Metric Temporal Answer Set Programming over Timed Traces (extended abstract)

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1 Introduction

The field of *Stream Reasoning* studies the ability to perform automated reasoning upon rapidly changing information. Stream reasoning is normally applied on challenging domains with a large amount of data, possibly coming from heterogeneous origins, but with the common feature that each individual piece of information is associated to a timestamp, reflecting the dynamic evolution of the environment. On the other hand, practical tools for temporal reasoning in the Artificial Intelligence area of Knowledge Representation (KR) are mostly oriented to transition systems where computation takes place in discrete instants or steps, while the real timestamp in which events occur is rarely relevant. In this paper we consider a metric extension of a KR temporal formalism, namely *Temporal Equilibrium Logic* (TEL [1, 6]) to deal with timed traces. This extension allows for keeping the usual temporal modal operators from *Linear Temporal Logic* (LTL [13]) to talk about the discrete steps in which events take place, but further specifies intervals for those operators, used to locate the timestamps associated to the discrete steps. The semantics is based on the idea of *timed trace*, that is, a sequence of states $S_0, S_1, \ldots$ where at each step i we not only have a state S_i specifying the truth of propositional variables, but also an associated metric *timestamp* $\tau(i)$, in (some fraction of) seconds, that reflects the moment in which this state occurred in real life using a universal time reference. In this first approach, we use positive integers to represent timestamps since, after all, this is what happens in practice with streams of digital data.

As said before, our approach is based on TEL, a temporal logic defined for the well-known syntax of LTL but with a non-monotonic semantics based on logic programming and the *answer sets* semantics [9]. In particular, TEL combines LTL with the intermediate logic of *Here-and-There* (HT [10]) and its non-monotonic extension, called *Equilibrium Logic* [12] which constitutes a full-blown logical characterization of Answer Set Programming (ASP [11]). Over the last years, this approach has led to other temporal extensions of ASP, like the introduction of linear-dynamic operators (DEL [3, 4]), and also gave rise to the temporal ASP system `telingo` [5] extending the full-featured ASP system `clingo` [8]. Recently, a metric extension of TEL was also considered in [7], but in that case, the metric intervals used in modal operators referred to the discrete steps in the trace, and not to an additional, external timestamp.

We proceed next to describe the basic syntax and semantics of our metric extension for timed traces.

2 Approach

Given $a \in \mathbb{N}$ and $b \in \mathbb{N} \cup \{\omega\}$, we let $[a..b]$ stand for the set $\{i \in \mathbb{N} \mid a \leq i \leq b\}$ and $[a..b)$ for $\{i \in \mathbb{N} \mid a \leq i < b\}$.

Given a set $\mathcal{A}$ of propositional variables (called *alphabet*), *metric formulas* φ are defined by the grammar:

$$\varphi ::= a \mid \bot \mid \varphi_1 \otimes \varphi_2 \mid \bullet_{\mathcal{I}}\varphi \mid \varphi_1 \mathbf{S}_{\mathcal{I}}\, \varphi_2 \mid \varphi_1 \mathbf{T}_{\mathcal{I}}\, \varphi_2 \mid \circ_{\mathcal{I}}\varphi \mid \varphi_1 \mathbb{U}_{\mathcal{I}}\, \varphi_2 \mid \varphi_1 \mathbb{R}_{\mathcal{I}}\, \varphi_2$$

where $a \in \mathcal{A}$ is an atom and $\otimes$ is any binary Boolean connective $\otimes \in \{\to, \wedge, \vee\}$, and where $\mathcal{I}$ is an interval of the form $[a..b]$ or $[a..\omega)$ with $a, b \in \mathbb{N}$. For intervals of the form $[0..b]$ or $[a..\omega)$ we use the shorthand notations $\leq b$ or $\geq a$. Intervals of length 0 can be denoted by the single point they contain, i.e. a for the interval $[a..a]$. If $\mathcal{I} = [0..\omega)$, we omit the index.

