See the proof for Theorem strorder_sublist_antisym in table 1

Table 1: Proof of Theorem strorder_sublist_antisym

Next step in Coq	Proof situation
$Proof.$	——— $\forall\ L\ K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$
$prove\_by\_induction.$	Hidden 2 goal(s)
$+_{1/2}$	——— $\forall\ K : List\ \mathbb{N},\ list\_in\_strorder\ [] \to list\_in\_strorder\ K \to [] \subseteq K \to K \subseteq [] \to [] = \{List\ \mathbb{N}\}\ K$
$prove\_all\_imp\_star.$	$\boxed{K}$: $List\ \mathbb{N}$ $\boxed{H}$: $list\_in\_strorder\ []$ $\boxed{H0}$: $list\_in\_strorder\ K$ $\boxed{H1}$: $[] \subseteq K$ $H2$: $K \subseteq []$ ——— $[] = \{List\ \mathbb{N}\}\ K$
$exception\ H2.$	$H2$: $K = \{List\ \mathbb{N}\}\ []$ ——— $[] = \{List\ \mathbb{N}\}\ K$
$switch\ in\ H2.$	$\boxed{H2}$: $[] = \{List\ \mathbb{N}\}\ K$ ——— $[] = \{List\ \mathbb{N}\}\ K$
$use\ H2.$	$+_{1/2}$ completed
$+_{2/2}$	a : $\mathbb{N}$ L : $List\ \mathbb{N}$ IHL : $\forall\ K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ ——— $\forall\ K : List\ \mathbb{N},\ list\_in\_strorder\ (a :: L) \to list\_in\_strorder\ K \to a :: L \subseteq K \to K \subseteq a :: L \to a :: L = \{List\ \mathbb{N}\}\ K$
$rename\ a\ into\ n.$	n : $\mathbb{N}$ L : $List\ \mathbb{N}$ IHL : $\forall\ K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ ——— $\forall\ K : List\ \mathbb{N},\ list\_in\_strorder\ (n :: L) \to list\_in\_strorder\ K \to n :: L \subseteq K \to K \subseteq n :: L \to n :: L = \{List\ \mathbb{N}\}\ K$
$prove\_by\_structure.$	Hidden 2 goal(s)
$*_{1/2}$	n : $\mathbb{N}$ L : $List\ \mathbb{N}$ IHL : $\forall\ K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ ——— $list\_in\_strorder\ (n :: L) \to list\_in\_strorder\ [] \to n :: L \subseteq [] \to [] \subseteq n :: L \to n :: L = \{List\ \mathbb{N}\}\ []$

Continuing proof of Theorem strorder_sublist_antisym on the next page

Table 1: Proof of Theorem strorder_sublist_antisym continued

Next step in Coq	Proof situation
$prove\_all\_imp\_star.$	$n\ : \mathbb{N}$ $L\ : List\ \mathbb{N}$ $IHL\ : \forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \rightarrow list\_in\_strorder\ K \rightarrow L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{List\ \mathbb{N}\}\ K$ $\boxed{H}\ : list\_in\_strorder\ (n :: L)$ $\boxed{H0}\ : list\_in\_strorder\ []$ $H1\ : n :: L \subseteq []$ $\boxed{H2}\ : [] \subseteq n :: L$ ——— $n :: L = \{List\ \mathbb{N}\}\ []$
$prove\_False.$	$n\ : \mathbb{N}$ $L\ : List\ \mathbb{N}$ $IHL\ : \forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \rightarrow list\_in\_strorder\ K \rightarrow L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{List\ \mathbb{N}\}\ K$ $H1\ : n :: L \subseteq []$ ——— $False$
$exception\ H1.$	$n\ : \mathbb{N}$ $L\ : List\ \mathbb{N}$ $IHL\ : \forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \rightarrow list\_in\_strorder\ K \rightarrow L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{List\ \mathbb{N}\}\ K$ $\boxed{H1}\ : n :: L = \{List\ \mathbb{N}\}\ []$ ——— $False$
$my\_contradiction\ H1.$	$*_{1/2}$ completed
$*_{2/2}$	$n\ : Basic.\mathbb{N}$ $L\ : List\ Basic.\mathbb{N}$ $IHL\ : \forall\ K : List\ Basic.\mathbb{N},\quad list\_in\_strorder\ L \rightarrow list\_in\_strorder\ K \rightarrow L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{List\ Basic.\mathbb{N}\}\ K$ $\mathbb{N}\ : Basic.\mathbb{N}$ $K\ : List\ Basic.\mathbb{N}$ ——— $list\_in\_strorder\ (n :: L) \rightarrow list\_in\_strorder\ (\mathbb{N} :: K) \rightarrow n :: L \subseteq \mathbb{N} :: K \rightarrow \mathbb{N} :: K \subseteq n :: L \rightarrow n :: L = \{List\ Basic.\mathbb{N}\}\ \mathbb{N} :: K$
$rename\ \mathbb{N}\ into\ m.$	$n\ : \mathbb{N}$ $L\ : List\ \mathbb{N}$ $IHL\ : \forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \rightarrow list\_in\_strorder\ K \rightarrow L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{List\ \mathbb{N}\}\ K$ $\boxed{m}\ : \mathbb{N}$ $\boxed{K}\ : List\ \mathbb{N}$ ——— $list\_in\_strorder\ (n :: L) \rightarrow list\_in\_strorder\ (m :: K) \rightarrow n :: L \subseteq m :: K \rightarrow m :: K \subseteq n :: L \rightarrow n :: L = \{List\ \mathbb{N}\}\ m :: K$

