# Answer Set Solving in Practice

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# Language Extensions: Overview

1. Two kinds of negation
2. Disjunctive logic programs
3. Propositional theories
4. Aggregates
5. Gringo language

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# Outline



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# Motivation

- Classical versus default negation
  - Symbol $\neg$ and $\sim$
  - Idea
    - $\neg a \approx \neg a \in X$
    - $\sim a \approx a \notin X$
  - Example
    - $cross \leftarrow \neg train$
    - $cross \leftarrow \sim train$

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  - Example
    - $cross \leftarrow \neg train$
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# Classical negation

- We consider logic programs in negation normal form
  - That is, classical negation is applied to atoms only
- Given an alphabet $\mathcal{A}$ of atoms, let $\overline{\mathcal{A}} = \{\neg a \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \overline{\mathcal{A}} = \emptyset$
- Given a program $P$ over $\mathcal{A}$, classical negation is encoded by adding

$$P^{\neg} = \{a \leftarrow b, \neg b \mid a \in (\mathcal{A} \cup \overline{\mathcal{A}}), b \in \mathcal{A}\}$$

- A set $X$ of atoms is a stable model of a program $P$ over $\mathcal{A} \cup \overline{\mathcal{A}}$, if $X$ is a stable model of $P \cup P^{\neg}$

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# An example

- The program

$$P \;=\; \{a \leftarrow {\sim}b,\; b \leftarrow {\sim}a\} \cup \{c \leftarrow b,\; \neg c \leftarrow b\}$$

induces

$$P^{\neg} = \left\{ \begin{array}{rcl|rcl|rcl} a & \leftarrow & a, \neg a & a & \leftarrow & b, \neg b & a & \leftarrow & c, \neg c \\ \neg a & \leftarrow & a, \neg a & \neg a & \leftarrow & b, \neg b & \neg a & \leftarrow & c, \neg c \\ b & \leftarrow & a, \neg a & b & \leftarrow & b, \neg b & b & \leftarrow & c, \neg c \\ \neg b & \leftarrow & a, \neg a & \neg b & \leftarrow & b, \neg b & \neg b & \leftarrow & c, \neg c \\ c & \leftarrow & a, \neg a & c & \leftarrow & b, \neg b & c & \leftarrow & c, \neg c \\ \neg c & \leftarrow & a, \neg a & \neg c & \leftarrow & b, \neg b & \neg c & \leftarrow & c, \neg c \end{array} \right\}$$

- The stable models of $P$ are given by the ones of $P \cup P^{\neg}$, viz $\{a\}$

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# Properties

- The only inconsistent stable "model" is $X = \mathcal{A} \cup \overline{\mathcal{A}}$

  Strictly speaking, an inconsistemt set like $\mathcal{A} \cup \overline{\mathcal{A}}$ is not a model

- For a logic program $P$ over $\mathcal{A} \cup \overline{\mathcal{A}}$, exactly one of the following two cases applies:
  1. All stable models of $P$ are consistent or
  2. $X = \mathcal{A} \cup \overline{\mathcal{A}}$ is the only stable model of $P$

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# Train spotting

- $P_1 = \{cross \leftarrow \sim train\}$
  - stable model: $\{cross\}$
- $P_2 = \{cross \leftarrow \neg train\}$
  - stable model: $\emptyset$
- $P_3 = \{cross \leftarrow \neg train,\ \neg train \leftarrow\}$
  - stable model: $\{cross, \neg train\}$
- $P_4 = \{cross \leftarrow \neg train,\ \neg train \leftarrow,\ \neg cross \leftarrow\}$
  - stable model: $\{cross, \neg cross, train, \neg train\}$
- $P_5 = \{cross \leftarrow \neg train,\ \neg train \leftarrow \sim train\}$
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# Default negation in rule heads

- We consider logic programs with default negation in rule heads
- Given an alphabet $\mathcal{A}$ of atoms, let $\widetilde{\mathcal{A}} = \{\widetilde{a} \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \widetilde{\mathcal{A}} = \emptyset$
- Given a program $P$ over $\mathcal{A}$, consider the program

$$\begin{aligned}\widetilde{P} \quad = \quad & \{r \in P \mid h(r) \neq \sim a\} \\ & \cup \{\leftarrow B(r) \cup \{\sim\widetilde{a}\} \mid r \in P \text{ and } h(r) = \sim a\} \\ & \cup \{\widetilde{a} \leftarrow \sim a \mid r \in P \text{ and } h(r) = \sim a\}\end{aligned}$$

- A set $X$ of atoms is a stable model of a program $P$ (with default negation in rule heads) over $\mathcal{A}$,
  if $X = Y \cap \mathcal{A}$ for some stable model $Y$ of $\widetilde{P}$ over $\mathcal{A} \cup \widetilde{\mathcal{A}}$

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# Disjunctive logic programs