The last six cases correspond to the temporal connectives whose names are listed below:

Past	$\bullet_{\mathcal{I}}$ for *previous*	*Future*	$\circ_{\mathcal{I}}$ for *next*
	$\mathbf{S}_{\mathcal{I}}$ for *since*		$\mathbb{U}_{\mathcal{I}}$ for *until*
	$\mathbf{T}_{\mathcal{I}}$ for *trigger*		$\mathbb{R}_{\mathcal{I}}$ for *release*

We also define several common derived operators like the Boolean connectives $\top \overset{def}{=} \neg\bot$, $\neg\varphi \overset{def}{=} \varphi \to \bot$, $\varphi \leftrightarrow \psi \overset{def}{=} (\varphi \to \psi) \wedge (\psi \to \varphi)$, and the following temporal operators:

$\blacksquare_{\mathcal{I}}\varphi \overset{def}{=} \bot\, \mathbf{T}_{\mathcal{I}}\, \varphi$	*always before*		$\square_{\mathcal{I}}\varphi \overset{def}{=} \bot\, \mathbb{R}_{\mathcal{I}}\, \varphi$	*always afterward*	
$\blacklozenge_{\mathcal{I}}\varphi \overset{def}{=} \top\, \mathbf{S}_{\mathcal{I}}\, \varphi$	*eventually before*		$\lozenge_{\mathcal{I}}\varphi \overset{def}{=} \top\, \mathbb{U}_{\mathcal{I}}\, \varphi$	*eventually afterward*	
$\mathbf{I} \overset{def}{=} \neg\bullet\top$	*initial*		$\mathbb{F} \overset{def}{=} \neg\circ\top$	*final*	
$\widehat{\bullet}_{\mathcal{I}}\varphi \overset{def}{=} \bullet_{\mathcal{I}}\varphi \vee \neg\bullet_{\mathcal{I}}\top$	*weak previous*		$\widehat{\circ}_{\mathcal{I}}\varphi \overset{def}{=} \circ_{\mathcal{I}}\varphi \vee \neg\circ_{\mathcal{I}}\top$	*weak next*	

Note that *initially* and *finally* do only depend on the state of the trace, not on the actual time that this state is mapped to. Therefore, we do not annotate them with an interval.

In previous work, we defined the weak one-step-operators using finally and initially, i.e. we defined derived operators using other derived operators. We can replace that definition by replacing initially and finally with their definitions, i.e. $\widehat{\circ}_{\mathcal{I}}\varphi = \circ_{\mathcal{I}}\varphi \vee \neg\circ_{\mathcal{I}}\top$, so that all derived operators are defined in terms of basic operators. Then, the weak one-step-operators cover the non-existence of states not only at the end of the trace; this is important to ensure the usual dualities in the metric case.

The definition of *Metric Equilibrium Logic* (MEL for short) is done in two steps. We start with the definition of a monotonic logic called *Metric logic of*

Here-and-There (MHT), a temporal extension of the intermediate logic of Here-and-There [10]. We then select some models from THT that are said to be in equilibrium, obtaining in this way a non-monotonic entailment relation.

A *Here-and-There trace* (for short HT-*trace*) of length λ over alphabet $\mathcal{A}$ is a sequence of pairs $(\langle H_i, T_i\rangle)_{i\in[0..\lambda)}$ with $H_i \subseteq T_i$ for any $i \in [0..\lambda)$. For convenience, we usually represent an HT-trace as the pair $\langle \mathbf{H}, \mathbf{T}\rangle$ of traces $\mathbf{H} = (H_i)_{i\in[0..\lambda)}$ and $\mathbf{T} = (T_i)_{i\in[0..\lambda)}$.

Definition 1. *A* timed trace $\mathbf{M} = (\langle \mathbf{H}, \mathbf{T}\rangle, \tau)$ *over* $(\mathbb{N}, <)$ *is a pair consisting of*

- *an* HT-*trace* $\langle \mathbf{H}, \mathbf{T}\rangle = (\langle H_i, T_i\rangle)_{i\in[0..\lambda)}$ *and*
- *a function* $\tau : [0..\lambda) \to \mathbb{N}$ *such that* $\tau(i) \leq \tau(i{+}1)$*.*

A timed trace of length $\lambda > 1$ *is called* strict *if* $\tau(i) < \tau(i{+}1)$ *for all* $i \in [0..\lambda{-}1)$ *and* non-strict *otherwise. We assume w.l.o.g. that* $\tau(0) = 0$*.*

Given any timed HT-trace $\mathbf{M}$, satisfaction of formulas is defined as follows.

