Answer Set Solving in Practice

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Rough Roadmap

1. Introduction
2. Language
3. Modeling
4. Grounding
5. Foundations
6. Solving
7. Systems
8. Applications

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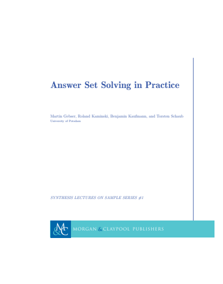
Resources

- Course material
  - `http://www.cs.uni-potsdam.de/wv/lehre`
  - `http://moodle.cs.uni-potsdam.de`
  - `http://potassco.sourceforge.net/teaching.html`
- Systems
  - clasp `http://potassco.sourceforge.net`
  - dlv `http://www.dlvsystem.com`
  - smodels `http://www.tcs.hut.fi/Software/smodels`
  - gringo `http://potassco.sourceforge.net`
  - lparse `http://www.tcs.hut.fi/Software/smodels`
  - clingo `http://potassco.sourceforge.net`
  - iclingo `http://potassco.sourceforge.net`
  - oclingo `http://potassco.sourceforge.net`
  - asparagus `http://asparagus.cs.uni-potsdam.de`

The Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



Resources

- http://potassco.sourceforge.net/book.html
- http://potassco.sourceforge.net/teaching.html

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Literature

Books [4], [29], [53]

Surveys [50], [2], [39], [21], [11]

Articles [41], [42], [6], [61], [54], [49], [40], etc.

Motivation: Overview

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Outline

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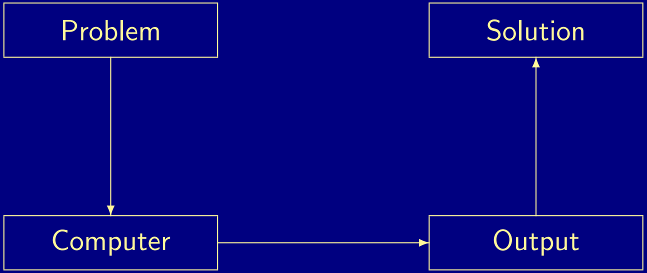
Informatics

"What is the problem?" versus *"How to solve the problem?"*



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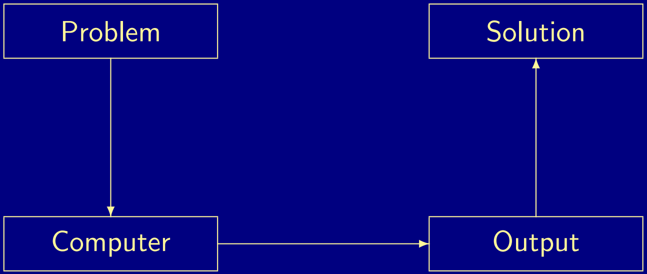

Traditional programming

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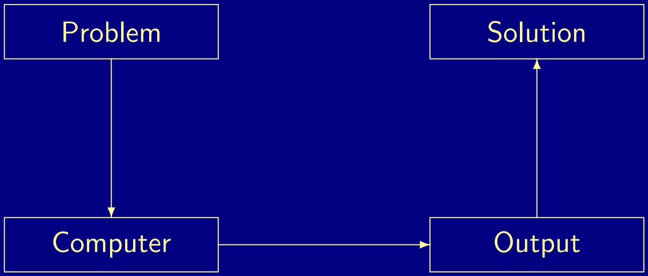

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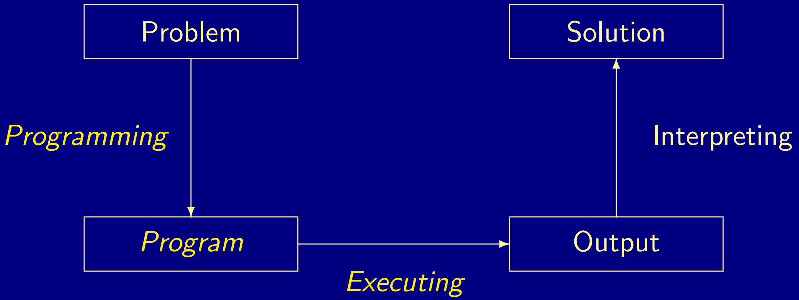
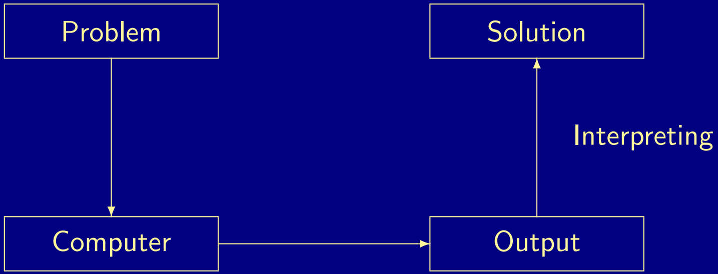


Declarative problem solving

“What is the problem?” versus *“How to solve the problem?”*



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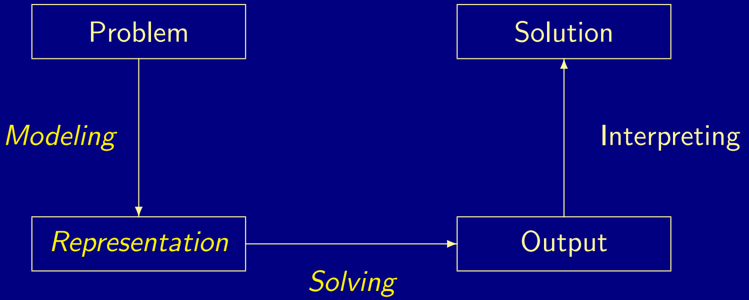

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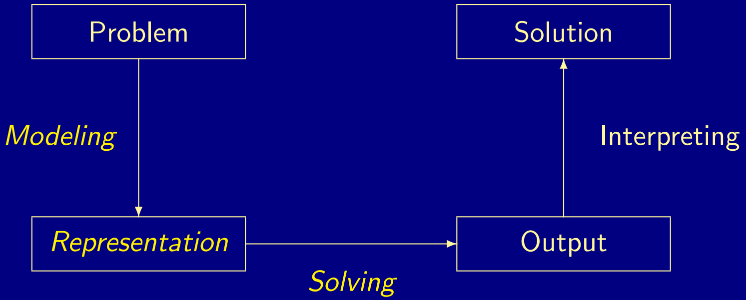

Outline

Answer Set Programming

in a Nutshell

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
- ASP has its roots in
  - (deductive) databases
  - logic programming (with negation)
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - constraint solving (in particular, SATisfiability testing)
- ASP allows for solving all search problems in NP (and NP^{NP}) in a uniform way
- ASP is versatile as reflected by the ASP solver *clasp*, winning first places at ASP, CASC, MISC, PB, and SAT competitions
- ASP embraces many emerging application areas

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Answer Set Programming

in a Hazelnutshell

- ASP is an approach to declarative problem solving, combining
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  tailored to Knowledge Representation and Reasoning

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$$\textbf{ASP} = \textbf{DB}+\textbf{LP}+\textbf{KR}+\textbf{SAT}$$

Outline

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KR's shift of paradigm

Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem
2. A solution is given by a derivation of a query

Model Generation based approach (eg. SATisfiability testing)

1. Provide a representation of the problem
2. A solution is given by a model of the representation

Automated planning, Kautz and Selman (ECAI'92)

Represent planning problems as propositional theories so that models not proofs describe solutions

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Model Generation based Problem Solving

Representation	Solution
constraint satisfaction problem	assignment
propositional horn theories	smallest model
propositional theories	models
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propositional programs	minimal models
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first-order theories	Herbrand models
auto-epistemic theories	expansions
default theories	extensions
⋮	⋮

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LP-style playing with blocks

Prolog program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Prolog queries

```
?- above(a,c).
true.

?- above(c,a).
no.
```

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Prolog queries (testing entailment)

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true.

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no.
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LP-style playing with blocks

Shuffled Prolog program

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Prolog queries

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Fatal Error: local stack overflow.
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Prolog queries (answered via fixed execution)

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SAT-style playing with blocks

Formula

$$\begin{array}{ll} & on(a,b) \\ \wedge & on(b,c) \\ \wedge & (on(X,Y) \rightarrow above(X,Y)) \\ \wedge & (on(X,Z) \wedge above(Z,Y) \rightarrow above(X,Y)) \end{array}$$

Herbrand model

$$\left\{ \begin{array}{ccccc} on(a,b), & on(b,c), & on(a,c), & on(b,b), & \\ above(a,b), & above(b,c), & above(a,c), & above(b,b), & above(c,b) \end{array} \right\}$$

SAT-style playing with blocks

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\end{array}
$$

Herbrand model

$$
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\end{array} \right\}
$$

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SAT-style playing with blocks

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Herbrand model (among 426!)

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➥ Answer Set Programming (ASP)

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Answer Set Programming *at large*

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Answer Set Programming *commonly*

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Answer Set Programming *in practice*

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ASP-style playing with blocks

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on(a,b).
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Stable Herbrand model

$$\{ \texttt{on}(a,b),\ \texttt{on}(b,c),\ \texttt{above}(b,c),\ \texttt{above}(a,b),\ \texttt{above}(a,c) \}$$

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Stable Herbrand model (and no others)

$\{\ \mathtt{on}(\mathtt{a},\mathtt{b}),\ \mathtt{on}(\mathtt{b},\mathtt{c}),\ \mathtt{above}(\mathtt{b},\mathtt{c}),\ \mathtt{above}(\mathtt{a},\mathtt{b}),\ \mathtt{above}(\mathtt{a},\mathtt{c})\ \}$

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Stable Herbrand model (and no others)

$\{$ on(a, b), on(b, c), above(b, c), above(a, b), above(a, c) $\}$

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ASP versus LP

ASP	Prolog
Model generation	Query orientation
Bottom-up	Top-down
Modeling language	Programming language
Rule-based format	
Instantiation Flat terms	Unification Nested terms
(Turing +) $NP(^{NP})$	Turing

ASP versus SAT

ASP	SAT
Model generation	
Bottom-up	
Constructive Logic	Classical Logic
Closed (and open) world reasoning	Open world reasoning
Modeling language	—
Complex reasoning modes	Satisfiability testing
Satisfiability	Satisfiability
Enumeration/Projection	—
Intersection/Union	—
Optimization	—
(Turing +) $NP(^{NP})$	NP

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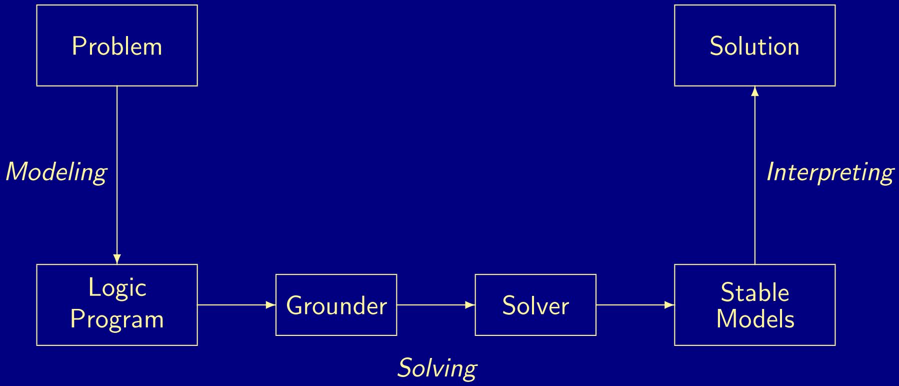
Outline

ASP solving



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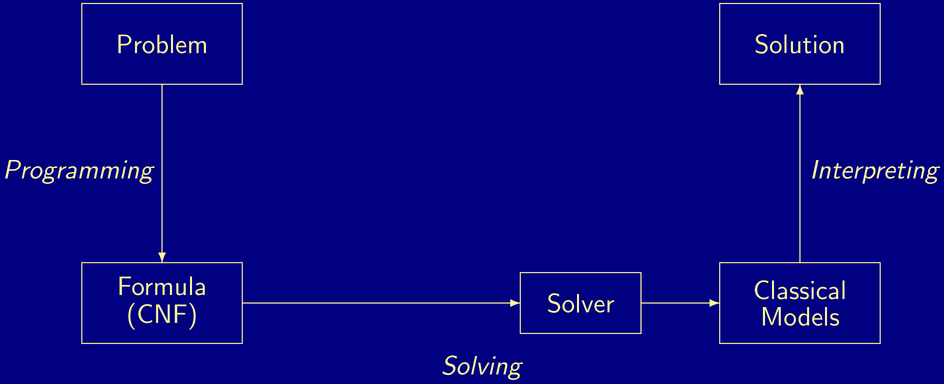
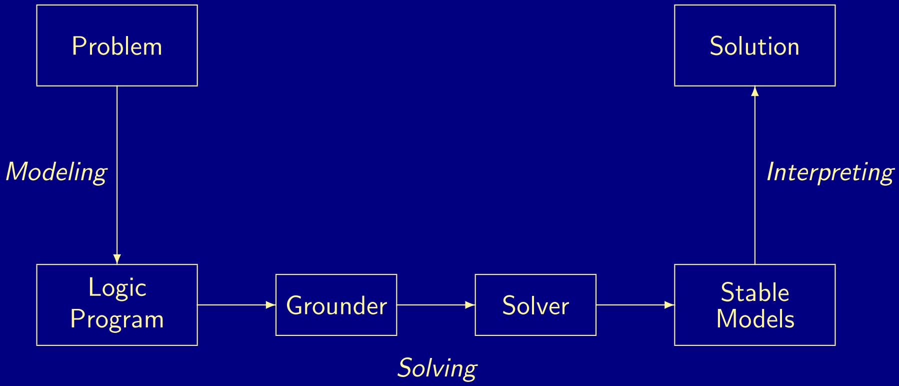
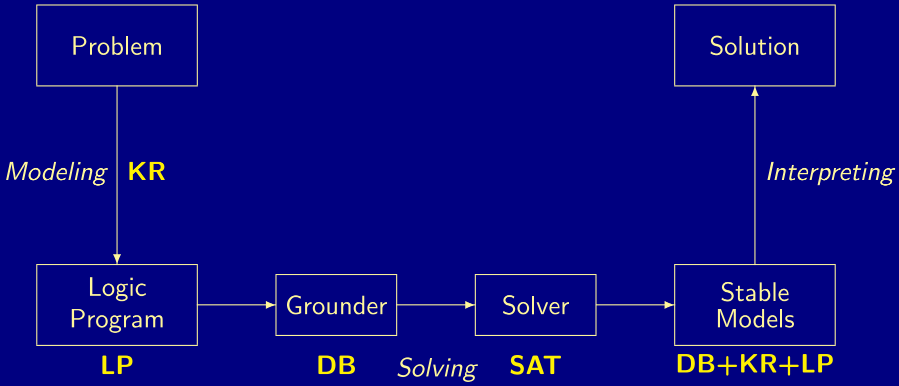
SAT solving


Problem
Solution
Programming
Interpreting
Formula (CNF)
Solver
Classical Models
Solving

Rooting ASP solving



Rooting ASP solving



Outline

Two sides of a coin

- ASP as High-level Language
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules
- ASP as Low-level Language
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation

What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated Planning
  - Code Optimization
  - Composition of Renaissance Music
  - Database Integration
  - Decision Support for NASA shuttle controllers
  - Model Checking
  - Product Configuration
  - Robotics
  - Systems Biology
  - System Synthesis
  - (industrial) Team-building
  - and many many more

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What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated Planning
  - Code Optimization
  - Composition of Renaissance Music
  - Database Integration
  - Decision Support for NASA shuttle controllers
  - Model Checking
  - Product Configuration
  - Robotics
  - Systems Biology
  - System Synthesis
  - (industrial) Team-building
  - and many many more

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What does ASP offer?

- Integration of DB, KR, and SAT techniques
- Succinct, elaboration-tolerant problem representations
  - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
  - including: data, frame axioms, exceptions, defaults, closures, etc

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ASP = DB+LP+KR+SMT

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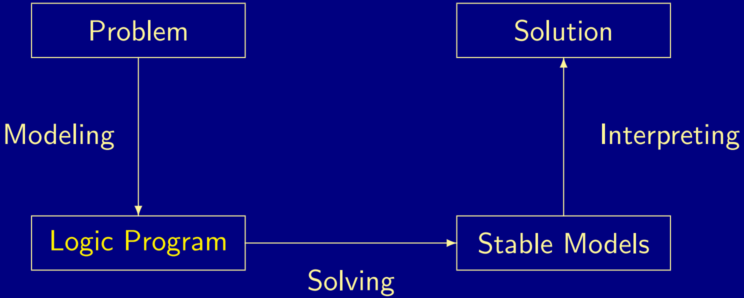
Introduction: Overview

Outline

Problem solving in ASP: Syntax



Normal logic programs

- A logic program, P, over a set $\mathcal{A}$ of atoms is a finite set of rules
- A (normal) rule, r, is of the form

$$a_0 \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

$$\begin{array}{rcl} head(r) & = & a_0 \\ body(r) & = & \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \\ body(r)^+ & = & \{a_1, \dots, a_m\} \\ body(r)^- & = & \{a_{m+1}, \dots, a_n\} \\ atom(P) & = & \bigcup_{r \in P} \left(\{head(r)\} \cup body(r)^+ \cup body(r)^-\right) \\ body(P) & = & \{body(r) \mid r \in P\} \end{array}$$

A program P is positive if $body(r)^- = \emptyset$ for all $r \in P$

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- Notation

$$\begin{aligned}
head(r) &= a_0 \\
body(r) &= \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \\
body(r)^+ &= \{a_1, \dots, a_m\} \\
body(r)^- &= \{a_{m+1}, \dots, a_n\} \\
atom(P) &= \textstyle\bigcup_{r \in P} \left(\{head(r)\} \cup body(r)^+ \cup body(r)^-\right) \\
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Rough notational convention

We sometimes use the following notation interchangeably in order to stress the respective view:

	true, false	if	and	or	iff	default negation	classical negation
source code		`:-`	`,`	`|`		`not`	`-`
logic program		$\leftarrow$	,	;		$\sim$	$\neg$
formula	$\bot, \top$	$\rightarrow$	$\wedge$	$\vee$	$\leftrightarrow$	$\sim$	$\neg$

Outline

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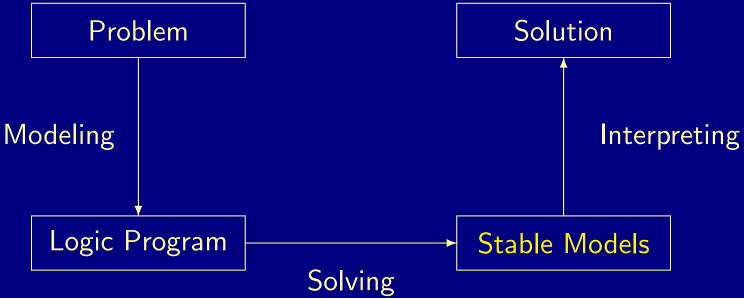
Problem solving in ASP: Semantics



Formal Definition

Stable models of positive programs

- A set of atoms X is closed under a positive program P iff for any $r \in P$, $head(r) \in X$ whenever $body(r)^+ \subseteq X$
  - X corresponds to a model of P (seen as a formula)
- The smallest set of atoms which is closed under a positive program P is denoted by $Cn(P)$
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of P (ditto)
- The set $Cn(P)$ of atoms is the stable model of a *positive* program P

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Some “logical” remarks

- Positive rules are also referred to as definite clauses
  - Definite clauses are disjunctions with exactly one positive atom:

    $$a_0 \vee \neg a_1 \vee \cdots \vee \neg a_m$$

  - A set of definite clauses has a (unique) smallest model
- Horn clauses are clauses with at most one positive atom
  - Every definite clause is a Horn clause but not vice versa
  - Non-definite Horn clauses can be regarded as integrity constraints
  - A set of Horn clauses has a smallest model or none
- This smallest model is the intended semantics of such sets of clauses
  - Given a positive program P, $Cn(P)$ corresponds to the smallest model of the set of definite clauses corresponding to P

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Basic idea

Consider the logical formula Φ and its three (classical) models:

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

Φ $\boxed{q \;\wedge\; (q \wedge \neg r \rightarrow p)}$

Formula Φ has one stable model, often called answer set:

$$\{p, q\}$$

P_Φ

q	$\leftarrow$	
p	$\leftarrow$	$q, \sim r$

Informally, a set X of atoms is a stable model of a logic program P

- if X is a (classical) model of P and
- if all atoms in X are justified by some rule in P

(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))

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p	$\mapsto$	1
q	$\mapsto$	1
r	$\mapsto$	0

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Consider the logical formula Φ and its three (classical) models:

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$$\Phi \quad \boxed{q \ \land \ (q \land \neg r \rightarrow p)}$$

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Stable model of normal programs

- The reduct, P^X, of a program P relative to a set X of atoms is defined by

$$P^X = \{head(r) \leftarrow body(r)^+ \mid r \in P \text{ and } body(r)^- \cap X = \emptyset\}$$

- A set X of atoms is a stable model of a program P, if $Cn(P^X) = X$

- Note $Cn(P^X)$ is the $\subseteq$–smallest (classical) model of P^X
- Note Every atom in X is justified by an *"applying rule from P"*

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A closer look at P^X

- In other words, given a set X of atoms from P,

  P^X is obtained from P by **deleting**

  1. each **rule** having $\sim a$ in its body with $a \in X$
     and then
  2. all **negative atoms** of the form $\sim a$
     in the bodies of the remaining rules

- Note Only negative body literals are evaluated wrt X

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Outline

A first example

$$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p\}$	$p \leftarrow p$	$\emptyset$
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p, q\}$	$p \leftarrow p$	$\emptyset$

A first example

$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✗
$\{p\}$	$p \leftarrow p$	$\emptyset$	
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✔
$\{p, q\}$	$p \leftarrow p$	$\emptyset$	✗

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A first example

$P = \{p \leftarrow p, \; q \leftarrow \neg p\}$

X	P^X	$Cn(P^X)$	
$\{\;\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✘
$\{p\}$	$p \leftarrow p$	$\emptyset$	✔
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✔
$\{p, q\}$	$p \leftarrow p$	$\emptyset$	✘

A second example

$P = \{p \leftarrow {\sim}q,\ q \leftarrow {\sim}p\}$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$
$\{p\}$	$p \leftarrow$	$\{p\}$
$\{q\}$	$q \leftarrow$	$\{q\}$
$\{p, q\}$		$\emptyset$

A second example

$$P = \{p \leftarrow \sim q, \;\; q \leftarrow \sim p\}$$

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A second example

$P = \{p \leftarrow \neg q,\ q \leftarrow \neg p\}$

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$\{\ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$	✘
$\{p\}$	$p \leftarrow$	$\{p\}$	✔
$\{q\}$	$q \leftarrow$	$\{q\}$	✔
$\{p, q\}$		$\emptyset$	✔

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A third example

$P = \{p \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$
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$\{p\}$		$\emptyset$

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X	P^X	$Cn(P^X)$	
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Some properties

- A logic program may have zero, one, or multiple stable models!
- If X is a stable model of a logic program P, then X is a model of P (seen as a formula)
- If X and Y are stable models of a *normal* program P, then $X \not\subset Y$

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Programs with Variables

Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \to \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground Instantiation of P: $\quad ground(P) = \bigcup_{r \in P} ground(r)$

Programs with Variables

Let P be a logic program

- Let $\mathcal{T}$ be a set of variable-free terms (also called Herbrand universe)
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$ (also called alphabet or Herbrand base)
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- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \rightarrow \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground Instantiation of P: $ground(P) = \bigcup_{r \in P} ground(r)$

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An example

$$P = \{\ r(a,b) \leftarrow,\ \ r(b,c) \leftarrow,\ \ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a,b,c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

$$ground(P) = \left\{ \begin{array}{lllllllll} r(a,b) & \leftarrow & , & & & & & & \\ r(b,c) & \leftarrow & , & & & & & & \\ t(a,a) & \leftarrow & r(a,a), & t(b,a) & \leftarrow & r(b,a), & t(c,a) & \leftarrow & r(c,a), \\ t(a,b) & \leftarrow & r(a,b), & t(b,b) & \leftarrow & r(b,b), & t(c,b) & \leftarrow & r(c,b), \\ t(a,c) & \leftarrow & r(a,c), & t(b,c) & \leftarrow & r(b,c), & t(c,c) & \leftarrow & r(c,c) \end{array} \right\}$$

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Stable models of programs with Variables

Let P be a normal logic program with variables

- A set X of (ground) atoms is a stable model of P, if $Cn(ground(P)^X) = X$

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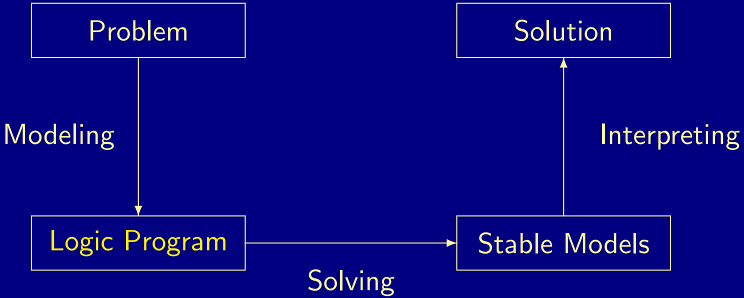
Outline

Problem solving in ASP: Extended Syntax



Language Constructs

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  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for
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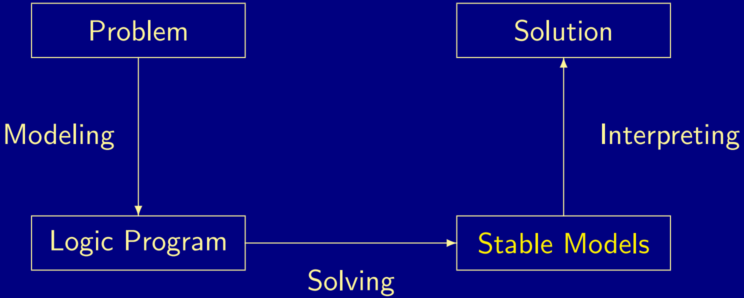
Outline

Problem solving in ASP: Reasoning Modes



Reasoning Modes

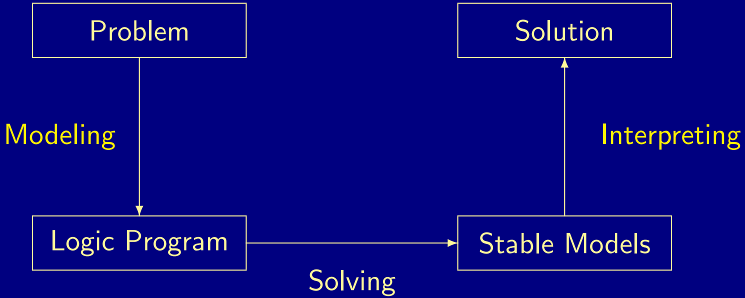
- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization
- and combinations of them

† without solution recording

‡ without solution enumeration

Basic Modeling: Overview

Modeling and Interpreting



Modeling

- For solving a problem class **C** for a problem instance **I**, encode
  1. the problem instance **I** as a set $P_{\mathbf{I}}$ of facts and
  2. the problem class **C** as a set $P_{\mathbf{C}}$ of rules

  such that the solutions to **C** for **I** can be (polynomially) extracted from the stable models of $P_{\mathbf{I}} \cup P_{\mathbf{C}}$

- $P_{\mathbf{I}}$ is (still) called problem instance
- $P_{\mathbf{C}}$ is often called the problem encoding

- An encoding $P_{\mathbf{C}}$ is uniform, if it can be used to solve all its problem instances
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Outline

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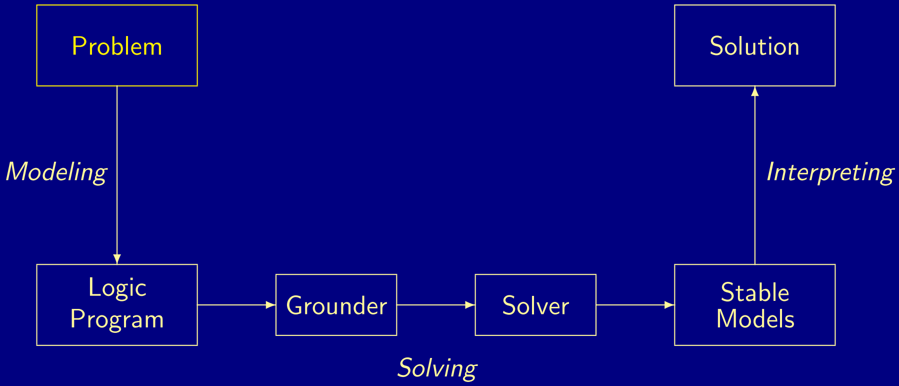

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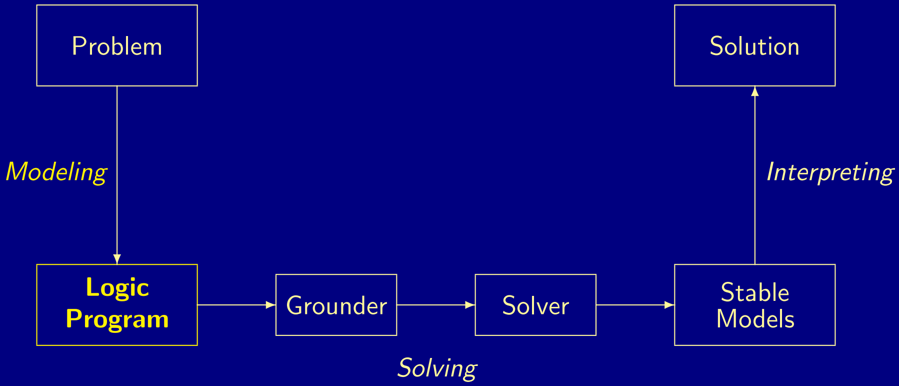

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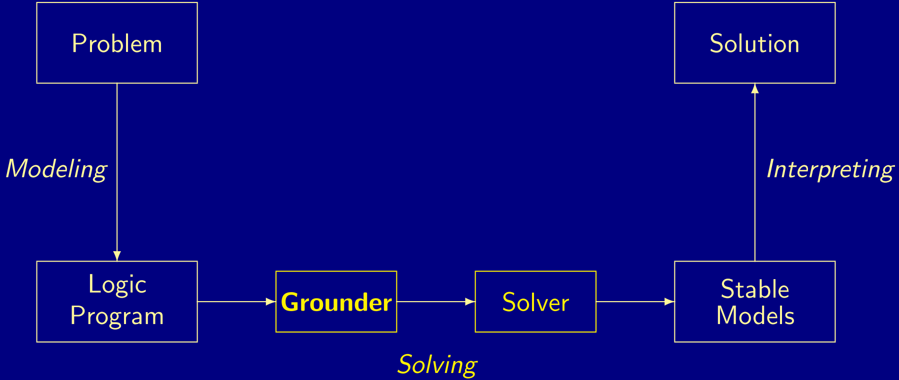

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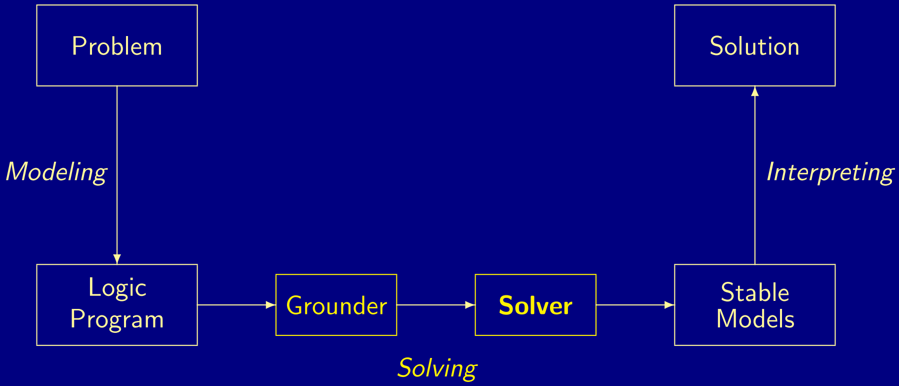

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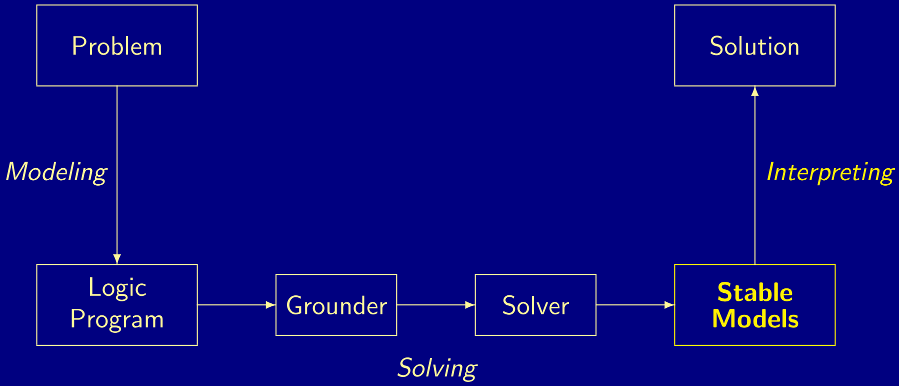

