

Answer Set Solving in Practice

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Advanced Modeling: Overview

- 1 Tweaking N -Queens
- 2 Do's and Dont's
- 3 Hints

Anything left to worry about?

- ASP offers

- rich yet easy modeling languages
- efficient instantiation procedures
- powerful search engines

- BUT The problem encoding (still) matters!

- Example Sort a list with 8 elements

- divide-and-conquer $\sim 8(\log_2 8) = 16$ “operations”
- permutation guessing $\sim 8!/2 = 20160$ “operations”

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Outline

- 1 Tweaking N -Queens
- 2 Do's and Dont's
- 3 Hints

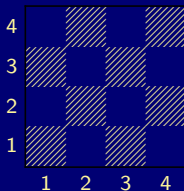
N -Queens Problem

Problem Specification

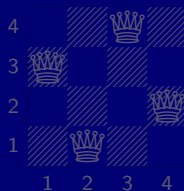
Given an $N \times N$ chessboard,
place N queens such that they do not attack each other
(neither horizontally, vertically, nor diagonally)

$$N = 4$$

Chessboard



Placement



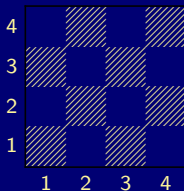
N -Queens Problem

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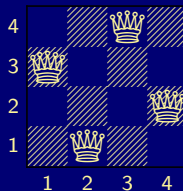
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Chessboard



Placement



A First Encoding

- 1 Each square may host a queen
- 2 No row, column, or diagonal hosts two queens
- 3 A placement is given by instances of `queen` in a stable model

```
queens_0.lp
```

```
% DOMAIN
```

```
#const n=4. square(1..n,1..n).
```

```
% GENERATE
```

```
0 { queen(X,Y) } 1 :- square(X,Y).
```

```
% TEST
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```
:- queen(X,Y), queen(X,Y'), Y < Y'.
```

```
% DISPLAY
```

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#show queen/2.
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[...]
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Anything missing?

A First Encoding

- 1 Each square may host a queen
- 2 No row, column, or diagonal hosts two queens
- 3 A placement is given by instances of `queen` in a stable model
- 4 We have to place (at least) N queens

```
queens_0.lp
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 { queen(X,Y) } 1 :- square(X,Y).

% TEST
[...]
:- not n { queen(X,Y) }.

% DISPLAY
#show queen/2.
```


A First Encoding

Let's Place 8 Queens!

```
gringo -c n=8 queens_0.lp | clasp --stats
```

```
Answer: 1
```

```
queen(1,6) queen(2,3) queen(3,1) queen(4,7)
```

```
queen(5,5) queen(6,8) queen(7,2) queen(8,4)
```

```
SATISFIABLE
```

```
Models      : 1+
```

```
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

```
CPU Time    : 0.000s
```

```
Choices     : 18
```

```
Conflicts   : 13
```

```
Restarts    : 0
```

```
Variables   : 793
```

```
Constraints : 729
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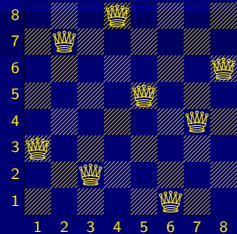
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Choices     : 18
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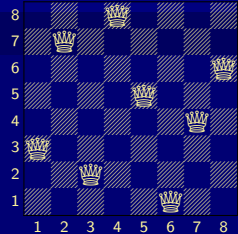
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Choices     : 18
```

```
Conflicts   : 13
```

```
Restarts    : 0
```

```
Variables   : 793
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```
Constraints : 729
```



A First Encoding

Let's Place **22** Queens!

```
gringo -c n=22 queens_0.lp | clasp --stats
```

```
Answer: 1
```

```
queen(1,10) queen(2,6) queen(3,16) queen(4,14) queen(5,8) ...
```

```
SATISFIABLE
```

```
Models      : 1+
```

```
Time        : 150.531s (Solving: 150.37s 1st Model: 150.34s Unsat: 0.00s)
```

```
CPU Time    : 147.480s
```

```
Choices     : 594960
```

```
Conflicts   : 574565
```

```
Restarts    : 19
```

```
Variables   : 17271
```

```
Constraints : 16787
```

A First Encoding

Let's Place **22** Queens!