Continuing proof of Theorem strorder_sublist_antisym on the next page

Table 1: Proof of Theorem strorder_sublist_antisym continued

Next step in Coq	Proof situation
$\mathit{prove\_all\_imp\_star}$.	$n\ : \mathbb{N}$ $L\ : \mathit{List}\ \mathbb{N}$ $\mathit{IHL}\ : \forall\ K : \mathit{List}\ \mathbb{N},\quad \mathit{list\_in\_strorder}\ L \rightarrow \mathit{list\_in\_strorder}\ K \rightarrow$ $L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{\mathit{List}\ \mathbb{N}\}\ K$ $\boxed{\text{H}}\ : \mathit{list\_in\_strorder}\ (n :: L)$ $\boxed{\text{H0}}\ : \mathit{list\_in\_strorder}\ (m :: K)$ $\boxed{\text{H1}}\ : n :: L \subseteq m :: K$ $\boxed{\text{H2}}\ : m :: K \subseteq n :: L$ ――――――――――――――― $n :: L = \{\mathit{List}\ \mathbb{N}\}\ m :: K$
fact ($\mathit{list\_in\_strorder\_n\_l\_to\_list\_in\_strorder\_l}$ H).	$n\ : \mathbb{N}$ $L\ : \mathit{List}\ \mathbb{N}$ $\mathit{IHL}\ : \forall\ K : \mathit{List}\ \mathbb{N},\quad \mathit{list\_in\_strorder}\ L \rightarrow \mathit{list\_in\_strorder}\ K \rightarrow$ $L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{\mathit{List}\ \mathbb{N}\}\ K$ $\boxed{\text{H3}}\ : \mathit{list\_in\_strorder}\ L$ ――――――――――――――― $n :: L = \{\mathit{List}\ \mathbb{N}\}\ m :: K$
fact ($\mathit{list\_in\_strorder\_n\_l\_to\_list\_in\_strorder\_l}$ $H0$).	$n\ : \mathbb{N}$ $L\ : \mathit{List}\ \mathbb{N}$ $\mathit{IHL}\ : \forall\ K : \mathit{List}\ \mathbb{N},\quad \mathit{list\_in\_strorder}\ L \rightarrow \mathit{list\_in\_strorder}\ K \rightarrow$ $L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{\mathit{List}\ \mathbb{N}\}\ K$ $\boxed{\text{H4}}\ : \mathit{list\_in\_strorder}\ K$ ――――――――――――――― $n :: L = \{\mathit{List}\ \mathbb{N}\}\ m :: K$
fact ($\mathit{strorder\_sublist\_antisym\_help}$ $H\ H0\ H1$).	$n\ : \mathbb{N}$ $L\ : \mathit{List}\ \mathbb{N}$ $\mathit{IHL}\ : \forall\ K : \mathit{List}\ \mathbb{N},\quad \mathit{list\_in\_strorder}\ L \rightarrow \mathit{list\_in\_strorder}\ K \rightarrow$ $L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{\mathit{List}\ \mathbb{N}\}\ K$ $\boxed{\text{H5}}\ : L \subseteq K$ ――――――――――――――― $n :: L = \{\mathit{List}\ \mathbb{N}\}\ m :: K$
fact ($\mathit{strorder\_sublist\_antisym\_help}$ $H0\ H\ H2$).	$n\ : \mathbb{N}$ $L\ : \mathit{List}\ \mathbb{N}$ $\mathit{IHL}\ : \forall\ K : \mathit{List}\ \mathbb{N},\quad \mathit{list\_in\_strorder}\ L \rightarrow \mathit{list\_in\_strorder}\ K \rightarrow$ $L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{\mathit{List}\ \mathbb{N}\}\ K$ $\boxed{\text{H6}}\ : K \subseteq L$ ――――――――――――――― $n :: L = \{\mathit{List}\ \mathbb{N}\}\ m :: K$
fact ($\mathit{IHL}\ K\ H3\ H4\ H5\ H6$).	$n\ : \mathbb{N}$ $L\ : \mathit{List}\ \mathbb{N}$ $\mathit{IHL}\ : \forall\ K : \mathit{List}\ \mathbb{N},\quad \mathit{list\_in\_strorder}\ L \rightarrow \mathit{list\_in\_strorder}\ K \rightarrow$ $L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{\mathit{List}\ \mathbb{N}\}\ K$ $\boxed{\text{H7}}\ : L = \{\mathit{List}\ \mathbb{N}\}\ K$ ――――――――――――――― $n :: L = \{\mathit{List}\ \mathbb{N}\}\ m :: K$

