See the proof for Theorem strorder_sublist_antisym in table 1

Table 1: Proof of Theorem strorder_sublist_antisym

Next step in Coq	Proof situation
Proof.	$\vdash \forall L\ K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K$
prove_by_induction.	Hidden 2 goal(s)
$+_{1/2}$	$\vdash \forall K : List\ \mathbb{N},\ list\_in\_strorder\ [] \to list\_in\_strorder\ K \to [] \subseteq K \to K \subseteq [] \to [] = \{List\ \mathbb{N}\}\ K$
prove_all_imp_star.	$\boxed{\text{K}}\ : List\ \mathbb{N}\ ;\ \boxed{\text{H}}\ : list\_in\_strorder\ []\ ;\ \boxed{\text{H0}}\ : list\_in\_strorder\ K\ ;\ \boxed{\text{H1}}\ : [] \subseteq K\ ;\ H2\ : K \subseteq [] \vdash\ [] = \{List\ \mathbb{N}\}\ K$
exception H2.	$H2\ : K = \{List\ \mathbb{N}\}\ [] \vdash\ [] = \{List\ \mathbb{N}\}\ K$
switch in H2.	$\boxed{\text{H2}}\ : [] = \{List\ \mathbb{N}\}\ K \vdash\ [] = \{List\ \mathbb{N}\}\ K$
use H2.	$+_{1/2}$ completed
$+_{2/2}$	$a\ : \mathbb{N}\ ;\ L\ : List\ \mathbb{N}\ ;\ IHL\ : \forall K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K \vdash \forall K : List\ \mathbb{N},\ list\_in\_strorder\ (a :: L) \to list\_in\_strorder\ K \to a :: L \subseteq K \to K \subseteq a :: L \to a :: L = \{List\ \mathbb{N}\}\ K$
rename a into n.	$n\ : \mathbb{N}\ ;\ L\ : List\ \mathbb{N}\ ;\ IHL\ : \forall K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K \vdash \forall K : List\ \mathbb{N},\ list\_in\_strorder\ (n :: L) \to list\_in\_strorder\ K \to n :: L \subseteq K \to K \subseteq n :: L \to n :: L = \{List\ \mathbb{N}\}\ K$
prove_by_structure.	Hidden 2 goal(s)
$*_{1/2}$	$n\ : \mathbb{N}\ ;\ L\ : List\ \mathbb{N}\ ;\ IHL\ : \forall K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K \vdash\ list\_in\_strorder\ (n :: L) \to list\_in\_strorder\ [] \to n :: L \subseteq [] \to [] \subseteq n :: L \to n :: L = \{List\ \mathbb{N}\}\ []$
prove_all_imp_star.	$n\ : \mathbb{N}\ ;\ L\ : List\ \mathbb{N}\ ;\ IHL\ : \forall K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H}}\ : list\_in\_strorder\ (n :: L)\ ;\ \boxed{\text{H0}}\ : list\_in\_strorder\ []\ ;\ H1\ : n :: L \subseteq []\ ;\ \boxed{\text{H2}}\ : [] \subseteq n :: L \vdash\ n :: L = \{List\ \mathbb{N}\}\ []$
prove_False.	$n\ : \mathbb{N}\ ;\ L\ : List\ \mathbb{N}\ ;\ IHL\ : \forall K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ H1\ : n :: L \subseteq [] \vdash\ False$
exception H1.	$n\ : \mathbb{N}\ ;\ L\ : List\ \mathbb{N}\ ;\ IHL\ : \forall K : List\ \mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H1}}\ : n :: L = \{List\ \mathbb{N}\}\ [] \vdash\ False$
my_contradiction H1.	$*_{1/2}$ completed
$*_{2/2}$	$n\ : Basic.\mathbb{N}\ ;\ L\ : List\ Basic.\mathbb{N}\ ;\ IHL\ : \forall K : List\ Basic.\mathbb{N},\ list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ Basic.\mathbb{N}\}\ K\ ;\ \mathbb{N}\ : Basic.\mathbb{N}\ ;\ K\ : List\ Basic.\mathbb{N} \vdash\ list\_in\_strorder\ (n :: L) \to list\_in\_strorder\ (\mathbb{N} :: K) \to n :: L \subseteq \mathbb{N} :: K \to \mathbb{N} :: K \subseteq n :: L \to n :: L = \{List\ Basic.\mathbb{N}\}\ \mathbb{N} :: K$