- A disjunctive rule, $r$, is of the form

$$a_1 \; ; \ldots \; ; a_m \leftarrow a_{m+1}, \ldots, a_n, {\sim}a_{n+1}, \ldots, {\sim}a_o$$

  where $0 \leq m \leq n \leq o$ and each $a_i$ is an atom for $0 \leq i \leq o$
- A disjunctive logic program is a finite set of disjunctive rules
- Notation

$$\begin{aligned} H(r) &= \{a_1, \ldots, a_m\} \\ B(r) &= \{a_{m+1}, \ldots, a_n, {\sim}a_{n+1}, \ldots, {\sim}a_o\} \\ B(r)^+ &= \{a_{m+1}, \ldots, a_n\} \\ B(r)^- &= \{a_{n+1}, \ldots, a_o\} \\ A(P) &= \bigcup_{r \in P} \left(H(r) \cup B(r)^+ \cup B(r)^-\right) \\ B(P) &= \{B(r) \mid r \in P\} \end{aligned}$$

- A program is called positive if $B(r)^- = \emptyset$ for all its rules

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# Stable models

- Positive programs
  - A set $X$ of atoms is closed under a positive program $P$ iff for any $r \in P$, $H(r) \cap X \neq \emptyset$ whenever $B(r)^+ \subseteq X$
    - $X$ corresponds to a model of $P$ (seen as a formula)
  - The set of all $\subseteq$-minimal sets of atoms being closed under a positive program $P$ is denoted by $\min_{\subseteq}(P)$
    - $\min_{\subseteq}(P)$ corresponds to the $\subseteq$-minimal models of $P$ (ditto)
- Disjunctive programs

  The reduct, $P^X$, of a disjunctive program $P$ relative to a set $X$ of atoms is defined by

  $$P^X = \{H(r) \leftarrow B(r)^+ \mid r \in P \text{ and } B(r)^- \cap X = \emptyset\}$$

  A set $X$ of atoms is a stable model of a disjunctive program $P$, if $X \in \min_{\subseteq}(P^X)$

# Stable models

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$$P^X = \{H(r) \leftarrow B(r)^+ \mid r \in P \text{ and } B(r)^- \cap X = \emptyset\}$$

  - A set $X$ of atoms is a stable model of a disjunctive program $P$, if $X \in \min_{\subseteq}(P^X)$

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# Stable models

- Positive programs
  - A set $X$ of atoms is closed under a positive program $P$ iff for any $r \in P$, $H(r) \cap X \neq \emptyset$ whenever $B(r)^+ \subseteq X$
    - $X$ corresponds to a model of $P$ (seen as a formula)
  - The set of all $\subseteq$-minimal sets of atoms being closed under a positive program $P$ is denoted by $\min_{\subseteq}(P)$
    - $\min_{\subseteq}(P)$ corresponds to the $\subseteq$-minimal models of $P$ (ditto)
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$$P = \left\{ \begin{array}{lcl} a & \leftarrow & \\ b\,;c & \leftarrow & a \end{array} \right\}$$

- The sets $\{a, b\}$, $\{a, c\}$, and $\{a, b, c\}$ are closed under $P$
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# Graph coloring (reloaded)

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node(1..6).

edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).


assign(X,r) ; assign(X,b) ; assign(X,g) :- node(X).


:- edge(X,Y), assign(X,C), assign(Y,C).
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assign(X,C) : color(C) :- node(X).

:- edge(X,Y), assign(X,C), assign(Y,C).
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# More Examples

- $P_1 = \{a\,;\,b\,;\,c \leftarrow\}$
  - stable models $\{a\}$, $\{b\}$, and $\{c\}$
- $P_2 = \{a\,;\,b\,;\,c \leftarrow\ ,\ \leftarrow a\}$

  stable models $\{b\}$ and $\{c\}$

  $P_3 = \{a\,;\,b\,;\,c \leftarrow\ ,\ \leftarrow a\ ,\ b \leftarrow c\ ,\ c \leftarrow b\}$

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  $P_4 = \{a\,;\,b \leftarrow c\ ,\ b \leftarrow {\sim}a, {\sim}c\ ,\ a\,;\,c \leftarrow {\sim}b\}$

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# Some properties

- A disjunctive logic program may have zero, one, or multiple stable models
- If $X$ is a stable model of a disjunctive logic program $P$, then $X$ is a model of $P$ (seen as a formula)
- If $X$ and $Y$ are stable models of a disjunctive logic program $P$, then $X \not\subseteq Y$

- If $a \in X$ for some stable model $X$ of a disjunctive logic program $P$, then there is a rule $r \in P$ such that $B(r)^+ \subseteq X$, $B(r)^- \cap X = \emptyset$, and $H(r) \cap X = \{a\}$

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# An example with variables

$$P = \left\{ \begin{array}{lcl} a(1,2) & \leftarrow & \\ b(X)\,;c(Y) & \leftarrow & a(X,Y), {\sim}c(Y) \end{array} \right\}$$

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For every stable model $X$ of $P$, we have

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# Default negation in rule heads

- Consider disjunctive rules of the form

$$a_1 \mid \ldots \mid a_m \mid \sim a_{m+1} \mid \ldots \mid \sim a_n \leftarrow a_{n+1}, \ldots, a_o, \sim a_{o+1}, \ldots, \sim a_p$$

  where $0 \leq m \leq n \leq o \leq p$ and each $a_i$ is an atom for $0 \leq i \leq p$

