Answer Set Solving in Practice

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Language Extensions: Overview

1 Two kinds of negation
2 Disjunctive logic programs
3 Propositional theories
4 Aggregates
5 Gringo language

Outline

Motivation

- Classical versus default negation
  - Symbol $\neg$ and $\sim$
  - Idea
    - $\neg a \quad \approx \quad \neg a \in X$
    - $\sim a \quad \approx \quad a \notin X$
  - Example
    - $cross \leftarrow \neg train$
    - $cross \leftarrow \sim train$

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Classical negation

- We consider logic programs in negation normal form
  - That is, classical negation is applied to atoms only
- Given an alphabet $\mathcal{A}$ of atoms, let $\overline{\mathcal{A}} = \{\neg a \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \overline{\mathcal{A}} = \emptyset$
- Given a program P over $\mathcal{A}$, classical negation is encoded by adding

$$P^{\neg} = \{a \leftarrow b, \neg b \mid a \in (\mathcal{A} \cup \overline{\mathcal{A}}), b \in \mathcal{A}\}$$

- A set X of atoms is a stable model of a program P over $\mathcal{A} \cup \overline{\mathcal{A}}$, if X is a stable model of $P \cup P^{\neg}$

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An example

- The program

$$P \ = \ \{a \leftarrow {\sim}b,\ b \leftarrow {\sim}a\} \cup \{c \leftarrow b,\ \neg c \leftarrow b\}$$

induces

$$P^{\neg} = \left\{ \begin{array}{rclcrclcrcl} a & \leftarrow & a, \neg a & & a & \leftarrow & b, \neg b & & a & \leftarrow & c, \neg c \\ \neg a & \leftarrow & a, \neg a & & \neg a & \leftarrow & b, \neg b & & \neg a & \leftarrow & c, \neg c \\ b & \leftarrow & a, \neg a & & b & \leftarrow & b, \neg b & & b & \leftarrow & c, \neg c \\ \neg b & \leftarrow & a, \neg a & & \neg b & \leftarrow & b, \neg b & & \neg b & \leftarrow & c, \neg c \\ c & \leftarrow & a, \neg a & & c & \leftarrow & b, \neg b & & c & \leftarrow & c, \neg c \\ \neg c & \leftarrow & a, \neg a & & \neg c & \leftarrow & b, \neg b & & \neg c & \leftarrow & c, \neg c \end{array} \right\}$$

- The stable models of P are given by the ones of $P \cup P^{\neg}$, viz $\{a\}$

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Properties

- The only inconsistent stable "model" is $X = \mathcal{A} \cup \overline{\mathcal{A}}$

  Strictly speaking, an inconsistemt set like $\mathcal{A} \cup \overline{\mathcal{A}}$ is not a model

- For a logic program P over $\mathcal{A} \cup \overline{\mathcal{A}}$, exactly one of the following two cases applies:
  1. All stable models of P are consistent or
  2. $X = \mathcal{A} \cup \overline{\mathcal{A}}$ is the only stable model of P

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Train spotting

- $P_1 = \{cross \leftarrow \sim train\}$
  - stable model: $\{cross\}$
- $P_2 = \{cross \leftarrow \neg train\}$
  - stable model: $\emptyset$
- $P_3 = \{cross \leftarrow \neg train, \ \neg train \leftarrow\}$
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Default negation in rule heads

- We consider logic programs with default negation in rule heads
- Given an alphabet $\mathcal{A}$ of atoms, let $\widetilde{\mathcal{A}} = \{\widetilde{a} \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \widetilde{\mathcal{A}} = \emptyset$
- Given a program P over $\mathcal{A}$, consider the program

$$\begin{array}{rcl}\widetilde{P} & = & \{r \in P \mid h(r) \neq {\sim}a\} \\ & & \cup\ \{\leftarrow B(r) \cup \{{\sim}\widetilde{a}\} \mid r \in P \text{ and } h(r) = {\sim}a\} \\ & & \cup\ \{\widetilde{a} \leftarrow {\sim}a \mid r \in P \text{ and } h(r) = {\sim}a\}\end{array}$$

- A set X of atoms is a stable model of a program P (with default negation in rule heads) over $\mathcal{A}$,
  if $X = Y \cap \mathcal{A}$ for some stable model Y of $\widetilde{P}$ over $\mathcal{A} \cup \widetilde{\mathcal{A}}$