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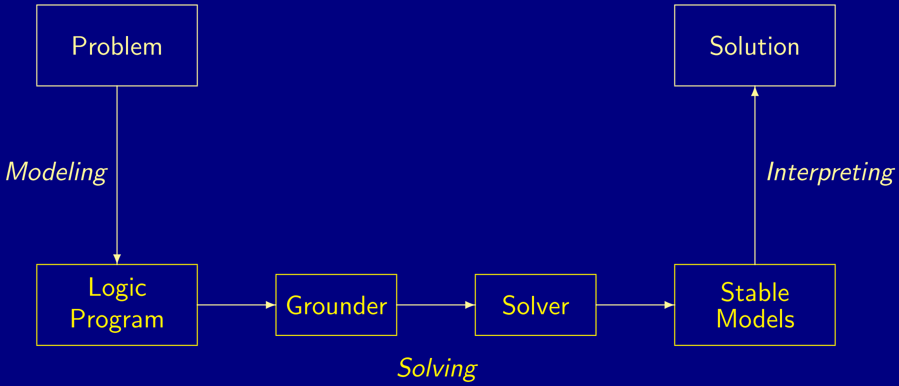

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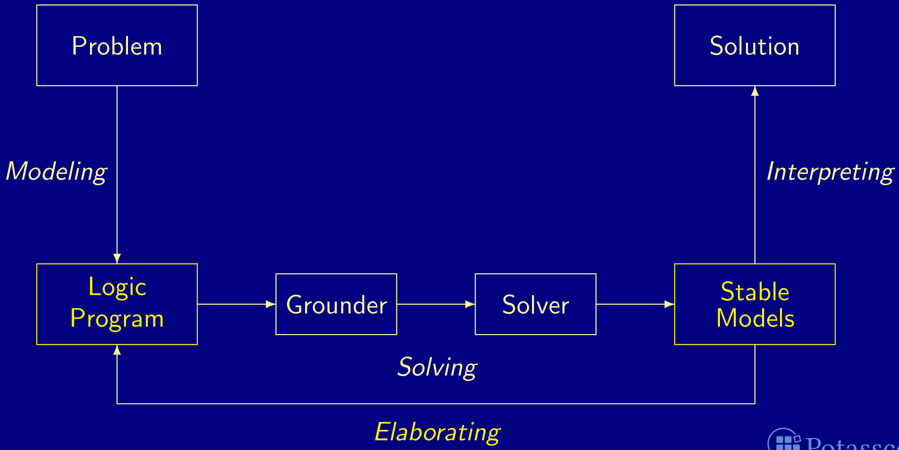
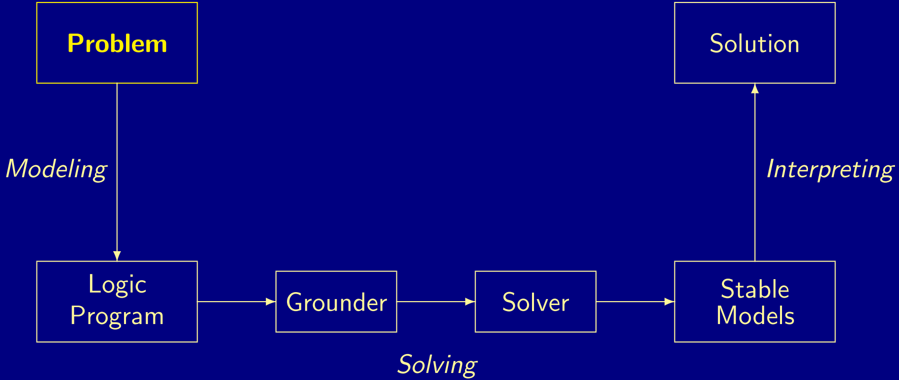


A case-study: Graph coloring



Graph coloring

- Problem instance A graph consisting of nodes and edges

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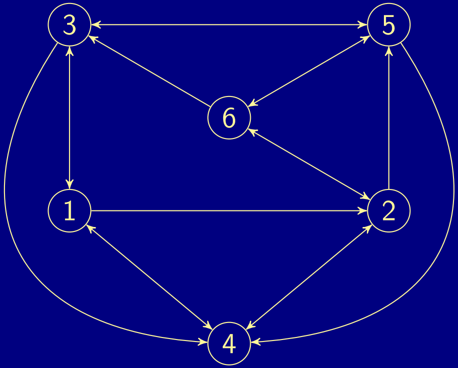

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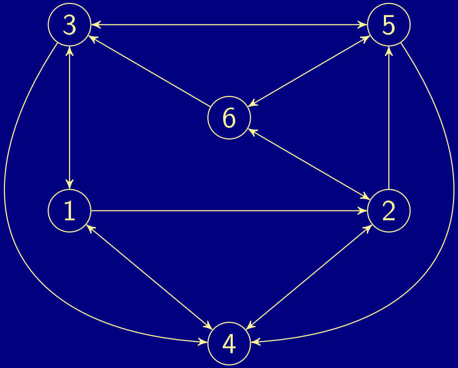

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  - facts formed by predicate `col/1`
- Problem class Assign each node one color such that no two nodes connected by an edge have the same color

  In other words,

  1. Each node has a unique color
  2. Two connected nodes must not have the same color

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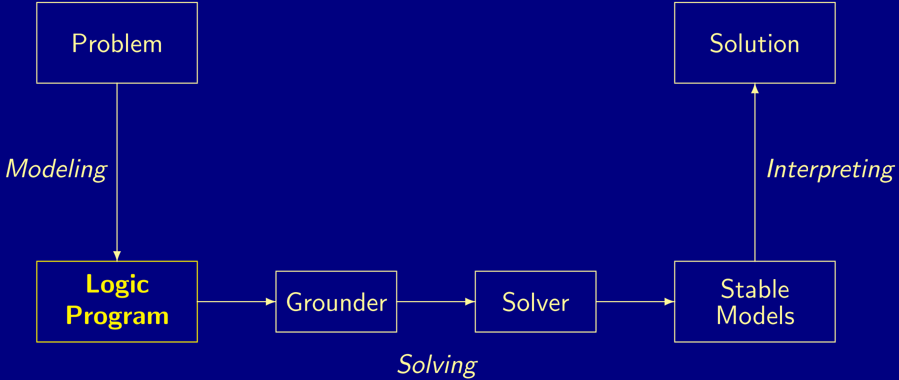
ASP solving process



Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
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col(r).   col(b).   col(g).
```

} Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
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edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

Potassco

Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

Potassco

color.lp

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

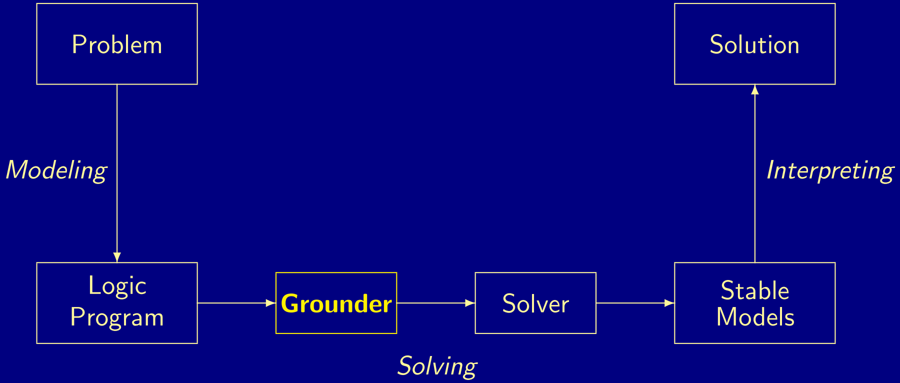
Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

Potassco

ASP solving process



Graph coloring: Grounding

$ gringo --text color.lp

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

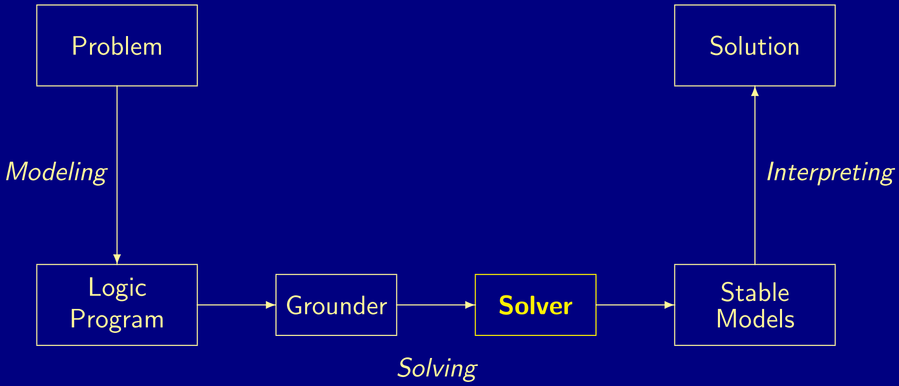
Graph coloring: Grounding

$ gringo --text color.lp

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

Potassco

ASP solving process



Graph coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

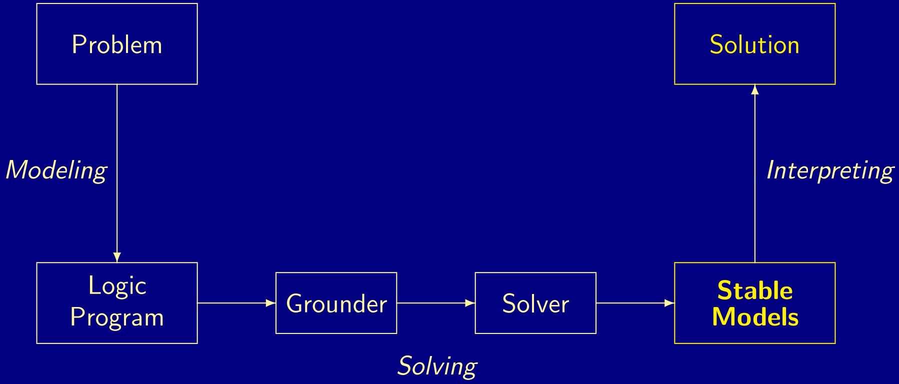
Potassco

Graph coloring: Solving

```
$ gringo color.lp | clasp 0
```

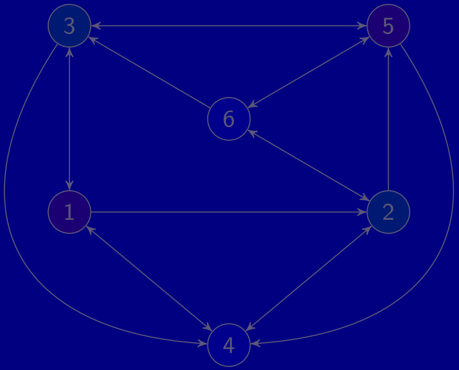
```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

ASP solving process



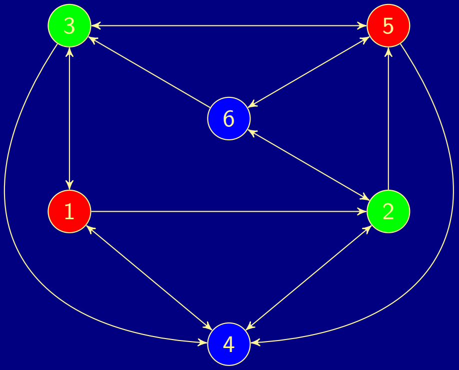
A coloring

```
Answer: 6
edge(1,2) ... col(r) ... node(1) ...                                          \
color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
```



A coloring

```
Answer: 6
edge(1,2) ... col(r) ... node(1) ...                                   \
color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
```



Outline

13 ASP solving process

14 Methodology
- Satisfiability
- Queens
- Traveling Salesperson
- Reviewer Assignment
- Planning

Potassco

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

- **Generator** Generate potential stable model candidates (typically through non-deterministic constructs)
- **Tester** Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

Generator: Generate potential stable model candidates (typically through non-deterministic constructs)

Tester: Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

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Outline

Satisfiability testing

- Problem Instance: A propositional formula ϕ in CNF
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program:

Generator	Tester	Stable models
$\{ a, b \} \leftarrow$	$\leftarrow \ {\sim}a, b$	$X_1 = \{a, b\}$
	$\leftarrow \ a, {\sim}b$	$X_2 = \{\}$

Satisfiability testing

- Problem Instance: A propositional formula ϕ in CNF
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program:

Generator	**Tester**	**Stable models**
$\{ a, b \} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
	$\leftarrow a, \sim b$	$X_2 = \{\}$

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- Logic Program:

Generator	**Tester**	**Stable models**
$\{ a, b \} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
	$\leftarrow a, \sim b$	$X_2 = \{\}$

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Satisfiability testing

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- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program:

Generator	**Tester**	**Stable models**
$\{ a, b \} \leftarrow$	$\leftarrow \ \sim a, b$	$X_1 = \{a, b\}$
	$\leftarrow \ a, \sim b$	$X_2 = \{\}$

Potassco

Satisfiability testing

- Problem Instance: A propositional formula ϕ in CNF
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

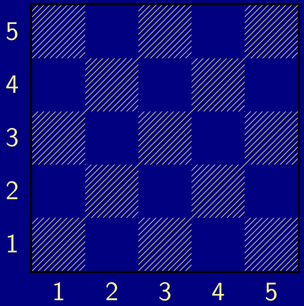
$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program:

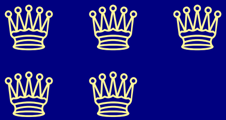
Generator	**Tester**	**Stable models**
$\{ a, b \} \leftarrow$	$\leftarrow \ \sim a, b$	$X_1 = \{a, b\}$
	$\leftarrow \ a, \sim b$	$X_2 = \{\}$

Outline

The n-Queens Problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another



Potassco

Defining the Field

queens.lp

```
row(1..n).
col(1..n).
```

- Create file `queens.lp`
- Define the field
  - n rows
  - n columns

Defining the Field

Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models      : 1
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Potassco

Placing some Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
```

- Guess a solution candidate
  by placing some queens on the board

Placing some Queens

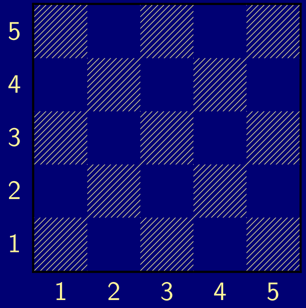
Running . . .

```
$ gringo queens.lp --const n=5 | clasp 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE

Models      : 3+
...
```

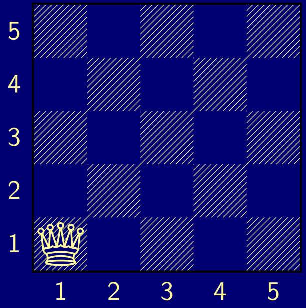
Placing some Queens: Answer 1

Answer 1



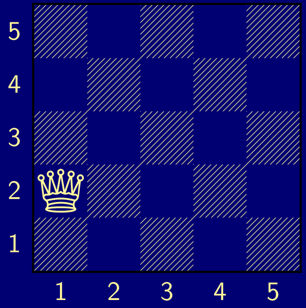
Placing some Queens: Answer 2

Answer 2



Placing some Queens: Answer 3

Answer 3



Placing n Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
```

- Place exactly n queens on the board

Potassco

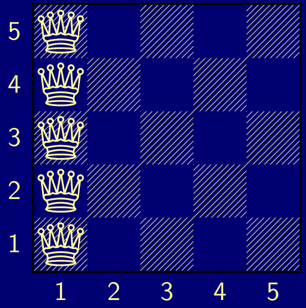
Placing n Queens

Running ...

```
$ gringo queens.lp --const n=5 | clasp 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
...
```

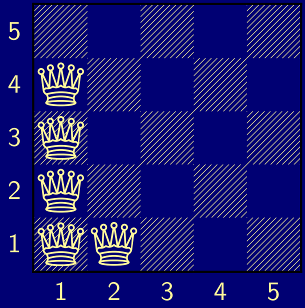
Placing n Queens: Answer 1

Answer 1



Placing *n* Queens: Answer 2

Answer 2



Horizontal and Vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

Horizontal and Vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

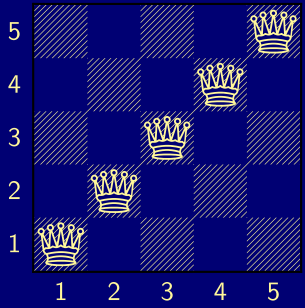
Horizontal and Vertical Attack

Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) \
queen(2,2) queen(1,1)
...
```

Horizontal and Vertical Attack: Answer 1

Answer 1



Diagonal Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I-J == II-JJ.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I+J == II+JJ.
```

- Forbid diagonal attacks

Diagonal Attack

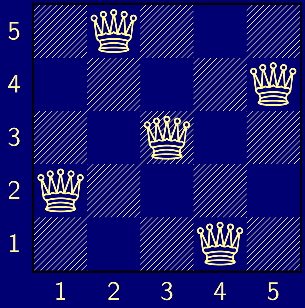
Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) queen(5,2) queen(2,1)
SATISFIABLE

Models      : 1+
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Diagonal Attack: Answer 1

Answer 1



Optimizing

queens-opt.lp

```
1 { queen(I,1..n) } 1 :- I = 1..n.
1 { queen(1..n,J) } 1 :- J = 1..n.
 :- 2 { queen(D-J,J) }, D = 2..2*n.
 :- 2 { queen(D+J,J) }, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=3
clingo version 4.1.0
Solving...
SATISFIABLE

Models       : 1+
Time         : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time     : 3758.320s

Choices      : 288594554
Conflicts    : 3442   (Analyzed: 3442)
Restarts     : 17     (Average: 202.47 Last: 3442)
Model-Level  : 7594728.0
Problems     : 1      (Average Length: 0.00 Splits: 0)
Lemmas       : 3442   (Deleted: 0)
  Binary     : 0      (Ratio:   0.00%)
  Ternary    : 0      (Ratio:   0.00%)
  Conflict   : 3442   (Average Length: 229056.5 Ratio: 100.00%)
  Loop       : 0      (Average Length:    0.0 Ratio:   0.00%)
  Other      : 0      (Average Length:    0.0 Ratio:   0.00%)

Atoms        : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules        : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies       : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight        : Yes
Variables    : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664  (Binary: 35.6% Ternary:  0.0% Other: 64.4%)

Backjumps    : 3442   (Average: 681.19 Max: 169512 Sum: 2344658)
  Executed   : 3442   (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
```

Outline

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
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```

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
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```

Potassco

Traveling Salesperson

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node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
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Traveling Salesperson

```
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize [ cycle(X,Y) = C : cost(X,Y,C) ].
```

Traveling Salesperson

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Potassco

Outline

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
...

3 { assigned(P,R) : reviewer(R) } 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { assignedB(P,R) : paper(P) : reviewer(R) }.
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Potassco

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Outline

Simplistic STRIPS Planning

```
time(1..k).    lasttime(T) :- time(T), not time(T+1).

fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).    query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

1 { occ(A,T) : action(A) } 1 :- time(T).
 :- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- holds(F,T-1), not nolds(F,T), time(T).
holds(F,T) :- occ(A,T), add(A,F).
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Language: Overview

Outline

Basic language extensions

- The expressiveness of a language can be enhanced by introducing new constructs
- To this end, we must address the following issues:
  - What is the syntax of the new language construct?
  - What is the semantics of the new language construct?
  - How to implement the new language construct?
- A way of providing semantics is to furnish a translation removing the new constructs, eg. classical negation
- This translation might also be used for implementing the language extension

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Outline

Outline

Integrity constraint

- Idea Eliminate unwanted solution candidates
- Syntax An integrity constraint is of the form

$$\leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n$$

where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$

- Example `:- edge(3,7), color(3,red), color(7,red).`
- Embedding The above integrity constraint can be turned into the normal rule

$$x \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n, \sim x$$

where x is a new symbol, that is, $x \notin \mathcal{A}$.

- Another example $P = \{a \leftarrow \sim b,\ b \leftarrow \sim a\}$
versus $P' = P \cup \{\leftarrow a\}$ and $P'' = P \cup \{\leftarrow \sim a\}$

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Choice rule

- Idea Choices over subsets
- Syntax A choice rule is of the form

$$\{a_1, \dots, a_m\} \leftarrow a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o$$

  where $0 \leq m \leq n \leq o$ and each a_i is an atom for $1 \leq i \leq o$

- Informal meaning If the body is satisfied by the stable model at hand, then any subset of $\{a_1, \dots, a_m\}$ can be included in the stable model

- Example `{ buy(pizza), buy(wine), buy(corn) } :- at(grocery).`
- Another Example $P = \{\{a\} \leftarrow b, \; b \leftarrow\}$ has two stable models: $\{b\}$ and $\{a, b\}$

Potassco

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Potassco

Embedding in normal rules

- A choice rule of form

$$\{a_1, \dots, a_m\} \leftarrow a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o$$

can be translated into $2m + 1$ normal rules

$$\begin{array}{lllllll} a' & \leftarrow & a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o & & & & \\ a_1 & \leftarrow & a', \sim\overline{a_1} & \dots & a_m & \leftarrow & a', \sim\overline{a_m} \\ \overline{a_1} & \leftarrow & \sim a_1 & \dots & \overline{a_m} & \leftarrow & \sim a_m \end{array}$$

by introducing new atoms $a', \overline{a_1}, \dots, \overline{a_m}$.

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Outline

Cardinality rule

- Idea Control (lower) cardinality of subsets
- Syntax A cardinality rule is the form

$$a_0 \leftarrow l \ \{ \ a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \ \}$$

where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$;
l is a non-negative integer.

- Informal meaning The head atom belongs to the stable model, if at least l elements of the body are included in the stable model
- Note l acts as a lower bound on the body
- Example `pass(c42) :- 2 { pass(a1), pass(a2), pass(a3) }.`
- Another Example $P = \{a \leftarrow 1\{b, c\}, \ b \leftarrow\}$ has stable model $\{a, b\}$

Potassco

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Embedding in normal rules

- Replace each cardinality rule

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \}$$

by $\quad a_0 \leftarrow ctr(1, l)$

where atom $ctr(i, j)$ represents the fact that at least j of the literals having an equal or greater index than i, are in a stable model

- The definition of $ctr/2$ is given for $0 \leq k \leq l$ by the rules

$$\begin{array}{rcll} ctr(i, k{+}1) & \leftarrow & ctr(i+1, k), a_i & \\ ctr(i, k) & \leftarrow & ctr(i+1, k) & \text{for } 1 \leq i \leq m \\ ctr(j, k{+}1) & \leftarrow & ctr(j+1, k), {\sim}a_j & \\ ctr(j, k) & \leftarrow & ctr(j+1, k) & \text{for } m+1 \leq j \leq n \\ ctr(n+1, 0) & \leftarrow & & \end{array}$$

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Embedding in normal rules

- Replace each cardinality rule

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n \; \}$$

by $\quad a_0 \leftarrow ctr(1, l)$

where atom $ctr(i, j)$ represents the fact that at least j of the literals having an equal or greater index than i, are in a stable model

- The definition of $ctr/2$ is given for $0 \leq k \leq l$ by the rules

$$\begin{array}{rcll} ctr(i, k{+}1) & \leftarrow & ctr(i+1, k), a_i & \\ ctr(i, k) & \leftarrow & ctr(i+1, k) & \text{for } 1 \leq i \leq m \\ ctr(j, k{+}1) & \leftarrow & ctr(j+1, k), \sim a_j & \\ ctr(j, k) & \leftarrow & ctr(j+1, k) & \text{for } m+1 \leq j \leq n \\ ctr(n+1, 0) & \leftarrow & & \end{array}$$

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An example

- Program $\{a \leftarrow,\; c \leftarrow 1\ \{a, b\}\}$ has the stable model $\{a, c\}$
- Translating the cardinality rule yields the rules

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. . . and vice versa

- A normal rule

$$a_0 \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n,$$

can be represented by the cardinality rule

$$a_0 \leftarrow n \; \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$$

Cardinality rules with upper bounds

- A rule of the form

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \} \; u$$

  where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$; l and u are non-negative integers

  stands for

$$\begin{array}{rcl} a_0 & \leftarrow & b, {\sim}c \\ b & \leftarrow & l \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \} \\ c & \leftarrow & u{+}1 \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \} \end{array}$$

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- The single constraint in the body of the above cardinality rule is referred to as a cardinality constraint

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Cardinality constraints

- Syntax A cardinality constraint is of the form

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- Informal meaning A cardinality constraint is satisfied by a stable model X, if the number of its contained literals satisfied by X is between l and u (inclusive)
- In other words, if

$$l \leq |\,(\{a_1, \ldots, a_m\} \cap X) \cup (\{a_{m+1}, \ldots, a_n\} \setminus X)\,| \leq u$$

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Cardinality constraints as heads

- A rule of the form

$$l\ \{a_1,\dots,a_m,\sim a_{m+1},\dots,\sim a_n\}\ u \leftarrow a_{n+1},\dots,a_o,\sim a_{o+1},\dots,\sim a_p$$

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  stands for

$$\begin{array}{rcl} b & \leftarrow & a_{n+1},\dots,a_o,\sim a_{o+1},\dots,\sim a_p \\ \{a_1,\dots,a_m\} & \leftarrow & b \\ c & \leftarrow & l\ \{a_1,\dots,a_m,,\sim a_{m+1},\dots,\sim a_n\}\ u \\ & \leftarrow & b,\sim c \end{array}$$

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- Example `1 { color(v42,red),color(v42,green),color(v42,blue) } 1.`

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Full-fledged cardinality rules

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$$l_0\ S_0\ u_0 \leftarrow l_1\ S_1\ u_1, \dots, l_n\ S_n\ u_n$$

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$$a \leftarrow b_1, \dots, b_n, \sim c_1, \dots, \sim c_n$$

$$\begin{array}{lcllcl} {S_0}^{+} & \leftarrow & a & & & \\ & \leftarrow & a, \sim b_0 & b_i & \leftarrow & l_i\ S_i \\ & \leftarrow & a, c_0 & c_i & \leftarrow & u_i{+}1\ S_i \end{array}$$

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Outline

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Weight rule

- Syntax A weight rule is the form

$$a_0 \leftarrow l \{ a_1 = w_1, \dots, a_m = w_m, \sim a_{m+1} = w_{m+1}, \dots, \sim a_n = w_n \}$$

  where $0 \leq m \leq n$ and each a_i is an atom;
  l and w_i are integers for $1 \leq i \leq n$

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- Meaning A weight constraint is satisfied by a stable model X, if

$$l \leq \left(\sum_{1 \leq i \leq m, a_i \in X} w_i + \sum_{m < i \leq n, a_i \notin X} w_i \right) \leq u$$

- Note (Cardinality and) weight constraints amount to constraints on (count and) sum aggregate functions

- Example `10 [course(db)=6,course(ai)=6,course(project)=8,course(xml)=3] 20`

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Outline

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Conditional literals (in *lparse* & *gringo* 3)

- Syntax A conditional literal is of the form

  $$\ell : \ell_1 : \cdots : \ell_n$$

  where ℓ and ℓ_i are literals for $0 \leq i \leq n$
- Informal meaning A conditional literal can be regarded as the list of elements in the set $\{\ell \mid \ell_1, \dots, \ell_n\}$
- Note The expansion of conditional literals is context dependent
- Example Given '`p(1). p(2). p(3). q(2).`'

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r(X):p(X):not q(X) :- r(X):p(X):not q(X), 1 {r(X):p(X):not q(X)}.
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is instantiated to

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Conditional literals (in *lparse* & *gringo* 3)

- Syntax A conditional literal is of the form

  $$\ell : \ell_1 : \cdots : \ell_n$$

  where ℓ and ℓ_i are literals for $0 \leq i \leq n$
- Informal meaning A conditional literal can be regarded as the list of elements in the set $\{\ell \mid \ell_1, \ldots, \ell_n\}$
- Note The expansion of conditional literals is context dependent
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Optimization statement

- Idea Express cost functions subject to minimization and/or maximization
- Syntax A minimize statement is of the form

$$minimize\{\ \ell_1 = w_1@p_1, \dots, \ell_n = w_n@p_n\ \}.$$

where each ℓ_i is a literal; and w_i and p_i are integers for $1 \leq i \leq n$

Priority levels, p_i, allow for representing lexicographically ordered minimization objectives

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- A maximize statement of the form

$$maximize\{ \ell_1 = w_1 @ p_1, \dots, \ell_n = w_n @ p_n \}$$

  stands for $minimize\{ \ell_1 = -w_1 @ p_1, \dots, \ell_n = -w_n @ p_n \}$

- Example When configuring a computer, we may want to maximize hard disk capacity, while minimizing price

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#maximize[ hd(1)=250@1, hd(2)=500@1, hd(3)=750@1, hd(4)=1000@1 ].
#minimize[ hd(1)=30@2, hd(2)=40@2, hd(3)=60@2, hd(4)=80@2 ].
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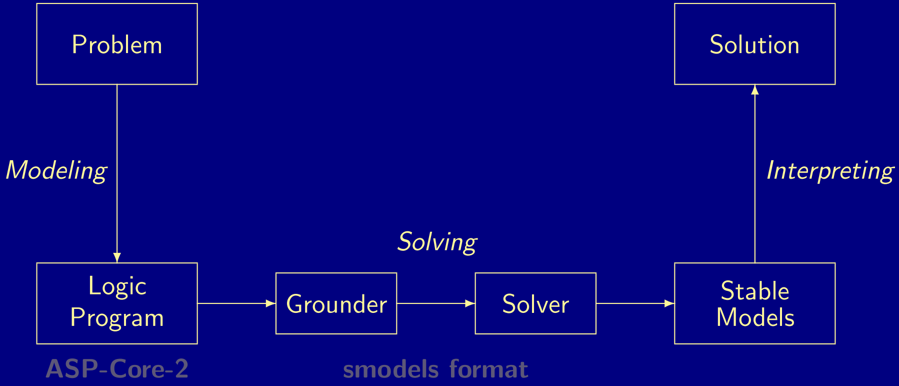
smodels format

- Logic programs in *smodels* format consist of
  - normal rules
  - choice rules
  - cardinality rules
  - weight rules
  - optimization statements
- Such a format is obtained by grounders *lparse* and *gringo*

Outline

ASP-Core-2



- smodels format is a machine-oriented standard for ground programs
- ASP-Core-2 is a user-oriented standard for (non-ground) programs, extending the input languages of *dlv* and *gringo* series 3

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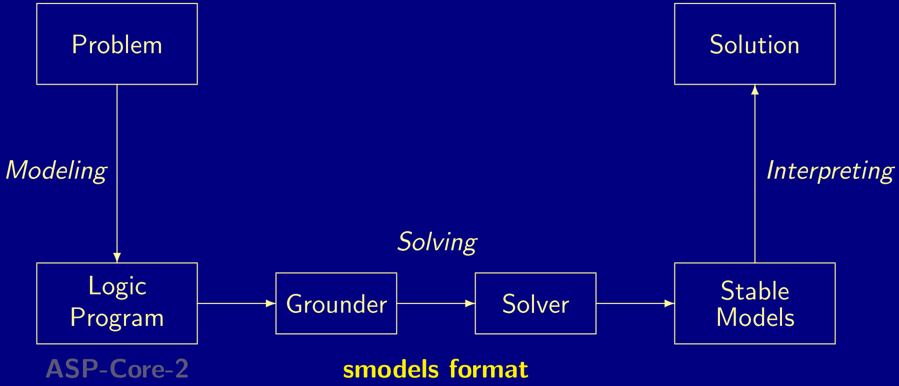

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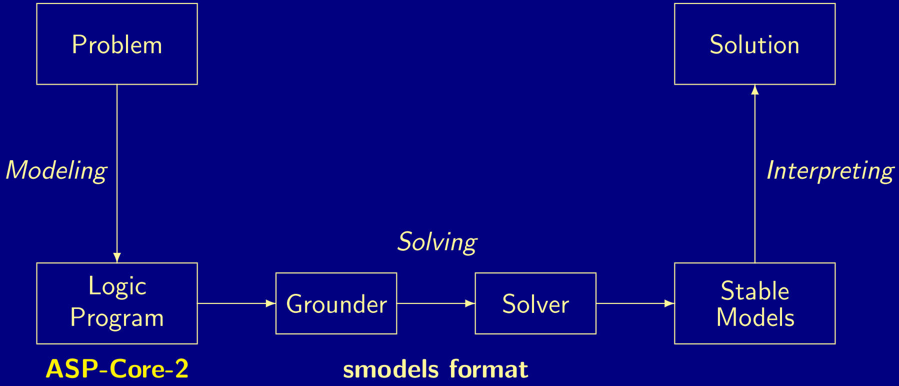

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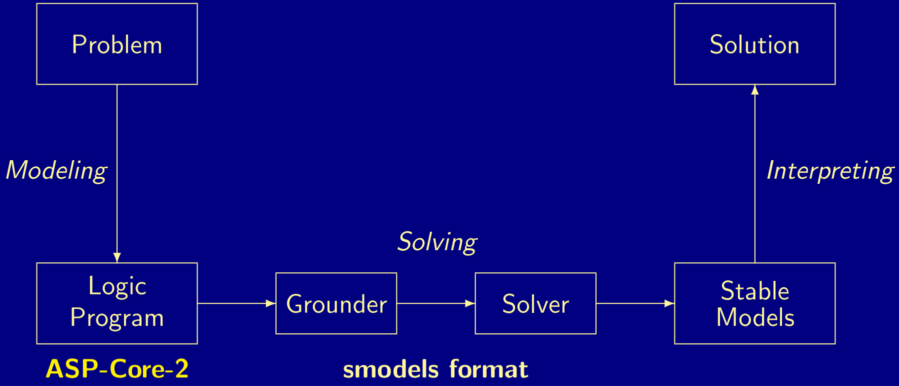

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Aggregates

- Syntax ASP-Core-2 aggregates are of the form

$$t_1 \prec_1 \#\mathtt{A}\{t_{1_1},\dots,t_{m_1} : \ell_{1_1},\dots,\ell_{n_1}\} \prec_2 t_2$$

  where
  - $\#\mathtt{A} \in \{\#\mathtt{count}, \#\mathtt{sum}, \#\mathtt{max}, \#\mathtt{min}\}$
  - $\prec_1, \prec_2 \in \{<, \leq, =, \neq, >, \geq\}$
  - $t_{1_1},\dots,t_{m_1}$ and t_1, t_2 are terms
  - $\ell_{1_1},\dots,\ell_{n_1}$ are literals

- Example Weight constraint