- Given a program $P$ over $\mathcal{A}$, consider the program

$$\begin{aligned}\widetilde{P} = & \{H(r)^+ \leftarrow B(r) \cup \{\sim\widetilde{a} \mid a \in H(r)^-\} \mid r \in P\} \\ & \cup \{\widetilde{a} \leftarrow \sim a \mid r \in P \text{ and } a \in H(r)^-\}\end{aligned}$$

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# An example

- The program

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yields

$$\widetilde{P} = \{a \leftarrow \sim\widetilde{a}\} \cup \{\widetilde{a} \leftarrow \sim a\}$$

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# Outline



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# Propositional theories

- Formulas are formed from
  - atoms in $\mathcal{A}$
  - $\bot$

  using
  - conjunction ($\wedge$)
  - disjunction ($\vee$)
  - implication ($\rightarrow$)
- Notation

$$\begin{aligned} \top &= (\bot \rightarrow \bot) \\ \sim\phi &= (\phi \rightarrow \bot) \end{aligned}$$

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# Reduct

- The satisfaction relation $X \models \phi$ between a set $X$ of atoms and a (set of) formula(s) $\phi$ is defined as in propositional logic
- The reduct, $\phi^X$, of a formula $\phi$ relative to a set $X$ of atoms is defined recursively as follows:

$$\begin{array}{ll} \phi^X = \bot & \text{if } X \not\models \phi \\ \phi^X = \phi & \text{if } \phi \in X \\ \phi^X = (\psi^X \circ \varphi^X) & \text{if } X \models \phi \text{ and } \phi = (\psi \circ \varphi) \text{ for } \circ \in \{\wedge, \vee, \rightarrow\} \end{array}$$

If $\phi = \sim\psi = (\psi \rightarrow \bot)$,
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Potassco

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# Stable models

- A set $X$ of atoms satisfies a propositional theory $\Phi$, written $X \models \Phi$, if $X \models \phi$ for each $\phi \in \Phi$
- The set of all $\subseteq$-minimal sets of atoms satisfying a propositional theory $\Phi$ is denoted by $\min_{\subseteq}(\Phi)$
- A set $X$ of atoms is a stable model of a propositional theory $\Phi$, if $X \in \min_{\subseteq}(\Phi^X)$
- If $X$ is a stable model of $\Phi$, then
  - $X \models \Phi$ and
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# Two examples

- $\Phi_1 = \{p \vee (p \rightarrow (q \wedge r))\}$
  - For $X = \{p, q, r\}$, we get
    $\Phi_1^{\{p,q,r\}} = \{p \vee (p \rightarrow (q \wedge r))\}$ and $\min_{\subseteq}(\Phi_1^{\{p,q,r\}}) = \{\emptyset\}$
    For $X = \emptyset$, we get
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# Relationship to logic programs

- The translation, $\tau[(\phi \leftarrow \psi)]$, of a rule $(\phi \leftarrow \psi)$ is defined as follows:
  - $\tau[(\phi \leftarrow \psi)] = (\tau[\psi] \rightarrow \tau[\phi])$
  - $\tau[\bot] = \bot$
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The translation of a logic program $P$ is $\tau[P] = \{\tau[r] \mid r \in P\}$

Given a logic program $P$ and a set $X$ of atoms,
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- Given a logic program $P$ and a set $X$ of atoms,
  $X$ is a stable model of $P$ iff $X$ is a stable model of $\tau[P]$

# Logic programs as propositional theories

- The normal logic program $P = \{p \leftarrow \sim q,\ q \leftarrow \sim p\}$ corresponds to $\tau[P] = \{\sim q \rightarrow p,\ \sim p \rightarrow q\}$
  - stable models: $\{p\}$ and $\{q\}$
- The disjunctive logic program $P = \{p\,;q \leftarrow\}$ corresponds to $\tau[P] = \{\top \rightarrow p \vee q\}$
  - stable models: $\{p\}$ and $\{q\}$
- The nested logic program $P = \{p \leftarrow \sim\sim p\}$ corresponds to $\tau[P] = \{\sim\sim p \rightarrow p\}$
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# Outline



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# Motivation

- Aggregates provide a general way to obtain a single value from a collection of input values
- Popular aggregate (functions)
  - average
  - count
  - maximum
  - minimum
  - sum
- Cardinality and weight constraints rely on count and sum aggregates

# Syntax

- An aggregate has the form:

$$\alpha\ \{w_1 : a_1, \ldots, w_m : a_m, w_{m+1} : {\sim}a_{m+1}, \ldots, w_n : {\sim}a_n\} \prec k$$

  where for $1 \leq i \leq n$
  - $\alpha$ stands for a function mapping multisets over $\mathbb{Z}$ to $\mathbb{Z} \cup \{+\infty, -\infty\}$
  - $\prec$ stands for a relation between $\mathbb{Z} \cup \{+\infty, -\infty\}$ and $\mathbb{Z}$
  - $k \in \mathbb{Z}$
  - $a_i$ are atoms and
  - $w_i$ are integers
- Example $\mathit{sum}\ \{30 : \mathit{hd}(a), \ldots, 50 : \mathit{hd}(m)\} \leq 300$