Continuing proof of Theorem strorder_sublist_antisym on the next page

Table 1: Proof of Theorem strorder_sublist_antisym continued

Next step in Coq	Proof situation
rename $\mathbb{N}$ *into* m.	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{m}}\ :\mathbb{N}\ ;\ \boxed{\text{K}}\ :List\ \mathbb{N} \vdash\ list\_in\_strorder\ (n :: L) \to list\_in\_strorder\ (m :: K) \to n :: L \subseteq m :: K \to m :: K \subseteq n :: L \to n :: L = \{List\ \mathbb{N}\}\ m :: K$
prove_all_imp_star.	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H}}\ :list\_in\_strorder\ (n :: L)\ ;\ \boxed{\text{H0}}\ :list\_in\_strorder\ (m :: K)\ ;\ \boxed{\text{H1}}\ :n :: L \subseteq m :: K\ ;\ \boxed{\text{H2}}\ :m :: K \subseteq n :: L \vdash\ n :: L = \{List\ \mathbb{N}\}\ m :: K$
fact (*list_in_strorder_n_l* *H*).	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H3}}\ :list\_in\_strorder\ L \vdash\ n :: L = \{List\ \mathbb{N}\}\ m :: K$
fact (*list_in_strorder_n_l* *H*0).	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H4}}\ :list\_in\_strorder\ K \vdash\ n :: L = \{List\ \mathbb{N}\}\ m :: K$
fact (*strorder_sublist_antisym_help* *H H*0 *H*1).	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H5}}\ :L \subseteq K \vdash\ n :: L = \{List\ \mathbb{N}\}\ m :: K$
fact (*strorder_sublist_antisym_help* *H*0 *H H*2).	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H6}}\ :K \subseteq L \vdash\ n :: L = \{List\ \mathbb{N}\}\ m :: K$
fact (*IHL K H*3 *H*4 *H*5 *H*6).	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H7}}\ :L = \{List\ \mathbb{N}\}\ K \vdash\ n :: L = \{List\ \mathbb{N}\}\ m :: K$
*use_uqe H*7.	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K \vdash\ n :: L = \{List\ \mathbb{N}\}\ m :: L$
have ($n \in n :: L$).	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H8}}\ :n \in n :: L \vdash\ n :: L = \{List\ \mathbb{N}\}\ m :: L$
*trans H*8 *H*1.	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H9}}\ :n \in m :: K \vdash\ n :: L = \{List\ \mathbb{N}\}\ m :: L$
use_or (*a_in_b_l* *H*9).	Hidden 2 goal(s)
$-_{1/2}$	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H10}}\ :n = \{\mathbb{N}\}\ m \vdash\ n :: L = \{List\ \mathbb{N}\}\ m :: L$
*use_uqe H*10.	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K : List\ \mathbb{N},\quad list\_in\_strorder\ L \to list\_in\_strorder\ K \to L \subseteq K \to K \subseteq L \to L = \{List\ \mathbb{N}\}\ K \vdash\ n :: L = \{List\ \mathbb{N}\}\ n :: L$
prove_equ.	$-_{1/2}$ completed