```
10 [course(db)=6,course(ai)=6,course(project)=8,course(xml)=3] 20
```

  is written as an ASP-Core-2 aggregate as

$$10 \leq \#\mathtt{sum}\{\mathtt{6,db:course(db);\ 6,ai:course(ai);\ 8,project:course(project);\ 3,xml:course(xml)}\} \leq 20$$

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$$:\sim a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n. \, [w@p, t_1, \dots, t_m]$$

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- Example Minimize statement

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#minimize[ hd(1)=30@2, hd(2)=40@2, hd(3)=60@2, hd(4)=80@2 ].
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gringo 4

- The input language of *gringo* series 4 comprises
  - ASP-Core-2
  - concepts from *gringo* 3 (conditional literals, `#show` directives, ...)
- Example The *gringo* 3 rule

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r(X):p(X):not q(X) :- r(X):p(X):not q(X), 1 {r(X):p(X):not q(X)}.
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can be written as follows in the language of *gringo* 4:

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Term-based `#show` directives as in

```
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Many example programs given in this tutorial are written for *gringo* 3

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Language Extensions: Overview

Outline

Motivation

- Classical versus default negation
  - Symbol $\neg$ and $\sim$
  - Idea
    - $\neg a \ \approx\ \neg a \in X$
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  - Example
    - $cross \leftarrow \neg train$
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Classical negation

- We consider logic programs in negation normal form
  - That is, classical negation is applied to atoms only
- Given an alphabet $\mathcal{A}$ of atoms, let $\overline{\mathcal{A}} = \{\neg a \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \overline{\mathcal{A}} = \emptyset$
- Given a program P over $\mathcal{A}$, classical negation is encoded by adding

$$P^{\neg} = \{a \leftarrow b, \neg b \mid a \in (\mathcal{A} \cup \overline{\mathcal{A}}), b \in \mathcal{A}\}$$

- A set X of atoms is a stable model of a program P over $\mathcal{A} \cup \overline{\mathcal{A}}$, if X is a stable model of $P \cup P^{\neg}$

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An example

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induces

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$$P^{\neg} = \left\{ \begin{array}{rclrclrcl} a & \leftarrow & a, \neg a & a & \leftarrow & b, \neg b & a & \leftarrow & c, \neg c \\ \neg a & \leftarrow & a, \neg a & \neg a & \leftarrow & b, \neg b & \neg a & \leftarrow & c, \neg c \\ b & \leftarrow & a, \neg a & b & \leftarrow & b, \neg b & b & \leftarrow & c, \neg c \\ \neg b & \leftarrow & a, \neg a & \neg b & \leftarrow & b, \neg b & \neg b & \leftarrow & c, \neg c \\ c & \leftarrow & a, \neg a & c & \leftarrow & b, \neg b & c & \leftarrow & c, \neg c \\ \neg c & \leftarrow & a, \neg a & \neg c & \leftarrow & b, \neg b & \neg c & \leftarrow & c, \neg c \end{array} \right\}$$

- The stable models of P are given by the ones of $P \cup P^{\neg}$, viz $\{a\}$

Properties

- The only inconsistent stable "model" is $X = \mathcal{A} \cup \overline{\mathcal{A}}$
- Note Strictly speaking, an inconsistemt set like $\mathcal{A} \cup \overline{\mathcal{A}}$ is not a model
- For a logic program P over $\mathcal{A} \cup \overline{\mathcal{A}}$, exactly one of the following two cases applies:
  1. All stable models of P are consistent or
  2. $X = \mathcal{A} \cup \overline{\mathcal{A}}$ is the only stable model of P

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Train spotting

- $P_1 = \{cross \leftarrow {\sim}train\}$
  - stable model: $\{cross\}$
- $P_2 = \{cross \leftarrow \neg train\}$
  - stable model: $\emptyset$
- $P_3 = \{cross \leftarrow \neg train,\ \neg train \leftarrow\}$
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Default negation in rule heads

- We consider logic programs with default negation in rule heads
- Given an alphabet $\mathcal{A}$ of atoms, let $\widetilde{\mathcal{A}} = \{\widetilde{a} \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \widetilde{\mathcal{A}} = \emptyset$
- Given a program P over $\mathcal{A}$, consider the program

$$\begin{array}{rcl}\widetilde{P} & = & \{r \in P \mid head(r) \neq {\sim}a\} \\ & & \cup\ \{\leftarrow body(r) \cup \{{\sim}\widetilde{a}\} \mid r \in P \text{ and } head(r) = {\sim}a\} \\ & & \cup\ \{\widetilde{a} \leftarrow {\sim}a \mid r \in P \text{ and } head(r) = {\sim}a\}\end{array}$$

- A set X of atoms is a stable model of a program P (with default negation in rule heads) over $\mathcal{A}$,
  if $X = Y \cap \mathcal{A}$ for some stable model Y of $\widetilde{P}$ over $\mathcal{A} \cup \widetilde{\mathcal{A}}$

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Outline

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Disjunctive logic programs

- A disjunctive rule, r, is of the form

$$a_1 \,;\dots;\, a_m \leftarrow a_{m+1},\dots,a_n,\sim a_{n+1},\dots,\sim a_o$$

where $0 \leq m \leq n \leq o$ and each a_i is an atom for $0 \leq i \leq o$

- A disjunctive logic program is a finite set of disjunctive rules
- Notation

$$\begin{aligned}
head(r) &= \{a_1,\dots,a_m\}\\
body(r) &= \{a_{m+1},\dots,a_n,\sim a_{n+1},\dots,\sim a_o\}\\
body(r)^+ &= \{a_{m+1},\dots,a_n\}\\
body(r)^- &= \{a_{n+1},\dots,a_o\}\\
atom(P) &= \bigcup_{r\in P}\left(head(r)\cup body(r)^+\cup body(r)^-\right)\\
body(P) &= \{body(r)\mid r\in P\}
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- A program is called positive if $body(r)^- = \emptyset$ for all its rules

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Stable models

- Positive programs
  - A set X of atoms is closed under a positive program P iff for any $r \in P$, $head(r) \cap X \neq \emptyset$ whenever $body(r)^+ \subseteq X$
    - X corresponds to a model of P (seen as a formula)
  - The set of all $\subseteq$-minimal sets of atoms being closed under a positive program P is denoted by $\min_\subseteq(P)$
    - $\min_\subseteq(P)$ corresponds to the $\subseteq$-minimal models of P (ditto)
- Disjunctive programs
  - The reduct, P^X, of a disjunctive program P relative to a set X of atoms is defined by

$$P^X = \{head(r) \leftarrow body(r)^+ \mid r \in P \text{ and } body(r)^- \cap X = \emptyset\}$$

  - A set X of atoms is a stable model of a disjunctive program P, if $X \in \min_\subseteq(P^X)$

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A "positive" example

$$P = \left\{ \begin{array}{lcr} a & \leftarrow & \\ b \,;\, c & \leftarrow & a \end{array} \right\}$$

- The sets $\{a, b\}$, $\{a, c\}$, and $\{a, b, c\}$ are closed under P
- We have $\min_{\subseteq}(P) = \{\{a, b\}, \{a, c\}\}$

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Graph coloring (reloaded)

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).


color(X,r) | color(X,b) | color(X,g) :- node(X).