```
gringo -c n=22 queens_0.lp | clasp --stats
```

Answer: 1

queen(1,10) queen(2,6) queen(3,16) queen(4,14) queen(5,8) ...

SATISFIABLE

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Time : 150.531s (Solving: 150.37s 1st Model: 150.34s Unsat: 0.00s)

CPU Time : 147.480s

Choices : 594960

Conflicts : 574565

Restarts : 19

Variables : 17271

Constraints : 16787

A First Refinement

At least N queens?

Exactly one queen per row and column!

queens_0.lp

% DOMAIN

#const n=4. square(1..n,1..n).

% GENERATE

0 { queen(X,Y) } 1 :- square(X,Y).

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:- queen(X,Y), queen(X,Y'), Y < Y'.

:- queen(X,Y), queen(X',Y), X < X'.

:- queen(X,Y), queen(X',Y'), X < X', X'-X = |Y'-Y|.

:- not n { queen(X,Y) }.

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#show queen/2.

A First Refinement

At least N queens?Exactly one queen per **row** and column!

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A First Refinement

At least N queens?

Exactly one queen per row and column!

queens_1.lp

% DOMAIN

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% TEST

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:- Y = 1..n, not 1 { queen(X,Y) } 1.

:- queen(X,Y), queen(X',Y'), X < X', X'-X = |Y'-Y|.

% DISPLAY

#show queen/2.

A First Refinement

Let's Place **22** Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

```
Answer: 1
```

```
queen(1,18) queen(2,10) queen(3,21) queen(4,3) queen(5,5) ...
```

```
SATISFIABLE
```

```
Models      : 1+
```

```
Time        : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

```
CPU Time    : 0.020s
```

```
Choices     : 132
```

```
Conflicts   : 105
```

```
Restarts    : 1
```

```
Variables   : 7238
```

```
Constraints : 6710
```

A First Refinement

Let's Place **22** Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

Answer: 1

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Models : 1+

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CPU Time : 0.020s

Choices : 132

Conflicts : 105

Restarts : 1

Variables : 7238

Constraints : 6710

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
```

```
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
```

```
SATISFIABLE
```

```
Models      : 1+
```

```
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
```

```
CPU Time    : 6.930s
```

```
Choices     : 1373
```

```
Conflicts   : 845
```

```
Restarts    : 4
```

```
Variables   : 1211338
```

```
Constraints : 1196210
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A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | clasp --stats)
```

```
1241358 7402724 24950848
```

```
real 1m15.468s
```

```
user 1m15.980s
```

```
sys 0m0.090s
```


A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)
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A First Refinement

Grounding Time \sim Space

```
queens_1.lp
```

```
% DOMAIN
```

```
#const n=4. square(1..n,1..n).
```

 $O(n \times n)$

```
% GENERATE
```

```
{ queen(X,Y) } :- square(X,Y).
```

 $O(n \times n)$

```
% TEST
```

```
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.
```

 $O(n \times n)$

```
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.
```

 $O(n \times n)$

```
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.
```

 $O(n^2 \times n^2)$

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% DISPLAY
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#show queen/2.
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```

```
#show queen/2.
```

A First Refinement

Grounding Time \sim Space

```
queens_1.lp
```

```
% DOMAIN
```

```
#const n=4. square(1..n,1..n).
```

 $O(n \times n)$

```
% GENERATE
```

```
{ queen(X,Y) } :- square(X,Y).
```

 $O(n \times n)$

```
% TEST
```

```
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.
```

 $O(n \times n)$

```
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.
```

 $O(n \times n)$

```
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.
```

 $O(n^2 \times n^2)$

```
% DISPLAY
```

```
#show queen/2.
```

A First Refinement

Grounding Time \sim Space

```
queens_1.lp
```

```
% DOMAIN
```

```
#const n=4. square(1..n,1..n).
```

 $O(n \times n)$

```
% GENERATE
```

```
{ queen(X,Y) } :- square(X,Y).
```

 $O(n \times n)$

```
% TEST
```

```
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.
```

 $O(n \times n)$

```
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.
```

 $O(n \times n)$

```
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.
```

 $O(n^2 \times n^2)$

```
% DISPLAY
```

```
#show queen/2.
```

A First Refinement

Grounding Time \sim Space

```
queens_1.lp
```

```
% DOMAIN
```

```
#const n=4. square(1..n,1..n).
```

 $O(n \times n)$

```
% GENERATE
```

```
{ queen(X,Y) } :- square(X,Y).
```

 $O(n \times n)$

```
% TEST
```

```
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.
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 $O(n \times n)$

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:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.
```

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```
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.
```

 $O(n^2 \times n^2)$

```
% DISPLAY
```

```
#show queen/2.
```

A First Refinement

Grounding Time \sim Space

```
queens_1.lp
```

```
% DOMAIN
```

```
#const n=4. square(1..n,1..n).
```

 $O(n \times n)$

```
% GENERATE
```

```
{ queen(X,Y) } :- square(X,Y).
```

 $O(n \times n)$

```
% TEST
```

```
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.
```

 $O(n \times n)$

```
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.
```

 $O(n \times n)$

```
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.
```

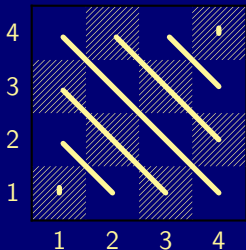
 $O(n^2 \times n^2)$

```
% DISPLAY
```

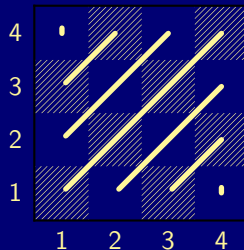
```
#show queen/2.
```

Diagonals make trouble!

