conservative extension of the stable model semantics and that satisfy the usual desired properties for an LP semantics (casual stable models are supported models, minimal models in case of normal programs and can be iteratively computed by split table programs). It worth to mention that, besides the syntactic approaches to justifications in LP, the more related approach to our semantics is (Damásio et al. 2013), for which a formal comparative can be found in (Cabalar and Fandinno 2016a) and that (Pontelli et al. 2009) allows a Prolog system to reason about justifications of an ASP program, but justifications cannot be inspected inside the ASP program.

Regarding complexity, it has been shown in (Cabalar et al. 2014b) that there may be an exponential number of causes for a given atom w.r.t. each causal stable model. Despite that, the existence of stable model for programs containing only monotonic queries evaluable in polynomial time is NP-complete (Fandinno 2015b). For programs containing only necessary causal literals we can prove NP-complete (NP-hard holds even for programs containing a single negated regular literal or positive programs containing a single constraint, see (Fandinno 2016)). The

complexity for programs including other non-monotonic causal literals (like contributory) is still an open question. A preliminary prototype extending the syntax of logic programs with causal literals capturing sufficient, necessary and contributory causal relation can be tested on-line at `http://kr.irlab.org/cgraphs-solver/nmsolver`.

In a companion paper (Cabalar and Fandinno 2016b), the causal semantics used here has been extended to disjunctive logic programs, which will be useful for representing non-deterministic causal laws. Interesting topics include a extend the complexity assessment for contributory causes, studying an extension to arbitrary theories as with Equilibrium Logic (Pearce 2006) for the non-causal case; and formalise the relation between our notions of necessary and contributory cause with the above definitions of the actual causation and, in particular, with (Vennekens 2011) who has studied it in the context of CP-logic. A promising approach seems to translate structural equations into logic programs in a similar way as it has been done to translate them into the causal theories (Giunchiglia et al. 2004; Bochman and Lifschitz 2015).

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