:- edge(X,Y), color(X,C), color(Y,C).
```

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col(r).   col(b).   col(g).

color(X,C) : col(C) :- node(X).

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```

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More Examples

- $P_1 = \{a \,;\, b \,;\, c \leftarrow\}$
  - stable models $\{a\}$, $\{b\}$, and $\{c\}$
- $P_2 = \{a \,;\, b \,;\, c \leftarrow \,,\, \leftarrow a\}$
  - stable models $\{b\}$ and $\{c\}$
- $P_3 = \{a \,;\, b \,;\, c \leftarrow \,,\, \leftarrow a \,,\, b \leftarrow c \,,\, c \leftarrow b\}$
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- $P_1 = \{a \,;\, b \,;\, c \leftarrow\}$
  - stable models $\{a\}$, $\{b\}$, and $\{c\}$
- $P_2 = \{a \,;\, b \,;\, c \leftarrow \;,\; \leftarrow a\}$
  - stable models $\{b\}$ and $\{c\}$
- $P_3 = \{a \,;\, b \,;\, c \leftarrow \;,\; \leftarrow a \;,\; b \leftarrow c \;,\; c \leftarrow b\}$
  - stable model $\{b, c\}$
- $P_4 = \{a \,;\, b \leftarrow c \;,\; b \leftarrow \sim a, \sim c \;,\; a \,;\, c \leftarrow \sim b\}$
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Some properties

- A disjunctive logic program may have zero, one, or multiple stable models
- If X is a stable model of a disjunctive logic program P, then X is a model of P (seen as a formula)
- If X and Y are stable models of a disjunctive logic program P, then $X \not\subset Y$
- If $A \in X$ for some stable model X of a disjunctive logic program P, then there is a rule $r \in P$ such that $body(r)^+ \subseteq X$, $body(r)^- \cap X = \emptyset$, and $head(r) \cap X = \{A\}$

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An example with variables

$$P = \left\{ \begin{array}{lcl} a(1,2) & \leftarrow & \\ b(X)\,;c(Y) & \leftarrow & a(X,Y), \sim c(Y) \end{array} \right\}$$

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For every stable model X of P, we have

- $a(1,2) \in X$ and
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Default negation in rule heads

- Consider disjunctive rules of the form

$$a_1 ; \dots ; a_m ; \sim a_{m+1} ; \dots ; \sim a_n \leftarrow a_{n+1}, \dots, a_o, \sim a_{o+1}, \dots, \sim a_p$$

  where $0 \leq m \leq n \leq o \leq p$ and each a_i is an atom for $0 \leq i \leq p$

- Given a program P over $\mathcal{A}$, consider the program

$$\begin{aligned} \widetilde{P} \quad = \quad & \{head(r)^+ \leftarrow body(r) \cup \{\sim \widetilde{a} \mid a \in head(r)^-\} \mid r \in P\} \\ & \cup \ \{\widetilde{a} \leftarrow \sim a \mid r \in P \text{ and } a \in head(r)^-\} \end{aligned}$$

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- The program

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yields

$$\widetilde{P} = \{a \leftarrow {\sim}\widetilde{a}\} \cup \{\widetilde{a} \leftarrow {\sim}a\}$$

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Outline

Propositional theories

- Formulas are formed from
  - atoms in $\mathcal{A}$
  - $\bot$

  using
  - conjunction ($\wedge$)
  - disjunction ($\vee$)
  - implication ($\rightarrow$)
- Notation

$$\top = (\bot \rightarrow \bot)$$
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Reduct

- The satisfaction relation $X \models \phi$ between a set X of atoms and a (set of) formula(s) ϕ is defined as in propositional logic
- The reduct, ϕ^X, of a formula ϕ relative to a set X of atoms is defined recursively as follows:

$$\phi^X = \bot \quad \text{if } X \not\models \phi$$
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If $\phi = \sim\psi = (\psi \rightarrow \bot)$,
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Stable models

- A set X of atoms satisfies a propositional theory Φ, written $X \models \Phi$, if $X \models \phi$ for each $\phi \in \Phi$
- The set of all $\subseteq$-minimal sets of atoms satisfying a propositional theory Φ is denoted by $\min_{\subseteq}(\Phi)$
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- If X is a stable model of Φ, then
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- A set X of atoms satisfies a propositional theory Φ, written $X \models \Phi$, if $X \models \phi$ for each $\phi \in \Phi$
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Two examples

- $\Phi_1 = \{p \vee (p \rightarrow (q \wedge r))\}$
  - For $X = \{p, q, r\}$, we get
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  - $\tau[\bot] = \bot$
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  - $\tau[\phi] = \phi$ if ϕ is an atom
  - $\tau[{\sim}\phi] = {\sim}\tau[\phi]$
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- The translation of a logic program P is $\tau[P] = \{\tau[r] \mid r \in P\}$
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Logic programs as propositional theories

- The normal logic program $P = \{p \leftarrow {\sim}q,\ q \leftarrow {\sim}p\}$ corresponds to $\tau[P] = \{{\sim}q \rightarrow p,\ {\sim}p \rightarrow q\}$
  - stable models: $\{p\}$ and $\{q\}$
- The disjunctive logic program $P = \{p\,;q \leftarrow\}$ corresponds to $\tau[P] = \{\top \rightarrow p \vee q\}$
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Grounding: Overview

Grounding by example

$d(a)$
$d(c)$
$d(d)$

$p(a, b)$
$p(b, c)$
$p(c, d)$
$p(X, Z) \leftarrow p(X, Y), p(Y, Z)$

$q(a)$
$q(b)$
$q(X) \leftarrow {\sim}r(X), d(X)$

$r(X) \leftarrow {\sim}q(X), d(X)$

$s(X) \leftarrow {\sim}r(X), p(X, Y), q(Y)$

Grounding by example

Safe ?

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$d(c)$
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Grounding by example

	Safe ?
$d(a)$	✔
$d(c)$	✔
$d(d)$	✔
$p(a,b)$	✔
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$q(a)$	✔
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Grounding by example

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$d(a)$	✔
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$s(X) \leftarrow \sim r(X), p(X, Y), q(Y)$	✔

Match

- A substitution is a mapping from variables to terms
- Given sets B and D of atoms, a substitution θ is a match of B in D, if $B\theta \subseteq D$
- Given a set B of atoms and a set D of ground atoms, define

$$\Theta(B, D) = \{ \theta \mid \theta \text{ is a } \subseteq\text{-minimal match of } B \text{ in } D \}$$

- Example $\{X \mapsto 1\}$ and $\{X \mapsto 2\}$ are $\subseteq$-minimal matches of $\{p(X)\}$ in $\{p(1), p(2), p(3)\}$, while match $\{X \mapsto 1, Y \mapsto 2\}$ is not

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$$\Theta(B, D) = \{ \theta \mid \theta \text{ is a } \subseteq\text{-minimal match of } B \text{ in } D \}$$

- Example $\{X \mapsto 1\}$ and $\{X \mapsto 2\}$ are $\subseteq$-minimal matches of $\{p(X)\}$ in $\{p(1), p(2), p(3)\}$, while match $\{X \mapsto 1, Y \mapsto 2\}$ is not

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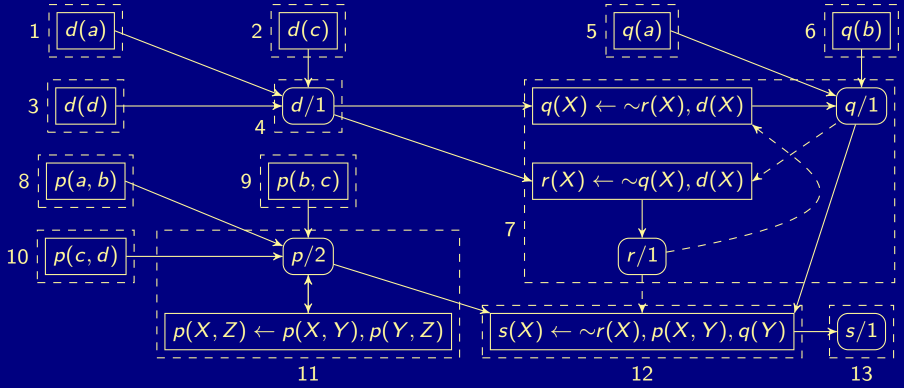
Naive instantiation

Algorithm 1: NAIVEINSTANTIATION

Input : A safe (first-order) logic program P
Output : A ground logic program P'

$D := \emptyset$
$P' := \emptyset$
repeat
  $D' := D$
  foreach $r \in P$ **do**
    $B := body(r)^+$
    foreach $\theta \in \Theta(B, D)$ **do**
      $D := D \cup \{head(r)\theta\}$
      $P' := P' \cup \{r\theta\}$
until $D = D'$

Predicate-rule dependency graph



Instantiation

SCC	$\Theta(B, D)$	D	P'
1	$\{\emptyset\}$	$d(a)$	$d(a) \leftarrow$
2	$\{\emptyset\}$	$d(c)$	$d(c) \leftarrow$
3	$\{\emptyset\}$	$d(d)$	$d(d) \leftarrow$
5	$\{\emptyset\}$	$q(a)$	$q(a) \leftarrow$
6	$\{\emptyset\}$	$q(b)$	$q(b) \leftarrow$
7	$\{\{X \mapsto a\},$		$q(a) \leftarrow \sim r(a), d(a)$
	$\{X \mapsto c\},$	$q(c)$	$q(c) \leftarrow \sim r(c), d(c)$
	$\{X \mapsto d\},$	$q(d)$	$q(d) \leftarrow \sim r(d), d(d)$
	$\{X \mapsto a\},$		$r(a) \leftarrow \sim q(a), d(a)$
	$\{X \mapsto c\},$	$r(c)$	$r(c) \leftarrow \sim q(c), d(c)$
	$\{X \mapsto d\}\}$	$r(d)$	$r(d) \leftarrow \sim q(d), d(d)$

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Instantiation

SCC	$\Theta(B,D)$	D	P'
8	$\{\emptyset\}$	$p(a,b)$	$p(a,b) \leftarrow$
9	$\{\emptyset\}$	$p(b,c)$	$p(b,c) \leftarrow$
10	$\{\emptyset\}$	$p(c,d)$	$p(c,d) \leftarrow$
11	$\{\{X \mapsto a, Y \mapsto b, Z \mapsto c\},$ $\{X \mapsto b, Y \mapsto c, Z \mapsto d\}\}$	$p(a,c)$ $p(b,d)$	$p(a,c) \leftarrow p(a,b), p(b,c)$ $p(b,d) \leftarrow p(b,c), p(c,d)$
	$\{\{X \mapsto a, Y \mapsto c, Z \mapsto d\},$ $\{X \mapsto a, Y \mapsto b, Z \mapsto d\}\}$	$p(a,d)$	$p(a,d) \leftarrow p(a,c), p(c,d)$ $p(a,d) \leftarrow p(a,b), p(b,d)$
12	$\{\{X \mapsto a, Y \mapsto b\},$ $\{X \mapsto a, Y \mapsto c\},$ $\{X \mapsto a, Y \mapsto d\},$ $\{X \mapsto b, Y \mapsto c\},$ $\{X \mapsto b, Y \mapsto d\},$ $\{X \mapsto c, Y \mapsto d\}\}$	$s(a)$ $s(b)$ $s(c)$	$s(a) \leftarrow \sim r(a), p(a,b), q(b)$ $s(a) \leftarrow \sim r(a), p(a,c), q(c)$ $s(a) \leftarrow \sim r(a), p(a,d), q(d)$ $s(b) \leftarrow \sim r(b), p(b,c), q(c)$ $s(b) \leftarrow \sim r(b), p(b,d), q(d)$ $s(c) \leftarrow \sim r(c), p(c,d), q(d)$

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Computational Aspects: Overview

23 Consequence operator

24 Computation from first principles

25 Complexity

Outline

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Consequence operator

- Let P be a positive program and X a set of atoms
  - The consequence operator T_P is defined as follows:

$$T_P X = \{head(r) \mid r \in P \text{ and } body(r) \subseteq X\}$$

  - Iterated applications of T_P are written as T_P^j for $j \geq 0$, where
    - $T_P^0 X = X$ and
    - $T_P^i X = T_P T_P^{i-1} X$ for $i \geq 1$
- For any positive program P, we have
  - $Cn(P) = \bigcup_{i \geq 0} T_P^i \emptyset$
  - $X \subseteq Y$ implies $T_P X \subseteq T_P Y$
  - $Cn(P)$ is the smallest fixpoint of T_P

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An example

- Consider the program

$$P = \{p \leftarrow,\ q \leftarrow,\ r \leftarrow p,\ s \leftarrow q, t,\ t \leftarrow r,\ u \leftarrow v\}$$

- We get

$$\begin{array}{lllllll}
T_P^0\emptyset & = & \emptyset & & & & \\
T_P^1\emptyset & = & \{p,q\} & = & T_P T_P^0\emptyset & = & T_P\emptyset \\
T_P^2\emptyset & = & \{p,q,r\} & = & T_P T_P^1\emptyset & = & T_P\{p,q\} \\
T_P^3\emptyset & = & \{p,q,r,t\} & = & T_P T_P^2\emptyset & = & T_P\{p,q,r\} \\
T_P^4\emptyset & = & \{p,q,r,t,s\} & = & T_P T_P^3\emptyset & = & T_P\{p,q,r,t\} \\
T_P^5\emptyset & = & \{p,q,r,t,s\} & = & T_P T_P^4\emptyset & = & T_P\{p,q,r,t,s\} \\
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\end{array}$$

- $Cn(P) = \{p,q,r,t,s\}$ is the smallest fixpoint of T_P because
  - $T_P\{p,q,r,t,s\} = \{p,q,r,t,s\}$ and
  - $T_P X \neq X$ for each $X \subset \{p,q,r,t,s\}$

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Outline

Approximating stable models

- First Idea Approximate a stable model X by two sets of atoms L and U such that $L \subseteq X \subseteq U$
  - L and U constitute lower and upper bounds on X
  - L and $(\mathcal{A} \setminus U)$ describe a three-valued model of the program
- Observation

$$X \subseteq Y \text{ implies } P^Y \subseteq P^X \text{ implies } Cn(P^Y) \subseteq Cn(P^X)$$

- Properties Let X be a stable model of normal logic program P
  - If $L \subseteq X$, then $X \subseteq Cn(P^L)$
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Approximating stable models

- Second Idea

  repeat

  replace L **by** $L \cup Cn(P^U)$
  replace U **by** $U \cap Cn(P^L)$

  until *L and U do not change anymore*

- Observations
  - At each iteration step
    - L becomes larger (or equal)
    - U becomes smaller (or equal)
  - $L \subseteq X \subseteq U$ is invariant for every stable model X of P

  If $L \not\subseteq U$, then P has no stable model
  If $L = U$, then L is a stable model of P

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The simplistic expand algorithm

expand$_P(L, U)$
repeat
$L' \leftarrow L$
$U' \leftarrow U$
$L \leftarrow L' \cup Cn(P^{U'})$
$U \leftarrow U' \cap Cn(P^{L'})$
if $L \not\subseteq U$ **then return**
until $L = L'$ and $U = U'$

An example

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, \sim c \\ d \leftarrow b, \sim e \\ e \leftarrow \ \sim d \end{array} \right\}$$

	L'	$Cn(P^{U'})$	L	U'	$Cn(P^{L'})$	U
1	$\emptyset$	$\{a\}$	$\{a\}$	$\{a,b,c,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$
2	$\{a\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$
3	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$

- Note We have $\{a,b\} \subseteq X$ and $(\mathcal{A} \setminus \{a,b,d,e\}) \cap X = (\{c\} \cap X) = \emptyset$ for every stable model X of P

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2	$\{a\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$
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The simplistic expand algorithm

- **expand_P**
  - tightens the approximation on stable models
  - is stable model preserving

Let's expand with d !

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, \sim c \\ d \leftarrow b, \sim e \\ e \leftarrow \ \sim d \end{array} \right\}$$

	L'	$Cn(P^{U'})$	L	U'	$Cn(P^{L'})$	U
1	$\{d\}$	$\{a\}$	$\{a, d\}$	$\{a, b, c, d, e\}$	$\{a, b, d\}$	$\{a, b, d\}$
2	$\{a, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$
3	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$

- Note $\{a, b, d\}$ is a stable model of P

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2	$\{a, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$
3	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$

- Note $\{a, b, d\}$ is a stable model of P

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Let's expand with $\sim d$!

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, \sim c \\ d \leftarrow b, \sim e \\ e \leftarrow \ \sim d \end{array} \right\}$$

	L'	$Cn(P^{U'})$	L	U'	$Cn(P^{L'})$	U
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- Note $\{a, b, e\}$ is a stable model of P

Let's expand with $\sim d$!

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A simplistic solving algorithm

$solve_P(L, U)$

$(L, U) \leftarrow expand_P(L, U)$ // *propagation*

if $L \not\subseteq U$ **then failure** // *failure*

if $L = U$ **then output** L // *success*

else choose $a \in U \setminus L$ // *choice*

$solve_P(L \cup \{a\}, U)$

$solve_P(L, U \setminus \{a\})$

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A simplistic solving algorithm

- Close to the approach taken by the ASP solver `smodels`, inspired by the Davis-Putman-Logemann-Loveland (DPLL) procedure
  - Backtracking search building a binary search tree
  - A node in the search tree corresponds to a three-valued interpretation
  - The search space is pruned by
    - deriving deterministic consequences and detecting conflicts (**expand**)
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Outline

Complexity

Let a be an atom and X be a set of atoms

- For a positive normal logic program P:
  - Deciding whether X is the stable model of P is P-complete
  - Deciding whether a is in the stable model of P is P-complete
- For a normal logic program P:
  - Deciding whether X is a stable model of P is P-complete
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- For a normal logic program P with optimization statements:
  - Deciding whether X is an optimal stable model of P is $co\text{-}NP$-complete
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Complexity

Let a be an atom and X be a set of atoms

- For a positive disjunctive logic program P:
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Axiomatic Characterization: Overview

Outline

Motivation

- Question Is there a propositional formula $F(P)$ such that the models of $F(P)$ correspond to the stable models of P ?
- Observation Although each atom is defined through a set of rules, each such rule provides only a sufficient condition for its head atom
- Idea The idea of program completion is to turn such implications into a definition by adding the corresponding necessary counterpart

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Program completion

Let P be a normal logic program

- The completion $CF(P)$ of P is defined as follows

$$CF(P) = \left\{ a \leftrightarrow \bigvee_{r \in P, head(r)=a} BF(body(r)) \mid a \in atom(P) \right\}$$

where

$$BF(body(r)) = \bigwedge_{a \in body(r)^+} a \wedge \bigwedge_{a \in body(r)^-} \neg a$$

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An example

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow \sim a \\ c \leftarrow a, \sim d \\ d \leftarrow \sim c, \sim e \\ e \leftarrow b, \sim f \\ e \leftarrow e \end{array} \right\} \qquad CF(P) = \left\{ \begin{array}{l} a \leftrightarrow \top \\ b \leftrightarrow \neg a \\ c \leftrightarrow a \wedge \neg d \\ d \leftrightarrow \neg c \wedge \neg e \\ e \leftrightarrow (b \wedge \neg f) \vee e \\ f \leftrightarrow \bot \end{array} \right\}$$

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A closer look

- $CF(P)$ is logically equivalent to $\overleftarrow{CF}(P) \cup \overrightarrow{CF}(P)$, where

$$\overleftarrow{CF}(P) = \left\{ a \leftarrow \bigvee_{B \in body_P(a)} BF(B) \mid a \in atom(P) \right\}$$
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$$body_P(a) = \{ body(r) \mid r \in P \text{ and } head(r) = a \}$$

- $\overleftarrow{CF}(P)$ characterizes the classical models of P
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Supported models

- Every stable model of P is a model of $CF(P)$, but not vice versa
- Models of $CF(P)$ are called the supported models of P

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Outline

The mismatch

- Question What causes the mismatch between supported models and stable models?
- Hint Consider the unstable yet supported model $\{a, c, e\}$ of P !
- Answer Cyclic derivations are causing the mismatch between supported and stable models

  Atoms in a stable model can be "derived" from a program in a finite number of steps

  Atoms in a cycle (not being "supported from outside the cycle") cannot be "derived" from a program in a finite number of steps

  But such atoms do not contradict the completion of a program and do thus not eliminate an unstable supported model

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Non-cyclic derivations

Let X be a stable model of normal logic program P

- For every atom $A \in X$, there is a finite sequence of positive rules

  $$\langle r_1, \dots, r_n \rangle$$

  such that
  1. $head(r_1) = A$
  2. $body(r_i)^+ \subseteq \{head(r_j) \mid i < j \leq n\}$ for $1 \leq i \leq n$
  3. $r_i \in P^X$ for $1 \leq i \leq n$

- That is, each atom of X has a non-cyclic derivation from P^X

- Example There is no finite sequence of rules providing a derivation for e from $P^{\{a,c,e\}}$

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Positive atom dependency graph

- The origin of (potential) circular derivations can be read off the positive atom dependency graph $G(P)$ of a logic program P given by

$$(atom(P), \{(a, b) \mid r \in P, a \in body(r)^+, head(r) = b\})$$

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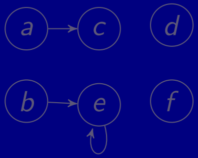

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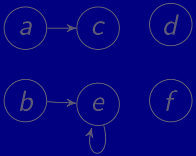

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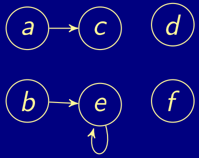

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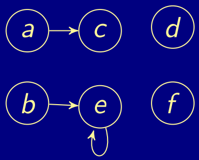

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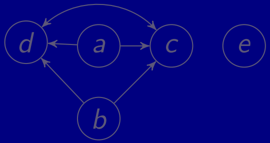

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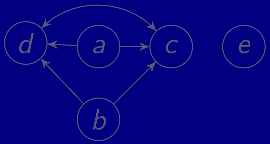

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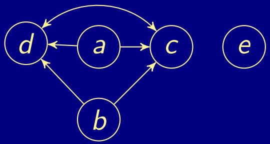

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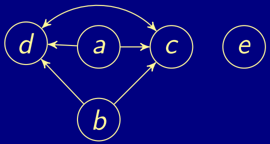

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Outline

Motivation

- Question Is there a propositional formula $F(P)$ such that the models of $F(P)$ correspond to the stable models of P ?
- Observation Starting from the completion of a program, the problem boils down to eliminating the circular support of atoms holding in the supported models of the program
- Idea Add formulas prohibiting circular support of sets of atoms
- Note Circular support between atoms a and b is possible, if a has a path to b and b has a path to a in the program's positive atom dependency graph

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Loops

Let P be a normal logic program, and
let $G(P) = (atom(P), E)$ be the positive atom dependency graph of P

- A set $\emptyset \subset L \subseteq atom(P)$ is a loop of P
  if it induces a non-trivial strongly connected subgraph of $G(P)$

  That is, each pair of atoms in L is connected by a path of non-zero length in $(L, E \cap (L \times L))$

- We denote the set of all loops of P by $loop(P)$

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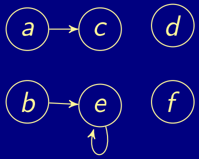

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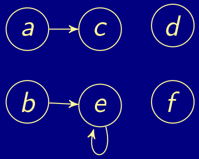

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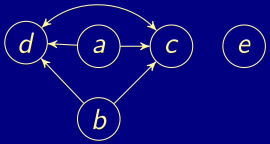

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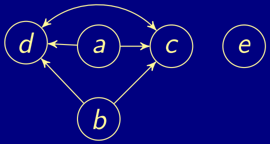

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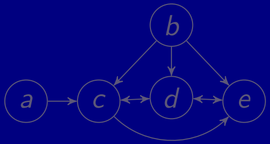

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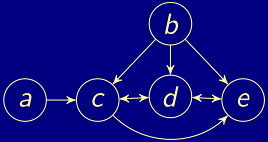

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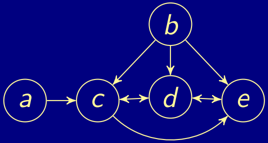

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Loop formulas

Let P be a normal logic program

- For $L \subseteq atom(P)$, define the external supports of L for P as

$$ES_P(L) = \{r \in P \mid head(r) \in L \text{ and } body(r)^+ \cap L = \emptyset\}$$

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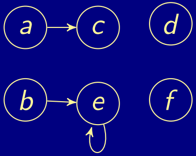

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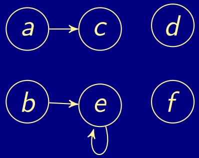

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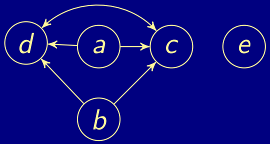

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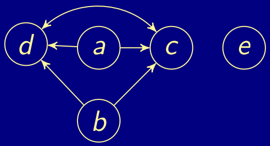

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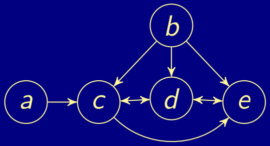

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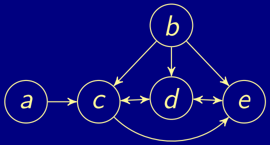

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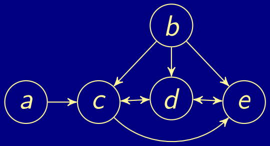

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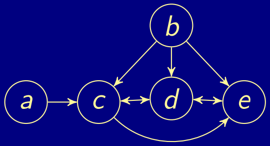

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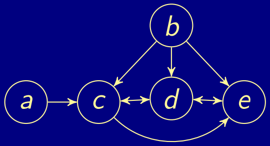

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Lin-Zhao Theorem

Theorem

Let P be a normal logic program and $X \subseteq atom(P)$
Then, X is a stable model of P iff $X \models CF(P) \cup LF(P)$

Loops and loop formulas: Properties

Let X be a supported model of normal logic program P

- Then, X is a stable model of P iff
  - $X \models \{LF_P(U) \mid U \subseteq atom(P)\}$;
  - $X \models \{LF_P(U) \mid U \subseteq X\}$;
  - $X \models \{LF_P(L) \mid L \in loop(P)\}$, that is, $X \models LF(P)$;
  - $X \models \{LF_P(L) \mid L \in loop(P)$ and $L \subseteq X\}$

- Note If X is not a stable model of P,
  then there is a loop $L \subseteq X \setminus Cn(P^X)$ such that $X \not\models LF_P(L)$

Loops and loop formulas: Properties

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- Then, X is a stable model of P iff
  - $X \models \{LF_P(U) \mid U \subseteq atom(P)\}$;
  - $X \models \{LF_P(U) \mid U \subseteq X\}$;
  - $X \models \{LF_P(L) \mid L \in loop(P)\}$, that is, $X \models LF(P)$;
  - $X \models \{LF_P(L) \mid L \in loop(P) \text{ and } L \subseteq X\}$

- Note If X is not a stable model of P, then there is a loop $L \subseteq X \setminus Cn(P^X)$ such that $X \not\models LF_P(L)$

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Loops and loop formulas: Properties (ctd)

If $\mathcal{P} \not\subseteq \mathcal{NC}^1/poly$,[1] then there is no translation $\mathcal{T}$ from logic programs to propositional formulas such that, for each normal logic program P, both of the following conditions hold:

1. The propositional variables in $\mathcal{T}[P]$ are a subset of $atom(P)$
2. The size of $\mathcal{T}[P]$ is polynomial in the size of P
   - Note Every vocabulary-preserving translation from normal logic programs to propositional formulas must be exponential (in the worst case).

Observations

- Translation $CF(P) \cup LF(P)$ preserves the vocabulary of P
- The number of loops in $loop(P)$ may be exponential in $|atom(P)|$

[1] A conjecture from the theory of complexity that is widely believed to be true.

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Operational Characterization: Overview

Outline

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Interlude: Partial interpretations

or: 3-valued interpretations

A partial interpretation maps atoms onto truth values *true*, *false*, and *unknown*

- Representation $\langle T, F\rangle$, where
  - T is the set of all *true* atoms and
  - F is the set of all *false* atoms
  - Truth of atoms in $\mathcal{A} \setminus (T \cup F)$ is *unknown*
- Properties
  - $\langle T, F\rangle$ is conflicting if $T \cap F \neq \emptyset$
  - $\langle T, F\rangle$ is total if $T \cup F = \mathcal{A}$ and $T \cap F = \emptyset$
- Definition For $\langle T_1, F_1\rangle$ and $\langle T_2, F_2\rangle$, define
  - $\langle T_1, F_1\rangle \sqsubseteq \langle T_2, F_2\rangle$ iff $T_1 \subseteq T_2$ and $F_1 \subseteq F_2$
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Outline

Basic idea

- Idea Extend T_P to normal logic programs
- Logical background The idea is to turn a program's completion into an operator such that
  - the head atom of a rule must be *true* if the rule's body is *true*
  - an atom must be *false* if the body of each rule having it as head is *false*

Definition

- Let P be a normal logic program
- Define

$$\mathbf{\Phi}_P\langle T,F\rangle = \langle \mathbf{T}_P\langle T,F\rangle, \mathbf{F}_P\langle T,F\rangle\rangle$$

where

$$\mathbf{T}_P\langle T,F\rangle = \{head(r) \mid r\in P, body(r)^+ \subseteq T, body(r)^- \subseteq F\}$$

$$\mathbf{F}_P\langle T,F\rangle = \{a \in atom(P) \mid \quad body(r)^+ \cap F \neq \emptyset \text{ or } body(r)^- \cap T \neq \emptyset \quad \text{for each } r \in P \text{ such that } head(r) = a\ \}$$

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Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- Let's iterate $\mathbf{\Phi}_P$ on $\langle\{a\},\{d\}\rangle$:

$$\begin{array}{rcl} \mathbf{\Phi}_P\langle\{a\},\{d\}\rangle & = & \langle\{a,c\},\{b,f\}\rangle \\ \mathbf{\Phi}_P\langle\{a,c\},\{b,f\}\rangle & = & \langle\{a\},\{b,d,f\}\rangle \\ \mathbf{\Phi}_P\langle\{a\},\{b,d,f\}\rangle & = & \langle\{a,c\},\{b,f\}\rangle \\ & \vdots & \end{array}$$

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Fitting semantics

- Define the iterative variant of $\mathbf{\Phi}_P$ analogously to T_P:

$$\mathbf{\Phi}_P^0\langle T, F\rangle = \langle T, F\rangle \qquad \mathbf{\Phi}_P^{i+1}\langle T, F\rangle = \mathbf{\Phi}_P\mathbf{\Phi}_P^i\langle T, F\rangle$$

- Define the Fitting semantics of a normal logic program P as the partial interpretation:

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Properties

Let P be a normal logic program

- $\mathbf{\Phi}_P\langle\emptyset,\emptyset\rangle$ is monotonic
  That is, $\mathbf{\Phi}^i_P\langle\emptyset,\emptyset\rangle \sqsubseteq \mathbf{\Phi}^{i+1}_P\langle\emptyset,\emptyset\rangle$
- The Fitting semantics of P is
  - not conflicting,
  - and generally not total

Fitting fixpoints

Let P be a normal logic program,
and let $\langle T, F \rangle$ be a partial interpretation

- Define $\langle T, F \rangle$ as a Fitting fixpoint of P if $\mathbf{\Phi}_P\langle T, F \rangle = \langle T, F \rangle$
  - The Fitting semantics is the $\sqsubseteq$-least Fitting fixpoint of P
  - Any other Fitting fixpoint extends the Fitting semantics
  - Total Fitting fixpoints correspond to supported models

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- P has three total Fitting fixpoints:
  - $\langle \{a, c\}, \{b, d, e\} \rangle$
  - $\langle \{a, d\}, \{b, c, e\} \rangle$
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P has three supported models, two of them are stable models

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Let P be a normal logic program,
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- Let $\mathbf{\Phi}_P\langle T, F\rangle = \langle T', F'\rangle$
- If X is a stable model of P such that $T \subseteq X$ and $X \cap F = \emptyset$, then $T' \subseteq X$ and $X \cap F' = \emptyset$

  That is, $\mathbf{\Phi}_P$ is stable model preserving

  Hence, $\mathbf{\Phi}_P$ can be used for approximating stable models and so for propagation in ASP-solvers

- However, $\mathbf{\Phi}_P$ is still insufficient, because total fixpoints correspond to supported models, not necessarily stable models

  Note The problem is the same as with program completion

  The missing piece is non-circularity of derivations !

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- That is, Fitting semantics cannot assign *false* to a and b, although they can never become *true* !

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Outline

Unfounded sets

Let P be a normal logic program, and let $\langle T, F \rangle$ be a partial interpretation

- A set $U \subseteq atom(P)$ is an unfounded set of P wrt $\langle T, F \rangle$, if we have for each rule $r \in P$ such that $head(r) \in U$ either
  1. $body(r)^+ \cap F \neq \emptyset$ or $body(r)^- \cap T \neq \emptyset$ or
  2. $body(r)^+ \cap U \neq \emptyset$
- Intuitively, $\langle T, F \rangle$ is what we already know about P
- Rules satisfying Condition 1 are not usable for further derivations
- Condition 2 is the unfounded set condition treating cyclic derivations: All rules still being usable to derive an atom in U require an(other) atom in U to be true

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- A set $U \subseteq atom(P)$ is an unfounded set of P wrt $\langle T, F\rangle$, if we have for each rule $r \in P$ such that $head(r) \in U$ either
  1. $body(r)^+ \cap F \neq \emptyset$ or $body(r)^- \cap T \neq \emptyset$ or
  2. $body(r)^+ \cap U \neq \emptyset$

  Intuitively, $\langle T, F\rangle$ is what we already know about P
- Rules satisfying Condition 1 are not usable for further derivations
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Example

$$P = \left\{ \begin{array}{lcl} a & \leftarrow & b \\ b & \leftarrow & a \end{array} \right\}$$

- $\emptyset$ is an unfounded set (by definition)
- $\{a\}$ is not an unfounded set of P wrt $\langle \emptyset, \emptyset \rangle$
- $\{a\}$ is an unfounded set of P wrt $\langle \emptyset, \{b\} \rangle$
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- Analogously for $\{b\}$
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Greatest unfounded sets

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Observation The union of two unfounded sets is an unfounded set
- The greatest unfounded set of P wrt $\langle T, F\rangle$ is the union of all unfounded sets of P wrt $\langle T, F\rangle$

  It is denoted by $\mathbf{U}_P\langle T, F\rangle$
- Alternatively, we may define

$$\mathbf{U}_P\langle T, F\rangle = atom(P) \setminus Cn(\{r \in P \mid body(r)^+ \cap F = \emptyset\}^T)$$

- Note $Cn(\{r \in P \mid body(r)^+ \cap F = \emptyset\}^T)$ contains all non-circularly derivable atoms from P wrt $\langle T, F\rangle$

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Outline

Well-founded operator

Let P be a normal logic program,
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- Observation Condition 1 (in the definition of an unfounded set) corresponds to $\mathbf{F}_P\langle T, F \rangle$ of Fitting's $\mathbf{\Phi}_P\langle T, F \rangle$
- Idea Extend (negative part of) Fitting's operator $\mathbf{\Phi}_P$
  That is,
  - keep definition of $\mathbf{T}_P\langle T, F \rangle$ from $\mathbf{\Phi}_P\langle T, F \rangle$ and
  - replace $\mathbf{F}_P\langle T, F \rangle$ from $\mathbf{\Phi}_P\langle T, F \rangle$ by $\mathbf{U}_P\langle T, F \rangle$
- In words, an atom must be *false*
  if it belongs to the greatest unfounded set
- Definition $\mathbf{\Omega}_P\langle T, F \rangle = \langle \mathbf{T}_P\langle T, F \rangle, \mathbf{U}_P\langle T, F \rangle \rangle$

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Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- Let's iterate $\mathbf{\Omega}_{P_1}$ on $\langle\{c\},\emptyset\rangle$:

$$\begin{array}{rcl} \mathbf{\Omega}_P\langle\{c\},\emptyset\rangle & = & \langle\{a\},\{d,f\}\rangle \\ \mathbf{\Omega}_P\langle\{a\},\{d,f\}\rangle & = & \langle\{a,c\},\{b,e,f\}\rangle \\ \mathbf{\Omega}_P\langle\{a,c\},\{b,e,f\}\rangle & = & \langle\{a\},\{b,d,e,f\}\rangle \\ \mathbf{\Omega}_P\langle\{a\},\{b,d,e,f\}\rangle & = & \langle\{a,c\},\{b,e,f\}\rangle \\ & \vdots & \end{array}$$

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Well-founded semantics

- Define the iterative variant of $\mathbf{\Omega}_P$ analogously to $\mathbf{\Phi}_P$:

$$\mathbf{\Omega}_P^0\langle T, F\rangle = \langle T, F\rangle \qquad \mathbf{\Omega}_P^{i+1}\langle T, F\rangle = \mathbf{\Omega}_P\mathbf{\Omega}_P^i\langle T, F\rangle$$

- Define the well-founded semantics of a normal logic program P as the partial interpretation:

$$\bigsqcup_{i\geq 0}\mathbf{\Omega}_P^i\langle\emptyset, \emptyset\rangle$$

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Properties

Let P be a normal logic program

- $\mathbf{\Omega}_P\langle\emptyset,\emptyset\rangle$ is monotonic
  That is, $\mathbf{\Omega}^i_P\langle\emptyset,\emptyset\rangle \sqsubseteq \mathbf{\Omega}^{i+1}_P\langle\emptyset,\emptyset\rangle$
- The well-founded semantics of P is
  - not conflicting,
  - and generally not total
- We have $\bigsqcup_{i\geq 0} \mathbf{\Phi}^i_P\langle\emptyset,\emptyset\rangle \sqsubseteq \bigsqcup_{i\geq 0} \mathbf{\Omega}^i_P\langle\emptyset,\emptyset\rangle$

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Well-founded fixpoints

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Define $\langle T, F\rangle$ as a well-founded fixpoint of P if $\mathbf{\Omega}_P\langle T, F\rangle = \langle T, F\rangle$
  - The well-founded semantics is the $\sqsubseteq$-least well-founded fixpoint of P
  - Any other well-founded fixpoint extends the well-founded semantics
  - Total well-founded fixpoints correspond to stable models

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- P has two total well-founded fixpoints:
  1. $\langle \{a, c\}, \{b, d, e\} \rangle$
  2. $\langle \{a, d\}, \{b, c, e\} \rangle$
- Both of them represent stable models

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Properties

Let P be a normal logic program,
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- Let $\mathbf{\Omega}_P\langle T, F\rangle = \langle T', F'\rangle$
- If X is a stable model of P such that $T \subseteq X$ and $X \cap F = \emptyset$, then $T' \subseteq X$ and $X \cap F' = \emptyset$

  That is, $\mathbf{\Omega}_P$ is stable model preserving

  Hence, $\mathbf{\Omega}_P$ can be used for approximating stable models and so for propagation in ASP-solvers
- In contrast to $\mathbf{\Phi}_P$, operator $\mathbf{\Omega}_P$ is sufficient for propagation because total fixpoints correspond to stable models
- Note In addition to $\mathbf{\Omega}_P$, most ASP-solvers apply backward propagation, originating from program completion (although this is unnecessary from a formal point of view)

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- If X is a stable model of P such that $T \subseteq X$ and $X \cap F = \emptyset$, then $T' \subseteq X$ and $X \cap F' = \emptyset$

  That is, $\mathbf{\Omega}_P$ is stable model preserving

  Hence, $\mathbf{\Omega}_P$ can be used for approximating stable models and so for propagation in ASP-solvers
- In contrast to $\mathbf{\Phi}_P$, operator $\mathbf{\Omega}_P$ is sufficient for propagation because total fixpoints correspond to stable models
- Note In addition to $\mathbf{\Omega}_P$, most ASP-solvers apply backward propagation, originating from program completion (although this is unnecessary from a formal point of view)

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Let P be a normal logic program,
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Proof-theoretic Characterization: Overview

Motivation

- Goal Analyze computations in ASP solvers
- Wanted A declarative and fine-grained instrument for characterizing operations as well as strategies of ASP solvers
- Idea View stable model computations as derivations in an inference system

  Consider Tableau-based proof systems for analyzing ASP solving

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Tableau calculi

- Traditionally, tableau calculi are used for
  - automated theorem proving and
  - proof theoretical analysis

  in classical as well as non-classical logics

- General idea Given an input, prove some property by decomposition
  Decomposition is done by applying deduction rules
- For details, see *Handbook of Tableau Methods*, Kluwer, 1999

General definitions

- A tableau is a (mostly binary) tree
- A branch in a tableau is a path from the root to a leaf
- A branch containing $\gamma_1, \dots, \gamma_m$ can be extended by applying tableau rules of form

$$\frac{\gamma_1, \dots, \gamma_m}{\begin{array}{c}\alpha_1\\ \vdots \\ \alpha_n\end{array}} \qquad\qquad \frac{\gamma_1, \dots, \gamma_m}{\beta_1 \mid \dots \mid \beta_n}$$

- Rules of the former format append entries $\alpha_1, \dots, \alpha_n$ to the branch
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Example

- A simple tableau calculus for proving unsatisfiability of propositional formulas, composed from $\neg$, $\wedge$, and $\vee$, consists of rules

$$\frac{\neg\neg\alpha}{\alpha} \qquad \frac{\alpha_1 \wedge \alpha_2}{\begin{array}{c}\alpha_1\\ \alpha_2\end{array}} \qquad \frac{\beta_1 \vee \beta_2}{\beta_1 \mid \beta_2}$$

- All rules are semantically valid, when interpreting entries in a branch conjunctively and distinct (sub-)branches as connected disjunctively
- A propositional formula φ is unsatisfiable iff there is a tableau with φ as the root node such that
  1. all other entries can be produced by tableau rules and
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(1)	$a \wedge ((\neg b \wedge (\neg a \vee b)) \vee \neg\neg\neg a)$	$[\varphi]$
(2)	a	[1]
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All three branches of the tableau are contradictory (cf 2, 5, 7, 8, 10)

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Tableaux and ASP

- A tableau rule captures an elementary inference scheme in an ASP solver
- A branch in a tableau corresponds to a successful or unsuccessful computation of a stable model
- An entire tableau represents a traversal of the search space

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ASP-specific definitions

- A (signed) tableau for a logic program P is a binary tree such that
  - the root node of the tree consists of the rules in P;
  - the other nodes in the tree are entries of the form $\mathbf{T}v$ or $\mathbf{F}v$, called signed literals, where v is a variable,
  - generated by extending a tableau using deduction rules (given below)
- An entry $\mathbf{T}v$ ($\mathbf{F}v$) reflects that variable v is *true* (*false*) in a corresponding variable assignment

  A set of signed literals constitutes a partial assignment
- For a normal logic program P,
  - atoms of P in $atom(P)$ and
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Tableau rules for ASP at a glance

$$\text{(FTB)}\quad \frac{\begin{array}{c} p \leftarrow l_1, \dots, l_n \\ \mathbf{t}l_1, \dots, \mathbf{t}l_n \end{array}}{\mathbf{T}\{l_1, \dots, l_n\}}$$

$$\text{(BFB)}\quad \frac{\begin{array}{c} \mathbf{F}\{l_1, \dots, l_i, \dots, l_n\} \\ \mathbf{t}l_1, \dots, \mathbf{t}l_{i-1}, \mathbf{t}l_{i+1}, \dots, \mathbf{t}l_n \end{array}}{\mathbf{f}l_i}$$

$$\text{(FTA)}\quad \frac{\begin{array}{c} p \leftarrow l_1, \dots, l_n \\ \mathbf{T}\{l_1, \dots, l_n\} \end{array}}{\mathbf{T}p}$$

$$\text{(BFA)}\quad \frac{\begin{array}{c} p \leftarrow l_1, \dots, l_n \\ \mathbf{F}p \end{array}}{\mathbf{F}\{l_1, \dots, l_n\}}$$

$$\text{(FFB)}\quad \frac{\begin{array}{c} p \leftarrow l_1, \dots, l_i, \dots, l_n \\ \mathbf{f}l_i \end{array}}{\mathbf{F}\{l_1, \dots, l_i, \dots, l_n\}}$$

$$\text{(BTB)}\quad \frac{\mathbf{T}\{l_1, \dots, l_i, \dots, l_n\}}{\mathbf{t}l_i}$$

$$\text{(FFA)}\quad \frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p}\quad (\S)$$

$$\text{(BTA)}\quad \frac{\begin{array}{c} \mathbf{T}p \\ \mathbf{F}B_1, \dots, \mathbf{F}B_{i-1}, \mathbf{F}B_{i+1}, \dots, \mathbf{F}B_m \end{array}}{\mathbf{T}B_i}\quad (\S)$$

$$\text{(WFN)}\quad \frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p}\quad (\dagger)$$

$$\text{(WFJ)}\quad \frac{\begin{array}{c} \mathbf{T}p \\ \mathbf{F}B_1, \dots, \mathbf{F}B_{i-1}, \mathbf{F}B_{i+1}, \dots, \mathbf{F}B_m \end{array}}{\mathbf{T}B_i}\quad (\dagger)$$

$$\text{(FL)}\quad \frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p}\quad (\ddagger)$$

$$\text{(BL)}\quad \frac{\begin{array}{c} \mathbf{T}p \\ \mathbf{F}B_1, \dots, \mathbf{F}B_{i-1}, \mathbf{F}B_{i+1}, \dots, \mathbf{F}B_m \end{array}}{\mathbf{T}B_i}\quad (\ddagger)$$

$$\text{(Cut}[X]\text{)}\quad \frac{}{\mathbf{T}v \mid \mathbf{F}v}\quad (\sharp[X])$$

More concepts

- A **tableau calculus** is a set of tableau rules
- A branch in a tableau is conflicting,
  if it contains both $\mathbf{T}v$ and $\mathbf{F}v$ for some variable v
- A branch in a tableau is total for a program P,
  if it contains either $\mathbf{T}v$ or $\mathbf{F}v$ for each $v \in atom(P) \cup body(P)$
- A branch in a tableau of some calculus $\mathcal{T}$ is closed,
  if no rule in $\mathcal{T}$ other than *Cut* can produce any new entries
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Example

- Consider the program

$$P = \left\{ \begin{array}{l} a \leftarrow \\ c \leftarrow {\sim}b, {\sim}d \\ d \leftarrow a, {\sim}c \end{array} \right\}$$

  having stable models $\{a, c\}$ and $\{a, d\}$

(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow {\sim}b, {\sim}d$$
$$d \leftarrow a, {\sim}c$$

(FTB)		$\mathbf{T}\emptyset$		
(FTA)		$\mathbf{T}a$		
(FFA)		$\mathbf{F}b$		
(Cut[$atom(P)$])	$\mathbf{T}c$			$\mathbf{F}c$
(BTA)	$\mathbf{T}\{{\sim}b, {\sim}d\}$		(BFA)	$\mathbf{F}\{{\sim}b, {\sim}d\}$
(BTB)	$\mathbf{F}d$		(BFB)	$\mathbf{T}d$
(FFB)	$\mathbf{F}\{a, {\sim}c\}$		(FTB)	$\mathbf{T}\{a, {\sim}c\}$

(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \sim b, \sim d$$
$$d \leftarrow a, \sim c$$

(FTB)	$\mathbf{T}\emptyset$		
(FTA)	$\mathbf{T}a$		
(FFA)	$\mathbf{F}b$		
(Cut[$atom(P)$])	$\mathbf{T}c$		$\mathbf{F}c$
(BTA)	$\mathbf{T}\{\sim b, \sim d\}$	(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
(BTB)	$\mathbf{F}d$	(BFB)	$\mathbf{T}d$
(FFB)	$\mathbf{F}\{a, \sim c\}$	(FTB)	$\mathbf{T}\{a, \sim c\}$

(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \sim b, \sim d$$
$$d \leftarrow a, \sim c$$

(FTB)	$\mathbf{T}\emptyset$		
(FTA)	$\mathbf{T}a$		
(FFA)	$\mathbf{F}b$		
(Cut[$atom(P)$])	$\mathbf{T}c$		$\mathbf{F}c$
(BTA)	$\mathbf{T}\{\sim b, \sim d\}$	(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
(BTB)	$\mathbf{F}d$	(BFB)	$\mathbf{T}d$
(FFB)	$\mathbf{F}\{a, \sim c\}$	(FTB)	$\mathbf{T}\{a, \sim c\}$

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(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \sim b, \sim d$$
$$d \leftarrow a, \sim c$$

(FTB)		$\mathbf{T}\emptyset$		
(FTA)		$\mathbf{T}a$		
(FFA)		$\mathbf{F}b$		
(Cut[$atom(P)$])		$\mathbf{T}c$		$\mathbf{F}c$
	(BTA)	$\mathbf{T}\{\sim b, \sim d\}$	(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
	(BTB)	$\mathbf{F}d$	(BFB)	$\mathbf{T}d$
	(FFB)	$\mathbf{F}\{a, \sim c\}$	(FTB)	$\mathbf{T}\{a, \sim c\}$

(Previewed) Example

$$
\begin{array}{r}
a \leftarrow \\
c \leftarrow \sim b, \sim d \\
d \leftarrow a, \sim c
\end{array}
$$

$$
\begin{array}{rccccc}
(FTB) & & & \mathbf{T}\emptyset & & \\
(FTA) & & & \mathbf{T}a & & \\
(FFA) & & & \mathbf{F}b & & \\
(\mathrm{Cut}[atom(P)]) & & \mathbf{T}c & & & \mathbf{F}c \\
& (BTA) & \mathbf{T}\{\sim b, \sim d\} & & (BFA) & \mathbf{F}\{\sim b, \sim d\} \\
& (BTB) & \mathbf{F}d & & (BFB) & \mathbf{T}d \\
& (FFB) & \mathbf{F}\{a, \sim c\} & & (FTB) & \mathbf{T}\{a, \sim c\}
\end{array}
$$

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(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \sim b, \sim d$$
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(FTB)		$\mathbf{T}\emptyset$		
(FTA)		$\mathbf{T}a$		
(FFA)		$\mathbf{F}b$		
(Cut[$atom(P)$])		$\mathbf{T}c$		$\mathbf{F}c$
	(BTA)	$\mathbf{T}\{\sim b, \sim d\}$	(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
	(BTB)	$\mathbf{F}d$	(BFB)	$\mathbf{T}d$
	(FFB)	$\mathbf{F}\{a, \sim c\}$	(FTB)	$\mathbf{T}\{a, \sim c\}$

Auxiliary definitions

- For a literal l, define conjugation functions $\mathbf{t}$ and $\mathbf{f}$ as follows

$$\mathbf{t}l = \begin{cases} \mathbf{T}l & \text{if } l \text{ is an atom} \\ \mathbf{F}a & \text{if } l = {\sim}a \text{ for an atom } a \end{cases}$$

$$\mathbf{f}l = \begin{cases} \mathbf{F}l & \text{if } l \text{ is an atom} \\ \mathbf{T}a & \text{if } l = {\sim}a \text{ for an atom } a \end{cases}$$

- Examples $\mathbf{t}a = \mathbf{T}a$, $\mathbf{f}a = \mathbf{F}a$, $\mathbf{t}{\sim}a = \mathbf{F}a$, and $\mathbf{f}{\sim}a = \mathbf{T}a$

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- Examples $\mathbf{t}a = \mathbf{T}a$, $\mathbf{f}a = \mathbf{F}a$, $\mathbf{t}{\sim}a = \mathbf{F}a$, and $\mathbf{f}{\sim}a = \mathbf{T}a$

Auxiliary definitions

- Some tableau rules require conditions for their application
- Such conditions are specified as provisos

$$\frac{\textit{prerequisites}}{\textit{consequence}}\ (\textit{proviso}) \qquad\qquad \textit{proviso}\text{: some condition(s)}$$

- Note All tableau rules given in the sequel are stable model preserving

Forward True Body (FTB)

- Prerequisites All of a body's literals are *true*
- Consequence The body is *true*
- Tableau Rule FTB

$$\frac{\begin{array}{c} p \leftarrow l_1, \dots, l_n \\ \mathbf{t}l_1, \dots, \mathbf{t}l_n \end{array}}{\mathbf{T}\{l_1, \dots, l_n\}}$$

- Example

$$\frac{\begin{array}{c} a \leftarrow b, {\sim}c \\ \mathbf{T}b \\ \mathbf{F}c \end{array}}{\mathbf{T}\{b, {\sim}c\}}$$

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Forward True Body (FTB)

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$$\frac{p \leftarrow l_1, \dots, l_n \quad \mathbf{t}l_1, \dots, \mathbf{t}l_n}{\mathbf{T}\{l_1, \dots, l_n\}}$$

- Example

$$\frac{a \leftarrow b, \sim c \quad \mathbf{T}b \quad \mathbf{F}c}{\mathbf{T}\{b, \sim c\}}$$

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Backward False Body (BFB)

- Prerequisites A body is *false*, and all its literals except for one are *true*
- Consequence The residual body literal is *false*
- Tableau Rule BFB

$$\frac{\mathbf{F}\{l_1,\dots,l_i,\dots,l_n\} \quad \mathbf{t}l_1,\dots,\mathbf{t}l_{i-1},\mathbf{t}l_{i+1},\dots,\mathbf{t}l_n}{\mathbf{f}l_i}$$

- Examples

$$\frac{\mathbf{F}\{b,\sim c\} \quad \mathbf{T}b}{\mathbf{T}c} \qquad \frac{\mathbf{F}\{b,\sim c\} \quad \mathbf{F}c}{\mathbf{F}b}$$

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$$\frac{\begin{array}{c}\mathbf{F}\{l_1, \dots, l_i, \dots, l_n\} \\ \mathbf{t}l_1, \dots, \mathbf{t}l_{i-1}, \mathbf{t}l_{i+1}, \dots, \mathbf{t}l_n\end{array}}{\mathbf{f}l_i}$$

- Examples

$$\frac{\begin{array}{c}\mathbf{F}\{b, \sim c\} \\ \mathbf{T}b\end{array}}{\mathbf{T}c} \qquad \frac{\begin{array}{c}\mathbf{F}\{b, \sim c\} \\ \mathbf{F}c\end{array}}{\mathbf{F}b}$$

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Forward False Body (FFB)

- Prerequisites Some literal of a body is *false*
- Consequence The body is *false*
- Tableau Rule FFB

$$\frac{\begin{array}{c} p \leftarrow l_1, \dots, l_i, \dots, l_n \\ \mathbf{f}\, l_i \end{array}}{\mathbf{F}\{l_1, \dots, l_i, \dots, l_n\}}$$

- Examples

$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{F}\, b \end{array}}{\mathbf{F}\{b, \sim c\}} \qquad \frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{T}\, c \end{array}}{\mathbf{F}\{b, \sim c\}}$$

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- Prerequisites Some literal of a body is *false*
- Consequence The body is *false*
- Tableau Rule FFB

$$\frac{\begin{array}{c} p \leftarrow l_1, \ldots, l_i, \ldots, l_n \\ \mathbf{f}\, l_i \end{array}}{\mathbf{F}\{l_1, \ldots, l_i, \ldots, l_n\}}$$

- Examples

$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{F}b \end{array}}{\mathbf{F}\{b, \sim c\}} \qquad \frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{T}c \end{array}}{\mathbf{F}\{b, \sim c\}}$$

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Backward True Body (BTB)

- Prerequisites A body is *true*
- Consequence The body's literals are *true*
- Tableau Rule BTB

$$\frac{\mathbf{T}\{l_1, \dots, l_i, \dots, l_n\}}{\mathbf{t}l_i}$$

- Examples

$$\frac{\mathbf{T}\{b, \sim c\}}{\mathbf{T}b} \qquad \frac{\mathbf{T}\{b, \sim c\}}{\mathbf{F}c}$$

Backward True Body (BTB)

- Prerequisites A body is *true*
- Consequence The body's literals are *true*
- Tableau Rule BTB

$$\frac{\mathbf{T}\{l_1,\dots,l_i,\dots,l_n\}}{\mathbf{t}l_i}$$

- Examples

$$\frac{\mathbf{T}\{b,\sim c\}}{\mathbf{T}b} \qquad \frac{\mathbf{T}\{b,\sim c\}}{\mathbf{F}c}$$

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Tableau rules for bodies

Consider rule body $B = \{l_1, \dots, l_n\}$

- Rules FTB and BFB amount to implication

  $$l_1 \wedge \cdots \wedge l_n \rightarrow B$$

- Rules FFB and BTB amount to implication

  $$B \rightarrow l_1 \wedge \cdots \wedge l_n$$

- Together they yield

  $$B \equiv l_1 \wedge \cdots \wedge l_n$$

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  $$B \rightarrow l_1 \wedge \cdots \wedge l_n$$

- Together they yield

  $$B \equiv l_1 \wedge \cdots \wedge l_n$$

Forward True Atom (FTA)

- Prerequisites Some of an atom's bodies is *true*
- Consequence The atom is *true*
- Tableau Rule FTA

$$\frac{p \leftarrow l_1, \dots, l_n \quad \mathbf{T}\{l_1, \dots, l_n\}}{\mathbf{T}p}$$

- Examples

$$\frac{a \leftarrow b, \sim c \quad \mathbf{T}\{b, \sim c\}}{\mathbf{T}a} \qquad \frac{a \leftarrow d, \sim e \quad \mathbf{T}\{d, \sim e\}}{\mathbf{T}a}$$

Forward True Atom (FTA)

- Prerequisites Some of an atom's bodies is *true*
- Consequence The atom is *true*
- Tableau Rule FTA

$$\frac{p \leftarrow l_1, \dots, l_n \quad \mathbf{T}\{l_1, \dots, l_n\}}{\mathbf{T}p}$$

- Examples

$$\frac{a \leftarrow b, {\sim}c \quad \mathbf{T}\{b, {\sim}c\}}{\mathbf{T}a} \qquad \frac{a \leftarrow d, {\sim}e \quad \mathbf{T}\{d, {\sim}e\}}{\mathbf{T}a}$$

Backward False Atom (BFA)

- Prerequisites An atom is *false*
- Consequence The bodies of all rules with the atom as head are *false*
- Tableau Rule BFA

$$\frac{p \leftarrow l_1, \ldots, l_n \quad \mathbf{F}p}{\mathbf{F}\{l_1, \ldots, l_n\}}$$

- Examples

$$\frac{a \leftarrow b, \sim c \quad \mathbf{F}a}{\mathbf{F}\{b, \sim c\}} \qquad \frac{a \leftarrow d, \sim e \quad \mathbf{F}a}{\mathbf{F}\{d, \sim e\}}$$

Backward False Atom (BFA)

- Prerequisites An atom is *false*
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$$\frac{p \leftarrow l_1, \ldots, l_n \qquad \mathbf{F}p}{\mathbf{F}\{l_1, \ldots, l_n\}}$$

- Examples

$$\frac{a \leftarrow b, \sim c \qquad \mathbf{F}a}{\mathbf{F}\{b, \sim c\}} \qquad \frac{a \leftarrow d, \sim e \qquad \mathbf{F}a}{\mathbf{F}\{d, \sim e\}}$$

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Forward False Atom (FFA)

- Prerequisites For some atom, the bodies of all rules with the atom as head are *false*
- Consequence The atom is *false*
- Tableau Rule FFA

$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \ (body_P(p) = \{B_1, \dots, B_m\})$$

- Example

$$\frac{\mathbf{F}\{b, \sim c\} \quad \mathbf{F}\{d, \sim e\}}{\mathbf{F}a} \ (body_P(a) = \{\{b, \sim c\}, \{d, \sim e\}\})$$

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Forward False Atom (FFA)

- Prerequisites For some atom, the bodies of all rules with the atom as head are *false*
- Consequence The atom is *false*
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$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \quad (body_P(p) = \{B_1, \dots, B_m\})$$

- Example

$$\frac{\begin{array}{c}\mathbf{F}\{b, {\sim}c\} \\ \mathbf{F}\{d, {\sim}e\}\end{array}}{\mathbf{F}a} \quad (body_P(a) = \{\{b, {\sim}c\}, \{d, {\sim}e\}\})$$

Backward True Atom (BTA)

- Prerequisites An atom is *true*, and the bodies of all rules with the atom as head except for one are *false*
- Consequence The residual body is *true*
- Tableau Rule BTA

$$\frac{\mathbf{T}p \quad \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m}{\mathbf{T}B_i} \quad (body_P(p) = \{B_1,\dots,B_m\})$$

- Examples

$$\frac{\mathbf{T}a \quad \mathbf{F}\{b,\sim c\}}{\mathbf{T}\{d,\sim e\}} \ (*) \qquad \frac{\mathbf{T}a \quad \mathbf{F}\{d,\sim e\}}{\mathbf{T}\{b,\sim c\}} \ (*)$$

$$(*) \quad body_P(a) = \{\{b,\sim c\},\{d,\sim e\}\}$$

Backward True Atom (BTA)

- Prerequisites An atom is *true*, and the bodies of all rules with the atom as head except for one are *false*
- Consequence The residual body is *true*
- Tableau Rule BTA

$$\frac{\begin{array}{c}\mathbf{T}p \\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i} \quad (body_P(p)=\{B_1,\dots,B_m\})$$

- Examples

$$\frac{\begin{array}{c}\mathbf{T}a \\ \mathbf{F}\{b,\sim c\}\end{array}}{\mathbf{T}\{d,\sim e\}}\ (*) \qquad \frac{\begin{array}{c}\mathbf{T}a \\ \mathbf{F}\{d,\sim e\}\end{array}}{\mathbf{T}\{b,\sim c\}}\ (*)$$

$$(*) \quad body_P(a)=\{\{b,\sim c\},\{d,\sim e\}\}$$

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Tableau rules for atoms

Consider an atom p such that $body_P(p) = \{B_1, \dots, B_m\}$

- Rules FTA and BFA amount to implication

  $$B_1 \vee \dots \vee B_m \rightarrow p$$

- Rules FFA and BTA amount to implication

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- Together they yield

  $$p \equiv B_1 \vee \dots \vee B_m$$

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Relationship with program completion

Let P be a normal logic program

- The eight tableau rules introduced so far essentially provide (straightforward) inferences from $CF(P)$

Preliminaries for unfounded sets

Let P be a normal logic program

- For $P' \subseteq P$, define the greatest unfounded set of P wrt P' as

$$\mathbf{U}_P(P') = atom(P) \setminus Cn((P')^{\emptyset})$$

- For a loop $L \in loop(P)$, define the external bodies of L as

$$EB_P(L) = \{body(r) \mid r \in P, head(r) \in L, body(r)^+ \cap L = \emptyset\}$$

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Well-Founded Negation (WFN)

- Prerequisites An atom is in the greatest unfounded set wrt rules whose bodies are *false*
- Consequence The atom is *false*
- Tableau Rule WFN

$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \quad (p \in \mathbf{U}_P(\{r \in P \mid body(r) \notin \{B_1, \dots, B_m\}\}))$$

- Examples

$$a \leftarrow \sim b \qquad\qquad \begin{array}{c} a \leftarrow a \\ a \leftarrow \sim b \end{array}$$

$$\frac{\mathbf{F}\{\sim b\}}{\mathbf{F}a}\ (*) \qquad\qquad \frac{\mathbf{F}\{\sim b\}}{\mathbf{F}a}\ (*)$$

$$(*) \quad a \in \mathbf{U}_P(P \setminus \{a \leftarrow \sim b\})$$

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- Examples

$$\frac{\begin{array}{c} a \leftarrow \sim b \\ \mathbf{F}\{\sim b\} \end{array}}{\mathbf{F}a}\ (*) \qquad \frac{\begin{array}{c} a \leftarrow a \\ a \leftarrow \sim b \\ \mathbf{F}\{\sim b\} \end{array}}{\mathbf{F}a}\ (*)$$

$$(*) \quad a \in \mathbf{U}_P(P \setminus \{a \leftarrow \sim b\})$$

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Well-Founded Justification (WFJ)

- Prerequisites A *true* atom is in the greatest unfounded set wrt rules whose bodies are *false*, if a particular body is made *false*
- Consequence The respective body is *true*
- Tableau Rule WFJ

$$\frac{\begin{array}{c}\mathbf{T}p\\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i}\quad(p\in\mathbf{U}_P(\{r\in P\mid body(r)\notin\{B_1,\dots,B_m\}\}))$$

- Examples

$$\frac{\begin{array}{c}a\leftarrow{\sim}b\\ \mathbf{T}a\end{array}}{\mathbf{T}\{{\sim}b\}}\;(*)\qquad\qquad\frac{\begin{array}{c}a\leftarrow a\\ a\leftarrow{\sim}b\\ \mathbf{T}a\end{array}}{\mathbf{T}\{{\sim}b\}}\;(*)$$

$$(*)\quad a\in\mathbf{U}_P(P\setminus\{a\leftarrow{\sim}b\})$$

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- Examples

$$\frac{\begin{array}{c}a \leftarrow {\sim}b\\ \mathbf{T}a\end{array}}{\mathbf{T}\{{\sim}b\}}\ (*) \qquad \frac{\begin{array}{c}a \leftarrow a\\ a \leftarrow {\sim}b\\ \mathbf{T}a\end{array}}{\mathbf{T}\{{\sim}b\}}\ (*)$$

$$(*) \quad a \in \mathbf{U}_P(P \setminus \{a \leftarrow {\sim}b\})$$

Well-founded tableau rules

- Tableau rules WFN and WFJ ensure non-circular support for *true* atoms
- Note
  1. WFN subsumes falsifying atoms via FFA,
  2. WFJ can be viewed as “backward propagation” for unfounded sets,
  3. WFJ subsumes backward propagation of *true* atoms via BTA

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  3. WFJ subsumes backward propagation of *true* atoms via BTA

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Relationship with well-founded operator

Let P be a normal logic program, $\langle T, F\rangle$ a partial interpretation, and $P' = \{r \in P \mid body(r)^+ \cap F = \emptyset \text{ and } body(r)^- \cap T = \emptyset\}$.

- The following conditions are equivalent
  1. $p \in \mathbf{U}_P\langle T, F\rangle$
  2. $p \in \mathbf{U}_P(P')$
- Hence, the well-founded operator $\mathbf{\Omega}$ and WFN coincide
- Note In contrast to $\mathbf{\Omega}$, WFN does not necessarily require a rule body to contain a *false* literal for the rule being inapplicable

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Forward Loop (FL)

- Prerequisites The external bodies of a loop are *false*
- Consequence The atoms in the loop are *false*
- Tableau Rule FL

$$\frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\ (p \in L, L \in loop(P), EB_P(L) = \{B_1,\dots,B_m\})$$

- Example

$$\frac{\begin{array}{c} a \leftarrow a \\ a \leftarrow \sim b \\ \mathbf{F}\{\sim b\} \end{array}}{\mathbf{F}a}\ (EB_P(\{a\}) = \{\{\sim b\}\})$$

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Forward Loop (FL)

- Prerequisites The external bodies of a loop are *false*
- Consequence The atoms in the loop are *false*
- Tableau Rule FL

$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \quad (p \in L, L \in loop(P), EB_P(L) = \{B_1, \dots, B_m\})$$

- Example

$$\begin{array}{c} a \leftarrow a \\ a \leftarrow \sim b \\ \hline \mathbf{F}\{\sim b\} \\ \hline \mathbf{F}a \end{array} \quad (EB_P(\{a\}) = \{\{\sim b\}\})$$

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Backward Loop (BL)

- Prerequisites An atom of a loop is *true*, and all external bodies except for one are *false*
- Consequence The residual external body is *true*
- Tableau Rule BL

$$\frac{\begin{array}{c}\mathbf{T}p\\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i}\;(p\in L, L\in loop(P), EB_P(L)=\{B_1,\dots,B_m\})$$

- Example

$$a \leftarrow a$$
$$a \leftarrow \sim b$$
$$\frac{\mathbf{T}a}{\mathbf{T}\{\sim b\}}\;(EB_P(\{a\})=\{\{\sim b\}\})$$

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Tableau rules for loops

- Tableau rules FL and BL ensure non-circular support for *true* atoms
- For a loop L such that $EB_P(L) = \{B_1, \dots, B_m\}$, they amount to implications of form

$$\bigvee_{p \in L} p \rightarrow B_1 \vee \dots \vee B_m$$

- Comparison to well-founded tableau rules yields
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Relationship with loop formulas

- Tableau rules FL and BL essentially provide (straightforward) inferences from loop formulas
  - Impractical to precompute exponentially many loop formulas
- In practice, ASP solvers such as *smodels* and *clasp*
  - exploit strongly connected components of positive atom dependency graphs
    - can be viewed as an interpolation of FL
  - do not directly implement BL (and neither WFJ)
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  - could simulate BL via FL/WFN by means of failed-literal detection (lookahead)

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Case analysis by *Cut*

- Up to now, all tableau rules are deterministic

  That is, rules extend a single branch but cannot create sub-branches
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- Prerequisites None
- Consequence Two alternative (complementary) entries
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$$\frac{}{\mathbf{T}v \mid \mathbf{F}v} \quad (v \in \mathcal{C})$$

- Examples

$$\frac{\begin{array}{c} a \leftarrow {\sim}b \\ b \leftarrow {\sim}a \end{array}}{\mathbf{T}a \mid \mathbf{F}a} \quad (\mathcal{C} = atom(P))$$

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Well-known tableau calculi

- Fitting's operator $\mathbf{\Phi}$ applies forward propagation without sophisticated unfounded set checks

$$\mathcal{T}_{\mathbf{\Phi}} = \{FTB, FTA, FFB, FFA\}$$

- Well-founded operator $\mathbf{\Omega}$ replaces negation of single atoms with negation of unfounded sets

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- "Local" propagation via a program's completion can be determined by elementary inferences on atoms and rule bodies

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Tableau calculi characterizing ASP solvers

- ASP solvers combine propagation with case analysis
- We obtain the following tableau calculi characterizing

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\mathcal{T}_{\mathit{cmodels\text{-}1}} &= \mathcal{T}_{\mathit{completion}} \cup \{\mathit{Cut}[\mathit{atom}(P) \cup \mathit{body}(P)]\} \\
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- SAT-based ASP solvers, *assat* and *cmodels*, incrementally add loop formulas to a program's completion
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Proof complexity

- Proof complexity is used for describing the relative efficiency of different proof systems
  - It compares proof systems based on minimal refutations
  - It is independent of heuristics
- A proof system $\mathcal{T}$ polynomially simulates a proof system $\mathcal{T}'$, if every refutation of $\mathcal{T}'$ can be polynomially mapped to a refutation of $\mathcal{T}$
  - Otherwise, $\mathcal{T}$ does not polynomially simulate $\mathcal{T}'$
- For showing that proof system $\mathcal{T}$ does not polynomially simulate $\mathcal{T}'$, we have to provide an infinite witnessing family of programs such that minimal refutations of $\mathcal{T}$ asymptotically are exponentially larger than minimal refutations of $\mathcal{T}'$
  - The size of tableaux is simply the number of their entries
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$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

- $\mathcal{T}_{smodels}$ restricts *Cut* to $atom(P)$ and $\mathcal{T}_{noMoRe}$ to $body(P)$
  Are both approaches similar or is one of them superior to the other?
- Let $\{P^n_a\}$, $\{P^n_b\}$, and $\{P^n_c\}$ be infinite families of programs where

$$P^n_a = \left\{ \begin{array}{c} x \leftarrow \sim x \\ x \leftarrow a_1, b_1 \\ \vdots \\ x \leftarrow a_n, b_n \end{array} \right\} \quad P^n_b = \left\{ \begin{array}{cc} \multicolumn{2}{c}{x \leftarrow c_1, \dots, c_n, \sim x} \\ c_1 \leftarrow a_1 & c_1 \leftarrow b_1 \\ \vdots & \vdots \\ c_n \leftarrow a_n & c_n \leftarrow b_n \end{array} \right\} \quad P^n_c = \left\{ \begin{array}{c} a_1 \leftarrow \sim b_1 \\ b_1 \leftarrow \sim a_1 \\ \vdots \\ a_n \leftarrow \sim b_n \\ b_n \leftarrow \sim a_n \end{array} \right\}$$

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$$P_a^n = \left\{ \begin{array}{c} x \leftarrow \sim x \\ x \leftarrow a_1, b_1 \\ \vdots \\ x \leftarrow a_n, b_n \end{array} \right\} \quad P_b^n = \left\{ \begin{array}{cc} \multicolumn{2}{c}{x \leftarrow c_1, \dots, c_n, \sim x} \\ c_1 \leftarrow a_1 & c_1 \leftarrow b_1 \\ \vdots & \vdots \\ c_n \leftarrow a_n & c_n \leftarrow b_n \end{array} \right\} \quad P_c^n = \left\{ \begin{array}{c} a_1 \leftarrow \sim b_1 \\ b_1 \leftarrow \sim a_1 \\ \vdots \\ a_n \leftarrow \sim b_n \\ b_n \leftarrow \sim a_n \end{array} \right\}$$

- In minimal refutations for $P_a^n \cup P_c^n$, the number of applications of $Cut[body(P_a^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $Cut[atom(P_a^n \cup P_c^n)]$
- Vice versa, minimal refutations for $P_b^n \cup P_c^n$ require linearly many applications of $Cut[atom(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $Cut[body(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$

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$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

- $\mathcal{T}_{smodels}$ restricts Cut to $\mathit{atom}(P)$ and $\mathcal{T}_{noMoRe}$ to $\mathit{body}(P)$
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- In minimal refutations for $P_a^n \cup P_c^n$, the number of applications of $\mathit{Cut}[\mathit{body}(P_a^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $\mathit{Cut}[\mathit{atom}(P_a^n \cup P_c^n)]$
- Vice versa, minimal refutations for $P_b^n \cup P_c^n$ require linearly many applications of $\mathit{Cut}[\mathit{atom}(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $\mathit{Cut}[\mathit{body}(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$

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Relative efficiency

- As witnessed by $\{P_a^n \cup P_c^n\}$ and $\{P_b^n \cup P_c^n\}$, respectively, $\mathcal{T}_{smodels}$ and $\mathcal{T}_{noMoRe}$ do not polynomially simulate one another
- Any refutation of $\mathcal{T}_{smodels}$ or $\mathcal{T}_{noMoRe}$ is a refutation of $\mathcal{T}_{nomore^{++}}$ (but not vice versa)
- Hence
  - both $\mathcal{T}_{smodels}$ and $\mathcal{T}_{noMoRe}$ are polynomially simulated by $\mathcal{T}_{nomore^{++}}$ and
  - $\mathcal{T}_{nomore^{++}}$ is polynomially simulated by neither $\mathcal{T}_{smodels}$ nor $\mathcal{T}_{noMoRe}$
- More generally, the proof system obtained with $Cut[atom(P) \cup body(P)]$ is exponentially stronger than the ones with either $Cut[atom(P)]$ or $Cut[body(P)]$
- Case analyses (at least) on atoms and bodies are mandatory in powerful ASP solvers

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$\mathcal{T}_{smodels}$: Example tableau

(r_1)	$a \leftarrow \sim b$	(r_2)	$b \leftarrow d, \sim a$	(r_3)	$c \leftarrow b, d$
(r_4)	$c \leftarrow g$	(r_5)	$d \leftarrow c$	(r_6)	$d \leftarrow g$
(r_7)	$e \leftarrow f, \sim c$	(r_8)	$f \leftarrow \sim g$	(r_9)	$g \leftarrow \sim a, \sim f$

(1)	$\mathbf{T}a$	$[Cut]$
(2)	$\mathbf{T}\{\sim b\}$	$[BTA\text{: } r_1, 1]$
(3)	$\mathbf{F}b$	$[BTB\text{: } 2]$
(4)	$\mathbf{F}\{d, \sim a\}$	$[BFA\text{: } r_2, 3]$
(5)	$\mathbf{F}\{\sim a, \sim f\}$	$[FFB\text{: } r_9, 1]$
(6)	$\mathbf{F}g$	$[FFA\text{: } r_9, 5]$
(7)	$\mathbf{T}\{\sim g\}$	$[FTB\text{: } r_8, 6]$
(8)	$\mathbf{T}f$	$[FTA\text{: } r_8, 7]$
(9)	$\mathbf{F}\{b, d\}$	$[FFB\text{: } r_3, 3]$
(10)	$\mathbf{F}\{g\}$	$[FFB\text{: } r_4, r_6, 6]$
(11)	$\mathbf{F}c$	$[FFA\text{: } r_3, r_4, 9, 10]$
(12)	$\mathbf{F}\{c\}$	$[FFB\text{: } r_5, 11]$
(13)	$\mathbf{F}d$	$[FFA\text{: } r_5, r_6, 10, 12]$
(14)	$\mathbf{T}\{f, \sim c\}$	$[FTB\text{: } r_7, 8, 11]$
(15)	$\mathbf{T}e$	$[FTA\text{: } r_7, 14]$

(16)	$\mathbf{F}a$	$[Cut]$
(17)	$\mathbf{F}\{\sim b\}$	$[BFA\text{: } r_1, 16]$
(18)	$\mathbf{T}b$	$[BFB\text{: } 17]$
(19)	$\mathbf{T}\{d, \sim a\}$	$[BTA\text{: } r_2, 18]$
(20)	$\mathbf{T}d$	$[BTB\text{: } 19]$
(21)	$\mathbf{T}\{b, d\}$	$[FTB\text{: } r_3, 18, 20]$
(22)	$\mathbf{T}c$	$[FTA\text{: } r_3, 21]$
(23)	$\mathbf{F}\{f, \sim c\}$	$[FFB\text{: } r_7, 22]$
(24)	$\mathbf{F}e$	$[FFA\text{: } r_7, 23]$
(25)	$\mathbf{T}\{c\}$	$[FTB\text{: } r_5, 22]$

(26)	$\mathbf{T}f$	$[Cut]$
(27)	$\mathbf{F}\{\sim a, \sim f\}$	$[FFB\text{: } r_9, 26]$
(28)	$\mathbf{F}c$	$[WFN\text{: } 27]$

(29)	$\mathbf{F}f$	$[Cut]$
(30)	$\mathbf{T}\{\sim a, \sim f\}$	$[FTB\text{: } r_9, 16, 29]$
(31)	$\mathbf{T}g$	$[FTA\text{: } r_9, 30]$
(32)	$\mathbf{T}\{g\}$	$[FTB\text{: } r_4, r_6, 31]$
(33)	$\mathbf{F}\{\sim g\}$	$[FFB\text{: } r_8, 31]$

$\mathcal{T}_{noMoRe}$: Example tableau

(r_1)	$a \leftarrow \sim b$	(r_2)	$b \leftarrow d, \sim a$	(r_3)	$c \leftarrow b, d$
(r_4)	$c \leftarrow g$	(r_5)	$d \leftarrow c$	(r_6)	$d \leftarrow g$
(r_7)	$e \leftarrow f, \sim c$	(r_8)	$f \leftarrow \sim g$	(r_9)	$g \leftarrow \sim a, \sim f$

(1)	$\mathbf{T}\{\sim b\}$	[*Cut*]
(2)	$\mathbf{T}a$	[*FTA*: r_1, 1]
(3)	$\mathbf{F}b$	[*BTB*: 1]
(4)	$\mathbf{F}\{d, \sim a\}$	[*BFA*: r_2, 3]
(5)	$\mathbf{F}\{\sim a, \sim f\}$	[*FFB*: r_9, 2]
(6)	$\mathbf{F}g$	[*FFA*: r_9, 5]
(7)	$\mathbf{T}\{\sim g\}$	[*FTB*: r_8, 6]
(8)	$\mathbf{T}f$	[*FTA*: r_8, 7]
(9)	$\mathbf{F}\{b, d\}$	[*FFB*: r_3, 3]
(10)	$\mathbf{F}\{g\}$	[*FFB*: r_4, r_6, 6]
(11)	$\mathbf{F}c$	[*FFA*: r_3, r_4, 9, 10]
(12)	$\mathbf{F}\{c\}$	[*FFB*: r_5, 11]
(13)	$\mathbf{F}d$	[*FFA*: r_5, r_6, 10, 12]
(14)	$\mathbf{T}\{f, \sim c\}$	[*FTB*: r_7, 8, 11]
(15)	$\mathbf{T}e$	[*FTA*: r_7, 14]

(16)	$\mathbf{F}\{\sim b\}$	[*Cut*]
(17)	$\mathbf{F}a$	[*FFA*: r_1, 16]
(18)	$\mathbf{T}b$	[*BFB*: 16]
(19)	$\mathbf{T}\{d, \sim a\}$	[*BTA*: r_2, 18]
(20)	$\mathbf{T}d$	[*BTB*: 19]
(21)	$\mathbf{T}\{b, d\}$	[*FTB*: r_3, 18, 20]
(22)	$\mathbf{T}c$	[*FTA*: r_3, 21]
(23)	$\mathbf{F}\{f, \sim c\}$	[*FFB*: r_7, 22]
(24)	$\mathbf{F}e$	[*FFA*: r_7, 23]
(25)	$\mathbf{T}\{c\}$	[*FTB*: r_5, 22]

(26)	$\mathbf{T}\{\sim g\}$	[*Cut*]
(27)	$\mathbf{F}g$	[*BTB*: 26]
(28)	$\mathbf{F}\{g\}$	[*FFB*: r_4, r_6, 27]
(29)	$\mathbf{F}c$	[*WFN*: 28]

(30)	$\mathbf{F}\{\sim g\}$	[*Cut*]
(31)	$\mathbf{T}g$	[*BFB*: 30]
(32)	$\mathbf{T}\{g\}$	[*FTB*: r_4, r_6, 31]
(33)	$\mathbf{F}f$	[*FFA*: r_8, 30]
(34)	$\mathbf{T}\{\sim a, \sim f\}$	[*FTB*: r_9, 17, 33]

$\mathcal{T}_{nomore^{++}}$: Example tableau

(r_1)	$a \leftarrow \sim b$	(r_2)	$b \leftarrow d, \sim a$	(r_3)	$c \leftarrow b, d$
(r_4)	$c \leftarrow g$	(r_5)	$d \leftarrow c$	(r_6)	$d \leftarrow g$
(r_7)	$e \leftarrow f, \sim c$	(r_8)	$f \leftarrow \sim g$	(r_9)	$g \leftarrow \sim a, \sim f$

(1)	$\mathbf{T}a$	[*Cut*]
(2)	$\mathbf{T}\{\sim b\}$	[*BTA*: r_1, 1]
(3)	$\mathbf{F}b$	[*BTB*: 2]
(4)	$\mathbf{F}\{d, \sim a\}$	[*BFA*: r_2, 3]
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(13)	$\mathbf{F}d$	[*FFA*: r_5, r_6, 10, 12]
(14)	$\mathbf{T}\{f, \sim c\}$	[*FTB*: r_7, 8, 11]
(15)	$\mathbf{T}e$	[*FTA*: r_7, 14]

(16)	$\mathbf{F}a$	[*Cut*]
(17)	$\mathbf{F}\{\sim b\}$	[*BFA*: r_1, 16]
(18)	$\mathbf{T}b$	[*BFB*: 17]
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(34)	$\mathbf{T}\{\sim a, \sim f\}$	[*FTB*: r_9, 16, 33]

Conflict-driven ASP Solving: Overview

Outline

Motivation

- Goal Approach to computing stable models of logic programs, based on concepts from
  - Constraint Processing (CP) and
  - Satisfiability Testing (SAT)
- Idea View inferences in ASP as unit propagation on nogoods
- Benefits
  - A uniform constraint-based framework for different kinds of inferences in ASP
  - Advanced techniques from the areas of CP and SAT
  - Highly competitive implementation

Outline

Assignments

- An assignment A over $dom(A) = atom(P) \cup body(P)$ is a sequence

$$(\sigma_1, \dots, \sigma_n)$$

  of signed literals σ_i of form $\mathbf{T}v$ or $\mathbf{F}v$ for $v \in dom(A)$ and $1 \leq i \leq n$
- $\mathbf{T}v$ expresses that v is *true* and $\mathbf{F}v$ that it is *false*
- The complement, $\overline{\sigma}$, of a literal σ is defined as $\overline{\mathbf{T}v} = \mathbf{F}v$ and $\overline{\mathbf{F}v} = \mathbf{T}v$
- $A \circ \sigma$ stands for the result of appending σ to A
- Given $A = (\sigma_1, \dots, \sigma_{k-1}, \sigma_k, \dots, \sigma_n)$, we let $A[\sigma_k] = (\sigma_1, \dots, \sigma_{k-1})$
- We sometimes identify an assignment with the set of its literals
- Given this, we access *true* and *false* propositions in A via

$$A^{\mathbf{T}} = \{v \in dom(A) \mid \mathbf{T}v \in A\} \text{ and } A^{\mathbf{F}} = \{v \in dom(A) \mid \mathbf{F}v \in A\}$$

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  $$A^{\mathbf{T}} = \{v \in dom(A) \mid \mathbf{T}v \in A\} \text{ and } A^{\mathbf{F}} = \{v \in dom(A) \mid \mathbf{F}v \in A\}$$

Assignments

- An assignment A over $dom(A) = atom(P) \cup body(P)$ is a sequence

$$(\sigma_1, \ldots, \sigma_n)$$

  of signed literals σ_i of form $\mathbf{T}v$ or $\mathbf{F}v$ for $v \in dom(A)$ and $1 \leq i \leq n$
- $\mathbf{T}v$ expresses that v is *true* and $\mathbf{F}v$ that it is *false*
- The complement, $\overline{\sigma}$, of a literal σ is defined as $\overline{\mathbf{T}v} = \mathbf{F}v$ and $\overline{\mathbf{F}v} = \mathbf{T}v$
- $A \circ \sigma$ stands for the result of appending σ to A
- Given $A = (\sigma_1, \ldots, \sigma_{k-1}, \sigma_k, \ldots, \sigma_n)$, we let $A[\sigma_k] = (\sigma_1, \ldots, \sigma_{k-1})$
- We sometimes identify an assignment with the set of its literals
- Given this, we access *true* and *false* propositions in A via

$$A^{\mathbf{T}} = \{v \in dom(A) \mid \mathbf{T}v \in A\} \text{ and } A^{\mathbf{F}} = \{v \in dom(A) \mid \mathbf{F}v \in A\}$$

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Nogoods, solutions, and unit propagation

- A nogood is a set $\{\sigma_1, \dots, \sigma_n\}$ of signed literals, expressing a constraint violated by any assignment containing $\sigma_1, \dots, \sigma_n$
- An assignment A such that $A^{\mathbf{T}} \cup A^{\mathbf{F}} = \mathit{dom}(A)$ and $A^{\mathbf{T}} \cap A^{\mathbf{F}} = \emptyset$ is a solution for a set Δ of nogoods, if $\delta \not\subseteq A$ for all $\delta \in \Delta$
- For a nogood δ, a literal $\sigma \in \delta$, and an assignment A, we say that $\overline{\sigma}$ is unit-resulting for δ wrt A, if
  1. $\delta \setminus A = \{\sigma\}$ and
  2. $\overline{\sigma} \notin A$
- For a set Δ of nogoods and an assignment A, unit propagation is the iterated process of extending A with unit-resulting literals until no further literal is unit-resulting for any nogood in Δ

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Nogoods, solutions, and unit propagation

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Nogoods, solutions, and unit propagation

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Outline

Outline

Nogoods from logic programs
via program completion

The completion of a logic program P can be defined as follows:

$$\{v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n \mid B \in body(P) \text{ and } B = \{a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n\}\}$$

$$\cup \quad \{a \leftrightarrow v_{B_1} \vee \cdots \vee v_{B_k} \mid a \in atom(P) \text{ and } body_P(a) = \{B_1, \ldots, B_k\}\} ,$$

where $body_P(a) = \{body(r) \mid r \in P \text{ and } head(r) = a\}$

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Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \land \cdots \land a_m \land \neg a_{m+1} \land \cdots \land \neg a_n$$

can be decomposed into two implications:

Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$$

can be decomposed into two implications:

1. $v_B \rightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$
   is equivalent to the conjunction of

$$\neg v_B \vee a_1, \; \ldots, \; \neg v_B \vee a_m, \; \neg v_B \vee \neg a_{m+1}, \; \ldots, \; \neg v_B \vee \neg a_n$$

   and induces the set of nogoods

$$\Delta(B) = \{ \{\mathbf{T}B, \mathbf{F}a_1\}, \ldots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \ldots, \{\mathbf{T}B, \mathbf{T}a_n\} \}$$

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Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$$

can be decomposed into two implications:

2. $a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n \rightarrow v_B$

   gives rise to the nogood

$$\delta(B) = \{\mathbf{F}B, \mathbf{T}a_1, \ldots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \ldots, \mathbf{F}a_n\}$$

Nogoods from logic programs
via program completion

- Analogously, the (atom-oriented) equivalence

  $$a \leftrightarrow v_{B_1} \vee \cdots \vee v_{B_k}$$

  yields the nogoods

  1. $\Delta(a) = \{\, \{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\} \,\}$ and
  2. $\delta(a) = \{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\}$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

$\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
$\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal
$\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
$\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\, \{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

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- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\begin{array}{lll} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}$$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

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$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\, \{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\} \,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\, \{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\} \,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
body-oriented nogoods

- For a body $B = \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$, we get

$$\{\mathbf{F}B, \mathbf{T}a_1, \dots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \dots, \mathbf{F}a_n\}$$
$$\{\,\{\mathbf{T}B, \mathbf{F}a_1\}, \dots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \dots, \{\mathbf{T}B, \mathbf{T}a_n\}\,\}$$

- Example Given Body $\{x, \sim y\}$, we obtain

$$\begin{array}{c} \dots \leftarrow x, \sim y \\ \vdots \\ \dots \leftarrow x, \sim y \end{array}$$

$$\{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$$
$$\{\,\{\mathbf{T}\{x, \sim y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, \sim y\}, \mathbf{T}y\}\,\}$$

For nogood $\delta(\{x, \sim y\}) = \{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$, the signed literal
- $\mathbf{T}\{x, \sim y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
- $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, \sim y\}, \mathbf{T}x)$

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Nogoods from logic programs
body-oriented nogoods

- For a body $B = \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$, we get

$$\{\mathbf{F}B, \mathbf{T}a_1, \dots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \dots, \mathbf{F}a_n\}$$
$$\{\, \{\mathbf{T}B, \mathbf{F}a_1\}, \dots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \dots, \{\mathbf{T}B, \mathbf{T}a_n\} \,\}$$

- Example Given Body $\{x, \sim y\}$, we obtain

$$\begin{array}{l} \dots \leftarrow x, \sim y \\ \quad\vdots \\ \dots \leftarrow x, \sim y \end{array}$$

$$\{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$$
$$\{\, \{\mathbf{T}\{x, \sim y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, \sim y\}, \mathbf{T}y\} \,\}$$

For nogood $\delta(\{x, \sim y\}) = \{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$, the signed literal
  - $\mathbf{T}\{x, \sim y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
  - $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, \sim y\}, \mathbf{T}x)$

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Nogoods from logic programs
body-oriented nogoods

- For a body $B = \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$, we get

$$\{\mathbf{F}B, \mathbf{T}a_1, \dots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \dots, \mathbf{F}a_n\}$$
$$\{\,\{\mathbf{T}B, \mathbf{F}a_1\}, \dots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \dots, \{\mathbf{T}B, \mathbf{T}a_n\}\,\}$$

- Example Given Body $\{x, \sim y\}$, we obtain

$$\begin{array}{l} \dots \leftarrow x, \sim y \\ \vdots \\ \dots \leftarrow x, \sim y \end{array}$$

$$\{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$$
$$\{\,\{\mathbf{T}\{x, \sim y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, \sim y\}, \mathbf{T}y\}\,\}$$

For nogood $\delta(\{x, \sim y\}) = \{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$, the signed literal
- $\mathbf{T}\{x, \sim y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
- $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, \sim y\}, \mathbf{T}x)$

Characterization of stable models

for tight logic programs

Let P be a logic program and

$$\begin{aligned}\Delta_P \quad = & \quad \{\delta(a) \mid a \in atom(P)\} \cup \{\delta \in \Delta(a) \mid a \in atom(P)\} \\ \cup & \quad \{\delta(B) \mid B \in body(P)\} \cup \{\delta \in \Delta(B) \mid B \in body(P)\}\end{aligned}$$

Theorem

Let P be a tight logic program. Then,
$X \subseteq atom(P)$ *is a stable model of P iff*
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Characterization of stable models

for tight logic programs, ie. free of positive recursion

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Outline

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Nogoods from logic programs
via loop formulas

Let P be a normal logic program and recall that:

- For $L \subseteq atom(P)$, the external supports of L for P are

$$ES_P(L) = \{r \in P \mid head(r) \in L \text{ and } body(r)^+ \cap L = \emptyset\}$$

- The (disjunctive) loop formula of L for P is

$$LF_P(L) = (\bigvee_{A \in L} A) \rightarrow (\bigvee_{r \in ES_P(L)} body(r))$$
$$\equiv (\bigwedge_{r \in ES_P(L)} \neg body(r)) \rightarrow (\bigwedge_{A \in L} \neg A)$$

  - Note The loop formula of L enforces all atoms in L to be *false* whenever L is not externally supported

- The external bodies of L for P are

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Nogoods from logic programs
loop nogoods

- For a logic program P and some $\emptyset \subset U \subseteq atom(P)$, define the loop nogood of an atom $a \in U$ as
$$\lambda(a, U) \quad = \quad \{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\}$$
where $EB_P(U) = \{B_1, \dots, B_k\}$

- We get the following set of loop nogoods for P:
$$\Lambda_P \quad = \quad \bigcup_{\emptyset \subset U \subseteq atom(P)} \{\lambda(a, U) \mid a \in U\}$$

- The set Λ_P of loop nogoods denies cyclic support among *true* atoms

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Example

- Consider the program

$$\left\{ \begin{array}{ll} & u \leftarrow x \\ x \leftarrow \sim y & u \leftarrow v \\ y \leftarrow \sim x & v \leftarrow u, y \end{array} \right\}$$

- For u in the set $\{u, v\}$, we obtain the loop nogood:

$$\lambda(u, \{u, v\}) \quad = \quad \{\mathbf{T}u, \mathbf{F}\{x\}\}$$

  Similarly for v in $\{u, v\}$, we get:

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Characterization of stable models

Theorem

Let P be a logic program. Then,

$X \subseteq atom(P)$ *is a stable model of* P *iff*

$X = A^{\mathbf{T}} \cap atom(P)$ *for a (unique) solution* A *for* $\Delta_P \cup \Lambda_P$

Some remarks

- Nogoods in Λ_P augment Δ_P with conditions checking for unfounded sets, in particular, those being loops
- While $|\Delta_P|$ is linear in the size of P, Λ_P may contain exponentially many (non-redundant) loop nogoods

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Outline

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Towards conflict-driven search

Boolean constraint solving algorithms pioneered for SAT led to:

- Traditional DPLL-style approach
  (DPLL stands for 'Davis-Putnam-Logemann-Loveland')
  - (Unit) propagation
  - (Chronological) backtracking
  - in ASP, eg *smodels*
- Modern CDCL-style approach
  (CDCL stands for 'Conflict-Driven Constraint Learning')
  - (Unit) propagation
  - Conflict analysis (via resolution)
  - Learning + Backjumping + Assertion
  - in ASP, eg *clasp*

DPLL-style solving

loop

propagate // deterministically assign literals

if no conflict **then**

if all variables assigned **then return** solution

else *decide* // non-deterministically assign some literal

else

if top-level conflict **then return** unsatisfiable

else

backtrack // unassign literals propagated after last decision

flip // assign complement of last decision literal

CDCL-style solving

loop

    propagate // deterministically assign literals

    if no conflict **then**

        if all variables assigned **then return** solution

        else *decide* // non-deterministically assign some literal

    else

        if top-level conflict **then return** unsatisfiable

        else

            analyze // analyze conflict and add conflict constraint

            backjump // unassign literals until conflict constraint is unit

Outline

Outline of CDNL-ASP algorithm

- Keep track of deterministic consequences by unit propagation on:
  - Program completion $[\Delta_P]$
  - Loop nogoods, determined and recorded on demand $[\Lambda_P]$
  - Dynamic nogoods, derived from conflicts and unfounded sets $[\nabla]$
- When a nogood in $\Delta_P \cup \nabla$ becomes violated:
  - Analyze the conflict by resolution (until reaching a Unique Implication Point, short: UIP)
  - Learn the derived conflict nogood δ
  - Backjump to the earliest (heuristic) choice such that the complement of the UIP is unit-resulting for δ
  - Assert the complement of the UIP and proceed (by unit propagation)
- Terminate when either:
  - Finding a stable model (a solution for $\Delta_P \cup \Lambda_P$)
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Potassco

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Algorithm 2: CDNL-ASP

Input : A normal program P
Output : A stable model of P or "no stable model"

$A := \emptyset$ // *assignment over atom(P) ∪ body(P)*
$\nabla := \emptyset$ // *set of recorded nogoods*
$dl := 0$ // *decision level*

loop
  $(A, \nabla) := \textsc{NogoodPropagation}(P, \nabla, A)$
  if $\varepsilon \subseteq A$ **for some** $\varepsilon \in \Delta_P \cup \nabla$ **then** // *conflict*
    if $\max(\{dlevel(\sigma) \mid \sigma \in \varepsilon\} \cup \{0\}) = 0$ **then return** no stable model
    $(\delta, dl) := \textsc{ConflictAnalysis}(\varepsilon, P, \nabla, A)$
    $\nabla := \nabla \cup \{\delta\}$ // *(temporarily) record conflict nogood*
    $A := A \setminus \{\sigma \in A \mid dl < dlevel(\sigma)\}$ // *backjumping*
  else if $A^{\mathbf{T}} \cup A^{\mathbf{F}} = atom(P) \cup body(P)$ **then** // *stable model*
    return $A^{\mathbf{T}} \cap atom(P)$
  else
    $\sigma_d := \textsc{Select}(P, \nabla, A)$ // *decision*
    $dl := dl + 1$
    $dlevel(\sigma_d) := dl$
    $A := A \circ \sigma_d$

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Observations

- Decision level dl, initially set to 0, is used to count the number of heuristically chosen literals in assignment A
- For a heuristically chosen literal $\sigma_d = \mathbf{T}a$ or $\sigma_d = \mathbf{F}a$, respectively, we require $a \in (atom(P) \cup body(P)) \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}})$
- For any literal $\sigma \in A$, $dl(\sigma)$ denotes the decision level of σ, viz. the value dl had when σ was assigned
- A conflict is detected from violation of a nogood $\varepsilon \subseteq \Delta_P \cup \nabla$
- A conflict at decision level 0 (where A contains no heuristically chosen literals) indicates non-existence of stable models
- A nogood δ derived by conflict analysis is asserting, that is, some literal is unit-resulting for δ at a decision level $k < dl$
  - After learning δ and backjumping to decision level k, at least one literal is newly derivable by unit propagation
  - No explicit flipping of heuristically chosen literals !

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Example: CDNL-ASP

Consider

$$P = \left\{ \begin{array}{llll} x \leftarrow \sim y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \sim x, \sim y \\ y \leftarrow \sim x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ
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$$P = \left\{ \begin{array}{llll} x \leftarrow \sim y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \sim x, \sim y \\ y \leftarrow \sim x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

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Outline of NogoodPropagation

- Derive deterministic consequences via:
  - Unit propagation on Δ_P and ∇;
  - Unfounded sets $U \subseteq atom(P)$
- Note that U is unfounded if $EB_P(U) \subseteq A^{\mathbf{F}}$
  - Note For any $a \in U$, we have $(\lambda(a, U) \setminus \{\mathbf{T}a\}) \subseteq A$
- An "interesting" unfounded set U satisfies:

$$\emptyset \subset U \subseteq (atom(P) \setminus A^{\mathbf{F}})$$

- Wrt a fixpoint of unit propagation,
  such an unfounded set contains some loop of P
  - Note Tight programs do not yield "interesting" unfounded sets !
- Given an unfounded set U and some $a \in U$, adding $\lambda(a, U)$ to ∇
  triggers a conflict or further derivations by unit propagation
  - Note Add loop nogoods atom by atom to eventually falsify all $a \in U$

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Algorithm 3: NogoodPropagation

Input : A normal program P, a set ∇ of nogoods, and an assignment A.
Output : An extended assignment and set of nogoods.

$U := \emptyset$ // *unfounded set*

loop
  repeat
    if $\delta \subseteq A$ **for some** $\delta \in \Delta_P \cup \nabla$ **then return** (A, ∇) // *conflict*
    $\Sigma := \{\delta \in \Delta_P \cup \nabla \mid \delta \setminus A = \{\overline{\sigma}\}, \sigma \notin A\}$ // *unit-resulting nogoods*
    if $\Sigma \neq \emptyset$ **then let** $\overline{\sigma} \in \delta \setminus A$ **for some** $\delta \in \Sigma$ **in**
      $dlevel(\sigma) := \max(\{dlevel(\rho) \mid \rho \in \delta \setminus \{\overline{\sigma}\}\} \cup \{0\})$
      $A := A \circ \sigma$
  until $\Sigma = \emptyset$
  if $loop(P) = \emptyset$ **then return** (A, ∇)
  $U := U \setminus A^{\mathbf{F}}$
  if $U = \emptyset$ **then** $U :=$ UnfoundedSet(P, A)
  if $U = \emptyset$ **then return** (A, ∇) // *no unfounded set* $\emptyset \subset U \subseteq atom(P) \setminus A^{\mathbf{F}}$
  let $a \in U$ **in**
    $\nabla := \nabla \cup \{\{\mathbf{T}a\} \cup \{\mathbf{F}B \mid B \in EB_P(U)\}\}$ // *record loop nogood*

Requirements for UNFOUNDEDSET

- Implementations of UNFOUNDEDSET must guarantee the following for a result U
  1. $U \subseteq (atom(P) \setminus A^{\mathbf{F}})$
  2. $EB_P(U) \subseteq A^{\mathbf{F}}$
  3. $U = \emptyset$ iff there is no nonempty unfounded subset of $(atom(P) \setminus A^{\mathbf{F}})$
- Beyond that, there are various alternatives, such as:
  - Calculating the greatest unfounded set
  - Calculating unfounded sets within strongly connected components of the positive atom dependency graph of P
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Outline

Outline of ConflictAnalysis

- Conflict analysis is triggered whenever some nogood $\delta \in \Delta_P \cup \nabla$ becomes violated, viz. $\delta \subseteq A$, at a decision level $dl > 0$
  - Note that all but the first literal assigned at dl have been unit-resulting for nogoods $\varepsilon \in \Delta_P \cup \nabla$
  - If $\sigma \in \delta$ has been unit-resulting for ε, we obtain a new violated nogood by resolving δ and ε as follows:

$$(\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$$

- Resolution is directed by resolving first over the literal $\sigma \in \delta$ derived last, viz. $(\delta \setminus A[\sigma]) = \{\sigma\}$
  - Iterated resolution progresses in inverse order of assignment
- Iterated resolution stops as soon as it generates a nogood δ containing exactly one literal σ assigned at decision level dl
  - This literal σ is called First Unique Implication Point (First-UIP)
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Algorithm 4: CONFLICTANALYSIS

Input : A non-empty violated nogood δ, a normal program P, a set ∇ of nogoods, and an assignment A.

Output : A derived nogood and a decision level.

loop
  let $\sigma \in \delta$ **such that** $\delta \setminus A[\sigma] = \{\sigma\}$ **in**
    $k := \max(\{dlevel(\rho) \mid \rho \in \delta \setminus \{\sigma\}\} \cup \{0\})$
    if $k = dlevel(\sigma)$ **then**
      let $\varepsilon \in \Delta_P \cup \nabla$ **such that** $\varepsilon \setminus A[\sigma] = \{\overline{\sigma}\}$ **in**
        $\delta := (\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$ // *resolution*
    else return (δ, k)

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			$\{\mathbf{T}u, \mathbf{F}\{x\}, \mathbf{F}\{x, y\}\} = \lambda(u, \{u, v\})$	✘

Example: ConflictAnalysis

Consider

$$P = \left\{ \begin{array}{llll} x \leftarrow \sim y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \sim x, \sim y \\ y \leftarrow \sim x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

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Remarks

- There always is a First-UIP at which conflict analysis terminates
  - In the worst, resolution stops at the heuristically chosen literal assigned at decision level *dl*
- The nogood δ containing First-UIP σ is violated by A, viz. $\delta \subseteq A$
- We have $k = max(\{dl(\rho) \mid \rho \in \delta \setminus \{\sigma\}\} \cup \{0\}) < dl$
  - After recording δ in ∇ and backjumping to decision level k, $\overline{\sigma}$ is unit-resulting for δ !
  - Such a nogood δ is called asserting
- Asserting nogoods direct conflict-driven search into a different region of the search space than traversed before, without explicitly flipping any heuristically chosen literal !

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Systems: Overview

Outline

Potassco

potassco.sourceforge.net

Potassco, the Potsdam Answer Set Solving Collection, bundles tools for ASP developed at the University of Potsdam, for instance:

- Grounder gringo, lingo, pyngo
- Solver clasp, {a,h,un}clasp, claspD, claspfolio, claspar, aspeed
- Grounder+Solver Clingo, iClingo, {ros}oClingo, Clingcon
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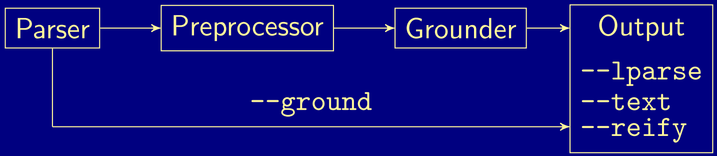
gringo

- Accepts safe programs with aggregates
- Tolerates unrestricted use of function symbols
  (as long as it yields a finite ground instantiation :)
- Expressive power of a Turing machine

- Basic architecture of *gringo*:



Outline

Potassco

clasp

- *clasp* is a native ASP solver combining conflict-driven search with sophisticated reasoning techniques:
  - advanced preprocessing, including equivalence reasoning
  - lookback-based decision heuristics
  - restart policies
  - nogood deletion
  - progress saving
  - dedicated data structures for binary and ternary nogoods
  - lazy data structures (watched literals) for long nogoods
  - dedicated data structures for cardinality and weight constraints
  - lazy unfounded set checking based on "source pointers"
  - tight integration of unit propagation and unfounded set checking
  - various reasoning modes
  - parallel search
  - ...

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Reasoning modes of *clasp*

- Beyond deciding (stable) model existence, *clasp* allows for:
  - Optimization
  - Enumeration (without solution recording)
  - Projective enumeration (without solution recording)
  - Intersection and Union (linear solution computation)
  - and combinations thereof
- *clasp* allows for
  - ASP solving (*smodels* format)
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Potassco

Parallel search in *clasp*

- *clasp*
  - pursues a coarse-grained, task-parallel approach to parallel search via shared memory multi-threading
    - up to 64 configurable (non-hierarchic) threads
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Sequential CDCL-style solving

loop

propagate // deterministically assign literals

if no conflict **then**

if all variables assigned **then return** solution

else *decide* // non-deterministically assign some literal

else

if top-level conflict **then return** unsatisfiable

else

analyze // analyze conflict and add conflict constraint

backjump // unassign literals until conflict constraint is unit

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Parallel CDCL-style solving in *clasp*