Enumerating Diagonals

 $N = 4$ 

$$\begin{aligned} \#diagonal_1 = \\ (\#row + \#column) - 1 \end{aligned}$$



$$\begin{aligned} \#diagonal_2 = \\ (\#row - \#column) + N \end{aligned}$$

- Note For each N , indexes $1, \dots, (2*N)-1$ refer to squares on $\#diagonal_{1/2}$

Enumerating Diagonals

 $N = 4$

4	4	5	6	7
3	3	4	5	6
2	2	3	4	5
1	1	2	3	4
	1	2	3	4

4	7	6	5	4
3	6	5	4	3
2	5	4	3	2
1	4	3	2	1
	1	2	3	4

$$\#diagonal_1 =$$

$$(\#row + \#column) - 1$$

$$\#diagonal_2 =$$

$$(\#row - \#column) + N$$

- Note For each N , indexes $1, \dots, (2*N)-1$ refer to squares on $\#diagonal_{1/2}$

Enumerating Diagonals

 $N = 4$

4	4	5	6	7
3	3	4	5	6
2	2	3	4	5
1	1	2	3	4
	1	2	3	4

$$\begin{aligned} \#diagonal_1 = \\ (\#row + \#column) - 1 \end{aligned}$$

4	7	6	5	4
3	6	5	4	3
2	5	4	3	2
1	4	3	2	1
	1	2	3	4

$$\begin{aligned} \#diagonal_2 = \\ (\#row - \#column) + N \end{aligned}$$

- Note For each N , indexes $1, \dots, (2*N)-1$ refer to squares on $\#diagonal_{1/2}$

Enumerating Diagonals

 $N = 4$

4	4	5	6	7
3	3	4	5	6
2	2	3	4	5
1	1	2	3	4
	1	2	3	4

$$\begin{aligned} \#diagonal_1 = \\ (\#row + \#column) - 1 \end{aligned}$$

4	7	6	5	4
3	6	5	4	3
2	5	4	3	2
1	4	3	2	1
	1	2	3	4

$$\begin{aligned} \#diagonal_2 = \\ (\#row - \#column) + N \end{aligned}$$

- Note For each N , indexes $1, \dots, (2*N)-1$ refer to squares on $\#diagonal_{1/2}$

A Second Refinement

Let's go for Diagonals!

```
queens_1.lp
```

```
% DOMAIN
```

```
#const n=4. square(1..n,1..n).
```

```
% GENERATE
```

```
0 { queen(X,Y) } 1 :- square(X,Y).
```

```
% TEST
```

```
:- X = 1..n, not 1 { queen(X,Y) } 1.
```

```
:- Y = 1..n, not 1 { queen(X,Y) } 1.
```

```
:- queen(X,Y), queen(X',Y'), X < X', X'-X = |Y'-Y|.
```

```
% DISPLAY
```

```
#show queen/2.
```

A Second Refinement

Let's go for Diagonals!

```
queens_1.lp
```

```
% DOMAIN
```

```
#const n=4. square(1..n,1..n).
```

```
% GENERATE
```

```
0 { queen(X,Y) } 1 :- square(X,Y).
```

```
% TEST
```

```
:- X = 1..n, not 1 { queen(X,Y) } 1.
```

```
:- Y = 1..n, not 1 { queen(X,Y) } 1.
```

```
:- D = 1..2*n-1, 2 { queen(X,Y) : D = (X+Y)-1 }. % Diagonal 1
```

```
% DISPLAY
```

```
#show queen/2.
```

A Second Refinement

Let's go for Diagonals!

```
queens_1.lp
```

```
% DOMAIN
```

```
#const n=4. square(1..n,1..n).
```

```
% GENERATE
```

```
0 { queen(X,Y) } 1 :- square(X,Y).
```

```
% TEST
```

```
:- X = 1..n, not 1 { queen(X,Y) } 1.
```

```
:- Y = 1..n, not 1 { queen(X,Y) } 1.
```

```
:- D = 1..2*n-1, 2 { queen(X,Y) : D = (X+Y)-1 }. % Diagonal 1
```

```
:- D = 1..2*n-1, 2 { queen(X,Y) : D = (X-Y)+n }. % Diagonal 2
```

```
% DISPLAY
```

```
#show queen/2.
```