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Outline

Disjunctive logic programs

- A disjunctive rule, r, is of the form

$$a_1 \,;\, \dots \,;\, a_m \leftarrow a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o$$

  where $0 \leq m \leq n \leq o$ and each a_i is an atom for $0 \leq i \leq o$
- A disjunctive logic program is a finite set of disjunctive rules
- Notation

$$\begin{array}{rcl} H(r) & = & \{a_1, \dots, a_m\} \\ B(r) & = & \{a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o\} \\ B(r)^+ & = & \{a_{m+1}, \dots, a_n\} \\ B(r)^- & = & \{a_{n+1}, \dots, a_o\} \\ A(P) & = & \bigcup_{r \in P} \left(H(r) \cup B(r)^+ \cup B(r)^-\right) \\ B(P) & = & \{B(r) \mid r \in P\} \end{array}$$

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Stable models

- Positive programs
  - A set X of atoms is closed under a positive program P iff for any $r \in P$, $H(r) \cap X \neq \emptyset$ whenever $B(r)^+ \subseteq X$
    - X corresponds to a model of P (seen as a formula)
  - The set of all $\subseteq$-minimal sets of atoms being closed under a positive program P is denoted by $\min_\subseteq(P)$
    - $\min_\subseteq(P)$ corresponds to the $\subseteq$-minimal models of P (ditto)
- Disjunctive programs
  - The reduct, P^X, of a disjunctive program P relative to a set X of atoms is defined by

$$P^X = \{H(r) \leftarrow B(r)^+ \mid r \in P \text{ and } B(r)^- \cap X = \emptyset\}$$

  - A set X of atoms is a stable model of a disjunctive program P, if $X \in \min_\subseteq(P^X)$

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  - The set of all $\subseteq$-minimal sets of atoms being closed under a positive program P is denoted by $\min_\subseteq(P)$
    - $\min_\subseteq(P)$ corresponds to the $\subseteq$-minimal models of P (ditto)
- Disjunctive programs
  - The reduct, P^X, of a disjunctive program P relative to a set X of atoms is defined by

$$P^X = \{H(r) \leftarrow B(r)^+ \mid r \in P \text{ and } B(r)^- \cap X = \emptyset\}$$

  - A set X of atoms is a stable model of a disjunctive program P, if $X \in \min_\subseteq(P^X)$

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A "positive" example

$$P = \left\{ \begin{array}{lcl} a & \leftarrow & \\ b\,;c & \leftarrow & a \end{array} \right\}$$

- The sets $\{a, b\}$, $\{a, c\}$, and $\{a, b, c\}$ are closed under P
- We have $\min_{\subseteq}(P) = \{\{a, b\}, \{a, c\}\}$

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Graph coloring (reloaded)

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node(1..6).

edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).


assign(X,r) ; assign(X,b) ; assign(X,g) :- node(X).


:- edge(X,Y), assign(X,C), assign(Y,C).
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color(r).   color(b).   color(g).

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More Examples

- $P_1 = \{a\,;b\,;c \leftarrow\}$
  - stable models $\{a\}$, $\{b\}$, and $\{c\}$
- $P_2 = \{a\,;b\,;c \leftarrow\ ,\ \leftarrow a\}$
  - stable models $\{b\}$ and $\{c\}$
- $P_3 = \{a\,;b\,;c \leftarrow\ ,\ \leftarrow a\ ,\ b \leftarrow c\ ,\ c \leftarrow b\}$
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Some properties

- A disjunctive logic program may have zero, one, or multiple stable models
- If X is a stable model of a disjunctive logic program P, then X is a model of P (seen as a formula)
- If X and Y are stable models of a disjunctive logic program P, then $X \not\subset Y$

- If $a \in X$ for some stable model X of a disjunctive logic program P, then there is a rule $r \in P$ such that $B(r)^+ \subseteq X$, $B(r)^- \cap X = \emptyset$, and $H(r) \cap X = \{a\}$