```
while work available
  while no (result) message to send
    communicate              // exchange information with other solver
    propagate                // deterministically assign literals
    if no conflict then
      if all variables assigned then send solution
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    else
      if root-level conflict then send unsatisfiable
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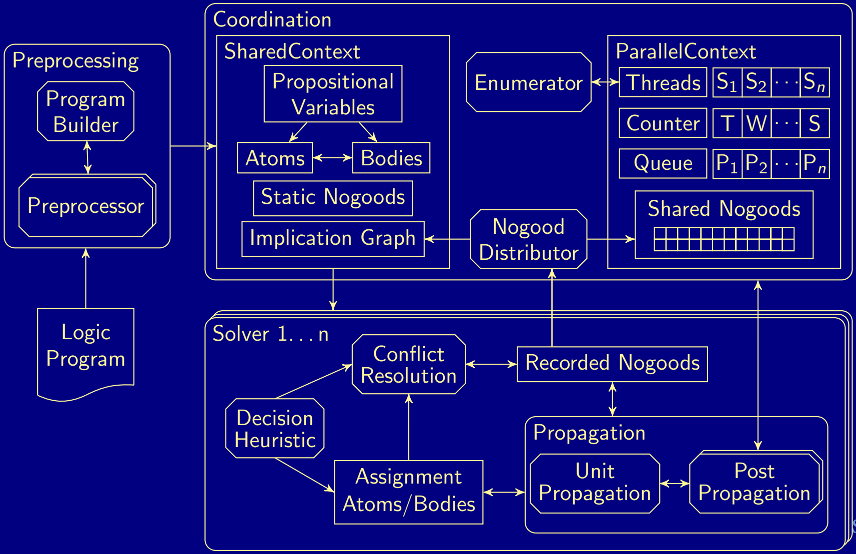
Multi-threaded architecture of *clasp*