A Second Refinement

Let's go for Diagonals!

```
queens_2.lp
```

```
% DOMAIN
```

```
#const n=4. square(1..n,1..n).
```

```
% GENERATE
```

```
0 { queen(X,Y) } 1 :- square(X,Y).
```

```
% TEST
```

```
:- X = 1..n, not 1 { queen(X,Y) } 1.
```

```
:- Y = 1..n, not 1 { queen(X,Y) } 1.
```

```
:- D = 1..2*n-1, 2 { queen(X,Y) : D = (X+Y)-1 }. % Diagonal 1
```

```
:- D = 1..2*n-1, 2 { queen(X,Y) : D = (X-Y)+n }. % Diagonal 2
```

```
% DISPLAY
```

```
#show queen/2.
```

A Second Refinement

Let's Place **122** Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
```

```
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
```

```
SATISFIABLE
```

```
Models      : 1+
```

```
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
```

```
CPU Time    : 0.210s
```

```
Choices     : 11036
```

```
Conflicts   : 499
```

```
Restarts    : 3
```

```
Variables   : 16098
```

```
Constraints : 970
```

A Second Refinement

Let's Place **122** Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

Answer: 1

queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...

SATISFIABLE

Models : 1+

Time : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)

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Choices : 11036

Conflicts : 499

Restarts : 3

Variables : 16098

Constraints : 970

A Second Refinement

Let's Place **122** Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

Answer: 1

queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...

SATISFIABLE

Models : 1+

Time : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)

CPU Time : 0.210s

Choices : 11036

Conflicts : 499

Restarts : 3

Variables : 16098

Constraints : 970

A Second Refinement

Let's Place **300** Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
```

```
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
```

```
SATISFIABLE
```

```
Models      : 1+
```

```
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
```

```
CPU Time    : 7.250s
```

```
Choices     : 141445
```

```
Conflicts   : 7488
```

```
Restarts    : 9
```

```
Variables   : 92994
```

```
Constraints : 2394
```

A Second Refinement

Let's Place **300** Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

Answer: 1

queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...

SATISFIABLE

Models : 1+

Time : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)

CPU Time : 7.250s

Choices : 141445

Conflicts : 7488

Restarts : 9

Variables : 92994

Constraints : 2394

A Second Refinement

Let's Place **300** Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
```

```
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
```

```
SATISFIABLE
```

```
Models      : 1+
```

```
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
```

```
CPU Time    : 7.250s
```

```
Choices     : 141445
```

```
Conflicts   : 7488
```

```
Restarts    : 9
```

```
Variables   : 92994
```

```
Constraints : 2394
```

A Third Refinement

Let's Precalculate Indexes!

```
queens_2.lp
```

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 { queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 { queen(X,Y) } 1.
:- Y = 1..n, not 1 { queen(X,Y) } 1.
:- D = 1..2*n-1, 2 { queen(X,Y) : D = (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 { queen(X,Y) : D = (X-Y)+n }. % Diagonal 2

% DISPLAY
#show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

```
queens_2.lp
```

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 { queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 { queen(X,Y) } 1.
:- Y = 1..n, not 1 { queen(X,Y) } 1.
:- D = 1..2*n-1, 2 { queen(X,Y) : D = (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 { queen(X,Y) : D = (X-Y)+n }. % Diagonal 2

% DISPLAY
#show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

```
queens_2.lp
```

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 { queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 { queen(X,Y) } 1.
:- Y = 1..n, not 1 { queen(X,Y) } 1.
:- D = 1..2*n-1, 2 { queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 { queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

```
queens_3.lp
```

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 { queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 { queen(X,Y) } 1.
:- Y = 1..n, not 1 { queen(X,Y) } 1.
:- D = 1..2*n-1, 2 { queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 { queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#show queen/2.
```


A Third Refinement

Let's Place **300** Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
```

```
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...  
SATISFIABLE
```

```
Models      : 1+  
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)  
CPU Time    : 7.320s  
Choices     : 141445  
Conflicts   : 7488  
Restarts    : 9
```

```
Variables   : 92994  
Constraints : 2394
```

A Third Refinement

Let's Place **300** Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
```

```
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
```

```
SATISFIABLE
```

```
Models      : 1+
```

```
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
```

```
CPU Time    : 7.320s
```

```
Choices     : 141445
```

```
Conflicts   : 7488
```

```
Restarts    : 9
```

```
Variables   : 92994
```

```
Constraints  : 2394
```