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An example with variables

$$P = \left\{ \begin{array}{lcl} a(1,2) & \leftarrow & \\ b(X)\,;c(Y) & \leftarrow & a(X,Y), \sim c(Y) \end{array} \right\}$$

$$ground(P) = \left\{ \begin{array}{lcl} a(1,2) & \leftarrow & \\ b(1)\,;c(1) & \leftarrow & a(1,1), \sim c(1) \\ b(1)\,;c(2) & \leftarrow & a(1,2), \sim c(2) \\ b(2)\,;c(1) & \leftarrow & a(2,1), \sim c(1) \\ b(2)\,;c(2) & \leftarrow & a(2,2), \sim c(2) \end{array} \right\}$$

For every stable model X of P, we have

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Default negation in rule heads

- Consider disjunctive rules of the form

$$a_1 \,;\ldots;\, a_m \,;\, \sim a_{m+1} \,;\ldots;\, \sim a_n \leftarrow a_{n+1},\ldots,a_o,\sim a_{o+1},\ldots,\sim a_p$$

  where $0 \leq m \leq n \leq o \leq p$ and each a_i is an atom for $0 \leq i \leq p$

- Given a program P over $\mathcal{A}$, consider the program

$$\begin{aligned}\widetilde{P} \;=\; & \{H(r)^+ \leftarrow B(r) \cup \{\sim\widetilde{a} \mid a \in H(r)^-\} \mid r \in P\} \\ & \cup\ \{\widetilde{a} \leftarrow \sim a \mid r \in P \text{ and } a \in H(r)^-\}\end{aligned}$$

- A set X of atoms is a stable model of a disjunctive program P (with default negation in rule heads) over $\mathcal{A}$, if $X = Y \cap \mathcal{A}$ for some stable model Y of $\widetilde{P}$ over $\mathcal{A} \cup \widetilde{\mathcal{A}}$

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- The program

$$P = \{a\ ;\ {\sim}a \leftarrow\}$$

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Outline

Propositional theories

- Formulas are formed from
  - atoms in $\mathcal{A}$
  - $\bot$

  using
  - conjunction ($\wedge$)
  - disjunction ($\vee$)
  - implication ($\rightarrow$)
- Notation

$$\begin{array}{rcl} \top & = & (\bot \rightarrow \bot) \\ {\sim}\phi & = & (\phi \rightarrow \bot) \end{array}$$

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- The satisfaction relation $X \models \phi$ between a set X of atoms and a (set of) formula(s) ϕ is defined as in propositional logic
- The reduct, ϕ^X, of a formula ϕ relative to a set X of atoms is defined recursively as follows:

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\begin{array}{ll}
\phi^X = \bot & \text{if } X \not\models \phi \\
\phi^X = \phi & \text{if } \phi \in X \\
\phi^X = (\psi^X \circ \varphi^X) & \text{if } X \models \phi \text{ and } \phi = (\psi \circ \varphi) \text{ for } \circ \in \{\land, \lor, \rightarrow\}
\end{array}
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If $\phi = {\sim}\psi = (\psi \rightarrow \bot)$,
then $\phi^X = (\bot \rightarrow \bot) = \top$, if $X \not\models \psi$, and $\phi^X = \bot$, otherwise

- The reduct, Φ^X, of a propositional theory Φ relative to a set X of atoms is defined as $\Phi^X = \{\phi^X \mid \phi \in \Phi\}$

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$\phi^X = (\psi^X \circ \varphi^X)$	if $X \models \phi$ and $\phi = (\psi \circ \varphi)$ for $\circ \in \{\wedge, \vee, \rightarrow\}$

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- The reduct, ϕ^X, of a formula ϕ relative to a set X of atoms is defined recursively as follows:
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  - $\phi^X = (\psi^X \circ \varphi^X)$ if $X \models \phi$ and $\phi = (\psi \circ \varphi)$ for $\circ \in \{\wedge, \vee, \rightarrow\}$
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Stable models

- A set X of atoms satisfies a propositional theory Φ, written $X \models \Phi$, if $X \models \phi$ for each $\phi \in \Phi$
- The set of all $\subseteq$-minimal sets of atoms satisfying a propositional theory Φ is denoted by $\min_{\subseteq}(\Phi)$
- A set X of atoms is a stable model of a propositional theory Φ, if $X \in \min_{\subseteq}(\Phi^X)$
- If X is a stable model of Φ, then
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Two examples

