Answer Set Solving in Practice

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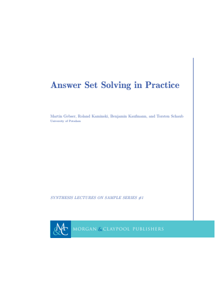
Rough Roadmap

Resources

- Course material
  - http://www.cs.uni-potsdam.de/wv/lehre
  - http://moodle.cs.uni-potsdam.de
  - http://potassco.sourceforge.net/teaching.html
- Systems
  - clasp http://potassco.sourceforge.net
  - dlv http://www.dlvsystem.com
  - smodels http://www.tcs.hut.fi/Software/smodels
  - gringo http://potassco.sourceforge.net
  - lparse http://www.tcs.hut.fi/Software/smodels
  - clingo http://potassco.sourceforge.net
  - iclingo http://potassco.sourceforge.net
  - oclingo http://potassco.sourceforge.net
  - asparagus http://asparagus.cs.uni-potsdam.de

The Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



Resources

- `http://potassco.sourceforge.net/book.html`
- `http://potassco.sourceforge.net/teaching.html`

Literature

Books	[4], [29], [53]
Surveys	[50], [2], [39], [21], [11]
Articles	[41], [42], [6], [61], [54], [49], [40], etc.

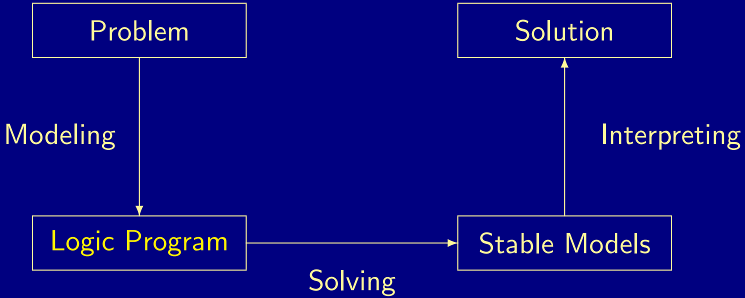
Introduction: Overview

Potassco

Outline

Potassco

Problem solving in ASP: Syntax



Normal logic programs

- A logic program, P, over a set $\mathcal{A}$ of atoms is a finite set of rules
- A (normal) rule, r, is of the form

$$a_0 \leftarrow a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

$$\begin{aligned} head(r) &= a_0 \\ body(r) &= \{a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n\} \\ body(r)^+ &= \{a_1, \ldots, a_m\} \\ body(r)^- &= \{a_{m+1}, \ldots, a_n\} \\ atom(P) &= \bigcup_{r \in P} \left(\{head(r)\} \cup body(r)^+ \cup body(r)^-\right) \\ body(P) &= \{body(r) \mid r \in P\} \end{aligned}$$

A program P is positive if $body(r)^- = \emptyset$ for all $r \in P$

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- Notation

$$\begin{array}{rcl} head(r) & = & a_0 \\ body(r) & = & \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\} \\ body(r)^+ & = & \{a_1, \dots, a_m\} \\ body(r)^- & = & \{a_{m+1}, \dots, a_n\} \\ atom(P) & = & \bigcup_{r \in P} \left(\{head(r)\} \cup body(r)^+ \cup body(r)^-\right) \\ body(P) & = & \{body(r) \mid r \in P\} \end{array}$$

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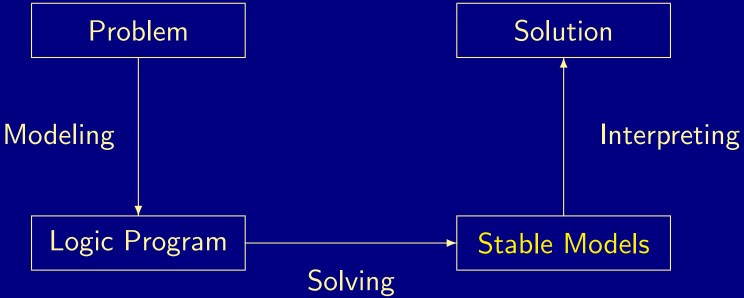
Rough notational convention