## Semantics

- A (positive) aggregate $\alpha\,\{w_1 : a_1, \ldots, w_n : a_n\} \prec k$ can be represented by the formula:

$$\bigwedge_{I \subseteq \{1,\ldots,n\}, \alpha\{w_i \mid i \in I\} \not\prec k} \left( \bigwedge_{i \in I} a_i \rightarrow \bigvee_{i \in \bar{I}} a_i \right)$$

  where $\bar{I} = \{1, \ldots, n\} \setminus I$ and $\not\prec$ is the complement of $\prec$
- Then, $\alpha\,\{w_1 : a_1, \ldots, w_n : a_n\} \prec k$ is true in $X$ iff the above formula is true in $X$

# Example

- Consider $sum\{1 : p, 1 : q\} \neq 1$
  That is, $a_1 = p$, $a_2 = q$ and $w_1 = 1$, $w_2 = 1$
- Calculemus!

| $I$ | $\{w_i \mid i \in I\}$ | $\sum\{w_i \mid i \in I\}$ | $\sum\{w_i \mid i \in I\} = 1$ |
|---|---|---|---|
| $\emptyset$ | $\{\}$ | 0 | *false* |
| $\{1\}$ | $\{1\}$ | 1 | *true* |
| $\{2\}$ | $\{1\}$ | 1 | *true* |
| $\{1, 2\}$ | $\{1, 1\}$ | 2 | *false* |

- We get $(p \rightarrow q) \wedge (q \rightarrow p)$
- Analogously, we obtain $(p \vee q) \wedge \neg(p \wedge q)$ for $sum\{1 : p, 1 : q\} = 1$

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# Monotonicity

- Monotone aggregates
  - For instance,
    - $B(r)^+$
    - $sum\{1:p, 1:q\} > 1$ amounts to $p \wedge q$
  - We get a simpler characterization: $\bigwedge_{I \subseteq \{1,\ldots,n\}, \alpha\{w_i \mid i \in I\} \not> k} \bigvee_{i \in \bar{I}} a_i$
- Anti-monotone aggregates
  - For instance,
    - $B(r)^-$
    - $sum\{1:p, 1:q\} < 1$ amounts to $\neg p \wedge \neg q$
  - We get a simpler characterization: $\bigwedge_{I \subseteq \{1,\ldots,n\}, \alpha\{w_i \mid i \in I\} \not< k} \neg \bigwedge_{i \in I} a_i$
- Non-monotone aggregates
  - For instance, $sum\{1:p, 1:q\} \neq 1$ is non-monotone.

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# Outline



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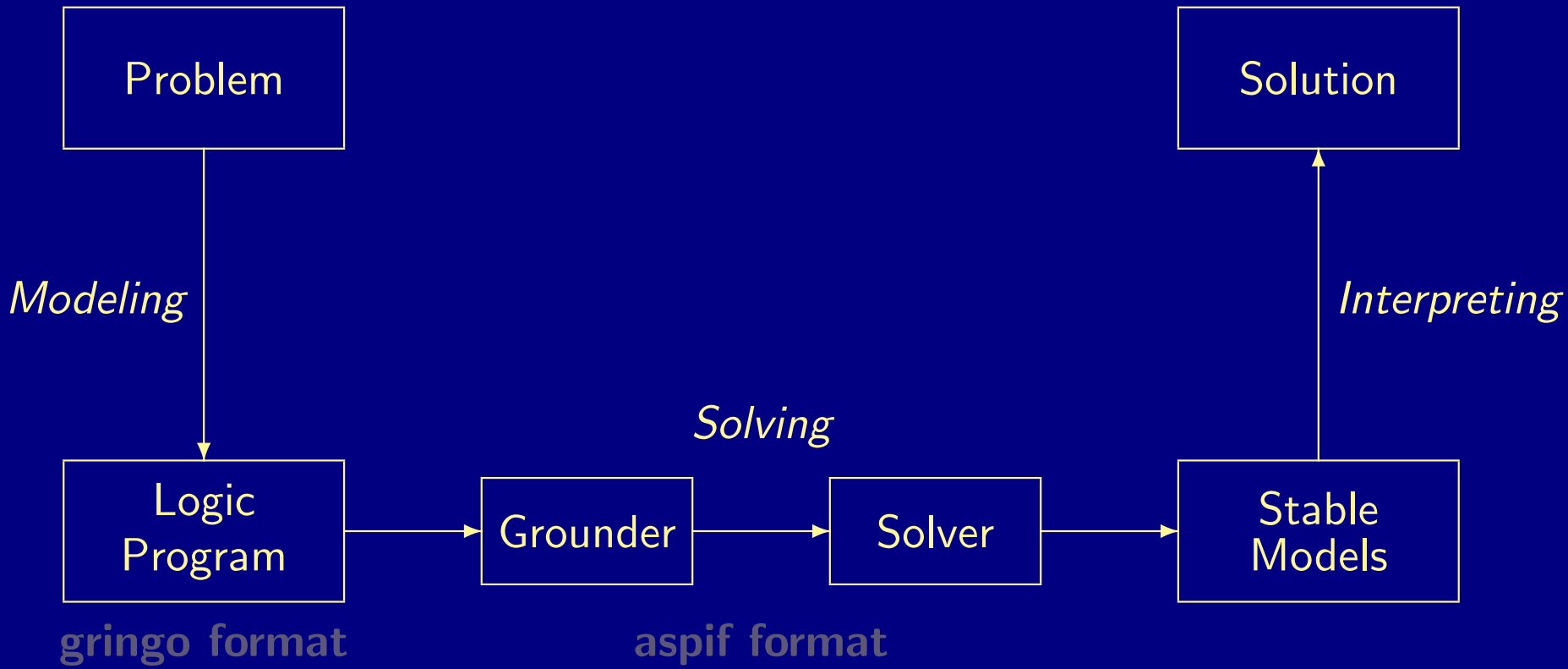
# Gringo language