Continuing proof of Theorem strorder_sublist_antisym on the next page

Table 1: Proof of Theorem strorder_sublist_antisym continued

Next step in Coq	Proof situation
$-_{2/2}$	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K:List\ \mathbb{N},\quad list\_in\_strorder\ L\rightarrow list\_in\_strorder\ K\rightarrow L\subseteq K\rightarrow K\subseteq L\rightarrow L=\{List\ \mathbb{N}\}\ K\ ;\ H10\ : n\in K\vdash\ n::L=\{List\ \mathbb{N}\}\ m::L$
prove_False.	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K:List\ \mathbb{N},\quad list\_in\_strorder\ L\rightarrow list\_in\_strorder\ K\rightarrow L\subseteq K\rightarrow K\subseteq L\rightarrow L=\{List\ \mathbb{N}\}\ K\ ;\ H10\ : n\in K\vdash\ False$
$use\_uqe\ H7\ in\ H10.$	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K:List\ \mathbb{N},\quad list\_in\_strorder\ L\rightarrow list\_in\_strorder\ K\rightarrow L\subseteq K\rightarrow K\subseteq L\rightarrow L=\{List\ \mathbb{N}\}\ K\ ;\ H10\ : n\in L\vdash\ False$
$assert\ (L\neq [\,]).$	Hidden 2 goal(s)
$++_{1/2}$	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K:List\ \mathbb{N},\quad list\_in\_strorder\ L\rightarrow list\_in\_strorder\ K\rightarrow L\subseteq K\rightarrow K\subseteq L\rightarrow L=\{List\ \mathbb{N}\}\ K\ ;\ H10\ : n\in L\vdash\ L\neq [\,]$
prove_not.	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K:List\ \mathbb{N},\quad list\_in\_strorder\ L\rightarrow list\_in\_strorder\ K\rightarrow L\subseteq K\rightarrow K\subseteq L\rightarrow L=\{List\ \mathbb{N}\}\ K\ ;\ H10\ : n\in L\ ;\ \boxed{\text{H11}}\ : L=\{List\ \mathbb{N}\}\ [\,]\vdash\ False$
$use\_equ\ \ H11\ \ in\ H10.$	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K:List\ \mathbb{N},\quad list\_in\_strorder\ L\rightarrow list\_in\_strorder\ K\rightarrow L\subseteq K\rightarrow K\subseteq L\rightarrow L=\{List\ \mathbb{N}\}\ K\ ;\ \boxed{\text{H10}}\ : n\in [\,]\vdash\ False$
$my\_contradiction\ H10.$	$++_{1/2}$ completed
$++_{2/2}$	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K:List\ \mathbb{N},\quad list\_in\_strorder\ L\rightarrow list\_in\_strorder\ K\rightarrow L\subseteq K\rightarrow K\subseteq L\rightarrow L=\{List\ \mathbb{N}\}\ K\ ;\ H10\ : n\in L\ ;\ \boxed{\text{H11}}\ : L\neq [\,]\vdash\ False$
$fact\ (list\_in\_strorder\_n\_l\_to\_n\_less\_min\_l\ H11\ H).$	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K:List\ \mathbb{N},\quad list\_in\_strorder\ L\rightarrow list\_in\_strorder\ K\rightarrow L\subseteq K\rightarrow K\subseteq L\rightarrow L=\{List\ \mathbb{N}\}\ K\ ;\ H10\ : n\in L\ ;\ \boxed{\text{H12}}\ : n<min\_list\ L\vdash\ False$
$fact\ (n\_in\_l\_to\_min\_l\_lequ\ H10).$	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K:List\ \mathbb{N},\quad list\_in\_strorder\ L\rightarrow list\_in\_strorder\ K\rightarrow L\subseteq K\rightarrow K\subseteq L\rightarrow L=\{List\ \mathbb{N}\}\ K\ ;\ H10\ : n\in L\ ;\ \boxed{\text{H13}}\ : min\_list\ L\leq n\vdash\ False$
$trans\ H12\ H13.$	$n\ :\mathbb{N}\ ;\ L\ :List\ \mathbb{N}\ ;\ IHL\ :\forall\ K:List\ \mathbb{N},\quad list\_in\_strorder\ L\rightarrow list\_in\_strorder\ K\rightarrow L\subseteq K\rightarrow K\subseteq L\rightarrow L=\{List\ \mathbb{N}\}\ K\ ;\ H10\ : n\in L\ ;\ \boxed{\text{H14}}\ : n<n\vdash\ False$
$my\_contradiction\ H14.$	$++_{2/2}$ completed, proof completed by Qed

End of proof of Theorem strorder_sublist_antisym