We sometimes use the following notation interchangeably in order to stress the respective view:

	true, false	if	and	or	iff	default negation	classical negation
source code		`:-`	`,`	`|`		`not`	`-`
logic program		$\leftarrow$	$,$	$;$		$\sim$	$\neg$
formula	$\bot, \top$	$\rightarrow$	$\wedge$	$\vee$	$\leftrightarrow$	$\sim$	$\neg$

Outline

Problem solving in ASP: Semantics



Formal Definition

Stable models of positive programs

- A set of atoms X is closed under a positive program P iff for any $r \in P$, $head(r) \in X$ whenever $body(r)^+ \subseteq X$
  - X corresponds to a model of P (seen as a formula)
- The smallest set of atoms which is closed under a positive program P is denoted by $Cn(P)$
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of P (ditto)
- The set $Cn(P)$ of atoms is the stable model of a *positive* program P

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Some "logical" remarks

- Positive rules are also referred to as definite clauses
  - Definite clauses are disjunctions with exactly one positive atom:

    $$a_0 \vee \neg a_1 \vee \cdots \vee \neg a_m$$

  - A set of definite clauses has a (unique) smallest model
- Horn clauses are clauses with at most one positive atom
  - Every definite clause is a Horn clause but not vice versa
  - Non-definite Horn clauses can be regarded as integrity constraints
  - A set of Horn clauses has a smallest model or none
- This smallest model is the intended semantics of such sets of clauses
  - Given a positive program P, $Cn(P)$ corresponds to the smallest model of the set of definite clauses corresponding to P

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Basic idea

Consider the logical formula Φ and its three (classical) models:

$\{p, q\}, \{q, r\}$, and $\{p, q, r\}$

Φ $\boxed{q \ \wedge \ (q \wedge \neg r \rightarrow p)}$

Formula Φ has one stable model, often called answer set:

$\{p, q\}$

P_Φ
$$\begin{array}{lll} q & \leftarrow & \\ p & \leftarrow & q, \ \sim r \end{array}$$

Informally, a set X of atoms is a stable model of a logic program P

- if X is a (classical) model of P and
- if all atoms in X are justified by some rule in P

(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))

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p	$\mapsto$	1
q	$\mapsto$	1
r	$\mapsto$	0

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q	$\leftarrow$	
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Stable model of normal programs

- The reduct, P^X, of a program P relative to a set X of atoms is defined by

$$P^X = \{head(r) \leftarrow body(r)^+ \mid r \in P \text{ and } body(r)^- \cap X = \emptyset\}$$

- A set X of atoms is a stable model of a program P, if $Cn(P^X) = X$

- Note $Cn(P^X)$ is the $\subseteq$–smallest (classical) model of P^X
- Note Every atom in X is justified by an *"applying rule from P"*

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A closer look at P^X

- In other words, given a set X of atoms from P,

  P^X is obtained from P by deleting
  1. each rule having $\sim a$ in its body with $a \in X$
     and then
  2. all negative atoms of the form $\sim a$
     in the bodies of the remaining rules

- Note Only negative body literals are evaluated wrt X

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Outline

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A first example

$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p\}$	$p \leftarrow p$	$\emptyset$
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p, q\}$	$p \leftarrow p$	$\emptyset$

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$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✗
$\{p\}$	$p \leftarrow p$	$\emptyset$	✗
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✓
$\{p, q\}$	$p \leftarrow p$	$\emptyset$	✗

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$P = \{p \leftarrow p,\ q \leftarrow \neg p\}$

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$\{p\}$	$p \leftarrow p$	$\emptyset$	✔
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$\{p, q\}$	$p \leftarrow p$	$\emptyset$	✘

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A second example

$P = \{p \leftarrow \sim q,\ q \leftarrow \sim p\}$

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$\{\ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$
$\{p\}$	$p \leftarrow$	$\{p\}$
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A second example