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- gringo format is a user-oriented language for (non-ground) programs extending the ASP language standard *ASP-Core-2*

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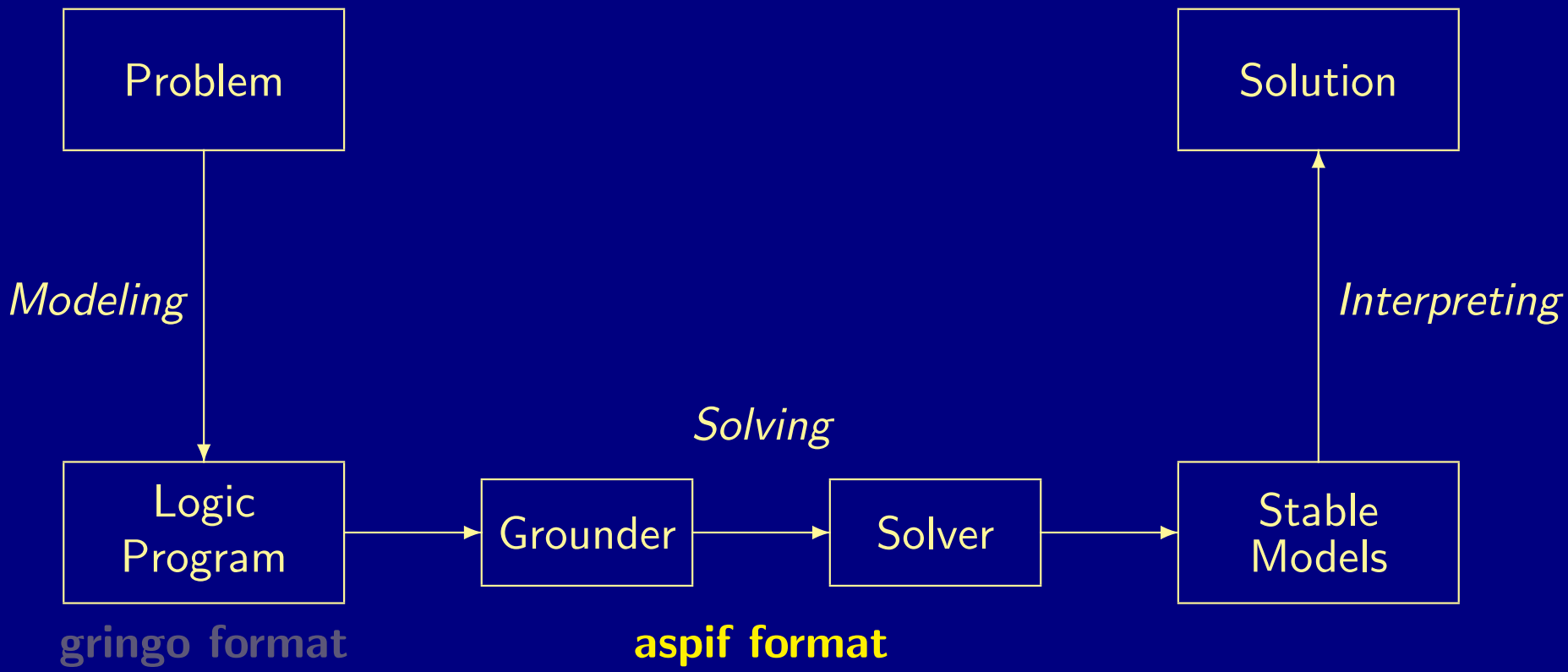
# Gringo language