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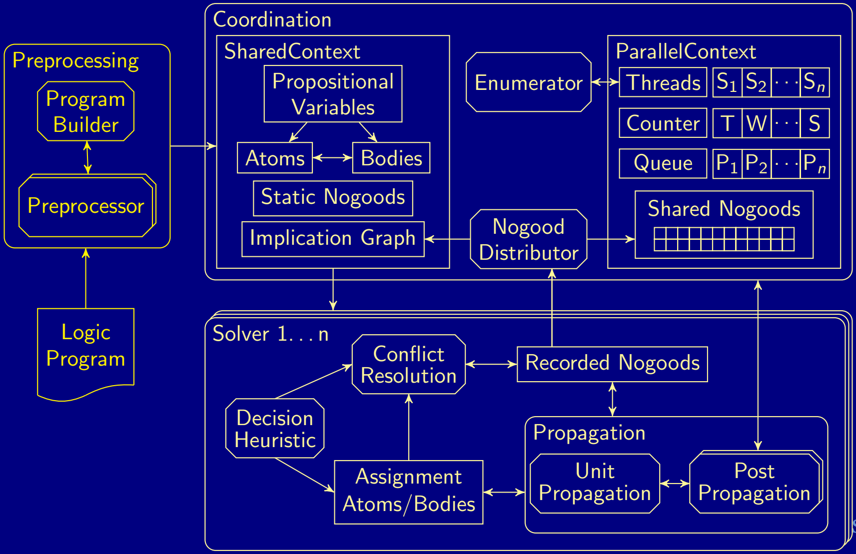

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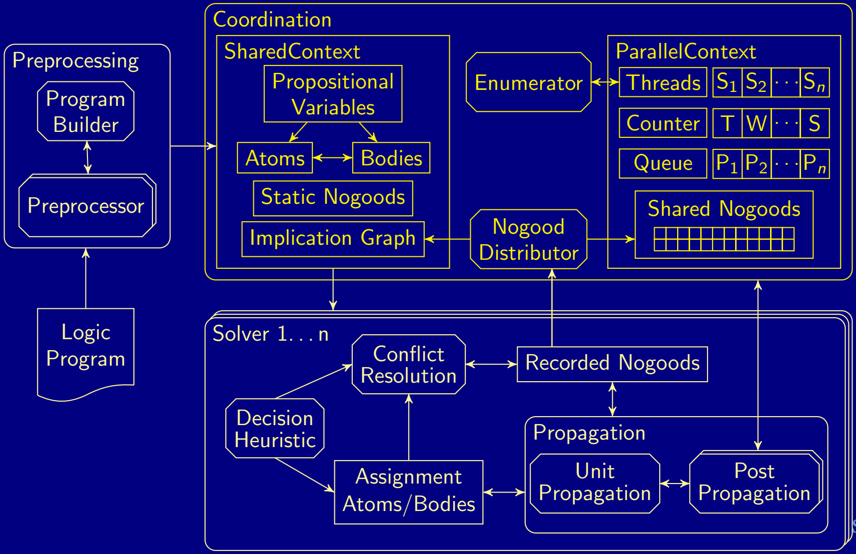

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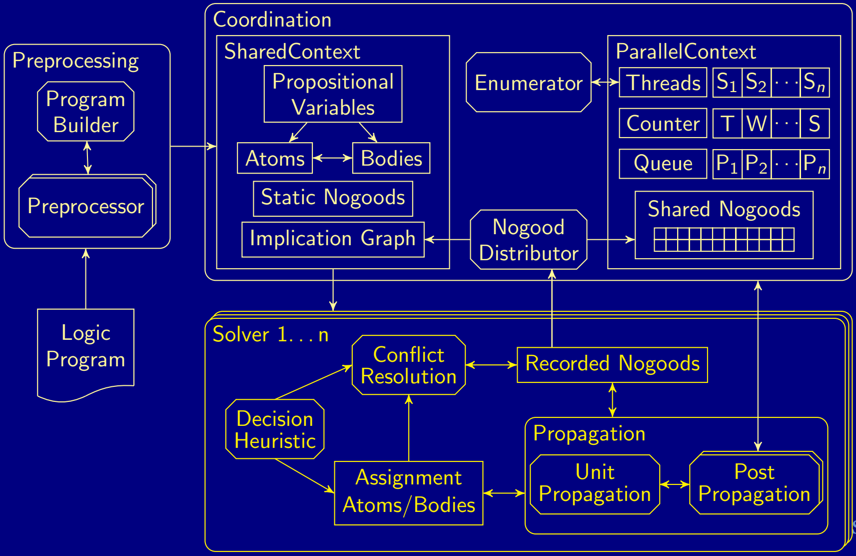

clasp in context

- Compare *clasp* (2.0.5) to the multi-threaded SAT solvers
  - *cryptominisat* (2.9.2)
  - *manysat* (1.1)
  - *miraxt* (2009)
  - *plingeling* (587f)

  all run with four and eight threads in their default settings
- 160/300 benchmarks from crafted category at SAT'11
  - all solvable by *ppfolio* in 1000 seconds
  - crafted SAT benchmarks are closest to ASP benchmarks

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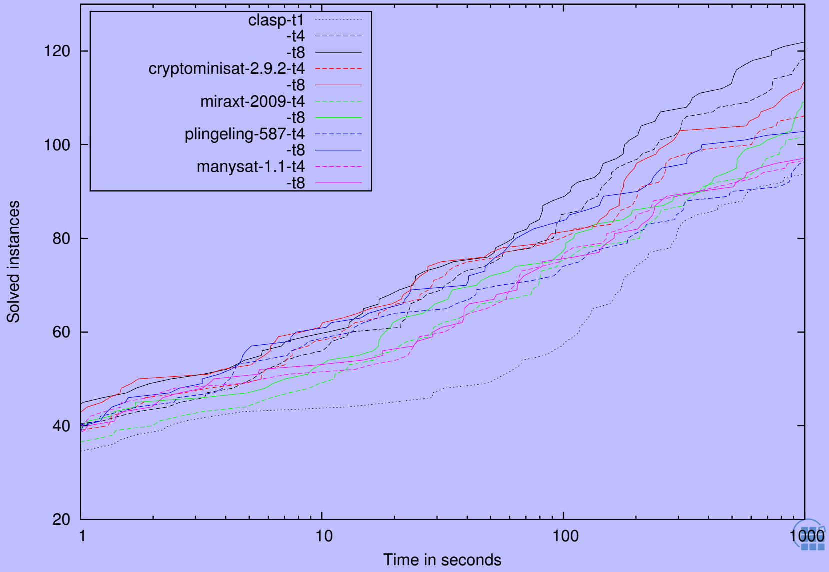
clasp in context



Using *clasp*

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--parallel-mode,-t <arg>: Run parallel search with given number of threads
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--configuration=<arg>   : Configure default configuration [frumpy]
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Outline

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Outline

hclasp

- *hclasp* allows for incorporating domain-specific heuristics into ASP solving
  - input language for expressing domain-specific heuristics
  - solving capacities for integrating domain-specific heuristics
- Example

```
_heuristics(occ(A,T),factor,T) :- action(A), time(T).
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Basic CDCL decision algorithm

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Inside *decide*

- Heuristic functions

$$h : \mathcal{A} \rightarrow [0, +\infty) \quad \text{and} \quad s : \mathcal{A} \rightarrow \{\mathbf{T}, \mathbf{F}\}$$

- Algorithmic scheme

1. $h(a) := \alpha \times h(a) + \beta(a)$ for each $a \in \mathcal{A}$
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Inside *decide*

- Heuristic functions

$$h : \mathcal{A} \rightarrow [0, +\infty) \quad \text{and} \quad s : \mathcal{A} \rightarrow \{\mathbf{T}, \mathbf{F}\}$$

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Heuristic language elements

- Heuristic predicate `_heuristic`
- Heuristic modifiers (atom, *a*, and integer, *v*)
  - `init` for initializing the heuristic value of *a* with *v*
  - `factor` for amplifying the heuristic value of *a* by factor *v*
  - `level` for ranking all atoms; the rank of *a* is *v*
  - `sign` for attributing the sign of *v* as truth value to *a*
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_heuristic(occurs(mv,5),factor,5) :- action(mv), time(5).
```

Potassco

Heuristic language elements

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`init`	for initializing the heuristic value of *a* with *v*
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```

Heuristic language elements

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  `level` for ranking all atoms; the rank of *a* is *v*

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```

Potassco

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  - `sign` for attributing the sign of *v* as truth value to *a*
- Heuristic atoms

```
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```

Potassco

Simple STRIPS planner

```
time(1..t).

holds(P,0) :- init(P).

1 { occurs(A,T) : action(A) } 1 :- time(T).
 :- occurs(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- holds(F,T-1), not nolds(F,T), time(T).
holds(F,T) :- occurs(A,T), add(A,F).
nolds(F,T) :- occurs(A,T), del(A,F).

 :- query(F), not holds(F,t).
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nolds(F,T) :- occurs(A,T), del(A,F).

 :- query(F), not holds(F,t).

_heuristic(occurs(A,T),factor,2) :- action(A), time(T).
```

Simple STRIPS planner

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nolds(F,T) :- occurs(A,T), del(A,F).

 :- query(F), not holds(F,t).

_heuristic(occurs(A,T),level,1) :- action(A), time(T).
```

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_heuristic(A,level,V) :- _heuristic(A,true, V).
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 :- query(F), not holds(F,t).

_heuristic(A,level,V) :- _heuristic(A,false,V).
_heuristic(A,sign,-1) :- _heuristic(A,false,V).
```

Simple STRIPS planner

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nolds(F,T) :- occurs(A,T), del(A,F).

 :- query(F), not holds(F,t).

_heuristic(holds(F,T-1),true, t-T+1) :- holds(F,T).
_heuristic(holds(F,T-1),false,t-T+1) :-
                fluent(F), time(T), not holds(F,T).
```

Heuristic modifications to functions h and s

- $\nu(V_{a,m}(A))$ — "*value for modifier m on atom a wrt assignment A*"
- init and

$$d_0(a) = \nu(V_{a,\texttt{init}}(A_0)) + h_0(a)$$

$$d_i(a) = \begin{cases} \nu(V_{a,\texttt{factor}}(A_i)) \times h_i(a) & \text{if } V_{a,\texttt{factor}}(A_i) \neq \emptyset \\ h_i(a) & \text{otherwise} \end{cases}$$

- sign

$$t_i(a) = \begin{cases} \mathbf{T} & \text{if } \nu(V_{a,\texttt{sign}}(A_i)) > 0 \\ \mathbf{F} & \text{if } \nu(V_{a,\texttt{sign}}(A_i)) < 0 \\ s_i(a) & \text{otherwise} \end{cases}$$

- level $\quad \ell_{A_i}(\mathcal{A}') = argmax_{a \in \mathcal{A}'} \nu(V_{a,\texttt{level}}(A_i)) \qquad \mathcal{A}' \subseteq \mathcal{A}$

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Potassco

Selected high scores from systematic experiments

Setting	Labyrinth	Sokoban	Hanoi Tower
base configuration	9,108*s* (14) 24,545,667	2,844*s* (3) 19,371,267	9,137*s* (11) 41,016,235
a, `init`, 2	95% (12) 94%	91% (1) 84%	85% (9) 89%
a, `factor`, 4	78% (8) 30%	120% (1) 107%	109% (11) 110%
a, `factor`, 16	78% (10) 23%	120% (1) 107%	109% (11) 110%
a, `level`, 1	90% (12) 5%	119% (2) 91%	126% (15) 120%
f, `init`, 2	103% (14) 123%	74% (2) 71%	97% (10) 109%
f, `factor`, 2	98% (12) 49%	116% (3) 134%	55% (6) 70%
f, `sign`, -1	94% (13) 89%	105% (1) 100%	92% (12) 92%

base configuration versus 38 (static) heuristic modifications (action, a, and fluent, f)

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Potassco

Abductive problems with optimization

Setting	*Diagnosis*	*Expansion*	*Repair (H)*	*Repair (S)*
base configuration	111.1*s* (115)	161.5*s* (100)	101.3*s* (113)	33.3*s* (27)
`sign,-1`	324.5*s* (407)	7.6*s* (3)	8.4*s* (5)	3.1*s* (0)
`sign,-1 factor,2`	310.1*s* (387)	7.4*s* (2)	3.5*s* (0)	3.2*s* (1)
`sign,-1 factor,8`	305.9*s* (376)	7.7*s* (2)	3.1*s* (0)	2.9*s* (0)
`sign,-1 level,1`	76.1*s* (83)	6.6*s* (2)	0.8*s* (0)	2.2*s* (1)
`level,1`	77.3*s* (86)	12.9*s* (5)	3.4*s* (0)	2.1*s* (0)

(abducibles subject to optimization)

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Potassco

Planning Competition Benchmarks

```
_heuristic(holds(F,T-1),true, t-T+1) :- holds(F,T).
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```

Problem	*base configuration*		`_heuristic`		*base c. (SAT)*		`_heur.` *(SAT)*	
Blocks'00	134.4*s*	(180/61)	9.2*s*	(239/3)	163.2*s*	(59)	2.6*s*	(0)
Elevator'00	3.1*s*	(279/0)	0.0*s*	(279/0)	3.4*s*	(0)	0.0*s*	(0)
Freecell'00	288.7*s*	(147/115)	184.2*s*	(194/74)	226.4*s*	(47)	52.0*s*	(0)
Logistics'00	145.8*s*	(148/61)	115.3*s*	(168/52)	113.9*s*	(23)	15.5*s*	(3)
Depots'02	400.3*s*	(51/184)	297.4*s*	(115/135)	389.0*s*	(64)	61.6*s*	(0)
Driverlog'02	308.3*s*	(108/143)	189.6*s*	(169/92)	245.8*s*	(61)	6.1*s*	(0)
Rovers'02	245.8*s*	(138/112)	165.7*s*	(179/79)	162.9*s*	(41)	5.7*s*	(0)
Satellite'02	398.4*s*	(73/186)	229.9*s*	(155/106)	364.6*s*	(82)	30.8*s*	(0)
Zenotravel'02	350.7*s*	(101/169)	239.0*s*	(154/116)	224.5*s*	(53)	6.3*s*	(0)
Total	252.8*s*	(1225/1031)	158.9*s*	(1652/657)	187.2*s*	(430)	17.1*s*	(3)

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Satellite'02	398.4*s*	(73/186)	229.9*s*	(155/106)	364.6*s*	(82)	30.8*s*	(0)
Zenotravel'02	350.7*s*	(101/169)	239.0*s*	(154/116)	224.5*s*	(53)	6.3*s*	(0)
Total	252.8*s*	(1225/1031)	158.9*s*	(1652/657)	187.2*s*	(430)	17.1*s*	(3)

Planning Competition Benchmarks

```
_heuristic(holds(F,T-1),true, t-T+1) :- holds(F,T).
_heuristic(holds(F,T-1),false,t-T+1) :-
                fluent(F), time(T), not holds(F,T).
```

Problem	*base configuration*		`_heuristic`		*base c.*	*(SAT)*	`_heur.`	*(SAT)*
Blocks'00	134.4*s*	(180/61)	9.2*s*	(239/3)	163.2*s*	(59)	2.6*s*	(0)
Elevator'00	3.1*s*	(279/0)	0.0*s*	(279/0)	3.4*s*	(0)	0.0*s*	(0)
Freecell'00	288.7*s*	(147/115)	184.2*s*	(194/74)	226.4*s*	(47)	52.0*s*	(0)
Logistics'00	145.8*s*	(148/61)	115.3*s*	(168/52)	113.9*s*	(23)	15.5*s*	(3)
Depots'02	400.3*s*	(51/184)	297.4*s*	(115/135)	389.0*s*	(64)	61.6*s*	(0)
Driverlog'02	308.3*s*	(108/143)	189.6*s*	(169/92)	245.8*s*	(61)	6.1*s*	(0)
Rovers'02	245.8*s*	(138/112)	165.7*s*	(179/79)	162.9*s*	(41)	5.7*s*	(0)
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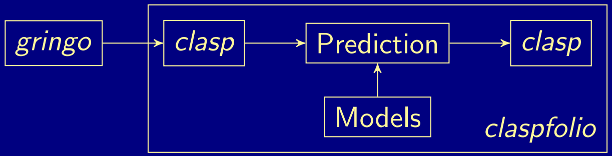
Total	252.8*s*	(1225/1031)	158.9*s*	(1652/657)	187.2*s*	(430)	17.1*s*	(3)

Potassco

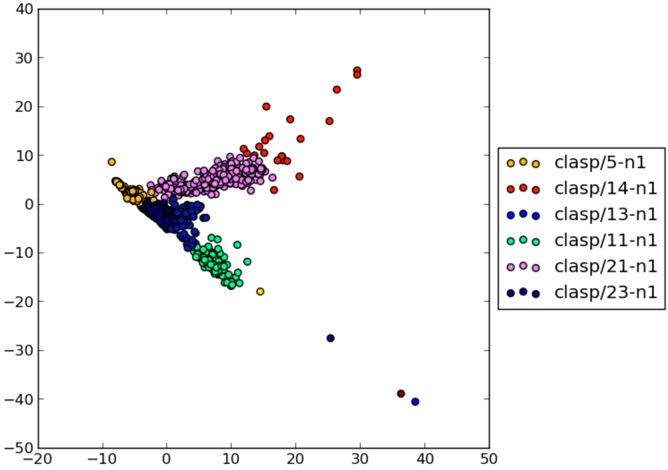
Outline

claspfolio

- Automatic selection of some *clasp* configuration among several predefined ones via (learned) classifiers
- Basic architecture of *claspfolio*:



Instance Feature Clusters (after PCA)



Potassco

Solving with *clasp* (as usual)

```
$ clasp queens500 --quiet

clasp version 2.0.2
Reading from queens500
Solving...
SATISFIABLE

Models       : 1+
Time         : 11.445s (Solving: 10.58s 1st Model: 10.55s Unsat: 0.00s)
CPU Time     : 11.410s
```

Potassco

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clasp version 2.0.2
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Potassco

Solving with *claspfolio*