- $\Phi_1 = \{p \vee (p \rightarrow (q \wedge r))\}$
  - For $X = \{p, q, r\}$, we get
    $\Phi_1^{\{p,q,r\}} = \{p \vee (p \rightarrow (q \wedge r))\}$ and $\min_{\subseteq}(\Phi_1^{\{p,q,r\}}) = \{\emptyset\}$
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- $\Phi_2 = \{p \vee ({\sim}p \rightarrow (q \wedge r))\}$
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Logic programs as propositional theories

- The normal logic program $P = \{p \leftarrow {\sim}q,\ q \leftarrow {\sim}p\}$ corresponds to $\tau[P] = \{{\sim}q \rightarrow p,\ {\sim}p \rightarrow q\}$
  - stable models: $\{p\}$ and $\{q\}$
- The disjunctive logic program $P = \{p\,;q \leftarrow\}$ corresponds to $\tau[P] = \{\top \rightarrow p \vee q\}$
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- The nested logic program $P = \{p \leftarrow \sim\sim p\}$ corresponds to $\tau[P] = \{\sim\sim p \rightarrow p\}$
  - stable models: $\emptyset$ and $\{p\}$

Logic programs as propositional theories

- The normal logic program $P = \{p \leftarrow \sim q,\ q \leftarrow \sim p\}$ corresponds to $\tau[P] = \{\sim q \rightarrow p,\ \sim p \rightarrow q\}$
  - stable models: $\{p\}$ and $\{q\}$
- The disjunctive logic program $P = \{p \,;\, q \leftarrow\}$ corresponds to $\tau[P] = \{\top \rightarrow p \vee q\}$
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Logic programs as propositional theories

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Outline

Motivation

- Aggregates provide a general way to obtain a single value from a collection of input values
- Popular aggregate (functions)
  - average
  - count
  - maximum
  - minimum
  - sum
- Cardinality and weight constraints rely on count and sum aggregates

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Syntax

- An aggregate has the form:

$$\alpha\ \{w_1 : a_1, \dots, w_m : a_m, w_{m+1} : \sim a_{m+1}, \dots, w_n : \sim a_n\} \prec k$$

  where for $1 \leq i \leq n$
  - α stands for a function mapping multisets over $\mathbb{Z}$ to $\mathbb{Z} \cup \{+\infty, -\infty\}$
  - $\prec$ stands for a relation between $\mathbb{Z} \cup \{+\infty, -\infty\}$ and $\mathbb{Z}$
  - $k \in \mathbb{Z}$
  - a_i are atoms and
  - w_i are integers
- Example $sum\ \{30 : hd(a), \dots, 50 : hd(m)\} \leq 300$

Semantics

- A (positive) aggregate $\alpha \{w_1 : a_1, \dots, w_n : a_n\} \prec k$ can be represented by the formula:

$$\bigwedge_{I \subseteq \{1,\dots,n\}, \alpha\{w_i \mid i \in I\} \not\prec k} \left(\bigwedge_{i \in I} a_i \rightarrow \bigvee_{i \in \overline{I}} a_i \right)$$

  where $\overline{I} = \{1, \dots, n\} \setminus I$ and $\not\prec$ is the complement of $\prec$
- Then, $\alpha \{w_1 : a_1, \dots, w_n : a_n\} \prec k$ is true in X iff the above formula is true in X

Example

- Consider $sum\{1 : p, 1 : q\} \neq 1$
  That is, $a_1 = p$, $a_2 = q$ and $w_1 = 1$, $w_2 = 1$
- Calculemus!

I	$\{w_i \mid i \in I\}$	$\sum\{w_i \mid i \in I\}$	$\sum\{w_i \mid i \in I\} = 1$
$\emptyset$	$\{\}$	0	*false*
$\{1\}$	$\{1\}$	1	*true*
$\{2\}$	$\{1\}$	1	*true*
$\{1, 2\}$	$\{1, 1\}$	2	*false*

- We get $(p \rightarrow q) \wedge (q \rightarrow p)$
- Analogously, we obtain $(p \vee q) \wedge \neg(p \wedge q)$ for $sum\{1 : p, 1 : q\} = 1$

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Monotonicity

- Monotone aggregates
  - For instance,
    - $B(r)^+$
    - $sum\{1:p,1:q\} > 1$ amounts to $p \wedge q$
  - We get a simpler characterization: $\bigwedge_{I\subseteq\{1,\dots,n\},\alpha\{w_i|i\in I\}\not\prec k} \bigvee_{i\in\bar{I}} a_i$
- Anti-monotone aggregates
  - For instance,
    - $B(r)^-$
    - $sum\{1:p,1:q\} < 1$ amounts to $\neg p \wedge \neg q$
  - We get a simpler characterization: $\bigwedge_{I\subseteq\{1,\dots,n\},\alpha\{w_i|i\in I\}\not\prec k} \neg \bigwedge_{i\in I} a_i$
- Non-monotone aggregates
  - For instance, $sum\{1:p,1:q\} \neq 1$ is non-monotone.