A Third Refinement

Let's Place **300** Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
```

```
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
```

```
SATISFIABLE
```

```
Models      : 1+
```

```
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
```

```
CPU Time    : 7.320s
```

```
Choices     : 141445
```

```
Conflicts   : 7488
```

```
Restarts    : 9
```

```
Variables   : 92994
```

```
Constraints : 2394
```

A Third Refinement

Let's Place **600** Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
```

```
Answer: 1  
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...  
SATISFIABLE
```

```
Models      : 1+  
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)  
CPU Time    : 68.620s  
Choices     : 869379  
Conflicts   : 25746  
Restarts    : 12
```

```
Variables   : 365994  
Constraints  : 4794
```

A Third Refinement

Let's Place **600** Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
```

Answer: 1

queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...

SATISFIABLE

Models : 1+

Time : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)

CPU Time : 68.620s

Choices : 869379

Conflicts : 25746

Restarts : 12

Variables : 365994

Constraints : 4794

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats  
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
```

```
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...  
SATISFIABLE
```

```
Models      : 1+  
Time       : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)  
CPU Time   : 68.620s  
Choices    : 869379  
Conflicts  : 25746  
Restarts   : 12  
  
Variables  : 365994  
Constraints : 4794
```

A Case for Oracles

Let's Place **600** Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats  
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
```

```
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...  
SATISFIABLE
```

```
Models      : 1+  
Time       : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)  
CPU Time   : 68.620s  
Choices    : 869379  
Conflicts  : 25746  
Restarts   : 12  
  
Variables  : 365994  
Constraints: 4794
```

A Case for Oracles

Let's Place **600** Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats  
--heuristic=vsids --trans-ext=dynamic
```

Answer: 1

```
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...  
SATISFIABLE
```

```
Models      : 1+  
Time       : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)  
CPU Time   : 29.580s  
Choices    : 961315  
Conflicts  : 3222  
Restarts   : 7  
  
Variables  : 365994  
Constraints : 4794
```


A Case for Oracles

Let's Place **600** Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats  
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
```

```
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...  
SATISFIABLE
```

```
Models      : 1+  
Time       : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)  
CPU Time   : 29.580s  
Choices    : 961315  
Conflicts  : 3222  
Restarts   : 7  
  
Variables  : 365994  
Constraints: 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats  
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
```

```
queen(1,90) queen(2,452) queen(3,494) queen(4,145) queen(5,84) ...  
SATISFIABLE
```

```
Models      : 1+  
Time       : 22.654s (Solving: 10.53s 1st Model: 10.47s Unsat: 0.00s)  
CPU Time   : 15.750s  
Choices    : 1058729  
Conflicts  : 2128  
Restarts   : 6  
  
Variables  : 403123  
Constraints: 49636
```

Outline

- 1 Tweaking N -Queens
- 2 Do's and Dont's
- 3 Hints

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a “list” hold

- ❶ check all properties explicitly ... obsolete if properties change
- ❷ use variable-sized conjunction (via ':') ... adapts to changing facts
- ❸ use negation of complement ... adapts to changing facts

Example: vegetables to buy

```

veg(asparagus).      veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap).
pro(asparagus,fresh). pro(cucumber,fresh).
pro(asparagus,tasty). pro(cucumber,tasty).
  
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a “list” hold

- 1 check all properties explicitly ... obsolete if properties change
- 2 use variable-sized conjunction (via ':') ... adapts to changing facts
- 3 use negation of complement ... adapts to changing facts

Example: vegetables to buy

```

veg(asparagus).      veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap).
pro(asparagus,fresh). pro(cucumber,fresh).
pro(asparagus,tasty). pro(cucumber,tasty).
  
```

```
buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a “list” hold

- 1 check all properties explicitly ... **obsolete if properties change**
- 2 use variable-sized conjunction (via ':') ... adapts to changing facts
- 3 use negation of complement ... adapts to changing facts

Example: vegetables to buy

```

veg(asparagus).      veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap).
pro(asparagus,fresh). pro(cucumber,fresh).
pro(asparagus,tasty). pro(cucumber,tasty).
pro(asparagus,clean).
```

```
buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty), pro(X,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a “list” hold

- 1 ~~check all properties explicitly~~ ... obsolete if properties change
- 2 use variable-sized conjunction (via ':') ... adapts to changing facts ✓
- 3 use negation of complement ... adapts to changing facts

Example: vegetables to buy

```

veg(asparagus).      veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap).
pro(asparagus,fresh). pro(cucumber,fresh).
pro(asparagus,tasty). pro(cucumber,tasty).
pro(asparagus,clean).