Definition 2 (MHT-satisfaction). *A timed* HT-*trace* $\mathbf{M} = (\langle \mathbf{H}, \mathbf{T}\rangle, \tau)$ *of length* λ *over alphabet* $\mathcal{A}$ satisfies *a temporal formula* φ *at step* $k \in [0..\lambda)$*, written* $\mathbf{M}, k \models \varphi$*, if the following conditions hold:*

1. $\mathbf{M}, k \not\models \bot$
2. $\mathbf{M}, k \models a$ *if* $a \in H_k$ *for any atom* $a \in \mathcal{A}$
3. $\mathbf{M}, k \models \varphi \wedge \psi$ *iff* $\mathbf{M}, k \models \varphi$ *and* $\mathbf{M}, k \models \psi$
4. $\mathbf{M}, k \models \varphi \vee \psi$ *iff* $\mathbf{M}, k \models \varphi$ *or* $\mathbf{M}, k \models \psi$
5. $\mathbf{M}, k \models \varphi \to \psi$ *iff* $\langle \mathbf{H}', \mathbf{T}\rangle, k \not\models \varphi$ *or* $\langle \mathbf{H}', \mathbf{T}\rangle, k \models \psi$*, for all* $\mathbf{H}' \in \{\mathbf{H}, \mathbf{T}\}$
6. $\mathbf{M}, k \models \bullet_{\mathcal{I}}\varphi$ *iff* $k > 0$ *and* $\mathbf{M}, k{-}1 \models \varphi$ *and* $\tau(k) - \tau(k{-}1) \in \mathcal{I}$
7. $\mathbf{M}, k \models \varphi \,\mathbf{S}_{\mathcal{I}}\, \psi$ *iff for some* $j \in [0..k]$ *with* $\tau(k) - \tau(j) \in \mathcal{I}$*, we have* $\mathbf{M}, j \models \psi$ *and* $\mathbf{M}, i \models \varphi$ *for all* $i \in (j..k]$
8. $\mathbf{M}, k \models \varphi \,\mathbf{T}_{\mathcal{I}}\, \psi$ *iff for all* $j \in [0..k]$ *with* $\tau(k) - \tau(j) \in \mathcal{I}$*, we have* $\mathbf{M}, j \models \psi$ *or* $\mathbf{M}, i \models \varphi$ *for some* $i \in (j..k]$
9. $\mathbf{M}, k \models \circ_{\mathcal{I}}\varphi$ *iff* $k + 1 < \lambda$ *and* $\mathbf{M}, k{+}1 \models \varphi$ *and* $\tau(k{+}1) - \tau(k) \in \mathcal{I}$
10. $\mathbf{M}, k \models \varphi \,\mathbb{U}_{\mathcal{I}}\, \psi$ *iff for some* $j \in [k..\lambda)$ *with* $\tau(j) - \tau(k) \in \mathcal{I}$*, we have* $\mathbf{M}, j \models \psi$ *and* $\mathbf{M}, i \models \varphi$ *for all* $i \in [k..j)$
11. $\mathbf{M}, k \models \varphi \,\mathbb{R}_{\mathcal{I}}\, \psi$ *iff for all* $j \in [k..\lambda)$ *with* $\tau(j) - \tau(k) \in \mathcal{I}$*, we have* $\mathbf{M}, j \models \psi$ *or* $\mathbf{M}, i \models \varphi$ *for some* $i \in [k..j)$
12. $\mathbf{M}, k \models \varphi \,\mathbb{W}_{\mathcal{I}}\, \psi$ *iff for all* $j \in [k..\lambda)$ *with* $\tau(j) - \tau(k) \in \mathcal{I}$*, we have* $\langle \mathbf{H}', \mathbf{T}\rangle, j \models \varphi$ *or* $\langle \mathbf{H}', \mathbf{T}\rangle, i \not\models \psi$ *for some* $i \in [k..j)$ *and for all* $\mathbf{H}' \in \{\mathbf{H}, \mathbf{T}\}$

⊠

Satisfaction of derived operators can be easily deduced:

Proposition 1. *Let* $\mathbf{M} = (\langle \mathbf{H}, \mathbf{T}\rangle, \tau)$ *be a timed* HT-*trace of length* λ *over* $\mathcal{A}$*. Given the respective definitions of derived operators, we get the following satisfaction conditions:*