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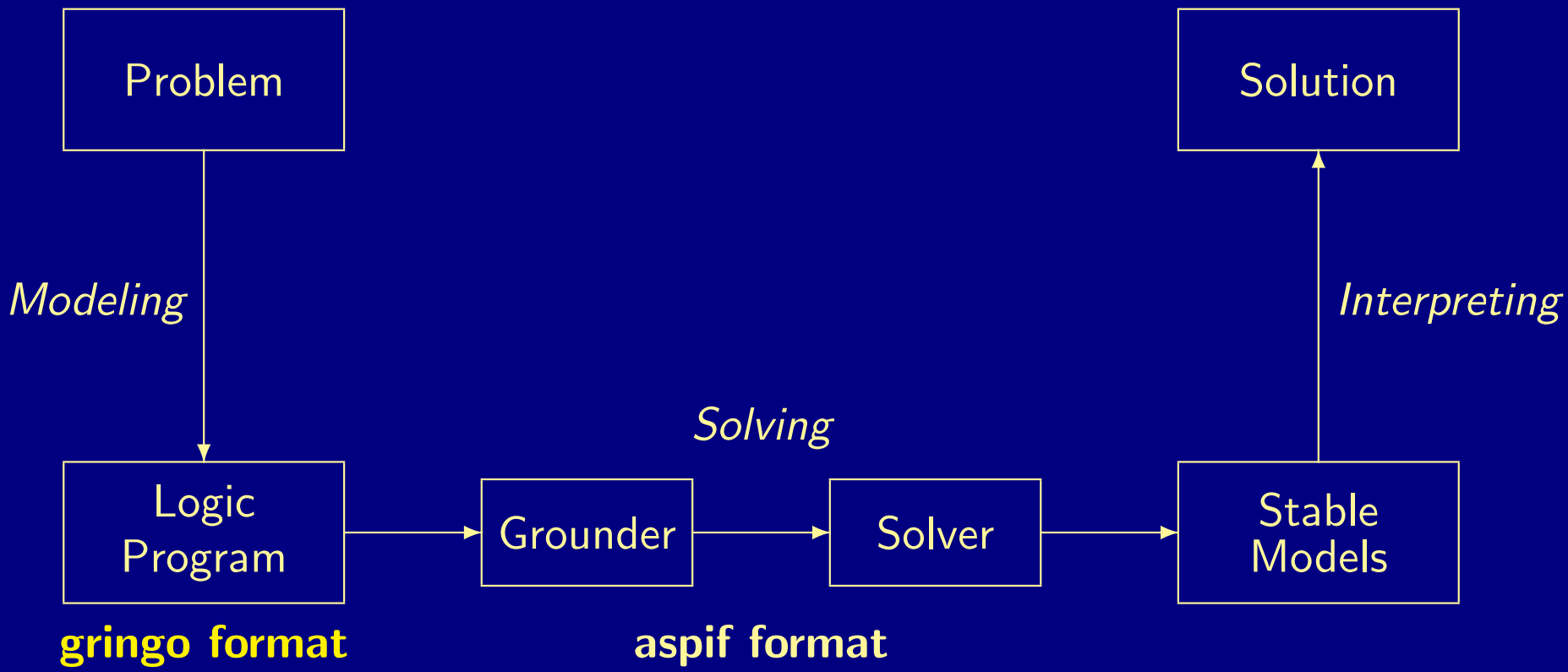


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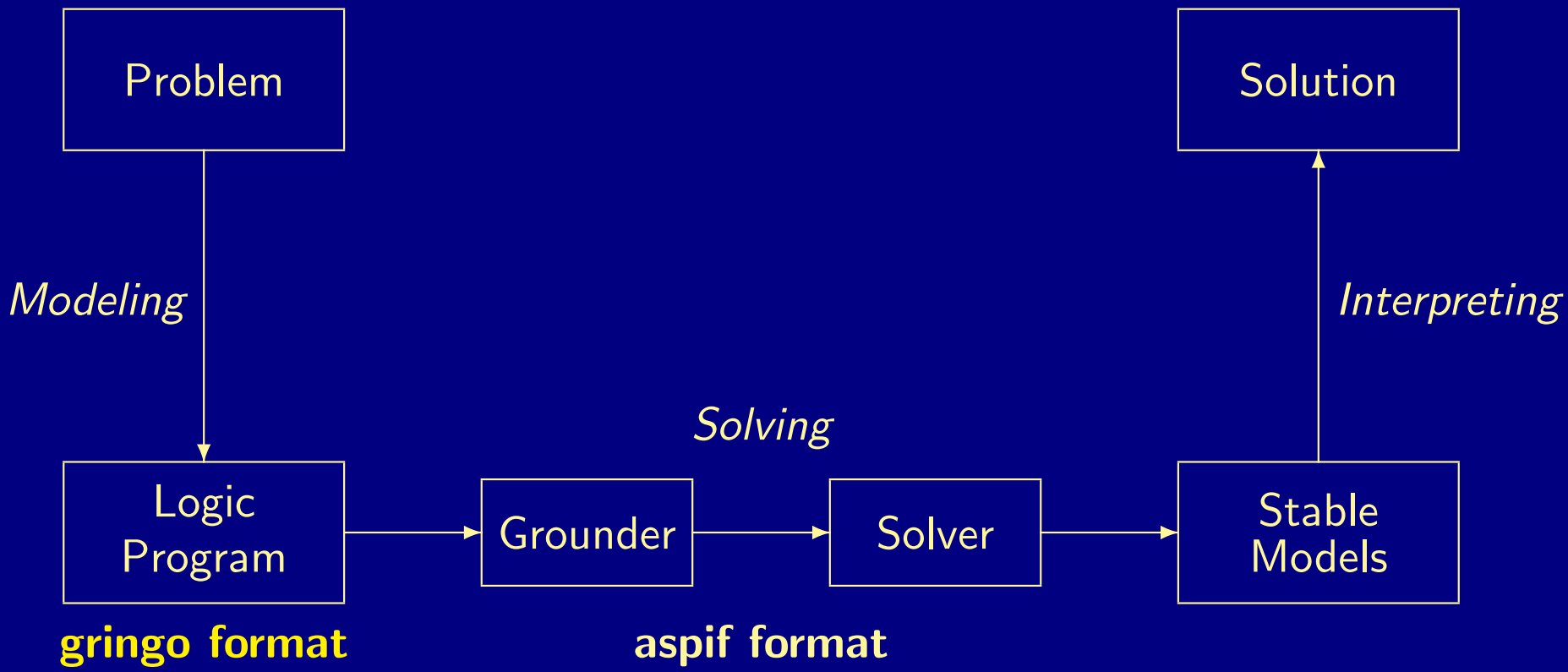