```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a “list” hold

- 1 ~~check all properties explicitly~~ ... obsolete if properties change
- 2 use variable-sized conjunction (via ':') ... adapts to changing facts ✓
- 3 use negation of complement ... adapts to changing facts

Example: vegetables to buy

```

veg(asparagus).      veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap). pre(cheap).
pro(asparagus,fresh). pro(cucumber,fresh). pre(fresh).
pro(asparagus,tasty). pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).
  
```

```
buy(X) :- veg(X), pro(X,P) : pre(P).
```


Implementing Universal Quantification

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Example: vegetables to buy

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pro(asparagus,tasty). pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).      pre(clean).
  
```

```
buy(X) :- veg(X), pro(X,P) : pre(P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a “list” hold

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```

veg(asparagus).      veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap). pre(cheap).
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pro(asparagus,tasty). pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).
  
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a “list” hold

- 1 ~~check all properties explicitly~~ ... obsolete if properties change ✗
- 2 use variable-sized conjunction (via ':') ... adapts to changing facts ✓
- 3 use negation of complement ... adapts to changing facts ✓

Example: vegetables to buy

```
veg(asparagus).      veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap). pre(cheap).
pro(asparagus,fresh). pro(cucumber,fresh). pre(fresh).
pro(asparagus,tasty). pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).
```

```
buy(X) :- veg(X), not bye(X).      bye(X) :- veg(X), pre(P), not pro(X,P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a “list” hold

- 1 ~~check all properties explicitly~~ ... obsolete if properties change ✗
- 2 use variable-sized conjunction (via ':') ... adapts to changing facts ✓
- 3 use negation of complement ... adapts to changing facts ✓

Example: vegetables to buy

```

veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap). pre(cheap).
pro(asparagus,fresh).  pro(cucumber,fresh). pre(fresh).
pro(asparagus,tasty).  pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).  pre(clean).
  
```

```

buy(X) :- veg(X), not bye(X).      bye(X) :- veg(X), pre(P), not pro(X,P).
  
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a “list” hold

- 1 ~~check all properties explicitly~~ ... obsolete if properties change ❌
- 2 use variable-sized conjunction (via ':') ... adapts to changing facts ✅
- 3 use negation of complement ... adapts to changing facts ✅

Example: vegetables to buy

```

veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap). pre(cheap).
pro(asparagus,fresh).  pro(cucumber,fresh). pre(fresh).
pro(asparagus,tasty).  pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).  pre(clean).
  
```

```

buy(X) :- veg(X), not bye(X).      bye(X) :- veg(X), pre(P), not pro(X,P).
  
```

Running Example: Latin Square

Given: an $N \times N$ board

1						
2						
3						
4						
5						
6						
	1	2	3	4	5	6

represented by facts:

```

square(1,1). ... square(1,6).
square(2,1). ... square(2,6).
square(3,1). ... square(3,6).
square(4,1). ... square(4,6).
square(5,1). ... square(5,6).
square(6,1). ... square(6,6).

```

Wanted: assignment of $1, \dots, N$

1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	1	2
4	4	5	6	1	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5
	1	2	3	4	5	6

represented by atoms:

```

num(1,1,1) num(1,2,2) ... num(1,6,6)
num(2,1,2) num(2,2,3) ... num(2,6,1)
num(3,1,3) num(3,2,4) ... num(3,6,2)
num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6,5)

```

CO

Running Example: Latin Square

Given: an $N \times N$ board

1						
2						
3						
4						
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represented by facts:

```

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square(5,1). ... square(5,6).
square(6,1). ... square(6,6).

```

Wanted: assignment of $1, \dots, N$

1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	1	2
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5	5	6	1	2	3	4
6	6	1	2	3	4	5
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```

num(1,1,1) num(1,2,2) ... num(1,6,6)
num(2,1,2) num(2,2,3) ... num(2,6,1)
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num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6,5)

```

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X,Y), N = 1..n, not num(X,Y',N) : square(X,Y').
:- square(X,Y), N = 1..n, not num(X',Y,N) : square(X',Y).
```

- Note unreused “singleton variables”

```
gringo latin_0.lp | wc
```

```
105480 2558984 14005258
```

```
gringo latin_1.lp | wc
```

```
42056 273672 1690522
```

```
-----CO
```


Projecting Irrelevant Details Out

A Latin square encoding

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% DOMAIN
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Projecting Irrelevant Details Out

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```

```
% GENERATE
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```
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).
```

```
% TEST
```

```
:- square(X,Y), N = 1..n, not num(X,Y',N) : square(X,Y').
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:- square(X,Y), N = 1..n, not num(X',Y,N) : square(X',Y).
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```
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Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
squareX(X) :- square(X,Y).      squareY(Y) :- square(X,Y).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- squareX(X), N = 1..n, not num(X,Y',N) : square(X,Y').
:- squareY(Y), N = 1..n, not num(X',Y,N) : square(X',Y).
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% TEST
:- squareX(X), N = 1..n, not num(X,Y',N) : square(X,Y').
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```