13. $\mathbf{M}, k \models \mathbf{I}$ *iff* $k = 0$

14. $\mathbf{M}, k \models \widehat{\bullet}_{\mathcal{I}} \varphi$ *iff* $k = 0$ *or* $\mathbf{M}, k{-}1 \models \varphi$ *or* $\tau(k) - \tau(k{-}1) \notin \mathcal{I}$
15. $\mathbf{M}, k \models \blacklozenge_{\mathcal{I}} \varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for some* $i \in [0..k]$ *with* $\tau(k) - \tau(i) \in \mathcal{I}$
16. $\mathbf{M}, k \models \blacksquare_{\mathcal{I}} \varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for all* $i \in [0..k]$ *with* $\tau(k) - \tau(i) \in \mathcal{I}$
17. $\mathbf{M}, k \models \mathbb{F}$ *iff* $k = \lambda{-}1$
18. $\mathbf{M}, k \models \widehat{\circ}_{\mathcal{I}} \varphi$ *iff* $k + 1 < \lambda$ *or* $\mathbf{M}, k{+}1 \models \varphi$ *or* $\tau(k{+}1) - \tau(k) \notin \mathcal{I}$
19. $\mathbf{M}, k \models \Diamond_{\mathcal{I}} \varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for some* $i \in [k..\lambda)$ *with* $\tau(i) - \tau(k) \in \mathcal{I}$
20. $\mathbf{M}, k \models \Box_{\mathcal{I}} \varphi$ *iff* $\mathbf{M}, i \models \varphi$ *for all* $i \in [k..\lambda)$ *with* $\tau(i) - \tau(k) \in \mathcal{I}$

$\boxtimes$

A formula φ is a *tautology* (or is valid), written $\models \varphi$, iff $\mathbf{M}, k \models \varphi$ for any timed HT-trace $\mathbf{M}$ and any $k \in [0..\lambda)$. MHT is the logic induced by the set of all tautologies. For two formulas φ, ψ we write $\varphi \equiv \psi$, iff $\models \varphi \leftrightarrow \psi$, that is, $\mathbf{M}, k \models \varphi \leftrightarrow \psi$ for any timed HT-trace $\mathbf{M}$ of length λ and any $k \in [0..\lambda)$.

2.1 Properties

Proposition 2 (Persistence). *Let* $\mathbf{M} = (\langle \mathbf{H}, \mathbf{T} \rangle, \tau)$ *be a timed* HT*-trace of length* λ *over* $\mathcal{A}$ *and let* φ *be a metric formula. Then, for any* $k \in [0..\lambda)$*, if* $(\langle \mathbf{H}, \mathbf{T} \rangle, \tau), k \models \varphi$ *then* $(\langle \mathbf{T}, \mathbf{T} \rangle, \tau), k \models \varphi$*.*

An interesting subset of MHT is the one formed by total timed traces $(\langle \mathbf{T}, \mathbf{T} \rangle, \tau)$. In the non-metric version of temporal HT, the restriction to total models corresponds to Linear Temporal Logic (LTL). In our case, the restriction to total traces defines a metric version of LTL that we call Metric Temporal Logic (MTL for short). It can be proved that MTL are those models of MHT satisfying the excluded middle axiom schema: $\Box(p \vee \neg p)$ for any atom $p \in \mathcal{A}$.

Proposition 3. *Let* φ *and* ψ *be metric formulas without implications (and so, without negations either). Then,* $\varphi \equiv \psi$ *in* MTL *iff* $\varphi \equiv \psi$ *in MHT.*

We define all the pairs of dual connectives as follows: $\wedge_{\mathcal{I}}/\vee_{\mathcal{I}}$, $\top_{\mathcal{I}}/\bot_{\mathcal{I}}$, $\mathbb{U}_{\mathcal{I}}/\mathbb{R}_{\mathcal{I}}$, $\circ_{\mathcal{I}}/\widehat{\circ}_{\mathcal{I}}$, $\Box_{\mathcal{I}}/\Diamond_{\mathcal{I}}$, $\mathbf{S}_{\mathcal{I}}/\mathbf{T}_{\mathcal{I}}$, $\bullet_{\mathcal{I}}/\widehat{\bullet}_{\mathcal{I}}$, $\blacksquare_{\mathcal{I}}/\blacklozenge_{\mathcal{I}}$, For any formula φ without implications we define $\delta(\varphi)$ as the result of replacing each connective by its dual operator. Then, we get the following corollary of Proposition 3.