```
$ claspfolio queens500 --quiet

PRESOLVING
Reading from queens500
Solving...
claspfolio version 1.0.1 (based on clasp version 2.0.2)
Reading from queens500
Solving...
SATISFIABLE

Models       : 1+
Time         : 4.785s (Solving: 3.96s 1st Model: 3.92s Unsat: 0.00s)
CPU Time     : 4.780s
```

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Reading from queens500
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claspfolio version 1.0.1 (based on clasp version 2.0.2)
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```

Feature-extraction with *claspfolio*

```
$ claspfolio --features queens500

PRESOLVING
Reading from queens500
Solving...
UNKNOWN
Features    : 84998,3994,0,250000,1.020,62.594,63.844,21.281,84998, \
 3994,100,250000,1.020,62.594,63.844,21.281,84998,3994,250,250000,  \
 1.020,62.594,63.844,21.281,84998,3994,475,250000,1.020,62.594,     \
 63.844,21.281,757989,757989,0,510983,506992,3990,1,0,127.066,9983, \
 1023958,502993,1994,518971,1,0,0,254994,0,3990,0.100,0.000,99.900, \
 0,270303,812,4,0,812,2223,2223,262,262,2.738,2.738,0.000,812,812,  \
 2270.982,0,0.000

$ claspfolio --list-features

maxLearnt,Constraints,LearntConstraints,FreeVars,Vars/FreeVars,...
```

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PRESOLVING
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Solving...
UNKNOWN
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 2270.982,0,0.000

$ claspfolio --list-features

maxLearnt,Constraints,LearntConstraints,FreeVars,Vars/FreeVars, ...
```

Prediction with *claspfolio*

```
$ claspfolio queens500 --decisionvalues

PRESOLVING
Reading from queens500
Solving...

Portfolio Decision Values:
[1] : 3.437538   [10] : 3.639444   [19] : 3.726391
[2] : 3.501728   [11] : 3.483334   [20] : 3.020325
[3] : 3.784733   [12] : 3.271890   [21] : 3.220219
[4] : 3.672955   [13] : 3.344085   [22] : 3.998709
[5] : 3.557408   [14] : 3.315235   [23] : 3.961214
[6] : 3.942037   [15] : 3.620479   [24] : 3.512924
[7] : 3.335304   [16] : 3.396838   [25] : 3.078143
[8] : 3.375315   [17] : 3.238764
[9] : 3.432931   [18] : 3.403484

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UNKNOWN
```

Solving with *claspfolio* (slightly verbosely)

```
$ claspfolio queens500 --quiet --autoverbose=1

PRESOLVING
Reading from queens500
Solving...

Chosen configuration: [20]
clasp --configurations=./models/portfolio.txt           \
      --modelpath=./models/                             \
      queens500 --quiet --autoverbose=1                 \
      --heu=VSIDS --sat-pre=20,25,120 --trans-ext=integ

claspfolio version 1.0.1 (based on clasp version 2.0.2)
Reading from queens500
Solving...
SATISFIABLE

Models       : 1+
Time         : 4.783s (Solving: 3.96s 1st Model: 3.93s Unsat: 0.00s)
CPU Time     : 4.760s
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SATISFIABLE

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```

Outline

claspD

- *claspD* is a multi-threaded solver for disjunctive logic programs
- aiming at an equitable interplay between "generating" and "testing" solver units
- allowing for a bidirectional dynamic information exchange between solver units for orthogonal tasks

claspD

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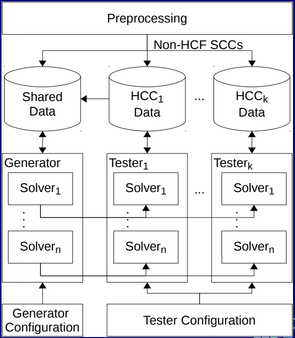


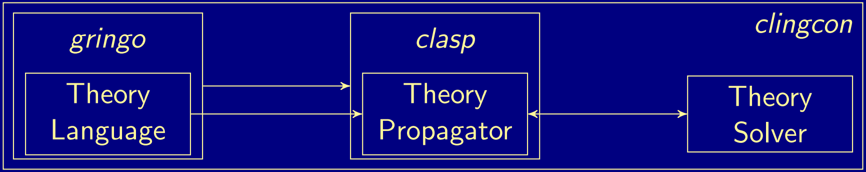
Potassco

Outline

clingcon

- Hybrid grounding and solving
- Solving in hybrid domains, like Bio-Informatics

- Basic architecture of *clingcon*:



Pouring Water into Buckets on a Scale

```
time(0..t).               $domain(0..500).
bucket(a).                volume(a,0) $== 0.
bucket(b).                volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Potassco

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Potassco

Pouring Water into Buckets on a Scale

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time(0..t).                $domain(0..500).
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bucket(b).                 volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, not (1 $<= amount(B,T)).
amount(B,T) $<= 30             :- pour(B,T), T < t.
amount(B,T) $==  0             :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

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```

Potassco

Pouring Water into Buckets on a Scale

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 :- pour(B,T), T < t, 1 $> amount(B,T).
amount(B,T) $<= 30            :- pour(B,T), T < t.
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 :- pour(B,T), T < t, amount(B,T) $> 30.
 :- not pour(B,T), bucket(B), time(T), T < t, amount(B,T) $!=  0.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
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Potassco

Pouring Water into Buckets on a Scale

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 :- pour(B,T), T < t, 1 $> amount(B,T).
 :- pour(B,T), T < t, amount(B,T) $> 30.
 :- not pour(B,T), bucket(B), time(T), T < t, amount(B,T) $!=  0.

 :- bucket(B), time(T), T < t, volume(B,T+1) $!= volume(B,T)$+amount(B,T).

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                  $domain(0..500).
bucket(a).                                              :- volume(a,0) $!= 0.
bucket(b).                                              :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                       ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                    ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                    ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                   ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                   ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.               ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.               ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).  ...  :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).  ...  :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).            ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).            ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).            ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).            ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                           ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                           ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                $domain(0..500).
bucket(a).                                            :- volume(a,0) $!= 0.
bucket(b).                                            :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                    ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                 ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                 ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.            ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.            ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)). ...  :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)). ...  :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).         ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).         ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).         ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).         ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                        ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                        ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                 $domain(0..500).
bucket(a).                                             :- volume(a,0) $!= 0.
bucket(b).                                             :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                     ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                  ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                  ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                 ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                 ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.             ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.             ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)). ...  :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)). ...  :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).          ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).          ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).          ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).          ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                         ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                         ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                 $domain(0..500).
bucket(a).                                             :- volume(a,0) $!= 0.
bucket(b).                                             :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                     ...  1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                  ...   :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                  ...   :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                 ...   :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                 ...   :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.             ...   :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.             ...   :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)). ...   :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)). ...   :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).          ...  down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).          ...  down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).          ...  down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).          ...  down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                         ...  up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                         ...  up(b,4) :- not down(b,4).

 :- up(a,4).
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                  $domain(0..500).
bucket(a).                                              :- volume(a,0) $!= 0.
bucket(b).                                              :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                      ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                   ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                   ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                  ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                  ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.              ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.              ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).  ...  :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).  ...  :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).           ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).           ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).           ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).           ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                          ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                          ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models       : 1
Time         : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models       : 1
Time         : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Boolean variables

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Non-Boolean variables

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11         amount(b,0)=0          1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30         amount(b,1)=0          1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30         amount(b,2)=0          1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30         amount(b,3)=0          1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100        volume(a,0) $< volume(b,0)
volume(a,1)=11         volume(b,1)=100        volume(a,1) $< volume(b,1)
volume(a,2)=41         volume(b,2)=100        volume(a,2) $< volume(b,2)
volume(a,3)=71         volume(b,3)=100        volume(a,3) $< volume(b,3)
volume(a,4)=101        volume(b,4)=100        volume(b,4) $< volume(a,4)

SATISFIABLE

Models       : 1+
Time         : 0.000
```

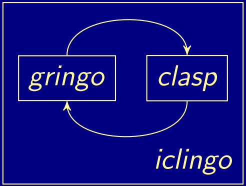
Potassco

Outline

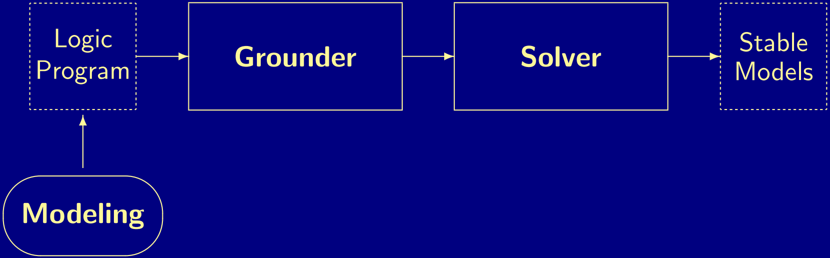
iclingo

- Incremental grounding and solving
- Offline solving in dynamic domains, like Automated Planning

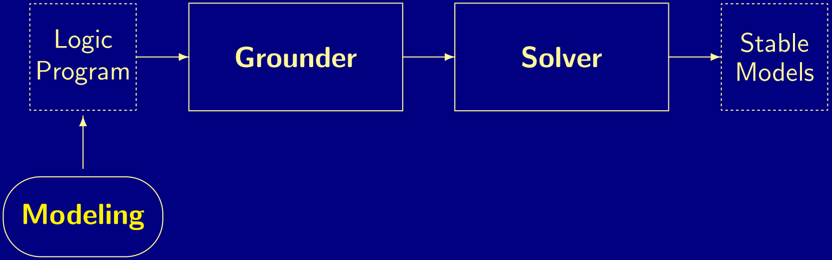
- Basic architecture of *iclingo*:



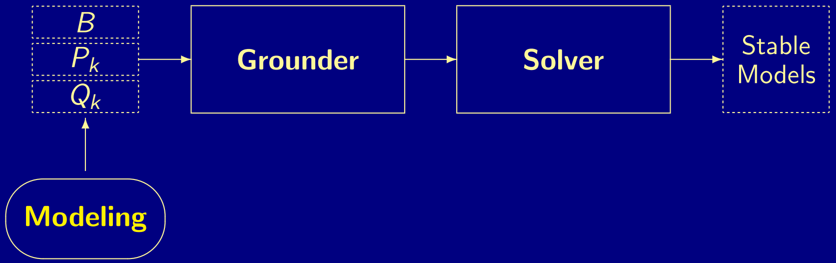
Incremental ASP Solving Process



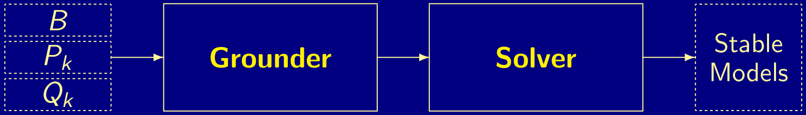
Incremental ASP Solving Process



Incremental ASP Solving Process



Incremental ASP Solving Process



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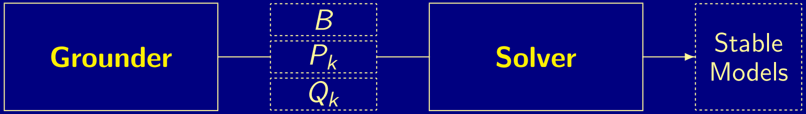

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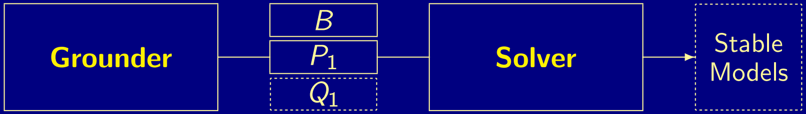

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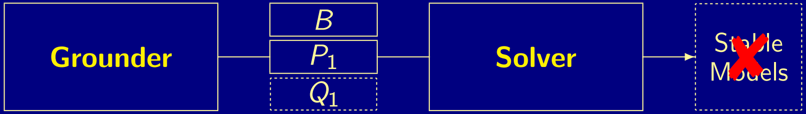

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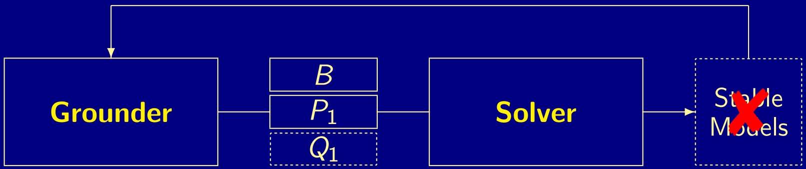

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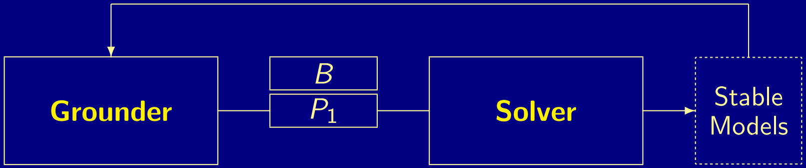

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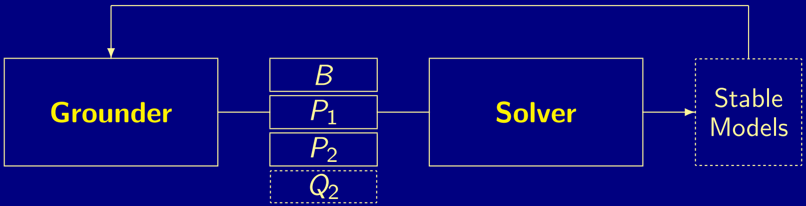

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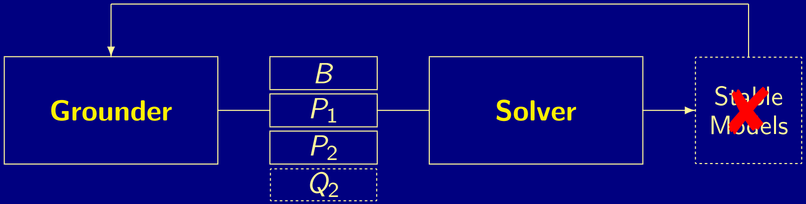

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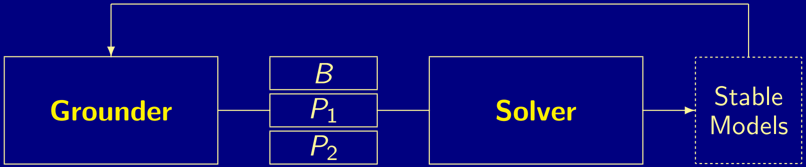

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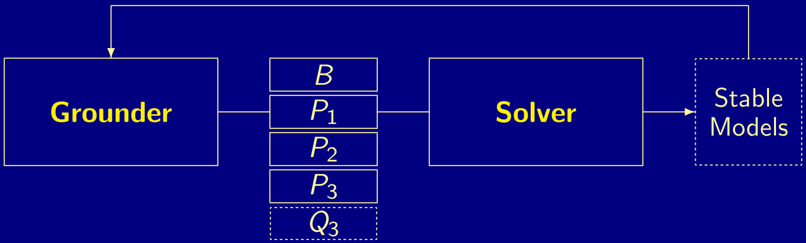

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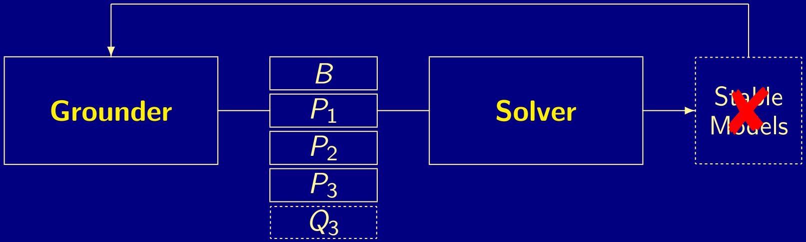

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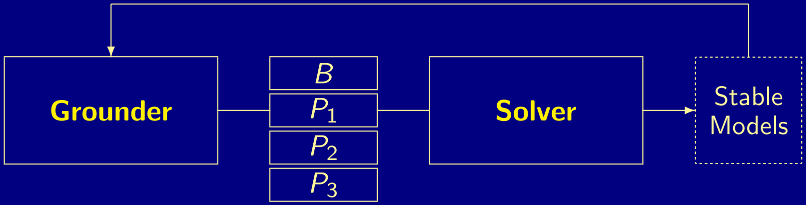

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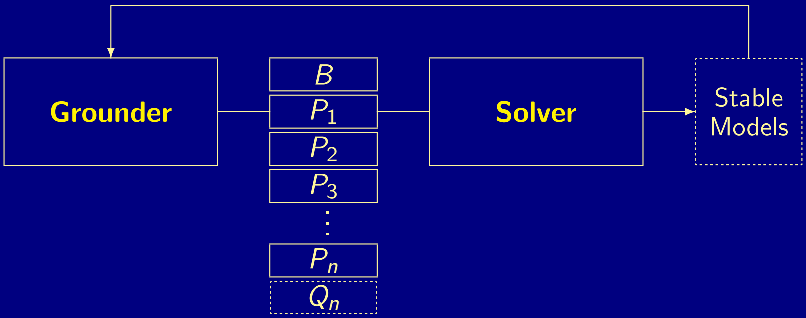

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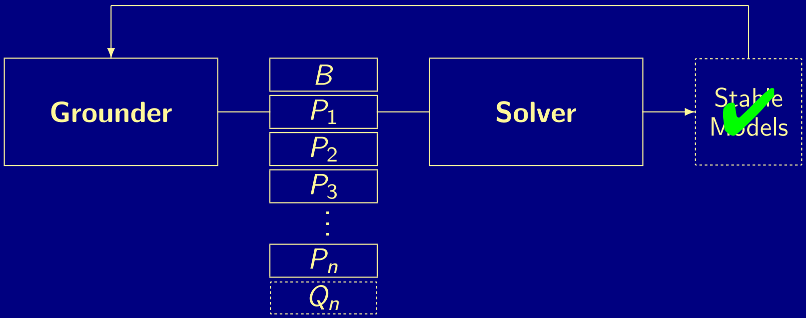

Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).      query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

holds(F,t) :- holds(F,t-1), not nolds(F,t).
holds(F,t) :- occ(A,t), add(A,F).
nolds(F,t) :- occ(A,t), del(A,F).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

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```
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fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
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#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp

Answer: 1
occ(a,1) occ(b,2)
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp

Answer: 1
occ(a,1) occ(b,2)
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp --istats

=============== step 1 ===============

Models   : 0
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 27
Choices  : 0
Conflicts: 0
=============== step 2 ===============
Answer: 1
occ(a,1) occ(b,2)

Models   : 1
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 16
Choices  : 0
Conflicts: 0
=============== Summary ===============
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp --istats

=============== step 1 ===============

Models   : 0
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 27
Choices  : 0
Conflicts: 0
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Answer: 1
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Models   : 1
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Rules    : 16
Choices  : 0
Conflicts: 0
=============== Summary ===============
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```
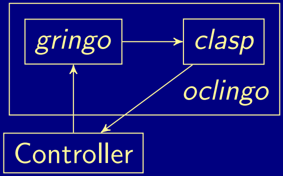

Outline

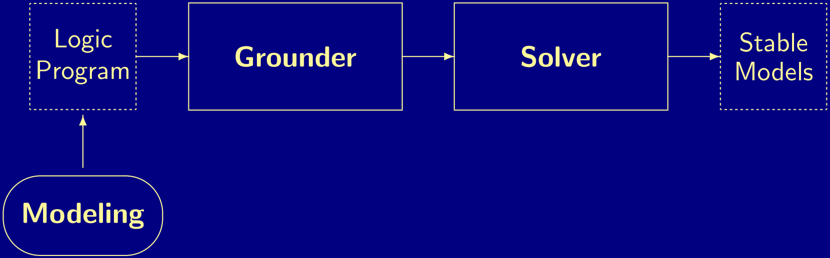
oclingo

- Reactive grounding and solving
- Online solving in dynamic domains, like Robotics

- Basic architecture of *oclingo*:



Reactive ASP Solving Process



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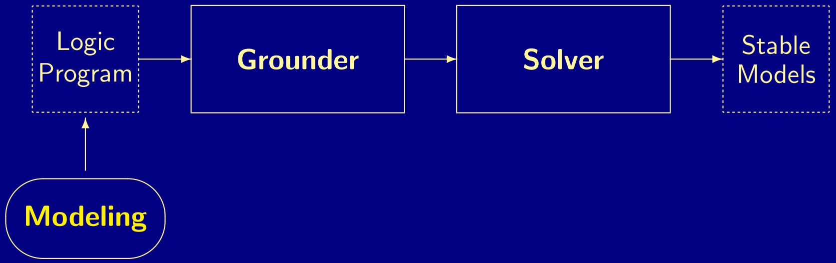

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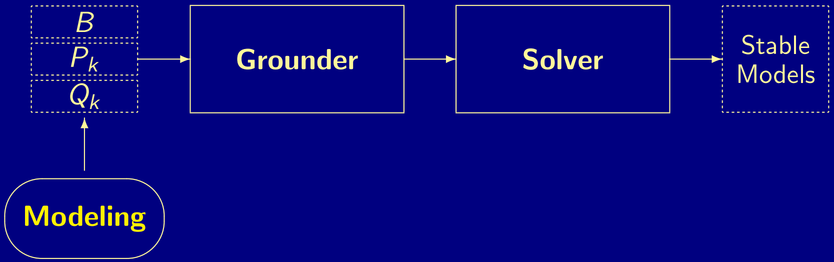
Reactive ASP Solving Process



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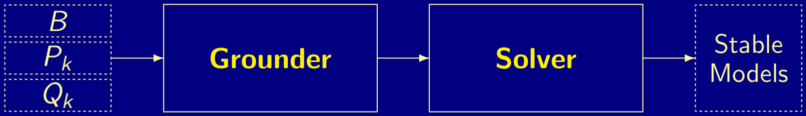

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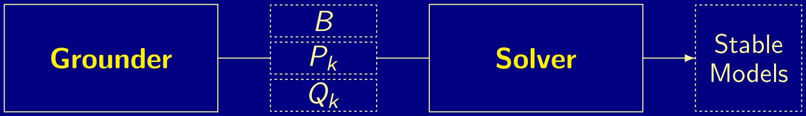

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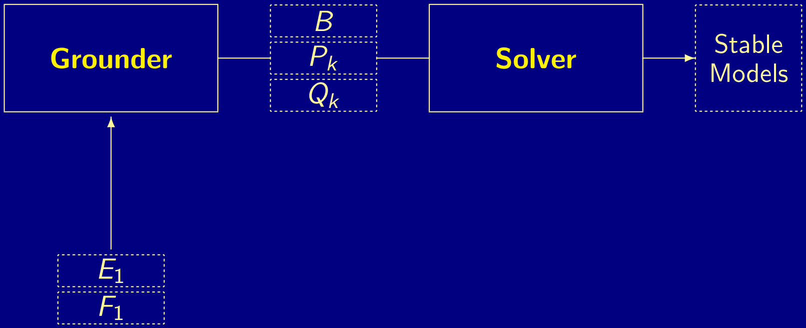

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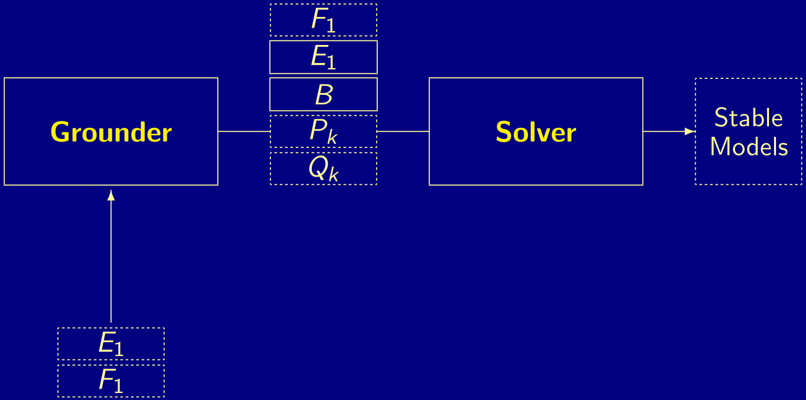

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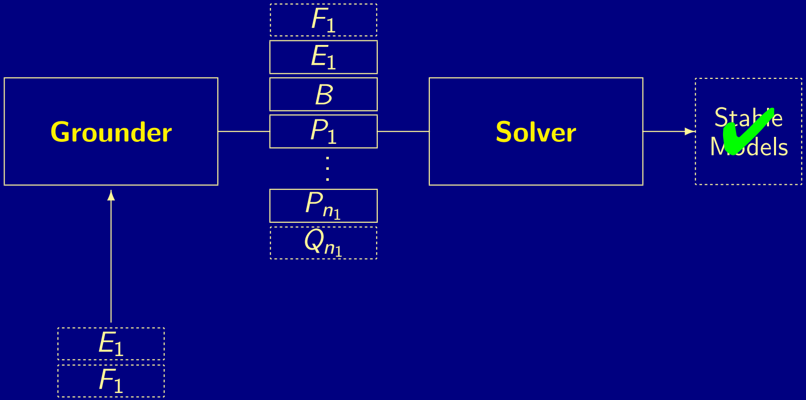

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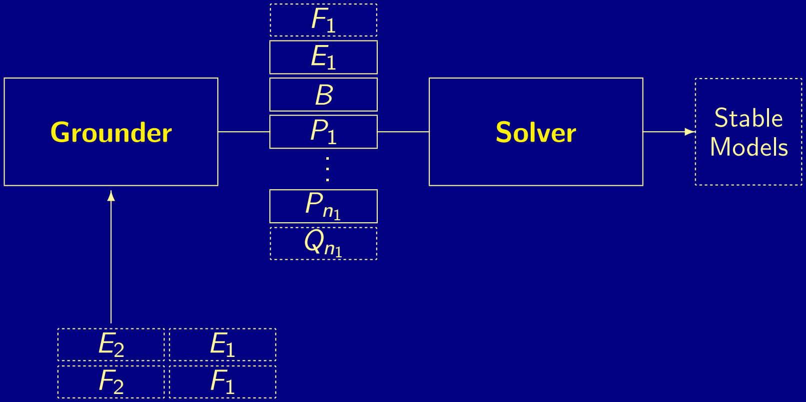

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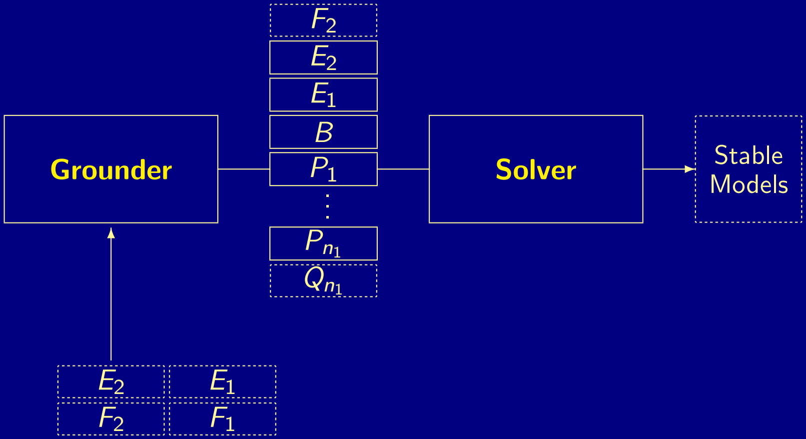

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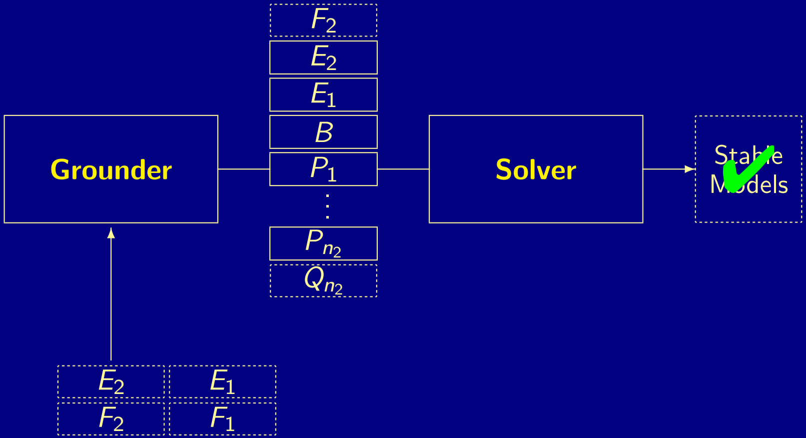

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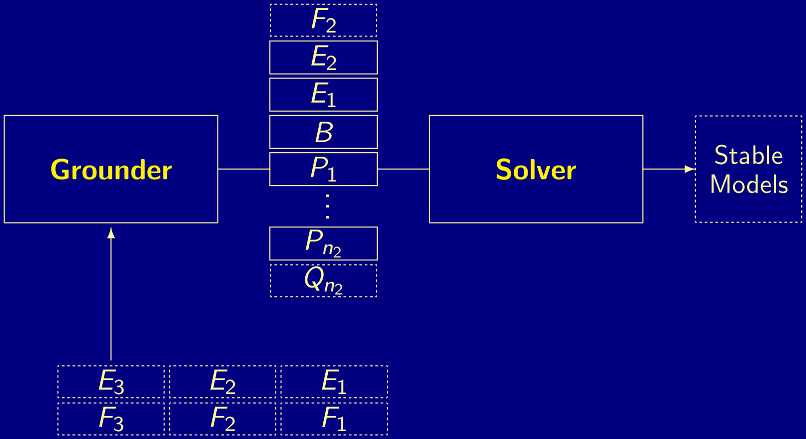

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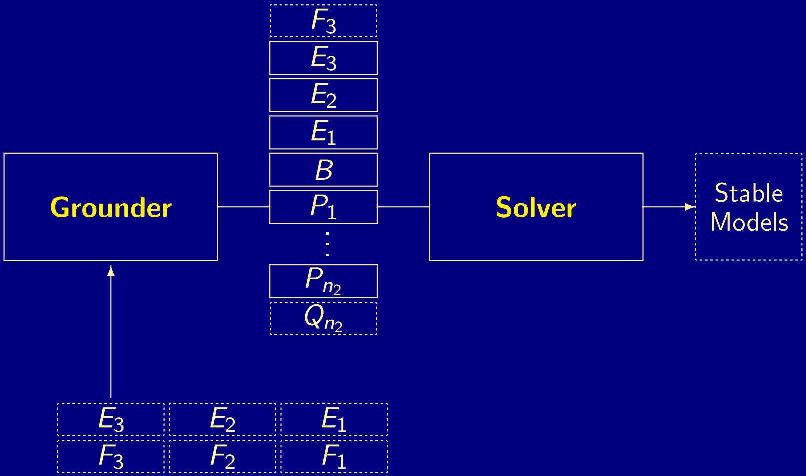

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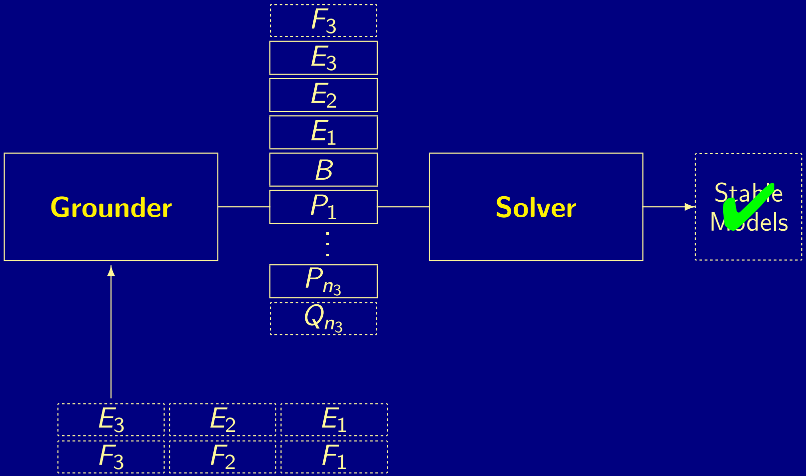

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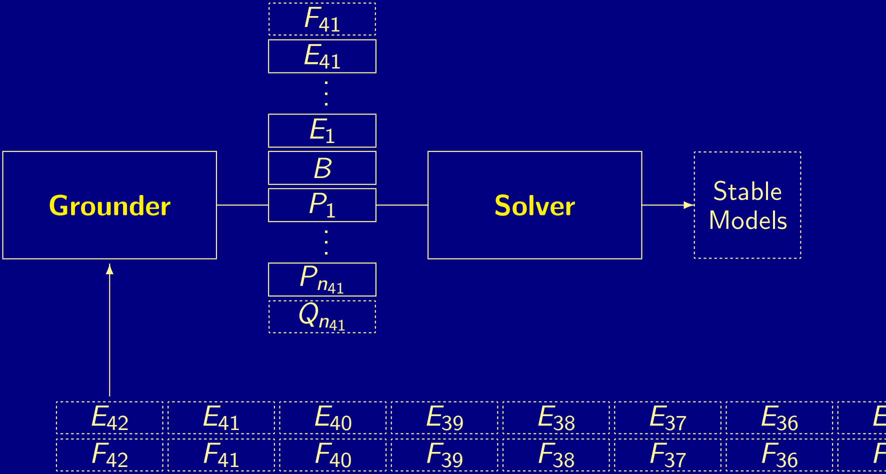

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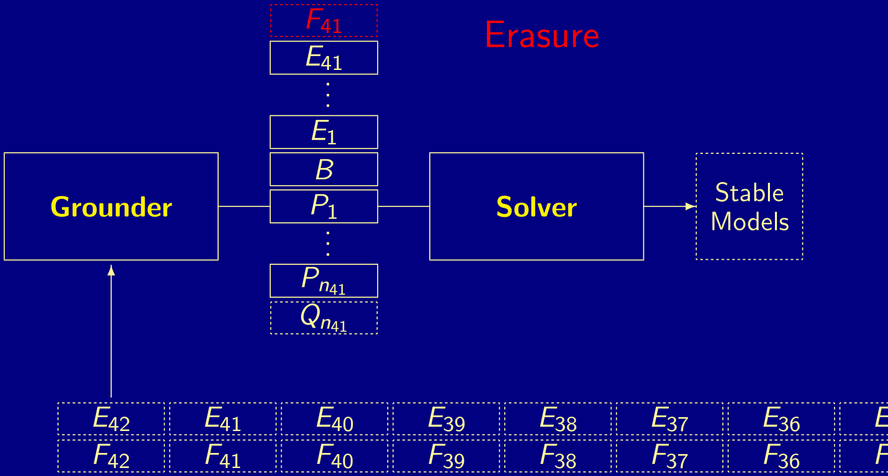

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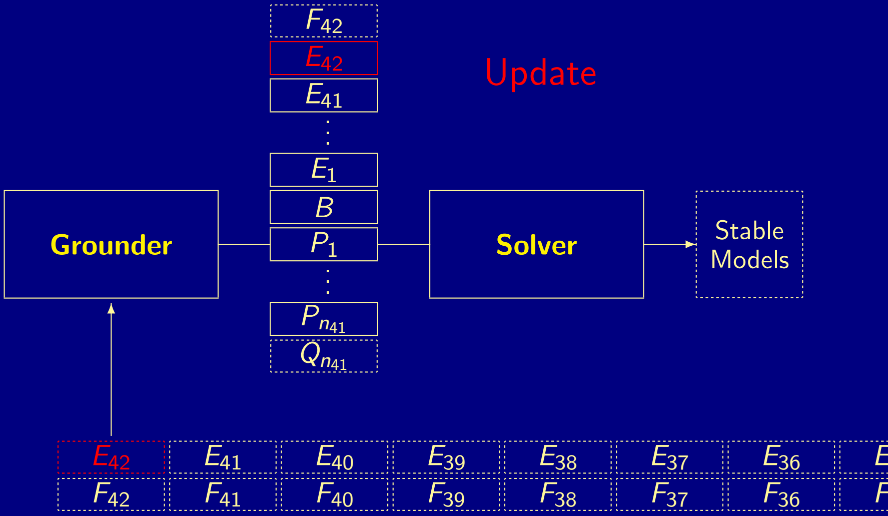

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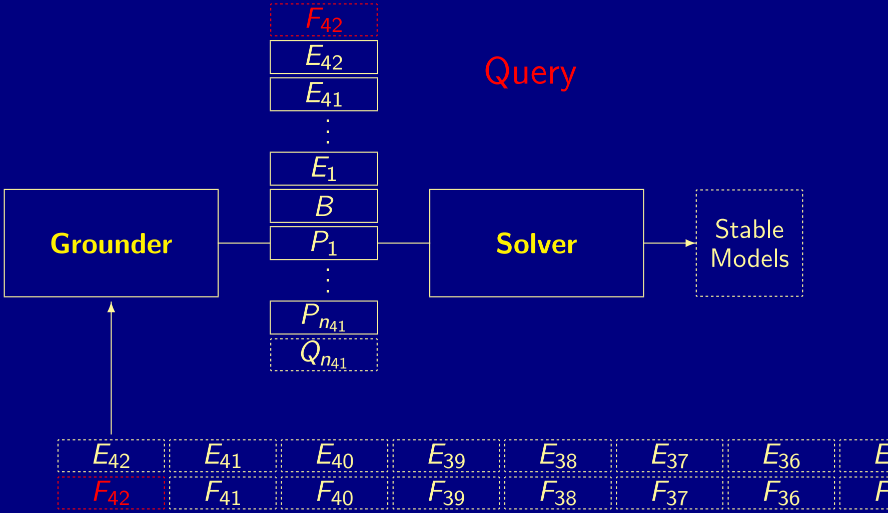

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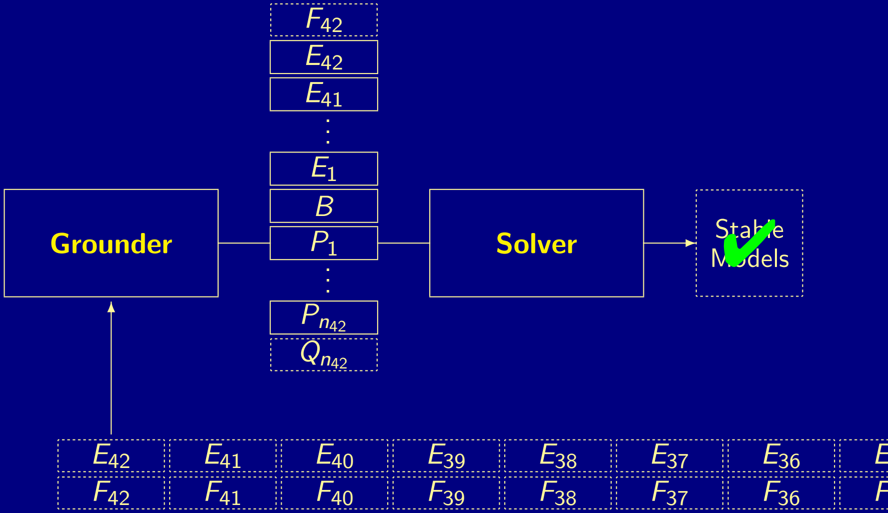

Elevator Control