```
42056 273672 1690522
```

```
-----CO
```

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X,Y,N), num(X,Y',N), Y != Y'.
:- num(X,Y,N), num(X',Y,N), X != X'.
```

- Note duplicate ground rules
(swapping Y/Y' and X/X' gives the “same”)

```
gringo latin_2.lp | wc
```

```
2071560 12389384 40906946
```

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

```
-----CO
```

Unraveling Symmetric Inequalities

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```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X,Y,N), num(X,Y',N), Y != Y'.
:- num(X,Y,N), num(X',Y,N), X != X'.
```

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```
1055752 6294536 21099558
```

```
-----CO
```

Unraveling Symmetric Inequalities

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#const n=32. square(1..n,1..n).

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1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X,Y,N), num(X,Y',N), Y != Y'.
:- num(X,Y,N), num(X',Y,N), X != X'.
```

- Note duplicate ground rules
(swapping Y/Y' and X/X' gives the “same”)

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```
-----CO
```

Unraveling Symmetric Inequalities

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```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X,Y,N), num(X,Y',N), Y < Y'.
:- num(X,Y,N), num(X',Y,N), X < X'.
```

- Note duplicate ground rules
(swapping Y/Y' and X/X' gives the "same")

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:- num(X,Y,N), num(X,Y',N), Y < Y'.
:- num(X,Y,N), num(X',Y,N), X < X'.
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- Note duplicate ground rules
(swapping Y/Y' and X/X' gives the “same”)

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gringo latin_2.lp | wc
```

```
2071560 12389384 40906946
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gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

```
-----CO
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X,Y,N), num(X,Y',N), Y < Y'.
:- num(X,Y,N), num(X',Y,N), X < X'.
```

- Note uniqueness of N in a row/column checked by enumerating pairs!

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

```
gringo latin_4.lp | wc
```

```
228360 1205256 4780744
```

```
-----CO
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X,Y,N), num(X,Y',N), Y < Y'.
:- num(X,Y,N), num(X',Y,N), X < X'.
```

- Note uniqueness of N in a row/column checked by enumerating pairs!

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

```
gringo latin_4.lp | wc
```

```
228360 1205256 4780744
```

```
-----CO
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X,Y,N), num(X,Y',N), Y < Y'.
:- num(X,Y,N), num(X',Y,N), X < X'.
```

- Note uniqueness of N in a row/column checked by enumerating pairs!

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

```
gringo latin_4.lp | wc
```

```
228360 1205256 4780744
```

```
-----CO
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
:- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of N in a row/column checked by enumerating pairs!

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

```
gringo latin_4.lp | wc
```

```
228360 1205256 4780744
```

```
-----CO
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
:- num(X,Y,N), gtX(X,Y,N).                :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of N in a row/column checked by enumerating pairs!

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

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228360 1205256 4780744
```

```
-----CO
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
:- num(X,Y,N), gtX(X,Y,N).                :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of N in a row/column checked by enumerating pairs!

```
gringo latin_3.lp | wc
```

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1055752 6294536 21099558
```

```
gringo latin_4.lp | wc
```

```
228360 1205256 4780744
```

```
-----CO
```

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum { X:square(X,n) }.

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = { num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = { num(X,Y,N) }.
:- occX(X,N,C), C != 1. :- occY(Y,N,C), C != 1.

% DISPLAY
#show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc
```

```
gringo latin_6.lp | wc
```


Assigning Aggregate Values

Yet another Latin square encoding

```

% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum { X:square(X,n) }.

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = { num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = { num(X,Y,N) }.
:- occX(X,N,C), C != 1. :- occY(Y,N,C), C != 1.

% DISPLAY
#show num/3. #show sigma/1.

```

```
gringo latin_5.lp | wc
```

```
gringo latin_6.lp | wc
```

Assigning Aggregate Values

Yet another Latin square encoding

% DOMAIN

```
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum { X:square(X,n) }.
```



% GENERATE

```
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).
```

% DEFINE + TEST

```
occX(X,N,C) :- X = 1..n, N = 1..n, C = { num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = { num(X,Y,N) }.
:- occX(X,N,C), C != 1. :- occY(Y,N,C), C != 1.
```

% DISPLAY

```
#show num/3. #show sigma/1.
```

gringo latin_5.lp | wc

gringo latin_6.lp | wc

Assigning Aggregate Values

Yet another Latin square encoding

```

% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum { X:square(X,n) }.