Monotonicity

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Outline

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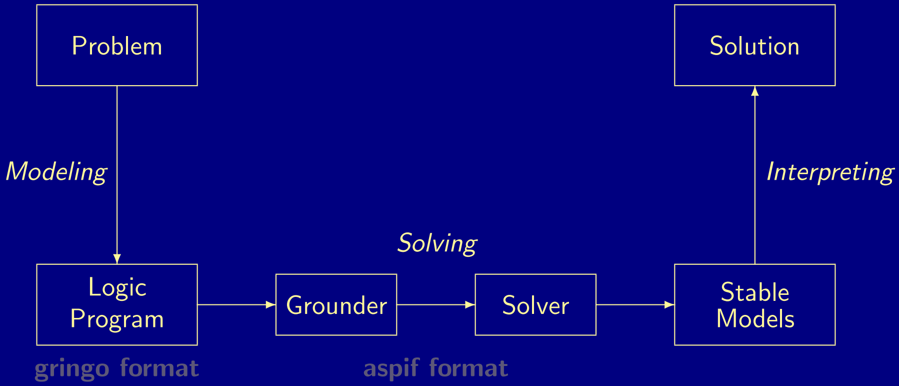
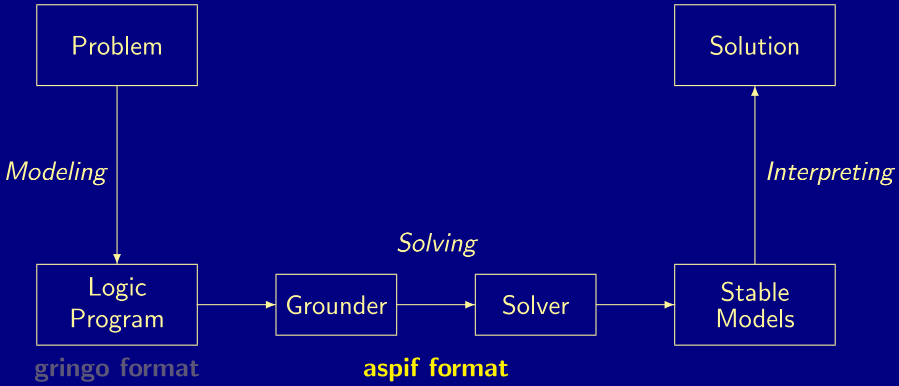
Gringo language



- aspif format is a machine-oriented standard for ground programs
- gringo format is a user-oriented language for (non-ground) programs extending the ASP language standard *ASP-Core-2*

Gringo language



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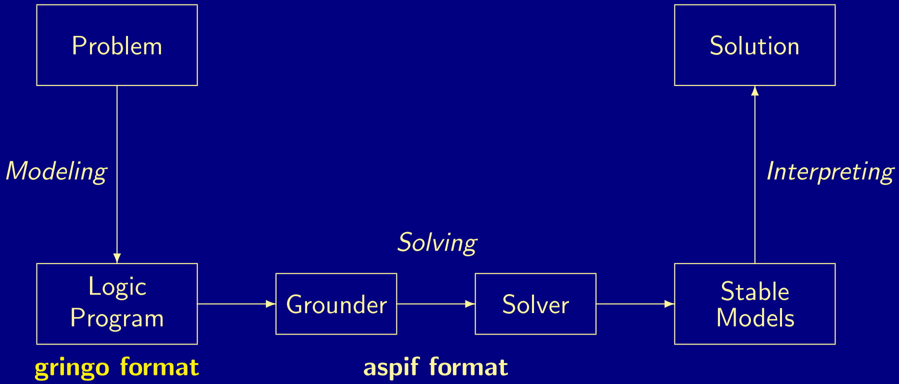