Corollary 1 (Boolean Duality). *Let* φ *and* ψ *be formulas without implication. Then, MHT satisfies:* $\varphi \equiv \psi$ *iff* $\delta(\varphi) \equiv \delta(\psi)$*.* $\boxtimes$

We indicate restriction to finite traces by usage of a subscript f, i.e. MHT_f if MHT is restricted to finite traces. Let $\mathbb{U}_{\mathcal{I}}/\mathbf{S}_{\mathcal{I}}$, $\mathbb{R}_{\mathcal{I}}/\mathbf{T}_{\mathcal{I}}$, $\circ_{\mathcal{I}}/\bullet_{\mathcal{I}}$, $\widehat{\circ}_{\mathcal{I}}/\widehat{\bullet}_{\mathcal{I}}$, $\Box_{\mathcal{I}}/\blacksquare_{\mathcal{I}}$, and $\Diamond_{\mathcal{I}}/\blacklozenge_{\mathcal{I}}$ denote all pairs of swapped-time connectives and let $\sigma(\varphi)$ denote the replacement in φ of each connective by its swapped-time version. Then, we have the following result.

Lemma 1. *There exists a mapping* ϱ *on finite timed* HT*-traces of the same length* $\lambda \geq 0$ *such that for any* $k \in [0..\lambda)$*,* $\mathbf{M}, k \models \varphi$ *iff* $\varrho(\mathbf{M}), \lambda{-}1{-}k \models \sigma(\varphi)$*.*

Theorem 1 (Temporal Duality Theorem). *A metric formula φ is a MHT$_f$-tautology iff $\sigma(\varphi)$ is a MHT$_f$-tautology.*

We write $\mathrm{MHT}(\Gamma, \lambda)$ to stand for the set of MHT models of length λ of a theory Γ, and define $\mathrm{MHT}(\Gamma) \stackrel{def}{=} \bigcup_{\lambda=0}^{\omega} \mathrm{MHT}(\Gamma, \lambda)$, that is, the whole set of models of Γ of any length. Given a set of MHT models, we define the ones in equilibrium as follows.

Definition 3 (Metric Equilibrium/Stable Model). *Let S be some set of timed* HT-*traces. A total timed* HT-*trace $(\langle \mathbf{T}, \mathbf{T}\rangle, \tau) \in S$ is a metric equilibrium model of $\mathfrak{S}$ iff there is no other $H < T$ such that $(\langle \mathbf{H}, \mathbf{T}\rangle, \tau) \in \mathfrak{S}$. The timed trace $(\mathbf{T}, \tau)$ is called a metric stable model of $\mathfrak{S}$.*

We talk about metric equilibrium or metric stable models of a theory Γ when $S = \mathrm{MHT}(\Gamma)$, and we write $\mathrm{MEL}(\Gamma, \lambda)$ and $\mathrm{MEL}(\Gamma)$ to stand for the metric equilibrium models of $\mathrm{MHT}(\Gamma, \lambda)$ and $\mathrm{MHT}(\Gamma)$ respectively.

2.2 Properties under strict semantics

When allowing only strict traces (as defined in Definition 1), we talk about satisfaction under strict semantics. In this case, we have some tautologies that are not valid in the non-strict case. In the following, we only consider strict semantics.

Let φ be an arbitrary metric formula and $n, m \in \mathbb{N}$. Then, $\square_{[n..m]}\bot$ means that there is no state in that interval and $\Diamond_{[n..m]}\top$ means that there is at least one state in that interval. The formula $\square_{[n..m]}\top$ is a tautology, whereas $\Diamond_{[n..m]}\bot$ can never be satisfied.

Proposition 4. *For metric formulas ψ and φ, we have the following tautologies:*

$$\psi \, \mathbf{S}_0 \, \varphi \equiv \psi \, \mathbf{T}_0 \, \varphi \equiv \psi \, \mathbb{U}_0 \, \varphi \equiv \psi \, \mathbb{R}_0 \, \varphi \equiv \varphi \tag{1}$$