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# Terms and literals

- Terms $t$
- Tuples $\boldsymbol{t}$
- Atoms $a$, $\neg a$
- Symbolic literals $a$, $\sim a$, $\sim\sim a$
- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
- Aggregate literals $a$, $\sim a$, $\sim\sim a$
- Literals

Potassco

# Terms and literals

- Terms $t$ are formed from
  - constant symbols, eg `c`, `d`, ...
  - function symbols, eg `f`, `g`, ...
  - numeric symbols, eg 1, 2, ...
  - variable symbols, eg `X`, `Y`, ..., `_`
  - parentheses (, )
  - tuple delimiters $\langle$, $\rangle$ (omitted whenever possible)
- Tuples $\boldsymbol{t}$
- Atoms $a, \neg a$
- Symbolic literals $a, \sim a, \sim\sim a$
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  - variables, eg `X`, `Y`, ..., `_`
  - parentheses (, )
  - tuple delimiters $\langle$, $\rangle$

  eg `f(3,c,Z)`, `g(42,_,_)`, or `f((3,c),X)`
- Tuples $\boldsymbol{t}$
- Atoms $a$, $\neg a$
- Symbolic literals $a$, $\sim a$, $\sim\sim a$
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# Terms and literals

- Terms $t$
- Tuples $\boldsymbol{t}$ of terms
- Atoms $a, \neg a$
- Symbolic literals $a, \sim a, \sim\sim a$
- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
- Aggregate literals $a, \sim a, \sim\sim a$
- Literals

# Terms and literals

- Terms $t$
- Tuples $\boldsymbol{t}$
- (Negated) Atoms $a$, $\neg a$ are formed from
  - predicate symbols, eg $\mathtt{p}$, $\mathtt{q}$, ...
  - parentheses (, )
  - tuples of terms
- Symbolic literals $a$, $\sim a$, $\sim\sim a$
- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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- Terms $t$
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- Atoms $a$, $\neg a$ are formed from
  - predicates, eg p, q, ...
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- Atoms $a$, $\neg a$ are formed from
  - predicates, eg `p`, `q`, ...
  - parentheses (, )
  - tuples of terms

  eg `-p(f(3,c,Z),g(42,_,_))` or `q()` written as `q`
- Symbolic literals $a$, $\sim a$, $\sim\sim a$
- Arithmetic literals $t_1 \prec t_2$
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# Terms and literals

- Terms $t$
- Tuples $\boldsymbol{t}$
- Atoms $a$, $\neg a$, $\bot$, $\top$
  viz `#false` and `#true`
- Symbolic literals $a$, $\sim a$, $\sim\sim a$
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- Atoms $a$, $\neg a$, $\bot$, $\top$
- Symbolic literals $a$, $\sim a$, $\sim\sim a$
  eg `p(a,X)`, 'not `p(a,X)`', 'not not `p(a,X)`'
- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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- Symbolic literals $a$, $\sim a$, $\sim\sim a$
- Arithmetic literals $t_1 \prec t_2$ where
  - $t_1$ and $t_2$ are terms
  - $\prec$ is a comparison symbol
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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  eg `3<1` or `f(42)=X`
- Conditional literals $l : \boldsymbol{L}$
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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- Literals

# Terms and literals

- Terms $t$
- Tuples $\boldsymbol{t}$, $\boldsymbol{L}$ of literals
- Atoms $a$, $\neg a$, $\bot$, $\top$
- Symbolic literals $a$, $\sim a$, $\sim\sim a$
- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$ where
  - $l$ is a symbolic or arithmetic literal
  - $\boldsymbol{L}$ is a tuple of symbol or arithmetic literals
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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- Arithmetic literals $t_1 \prec t_2$
- Conditional literals $l : \boldsymbol{L}$ where
  - $l$ is a symbolic or arithmetic literal
  - $\boldsymbol{L}$ is a tuple of symbol or arithmetic literals
  - $l : \boldsymbol{L}$ is written as $l$ whenever $\boldsymbol{L}$ is empty
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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  eg ‘`p(X,Y):q(X),r(Y)`’ or `p(42)` or ‘`#false:q`’
- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$
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  - $\alpha$ is an aggregate name
  - $\boldsymbol{t}_1 : \boldsymbol{L}_1, \ldots, \boldsymbol{t}_n : \boldsymbol{L}_n$ are conditional literals
  - $\prec_1$ and $\prec_2$ are comparison symbols
  - $s_1$ and $s_2$ are terms
- Aggregate literals $a$, $\sim a$, $\sim\sim a$
- Literals