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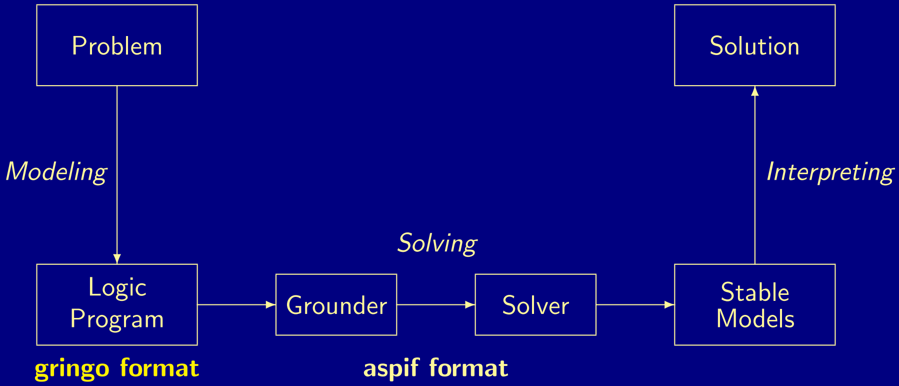

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Terms and literals

- Terms t
- Tuples $\boldsymbol{t}$
- Atoms a, $\neg a$
- Symbolic literals a, $\sim a$, $\sim\sim a$
- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \dots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
- Aggregate literals a, $\sim a$, $\sim\sim a$
- Literals

Terms and literals

- Terms t are formed from
  - constant symbols, eg `c`, `d`, ...
  - function symbols, eg `f`, `g`, ...
  - numeric symbols, eg 1, 2, ...
  - variable symbols, eg `X`, `Y`, ..., `_`
  - parentheses (,)
  - tuple delimiters $\langle, \rangle$ (omitted whenever possible)
- Tuples $\boldsymbol{t}$
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  - numerics, eg `1`, `2`, ...
  - variables, eg `X`, `Y`, ..., `_`
  - parentheses (,)
  - tuple delimiters $\langle, \rangle$

  eg `f(3,c,Z)`, `g(42,_,_)`, or `f((3,c),X)`
- Tuples $\boldsymbol{t}$
- Atoms a, $\neg a$
- Symbolic literals a, $\sim a$, $\sim\sim a$
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Terms and literals

- Terms t
- Tuples $\boldsymbol{t}$ of terms
- Atoms a, $\neg a$
- Symbolic literals a, $\sim a$, $\sim\sim a$
- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \dots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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- Literals

Terms and literals

- Terms t
- Tuples $\boldsymbol{t}$
- (Negated) Atoms a, $\neg a$ are formed from
  - predicate symbols, eg p, q, ...
  - parentheses (,)
  - tuples of terms
- Symbolic literals a, $\sim a$, $\sim\sim a$
- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \dots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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Terms and literals

- Terms t
- Tuples $\boldsymbol{t}$
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- Tuples $\boldsymbol{t}$
- Atoms a, $\neg a$ are formed from
  - predicates, eg p, q, ...
  - parentheses (,)
  - tuples of terms

  eg `-p(f(3,c,Z),g(42,_,_))` or `q()` written as `q`
- Symbolic literals a, $\sim a$, $\sim\sim a$
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Terms and literals

- Terms t
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- Atoms a, $\neg a$, $\bot$, $\top$
  viz `#false` and `#true`
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Terms and literals

- Terms t
- Tuples $\boldsymbol{t}$
- Atoms a, $\neg a$, $\bot$, $\top$
- Symbolic literals a, $\sim a$, $\sim\sim a$
  eg `p(a,X)`, '`not p(a,X)`', '`not not p(a,X)`'
- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$
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Terms and literals

- Terms t
- Tuples $\boldsymbol{t}$
- Atoms a, $\neg a$, $\bot$, $\top$
- Symbolic literals a, $\sim a$, $\sim\sim a$
- Arithmetic literals $t_1 \prec t_2$ where
  - t_1 and t_2 are terms
  - $\prec$ is a comparison symbol
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \dots ; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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  eg `3<1` or `f(42)=X`
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \dots ; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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Terms and literals