Continuing proof of Theorem strorder_sublist_antisym on the next page

Table 1: Proof of Theorem strorder_sublist_antisym continued

Next step in Coq	Proof situation
use_uqe $H7$.	n : $\mathbb{N}$ L : $List\ \mathbb{N}$ IHL : $\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ ——— $n :: L = \{List\ \mathbb{N}\}\ m :: L$
have $(n \in n :: L)$.	n : $\mathbb{N}$ L : $List\ \mathbb{N}$ IHL : $\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ H8 : $n \in n :: L$ ——— $n :: L = \{List\ \mathbb{N}\}\ m :: L$
trans $H8$ $H1$.	n : $\mathbb{N}$ L : $List\ \mathbb{N}$ IHL : $\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ H9 : $n \in m :: K$ ——— $n :: L = \{List\ \mathbb{N}\}\ m :: L$
use_or (*a_in_b_l* $H9$).	Hidden 2 goal(s)
$-_{1/2}$	n : $\mathbb{N}$ L : $List\ \mathbb{N}$ IHL : $\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ H10 : $n = \{\mathbb{N}\}\ m$ ——— $n :: L = \{List\ \mathbb{N}\}\ m :: L$
use_uqe $H10$.	n : $\mathbb{N}$ L : $List\ \mathbb{N}$ IHL : $\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ ——— $n :: L = \{List\ \mathbb{N}\}\ n :: L$
prove_equ.	$-_{1/2}$ completed
$-_{2/2}$	n : $\mathbb{N}$ L : $List\ \mathbb{N}$ IHL : $\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ $H10$: $n \in K$ ——— $n :: L = \{List\ \mathbb{N}\}\ m :: L$

Continuing proof of Theorem strorder_sublist_antisym on the next page

Table 1: Proof of Theorem strorder_sublist_antisym continued

Next step in Coq	Proof situation
$prove\_False.$	$n\ :\mathbb{N}$ $L\ :List\ \mathbb{N}$ $IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ $H10\ : n \in K$ ——— $False$
$use\_uqe\ H7\ in\ H10.$	$n\ :\mathbb{N}$ $L\ :List\ \mathbb{N}$ $IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ $H10\ : n \in L$ ——— $False$
$assert\ (L \neq []).$	Hidden 2 goal(s)
$++_{1/2}$	$n\ :\mathbb{N}$ $L\ :List\ \mathbb{N}$ $IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ $H10\ : n \in L$ ——— $L \neq []$
$prove\_not.$	$n\ :\mathbb{N}$ $L\ :List\ \mathbb{N}$ $IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ $H10\ : n \in L$ $\boxed{\text{H11}}\ : L = \{List\ \mathbb{N}\}\ []$ ——— $False$
$use\_equ\ H11\ in\ H10.$	$n\ :\mathbb{N}$ $L\ :List\ \mathbb{N}$ $IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ $\boxed{\text{H10}}\ : n \in []$ ——— $False$
$my\_contradiction\ H10.$	$++_{1/2}$ completed
$++_{2/2}$	$n\ :\mathbb{N}$ $L\ :List\ \mathbb{N}$ $IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$ $H10\ : n \in L$ $\boxed{\text{H11}}\ : L \neq []$ ——— $False$

Continuing proof of Theorem strorder_sublist_antisym on the next page

Table 1: Proof of Theorem strorder_sublist_antisym continued

Next step in Coq	Proof situation
$fact$ $(list\_in\_strorder\_n\_l\_to\_n\_less\_min\_l$ $H11\ H)$.	$n\ :\mathbb{N}$ $L\ :List\ \mathbb{N}$ $IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \rightarrow list\_in\_strorder\ K \rightarrow$ $L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{List\ \mathbb{N}\}\ K$ $H10\ : n \in L$ $\boxed{\text{H12}}\ : n < min\_list\ L$ ——————————— $False$
$fact$ $(n\_in\_l\_to\_min\_l\_lequ\_n$ $H10)$.	$n\ :\mathbb{N}$ $L\ :List\ \mathbb{N}$ $IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \rightarrow list\_in\_strorder\ K \rightarrow$ $L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{List\ \mathbb{N}\}\ K$ $H10\ : n \in L$ $\boxed{\text{H13}}\ : min\_list\ L \leq n$ ——————————— $False$
$trans\ H12\ H13$.	$n\ :\mathbb{N}$ $L\ :List\ \mathbb{N}$ $IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \rightarrow list\_in\_strorder\ K \rightarrow$ $L \subseteq K \rightarrow K \subseteq L \rightarrow L = \{List\ \mathbb{N}\}\ K$ $H10\ : n \in L$ $\boxed{\text{H14}}\ : n < n$ ——————————— $False$
$my\_contradiction$ $H14$.	$++_{2/2}$ completed, proof completed by Qed

End of proof of Theorem strorder_sublist_antisym