$P = \{p \leftarrow \neg q,\ q \leftarrow \neg p\}$

X	P^X	$Cn(P^X)$	
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$\{q\}$	$q \leftarrow$	$\{q\}$	✔
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A third example

$P = \{p \leftarrow {\sim}p\}$

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Potassco

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Some properties

- A logic program may have zero, one, or multiple stable models!
- If X is a stable model of a logic program P,
  then X is a model of P (seen as a formula)
- If X and Y are stable models of a *normal* program P,
  then $X \not\subset Y$

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Programs with Variables

Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \rightarrow \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground Instantiation of P: $\quad ground(P) = \bigcup_{r \in P} ground(r)$

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Let P be a logic program

- Let $\mathcal{T}$ be a set of variable-free terms (also called Herbrand universe)
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$ (also called alphabet or Herbrand base)
- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \rightarrow \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground Instantiation of P: $ground(P) = \bigcup_{r \in P} ground(r)$

Programs with Variables

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An example

$$P = \{\ r(a,b) \leftarrow,\ \ r(b,c) \leftarrow,\ \ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a,b,c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

$$ground(P) = \left\{ \begin{array}{lllllllll} r(a,b) & \leftarrow & , & & & & & & \\ r(b,c) & \leftarrow & , & & & & & & \\ t(a,a) & \leftarrow & r(a,a), & t(b,a) & \leftarrow & r(b,a), & t(c,a) & \leftarrow & r(c,a), \\ t(a,b) & \leftarrow & r(a,b), & t(b,b) & \leftarrow & r(b,b), & t(c,b) & \leftarrow & r(c,b), \\ t(a,c) & \leftarrow & r(a,c), & t(b,c) & \leftarrow & r(b,c), & t(c,c) & \leftarrow & r(c,c) \end{array} \right\}$$

Intelligent Grounding aims at reducing the ground instantiation

Potassco

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Stable models of programs with Variables

Let P be a normal logic program with variables

- A set X of (ground) atoms is a stable model of P,
  if $Cn(ground(P)^X) = X$

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Outline

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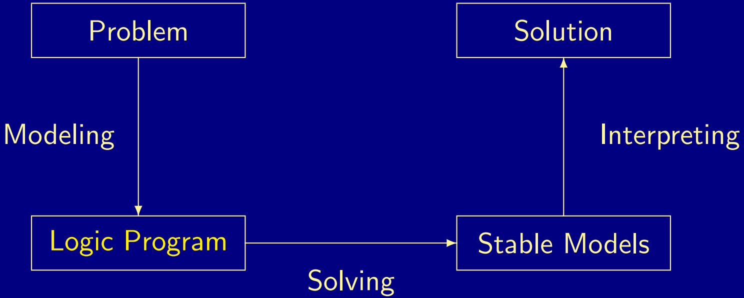
Problem solving in ASP: Extended Syntax



Language Constructs

- Variables (over the Herbrand Universe)
  - `p(X) :- q(X)` over constants $\{a, b, c\}$ stands for
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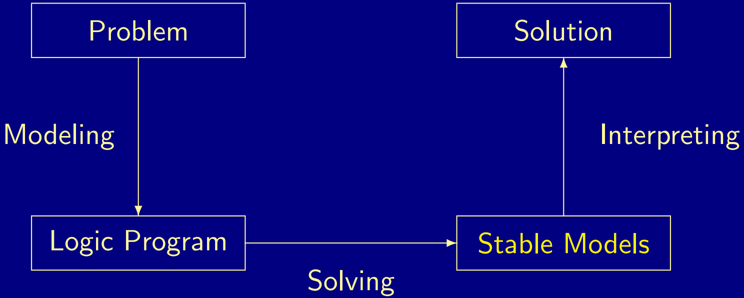
Outline

Problem solving in ASP: Reasoning Modes



Reasoning Modes

- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization
- and combinations of them

† without solution recording

‡ without solution enumeration

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