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- Terms $t$
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  - $\prec_1$ and $\prec_2$ are comparison symbols
  - $s_1$ and $s_2$ are terms
  - one (or both) of '$s_1 \prec_1$' and '$\prec_2 s_2$' can be omitted
- Aggregate literals $a$, $\sim a$, $\sim\sim a$
- Literals

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# Terms and literals

- Terms $t$
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- Aggregate atoms $s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_n : \boldsymbol{L}_n\} \prec_2 s_2$ where
  - $\alpha$ is an aggregate name
  - $\boldsymbol{t}_1 : \boldsymbol{L}_1, \ldots, \boldsymbol{t}_n : \boldsymbol{L}_n$ are conditional literals
  - $\prec_1$ and $\prec_2$ are comparison symbols
  - $s_1$ and $s_2$ are terms
  - omitting $\prec_1$ or $\prec_2$ defaults to $\leq$
- Aggregate literals $a$, $\sim a$, $\sim\sim a$
- Literals are conditional or aggregate literals

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# Terms and literals

- Terms $t$
- Tuples $\boldsymbol{t}$, $\boldsymbol{L}$
- Atoms $a$, $\neg a$, $\bot$, $\top$
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  eg `10 <= #sum {6,C:course(C); 3,S:seminar(S)} <= 20`
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  eg `not 10 #sum {6,C:course(C); 3,S:seminar(S)} 20`
- Literals are conditional or aggregate literals

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- For a detailed account please consult the user's guide!

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# Rules

- Rules are of the form

$$l_1 \,;\, \ldots \,;\, l_m \leftarrow l_{m+1}, \ldots, l_n \tag{2}$$

  where
  - $l_i$ is a conditional literal for $1 \leq i \leq m$ and
  - $l_i$ is a literal for $m+1 \leq i \leq n$

- Note Semicolons ';' must be used in (2) instead of commas ',' whenever some $l_i$ is a (genuine) conditional literal for $1 \leq i \leq n$
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# Shortcuts

- A rule of the form

$$s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : l_1 : \boldsymbol{L}_1; \ldots; \boldsymbol{t}_k : l_k : \boldsymbol{L}_k\} \prec_2 s_2 \leftarrow l_{m+1}, \ldots, l_n$$

  where
  - $\alpha$, $\prec_i$, $s_i$, $\boldsymbol{t}_j$ are as given above for $i = 1, 2$ and $1 \leq j \leq k$
  - $l_j : \boldsymbol{L}_j$ is a conditional literal for $1 \leq j \leq k$
  - $l_i$ is a literal for $m + 1 \leq i \leq n$ (as in (2))

  is a shorthand for the following $k + 1$ rules

$$\{l_j\} \leftarrow l_{m+1}, \ldots, l_n, \boldsymbol{L}_j \qquad \text{for } 1 \leq j \leq k$$
$$\leftarrow l_{m+1}, \ldots, l_n, \sim s_1 \prec_1 \alpha\{\boldsymbol{t}_1 : l_1, \boldsymbol{L}_1; \ldots; \boldsymbol{t}_k : l_k, \boldsymbol{L}_k\} \prec_2 s_2$$

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- `{a; b}`

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$ gringo --text <(echo "{a;b}.")
#count{1,0,a:a;1,0,b:b}.
```

*gringo* generates two distinct term tuples `1,0,a` and `1,0,b`

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1 = { q(X,Y): p(X), p(Y), X < Y; q(X,X): p(X) }
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# Weak constraints

- Syntax A weak constraint is of the form

  $$:\sim l_1, \ldots, l_n. [w@p, t_1, \ldots, t_m]$$

  where
  - $l_1, \ldots, l_n$ are literals
  - $t_1, \ldots, t_m$, $w$, and $p$ are terms
- $w$ and $p$ stand for a weight and priority level ($p = 0$ if '$@p$' is omitted)
- Example The weak constraint

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:~ hd(I,P,C). [C@2,I]
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amounts to the minimize statement

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# Some more directives

- Output

  `#show.` `#show` $p/n.$ `#show` $t$ : $l_1, \ldots, l_n.$

- Projection

  `#project` $p/n.$ `#project` $a$ : $l_1, \ldots, l_n.$

- Heuristics

  `#heuristic` $a$ : $l_1, \ldots, l_n.$ $[k@p, m]$

- Acyclicity

  `#edge` $(u, v)$ : $l_1, \ldots, l_n.$

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# *gringo* 3 versus 4/5

- The input language of *gringo* series 4/5 comprises
  - ASP-Core-2
  - concepts from *lparse* and *gringo* 3
- Example The *gringo* 3 rule
  - ```
    r(X) : p(X) : not q(X) :- r(X) : p(X) : not q(X),
          1 { r(X) : p(X) : not q(X) }.
    ```
    can be written as follows in the language of *gringo* 4/5:
  - ```
    r(X) : p(X), not q(X) :- r(X) : p(X), not q(X);
    ```
- Note Directives `#compute`, `#domain`, and `#hide` are discontinued
- Attention
  - The languages of *gringo* 3 and 4/5 are not fully compatible
  - Many example programs in the literature are written for *gringo* 3

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