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = { num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = { num(X,Y,N) }.
:- occX(X,N,C), C != 1. :- occY(Y,N,C), C != 1.

% DISPLAY
#show num/3. #show sigma/1.

```

```
gringo latin_5.lp | wc
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```
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Assigning Aggregate Values

Yet another Latin square encoding

```

% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum { X:square(X,n) }.

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C { num(X,Y,N) } C, C = 0..n.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C { num(X,Y,N) } C, C = 0..n.
:- occX(X,N,C), C != 1. :- occY(Y,N,C), C != 1.

% DISPLAY
#show num/3. #show sigma/1.

```

- Note internal transformation by gringo

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
```

```
#const n=32. square(1..n,1..n).
```

```
sigma(S) :- S = #sum { X:square(X,n) }.
```

```
% GENERATE
```

```
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).
```

```
% DEFINE + TEST
```

```
occX(X,N,C) :- X = 1..n, N = 1..n, C = { num(X,Y,N) }. ✗
```

```
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = { num(X,Y,N) }. ✗
```

```
:- occX(X,N,C), C != 1. :- occY(Y,N,C), C != 1.
```

```
% DISPLAY
```

```
#show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc
```

```
gringo latin_6.lp | wc
```

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = { num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = { num(X,Y,N) }.
:- occX(X,N,C), C != 1. :- occY(Y,N,C), C != 1.

% DISPLAY
#show num/3.
```

```
gringo latin_5.lp | wc
```

```
gringo latin_6.lp | wc
```

```
48136 373768 2185042
```

CO

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

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% DISPLAY
#show num/3.
```

```
gringo latin_5.lp | wc
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```
304136 5778440 30252505
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% GENERATE
1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 { num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 { num(X,Y,N) } 1.

% DISPLAY
#show num/3.
```

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Breaking Symmetries

The ultimate Latin square encoding?

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```

- Note many symmetric solutions
(mirroring, rotation, value permutation)

Breaking Symmetries

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:- Y = 1..n, N = 1..n, not 1 { num(X,Y,N) } 1.

% DISPLAY
#show num/3.

```

- Note **easy and safe** to fix a full row/column!

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:- Y = 1..n, N = 1..n, not 1 { num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#show num/3.
```

- Note easy and safe to fix a full row/column!

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% DISPLAY
#show num/3.
```

- Note Let's compare **enumeration** speed!

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% DISPLAY
#show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

Breaking Symmetries

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% DISPLAY
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```

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```
Models : 161280      Time : 2.078s
```


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:- Y = 1..n, N = 1..n, not 1 { num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 161280      Time : 2.078s
```

Breaking Symmetries

The ultimate Latin square encoding?

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% DOMAIN
#const n=32. square(1..n,1..n).

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1 { num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 { num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 { num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 1344          Time : 0.024s
```

Outline

- 1 Tweaking N -Queens
- 2 Do's and Dont's
- 3 Hints

Encode With Care!

1 Create a **working** encoding

- Q1: Do you need to modify the encoding if the facts change?
- Q2: Are all variables significant (or statically functionally dependent)?
- Q3: Can there be (many) identic ground rules?
- Q4: Do you enumerate pairs of values (to test uniqueness)?
- Q5: Do you assign dynamic aggregate values (to check a fixed bound)?
- Q6: Do you admit (obvious) symmetric solutions?
- Q7: Do you have additional domain knowledge simplifying the problem?
- Q8: Are you aware of anything else that, if encoded, would reduce grounding and/or solving efforts?

2 Revise until no “Yes” answer!

- Note If the format of facts makes encoding painful (for instance, abusing grounding for “scientific calculations”), revise the fact format as well.

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Some Hints on (Preventing) Debugging

Kinds of errors

- syntactic ... follow error messages by the grounder
- semantic ... (most likely) encoding/intention mismatch

Ways to identify semantic errors (early)

develop and test incrementally

prepare toy instances with “interesting features”

build the encoding bottom-up and verify additions (eg. new predicates)

compare the encoded to the intended meaning

check whether the grounding fits (use `gringo --text`)

if stable models are unintended, investigate conditions that fail to hold

if stable models are missing, examine integrity constraints (add heads)

ask tools (eg. <http://www.kr.tuwien.ac.at/research/projects/mmdasp/>)

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Overcoming Performance Bottlenecks

Grounding

- monitor **time** spent by and output **size** of gringo
 - 1 system tools (eg. `time(gringo [...] | wc)`)
 - 2 grounding info (eg. `gringo --text`)
- Note once identified, reformulate “critical” logic program parts

Solving

- check solving statistics (use `clasp --stats`)
 - if great search efforts (Conflicts/Choices/Restarts), then try prefabricated settings (using `clasp` option ‘`--configuration`’)
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