- Terms t
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- Atoms a, $\neg a$, $\bot$, $\top$
- Symbolic literals a, $\sim a$, $\sim\sim a$
- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$ where
  - l is a symbolic or arithmetic literal
  - $\boldsymbol{L}$ is a tuple of symbol or arithmetic literals
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \dots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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  - l is a symbolic or arithmetic literal
  - $\boldsymbol{L}$ is a tuple of symbol or arithmetic literals
  - $l : \boldsymbol{L}$ is written as l whenever $\boldsymbol{L}$ is empty
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \dots ; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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  eg '`p(X,Y):q(X),r(Y)`' or `p(42)` or '`#false:q`'
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \dots ; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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  - α is an aggregate name
  - $\mathbf{t}_1 : \mathbf{L}_1, \dots, \mathbf{t}_n : \mathbf{L}_n$ are conditional literals
  - $\prec_1$ and $\prec_2$ are comparison symbols
  - s_1 and s_2 are terms
- Aggregate literals a, $\sim a$, $\sim\sim a$
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  - $\prec_1$ and $\prec_2$ are comparison symbols
  - s_1 and s_2 are terms
  - one (or both) of '$s_1 \prec_1$' and '$\prec_2 s_2$' can be omitted
- Aggregate literals a, $\sim a$, $\sim\sim a$
- Literals

Terms and literals

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  - $\prec_1$ and $\prec_2$ are comparison symbols
  - s_1 and s_2 are terms
  - omitting $\prec_1$ or $\prec_2$ defaults to $\leq$
- Aggregate literals a, $\sim a$, $\sim\sim a$
- Literals are conditional or aggregate literals

Terms and literals

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  - s_1 and s_2 are terms

  eg `10 <= #sum {6,C:course(C); 3,S:seminar(S)} <= 20`
- Aggregate literals a, $\sim a$, $\sim\sim a$
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Terms and literals

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- Aggregate literals a, $\sim a$, $\sim\sim a$ where
  - a is an aggregate atom

  eg `not 10 #sum {6,C:course(C); 3,S:seminar(S)} 20`
- Literals are conditional or aggregate literals

Terms and literals

- Terms t
- Tuples $\boldsymbol{t}$, $\boldsymbol{L}$
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- For a detailed account please consult the user's guide!

Rules

- Rules are of the form

$$l_1 ; \dots ; l_m \leftarrow l_{m+1}, \dots, l_n \tag{2}$$

  where
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- Note Semicolons ';' must be used in (2) instead of commas ','
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Shortcuts

- A rule of the form

$$s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : l_1 : \boldsymbol{L}_1; \dots ; \boldsymbol{t}_k : l_k : \boldsymbol{L}_k\} \prec_2 s_2 \;\leftarrow\; l_{m+1}, \dots , l_n$$

where

  - α, $\prec_i$, s_i, $\boldsymbol{t}_j$ are as given above for $i = 1, 2$ and $1 \leq j \leq k$
  - $l_j : \boldsymbol{L}_j$ is a conditional literal for $1 \leq j \leq k$
  - l_i is a literal for $m + 1 \leq i \leq n$ (as in (2))

is a shorthand for the following $k + 1$ rules

$$\{l_j\} \leftarrow l_{m+1}, \dots , l_n, \boldsymbol{L}_j \qquad \text{for } 1 \leq j \leq k$$

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Examples

- {a; b}

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$ gringo --text <(echo "{a;b}.")
#count{1,0,a:a;1,0,b:b}.
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gringo generates two distinct term tuples `1,0,a` and `1,0,b`

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1 = { q(X,Y): p(X), p(Y), X < Y; q(X,X): p(X) }
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Weak constraints

- Syntax A weak constraint is of the form

$$:\sim l_1, \ldots, l_n. [w@p, t_1, \ldots, t_m]$$

  where
  - $l_1, \ldots, l_n$ are literals
  - $t_1, \ldots, t_m$, w, and p are terms
- w and p stand for a weight and priority level ($p = 0$ if '@p' is omitted)
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  amounts to the minimize statement

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- Projection

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gringo 3 versus 4/5

- The input language of *gringo* series 4/5 comprises
  - ASP-Core-2
  - concepts from *lparse* and *gringo* 3
- Example The *gringo* 3 rule
  - ```
    r(X) : p(X) : not q(X) :- r(X) : p(X) : not q(X),
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    can be written as follows in the language of *gringo* 4/5:
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