```
#base.
floor(1..3).
atFloor(1,0).

#cumulative t.
#external request(F,t) : floor(F).
1 { atFloor(F-1;F+1,t) } 1 :- atFloor(F,t-1), floor(F).
:- atFloor(F,t), not floor(F).
requested(F,t) :- request(F,t),     floor(F), not atFloor(F,t).
requested(F,t) :- requested(F,t-1), floor(F), not atFloor(F,t).
goal(t) :- not requested(F,t) : floor(F).

#volatile t.
:- not goal(t).
```

Pushing a button

- oClingo acts as a server listening on a port waiting for client requests
- To issue such requests, a separate controller program sends online progressions using network sockets
- For instance,

```
#step 1.
request(3,1).
#endstep.
```

- This process terminates when the client sends

```
#stop.
```

Potassco

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Outline

Potassco

clavis

- Analysis and visualization toolchain for clasp
- *clavis*
  - Event logger integrated in clasp
  - Records CDCL events like propagation, conflicts, restarts, ...
  - Generated logfiles readable with different backends
  - Easily configurable
  - Applicable to clasp variants like hclasp
- *insight*
  - Visualization backend for clavis
  - Combines information about problem structure and solving process
  - Networks for structural and aggregated information
  - Plots for temporal information and navigation

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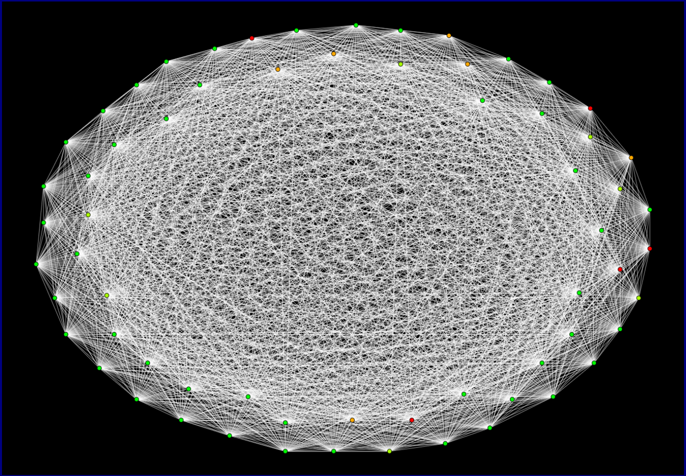
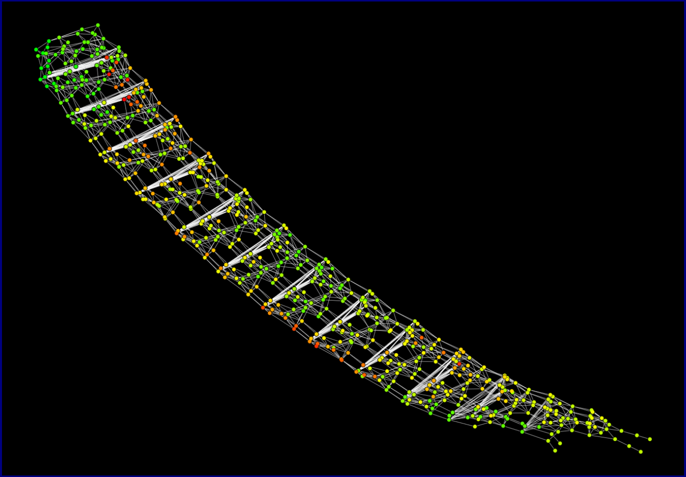
Visualization Examples

8-Queens: program interaction graph



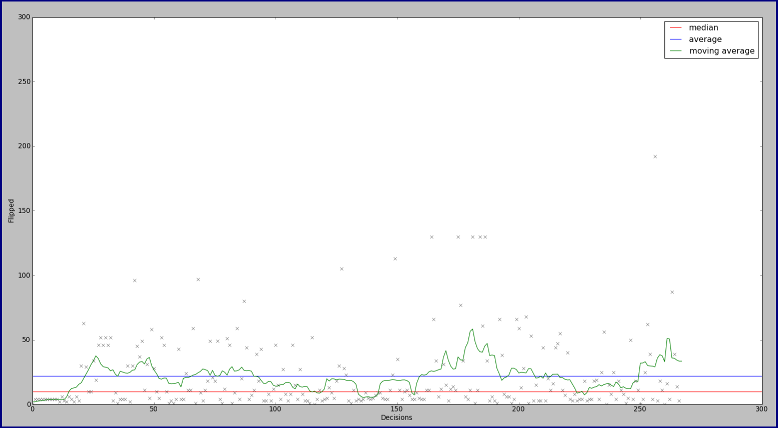
Visualization Examples

Towers of Hanoi: program interaction graph
Colors showing flipped assignments



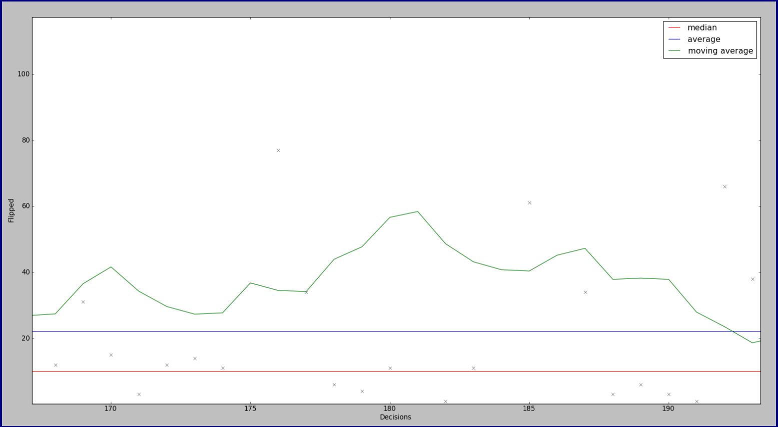
Visualization Examples

Towers of Hanoi: flipped assignments between decisions



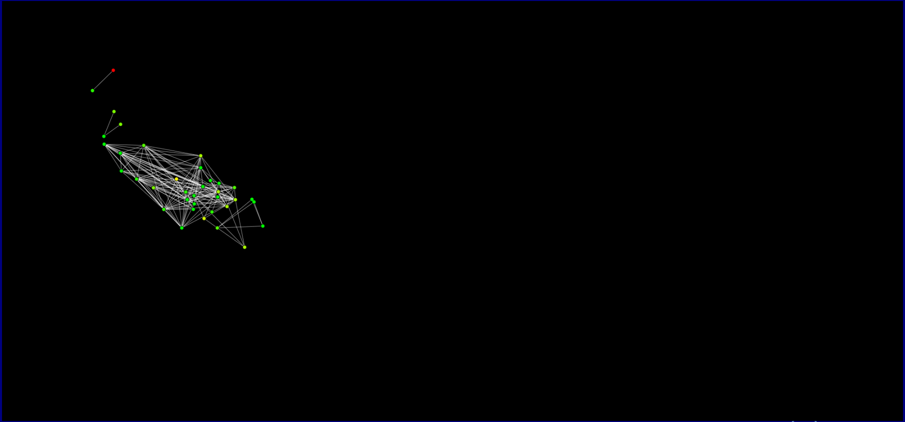
Visualization Examples

Towers of Hanoi: flipped assignments between decisions (zoomed in)



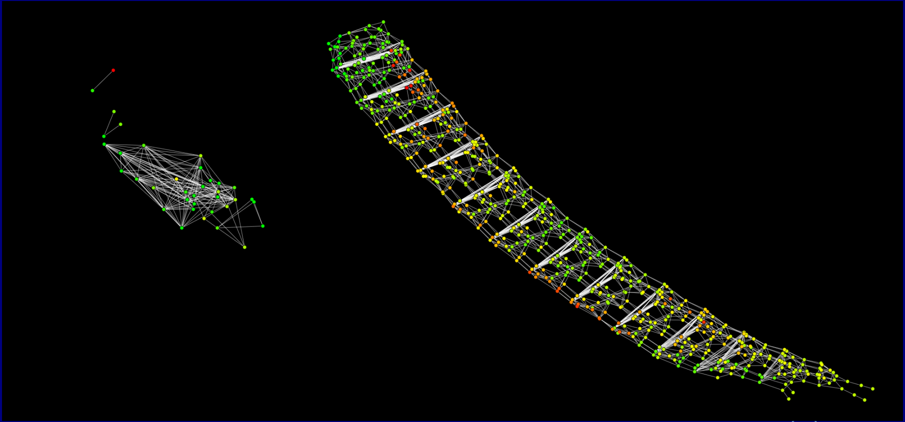
Visualization Examples

Towers of Hanoi: learned nogoods during zoomed in segment projected onto program interaction graph layout

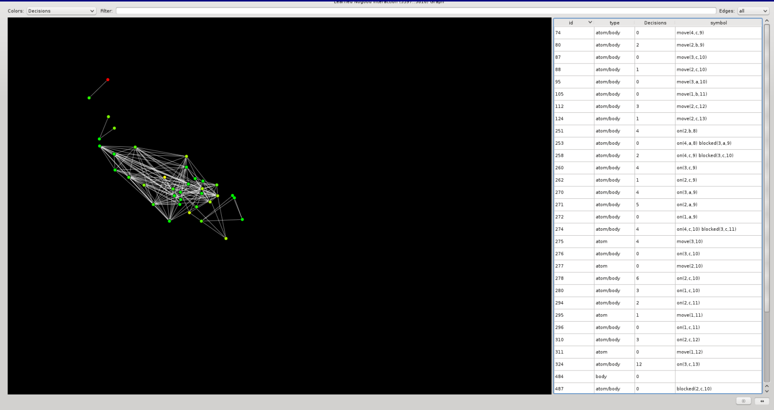


Visualization Examples

Towers of Hanoi: learned nogoods during zoomed in segment compared to program interaction graph



Interactive View



- Symbol table shows additional information about variables
- Search bar and symbol table allow for dynamic change of the view

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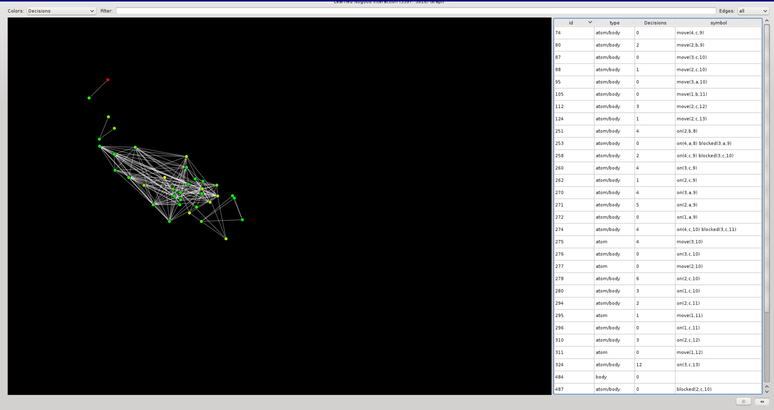
Interactive View



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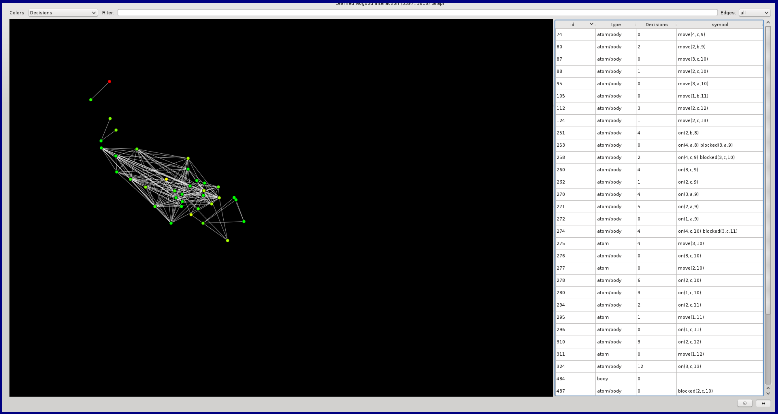

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Advanced Modeling: Overview

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Anything left to worry about?

- ASP offers
  - rich yet easy modeling languages
  - efficient instantiation procedures
  - powerful search engines

- BUT The problem encoding (still) matters!

- Example Sort a list with 8 elements
  - divide-and-conquer $\sim$ $8(\log_2 8) = 16$ "operations"
  - permutation guessing $\sim$ $8!/2 = 20160$ "operations"

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Outline

41 Tweaking N-Queens

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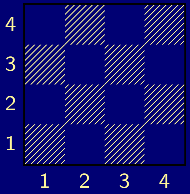
N-Queens Problem

Problem Specification

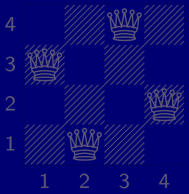
Given an $N \times N$ chessboard,
place N queens such that they do not attack each other
(neither horizontally, vertically, nor diagonally)

$N = 4$

Chessboard



Placement



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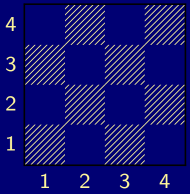

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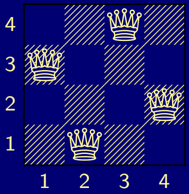

A First Encoding

1. Each square may host a queen
2. No row, column, or diagonal hosts two queens
3. A placement is given by instances of `queen` in a stable model

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- queen(X1,Y1), queen(X1,Y2), Y1 < Y2.

% DISPLAY
#hide. #show queen/2.
```

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[...]

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A First Encoding

1. Each square may host a queen
2. No row, column, or diagonal hosts two queens
3. A placement is given by instances of `queen` in a stable model

queens_0.lp

Anything missing?

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
[...]

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Encoding

1. Each square may host a queen
2. No row, column, or diagonal hosts two queens
3. A placement is given by instances of `queen` in a stable model
4. We have to place (at least) *N* queens

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
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% TEST
[...]
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% DISPLAY
#hide. #show queen/2.
```

A First Encoding

Let's Place 8 Queens!

```
gringo -c n=8 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,6) queen(2,3) queen(3,1) queen(4,7)
queen(5,5) queen(6,8) queen(7,2) queen(8,4)
SATISFIABLE

Models      : 1+
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 18
Conflicts   : 13
Restarts    : 0

Variables   : 793
Constraints : 729
```

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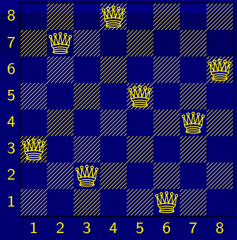


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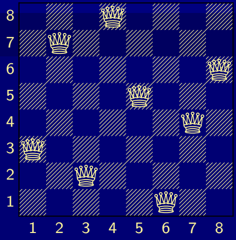


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Choices     : 18
Conflicts   : 13
Restarts    : 0

Variables   : 793
Constraints : 729
```

A First Encoding

Let's Place 22 Queens!

```
gringo -c n=22 queens_0.lp | clasp --stats

Answer: 1
queen(1,10) queen(2,6) queen(3,16) queen(4,14) queen(5,8) ...
SATISFIABLE

Models       : 1+
Time         : 150.531s (Solving: 150.37s 1st Model: 150.34s Unsat: 0.00s)
CPU Time     : 147.480s
Choices      : 594960
Conflicts    : 574565
Restarts     : 19

Variables    : 17271
Constraints  : 16787
```

A First Encoding

Let's Place 22 Queens!

```
gringo -c n=22 queens_0.lp | clasp --stats
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Answer: 1
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CPU Time     : 147.480s
Choices      : 594960
Conflicts    : 574565
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Variables    : 17271
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A First Refinement

At least N queens? Exactly one queen per row and column!

queens_0.lp

```
% DOMAIN
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:- not n #count{ queen(X,Y) }.

% DISPLAY
#hide. #show queen/2.
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A First Refinement

At least *N* queens? Exactly one queen per row and column!

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A First Refinement

At least *N* queens? Exactly one queen per row and column!

queens_1.lp

```
% DOMAIN
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% GENERATE
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% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Let's Place 22 Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats

Answer: 1
queen(1,18) queen(2,10) queen(3,21) queen(4,3) queen(5,5) ...
SATISFIABLE

Models      : 1+
Time        : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.020s
Choices     : 132
Conflicts   : 105
Restarts    : 1

Variables   : 7238
Constraints : 6710
```

A First Refinement

Let's Place 22 Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

```
Answer: 1
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Models      : 1+
Time        : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.020s
Choices     : 132
Conflicts   : 105
Restarts    : 1

Variables   : 7238
Constraints : 6710
```

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models      : 1+
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time    : 6.930s
Choices     : 1373
Conflicts   : 845
Restarts    : 4

Variables   : 1211338
Constraints : 1196210
```

A First Refinement

Let's Place 122 Queens!

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A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | clasp --stats

1241358 7402724 24950848

real  1m15.468s
user  1m15.980s
sys   0m0.090s
```

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)

1241358 7402724 24950848

real  1m15.468s
user  1m15.980s
sys   0m0.090s
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A First Refinement

Where Time Has Gone

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time(gringo -c n=122 queens_1.lp | wc)
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A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                  O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                  O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                      O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.        O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

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Grounding Time ∼ Space

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Grounding Time $\sim$ Space

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A First Refinement

Grounding Time ∼ Space

queens_1.lp

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A First Refinement

Grounding Time ∼ Space

queens_1.lp

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A First Refinement

Grounding Time ~ Space

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% DISPLAY
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Diagonals make trouble!

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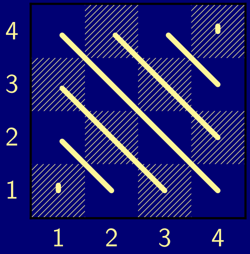
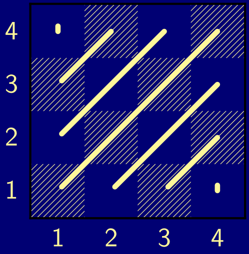
Enumerating Diagonals

$N = 4$





$\#\text{diagonal}_1 = (\#\text{row} + \#\text{column}) - 1$

$\#\text{diagonal}_2 = (\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2{*}N){-}1$ refer to squares on $\#\text{diagonal}_{1/2}$

Enumerating Diagonals

$N = 4$

4	4	5	6	7
3	3	4	5	6
2	2	3	4	5
1	1	2	3	4
	1	2	3	4

$\#\text{diagonal}_1 = (\#\text{row} + \#\text{column}) - 1$

4	7	6	5	4
3	6	5	4	3
2	5	4	3	2
1	4	3	2	1
	1	2	3	4

$\#\text{diagonal}_2 = (\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2*N)-1$ refer to squares on $\#\text{diagonal}_{1/2}$

Enumerating Diagonals

$N = 4$

4	**4**	**5**	**6**	**7**
3	**3**	**4**	**5**	**6**
2	**2**	**3**	**4**	**5**
1	**1**	**2**	**3**	**4**
	1	2	3	4

$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$

4	**7**	**6**	**5**	**4**
3	**6**	**5**	**4**	**3**
2	**5**	**4**	**3**	**2**
1	**4**	**3**	**2**	**1**
	1	2	3	4

$\#\text{diagonal}_2 =$
$(\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2*N)-1$ refer to squares on $\#\text{diagonal}_{1/2}$

Enumerating Diagonals

$N = 4$

4	**4**	**5**	**6**	**7**
3	**3**	**4**	**5**	**6**
2	**2**	**3**	**4**	**5**
1	**1**	**2**	**3**	**4**
	1	2	3	4

$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$

4	**7**	**6**	**5**	**4**
3	**6**	**5**	**4**	**3**
2	**5**	**4**	**3**	**2**
1	**4**	**3**	**2**	**1**
	1	2	3	4

$\#\text{diagonal}_2 =$
$(\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2*N)-1$ refer to squares on $\#\text{diagonal}_{1/2}$

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.


% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1


% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats

Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

Potassco

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats

Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time    : 7.250s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats

Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
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queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
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CPU Time    : 7.250s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Precalculate Indexes!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

queens_3.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats

Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats

Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
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Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats

Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Third Refinement

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...
SATISFIABLE

Models      : 1+
Time        : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)
CPU Time    : 29.580s
Choices     : 961315
Conflicts   : 3222
Restarts    : 7

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...
SATISFIABLE

Models      : 1+
Time        : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)
CPU Time    : 29.580s
Choices     : 961315
Conflicts   : 3222
Restarts    : 7

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic

Answer: 1
queen(1,90) queen(2,452) queen(3,494) queen(4,145) queen(5,84) ...
SATISFIABLE

Models      : 1+
Time        : 22.654s (Solving: 10.53s 1st Model: 10.47s Unsat: 0.00s)
CPU Time    : 15.750s
Choices     : 1058729
Conflicts   : 2128
Restarts    : 6

Variables   : 403123
Constraints : 49636
```

Outline

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change
2. use variable-sized conjunction (via ':') ... adapts to changing facts
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).    pro(cucumber,cheap).
pro(asparagus,fresh).    pro(cucumber,fresh).
pro(asparagus,tasty).    pro(cucumber,tasty).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).


buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty).
```

Implementing Universal Quantification

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Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).
pro(asparagus,clean).

buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty), pro(X,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✓
3. use negation of complement ... adapts to changing facts ✓

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).
pro(asparagus,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).   pre(cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).   pre(fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).   pre(tasty).
pro(asparagus,clean).

buy(X) :- veg(X), pro(X,P) : pre(P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✓
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).   pre(cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).   pre(fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).   pre(tasty).
pro(asparagus,clean).                          pre(clean).

buy(X) :- veg(X), pro(X,P) : pre(P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✓
3. use negation of complement ... adapts to changing facts ✓

Example: vegetables to buy

```
veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).    pro(cucumber,cheap).  pre(cheap).
pro(asparagus,fresh).    pro(cucumber,fresh).  pre(fresh).
pro(asparagus,tasty).    pro(cucumber,tasty).  pre(tasty).
pro(asparagus,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).   pre(cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).   pre(fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).   pre(tasty).
pro(asparagus,clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✓
3. use negation of complement ... adapts to changing facts ✓

Example: vegetables to buy

```
veg(asparagus).            veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).  pre(cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).  pre(fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).  pre(tasty).
pro(asparagus,clean).                        pre(clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✘
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).    pro(cucumber,cheap).    pre(cheap).
pro(asparagus,fresh).    pro(cucumber,fresh).    pre(fresh).
pro(asparagus,tasty).    pro(cucumber,tasty).    pre(tasty).
pro(asparagus,clean).                            pre(clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Running Example: Latin Square

Given: an $N \times N$ board

1						
2						
3						
4						
5						
6						
	1	2	3	4	5	6

represented by facts:

```
square(1,1). ... square(1,6).
square(2,1). ... square(2,6).
square(3,1). ... square(3,6).
square(4,1). ... square(4,6).
square(5,1). ... square(5,6).
square(6,1). ... square(6,6).
```

Wanted: assignment of $1, \ldots, N$

1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	1	2
4	4	5	6	1	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5
	1	2	3	4	5	6

represented by atoms:

```
num(1,1,1) num(1,2,2) ... num(1,6,6)
num(2,1,2) num(2,2,3) ... num(2,6,1)
num(3,1,3) num(3,2,4) ... num(3,6,2)
num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6.5)
```

Running Example: Latin Square

Given: an $N \times N$ board

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

represented by facts:

```
square(1,1). ... square(1,6).
square(2,1). ... square(2,6).
square(3,1). ... square(3,6).
square(4,1). ... square(4,6).
square(5,1). ... square(5,6).
square(6,1). ... square(6,6).
```

Wanted: assignment of $1, \ldots, N$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	1	2
4	4	5	6	1	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5

represented by atoms:

```
num(1,1,1) num(1,2,2) ... num(1,6,6)
num(2,1,2) num(2,2,3) ... num(2,6,1)
num(3,1,3) num(3,2,4) ... num(3,6,2)
num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6.5)
```

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).



% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

co

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).



% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

gringo latin_0.lp | wc

105480 2558984 14005258

gringo latin_1.lp | wc

42056 273672 1690522

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).



% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

co

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
squareX(X) :- square(X,Y).      squareY(Y) :- square(X,Y).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- squareX(X1)  , N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- squareY(Y1)  , N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

co

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
squareX(X) :- square(X,Y).      squareY(Y) :- square(X,Y).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- squareX(X1)  , N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- squareY(Y1)  , N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

co

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

- Note duplicate ground rules (swapping Y1/Y2 and X1/X2 gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

- Note duplicate ground rules (swapping Y1/Y2 and X1/X2 gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

co

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

- Note duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

gringo latin_2.lp | wc	gringo latin_3.lp | wc
2071560 12389384 40906946	1055752 6294536 21099558

co

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 <  Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 <  X2.
```

- Note duplicate ground rules (swapping Y1/Y2 and X1/X2 gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

co

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 <  Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 <  X2.
```

- Note duplicate ground rules (swapping Y1/Y2 and X1/X2 gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

co

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

- Note uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

co

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

co

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).               :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.     gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.     gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).             :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc

1055752 6294536 21099558
```

```
gringo latin_4.lp | wc

228360 1205256 4780744
```

co

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of N in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

co

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

gringo latin_5.lp | wc   gringo latin_6.lp | wc   co

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc
gringo latin_6.lp | wc
```

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc
```

```
gringo latin_6.lp | wc
```

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc          gringo latin_6.lp | wc
304136 5778440 30252505         48136 373768 2185042
```

co

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C #count{ num(X,Y,N) } C, C = 0..n.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C #count{ num(X,Y,N) } C, C = 0..n.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

- Note internal transformation by gringo

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.     ✗
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.     ✗
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc
304136 5778440 30252505
```

```
gringo latin_6.lp | wc
48136 373768 2185042
```

co

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3.
```

```
gringo latin_5.lp | wc
```

```
gringo latin_6.lp | wc
```

48136 373768 2185042

co

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3.
```

```
gringo latin_5.lp | wc
```

304136 5778440 30252505

```
gringo latin_6.lp | wc
```

48136 373768 2185042

co

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.



% DISPLAY
#hide. #show num/3.
```

```
gringo latin_5.lp | wc
```

```
304136 5778440 30252505
```

```
gringo latin_6.lp | wc
```

Potassco

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

```
gringo latin_5.lp | wc
```

304136 5778440 30252505

```
gringo latin_6.lp | wc
```

48136 373768 2185042

co

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

- Note many symmetric solutions (mirroring, rotation, value permutation)

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

- Note easy and safe to fix a full row/column!

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

- Note easy and safe to fix a full row/column!

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

- Note Let's compare enumeration speed!

Potassco

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.



% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

```
Models : 161280    Time : 2.078s
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 161280     Time : 2.078s
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 1344       Time : 0.024s
```

Outline

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Encode With Care!

1. Create a working encoding

   Q1: Do you need to modify the encoding if the facts change?
   Q2: Are all variables significant (or statically functionally dependent)?
   Q3: Can there be (many) identic ground rules?
   Q4: Do you enumerate pairs of values (to test uniqueness)?
   Q5: Do you assign dynamic aggregate values (to check a fixed bound)?
   Q6: Do you admit (obvious) symmetric solutions?
   Q7: Do you have additional domain knowledge simplifying the problem?
   Q8: Are you aware of anything else that, if encoded, would reduce grounding and/or solving efforts?

2. Revise until no "Yes" answer!

   - Note If the format of facts makes encoding painful (for instance, abusing grounding for "scientific calculations"), revise the fact format as well.

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Some Hints on (Preventing) Debugging

Kinds of errors

- syntactic . . . follow error messages by the grounder
- semantic . . . (most likely) encoding/intention mismatch

Ways to identify semantic errors (early)

- develop and test incrementally
  - prepare toy instances with "interesting features"
  - build the encoding bottom-up and verify additions (eg. new predicates)
- compare the encoded to the intended meaning
  - check whether the grounding fits (use `gringo -t`)
  - if answer sets are unintended, investigate conditions that fail to hold
  - if answer sets are missing, examine integrity constraints (add heads)
- ask tools (eg. `http://www.kr.tuwien.ac.at/research/projects/mmdasp/`)

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Overcoming Performance Bottlenecks

Grounding

- monitor time spent by and output size of `gringo`
  1. system tools (eg. `time(gringo [...] | wc)`)
  2. profiling info (eg. `gringo --gstats --verbose=3 [...] > /dev/null`)
- Note once identified, reformulate "critical" logic program parts

Solving

- check solving statistics (use `clasp --stats`)

  if great search efforts (`Conflicts`/`Choices`/`Restarts`), then

  try prefabricated settings (using `clasp` option '`--configuration`')
  try auto-configuration (offered by `claspfolio`)
  try manual fine-tuning (requires expert knowledge!)
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