Lemma 2. *For metric formulas ψ and φ and for $n > 0$ we have*

$$\psi \, \mathbb{U}_n \, \varphi \equiv \bigvee_{i=1}^{n} \circ_i (\psi \, \mathbb{U}_{n-i} \, \varphi) \tag{2}$$

$$\psi \, \mathbb{R}_n \, \varphi \equiv \bigwedge_{i=1}^{n} \widehat{\circ}_i (\psi \, \mathbb{R}_{n-i} \, \varphi) \tag{3}$$

$$\Diamond_n \varphi \equiv \bigvee_{i=1}^{n} \circ_i \Diamond_{n-i} \varphi \tag{4}$$

$$\square_n \varphi \equiv \bigwedge_{i=1}^{n} \widehat{\circ}_i \square_{n-i} \varphi \tag{5}$$

Lemma 3. *For metric formulas ψ and φ and for $n > 0$, the following are tautologies:*

$$\psi \, \mathbb{U}_{\leq n} \, \varphi \equiv \varphi \vee (\psi \wedge \bigvee_{i=1}^{n} \circ_i (\psi \, \mathbb{U}_{\leq (n-i)} \, \varphi)) \tag{6}$$

$$\psi \, \mathbb{R}_{\leq n} \, \varphi \equiv \varphi \wedge (\psi \vee \bigwedge_{i=1}^{n} \widehat{\circ}_i (\psi \, \mathbb{R}_{\leq (n-i)} \, \varphi)) \tag{7}$$

Lemma 4. *For metric formulas ψ and φ and for $m > n > 0$, the following are tautologies:*

$$\psi \, \mathbb{U}_{[n..m]} \, \varphi \equiv \bigvee_{i=1}^{n} \circ_i(\psi \, \mathbb{U}_{[(n-i)..(m-i)]} \, \varphi) \vee \bigvee_{i=n+1}^{m} \circ_i(\psi \, \mathbb{U}_{\leq(m-i)} \, \varphi) \quad (8)$$

$$\psi \, \mathbb{R}_{[n..m]} \, \varphi \equiv \bigwedge_{i=1}^{n} \widehat{\circ}_i(\psi \, \mathbb{R}_{[(n-i)..(m-i)]} \, \varphi) \wedge \bigwedge_{i=n+1}^{m} \widehat{\circ}_i(\psi \, \mathbb{R}_{\leq(m-i)} \, \varphi) \quad (9)$$

Example 1. For metric formulas ψ and φ, we have

$$\begin{aligned}
\psi \, \mathbb{U}_{[2..3]} \, \varphi &\equiv \bigvee_{i=1}^{2} \circ_i(\psi \, \mathbb{U}_{[(2-i)..(3-i)]} \, \varphi) \vee \bigvee_{i=2+1}^{3} \circ_i(\psi \, \mathbb{U}_{\leq(3-i)} \, \varphi) \\
&\equiv \circ_1(\psi \, \mathbb{U}_{[1..2]} \, \varphi) \vee \circ_2(\psi \, \mathbb{U}_{\leq 1} \, \varphi) \vee \circ_3(\psi \, \mathbb{U}_0 \, \varphi) \\
&\equiv \circ_1(\psi \, \mathbb{U}_{[1..2]} \, \varphi) \vee \circ_2(\varphi \vee (\psi \wedge \circ_1\varphi)) \vee \circ_3\varphi \\
&\equiv \circ_1(\circ_1(\varphi \vee (\psi \wedge \circ_1\varphi)) \vee \circ_2\varphi) \vee \circ_2(\varphi \vee (\psi \wedge \circ_1\varphi)) \vee \circ_3\varphi
\end{aligned}$$

Corollary 2. *For metric formulas ψ and φ, we have*

$$\psi \, \mathbb{U}_{[n..m]} \, \varphi \equiv \psi \, \mathbb{U}_{[n..p]} \, \varphi \vee \psi \, \mathbb{U}_{[p..m]} \, \varphi \text{ for all } p \in [n, m]$$
$$\psi \, \mathbb{R}_{[n..m]} \, \varphi \equiv \psi \, \mathbb{R}_{[n..p]} \, \varphi \wedge \psi \, \mathbb{R}_{[p..m]} \, \varphi \text{ for all } p \in [n, m]$$

3 Conclusion

We have presented a formalism allowing for non-monotonic reasoning over traces over time stamps. Compared to other non-monotonic temporal approaches this allows us to distinguish between the state and the time point at which this state is being observed. This distinction is well-suited for stream reasoning, as we typically receive data for certain time points, knowing that time passes between these different states with our system being unobserved. Further analysis of the non-monotonic aspect of our approach is yet to be done, in particular the representation of inertia is of special interest here.

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