# Answer Set Solving in Practice

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# Rough Roadmap

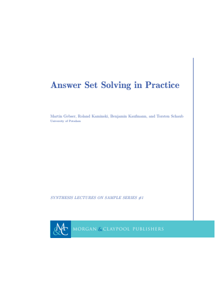

1 Introduction
2 Language
3 Modeling
4 Grounding
5 Foundations
6 Solving
7 Systems
8 Applications

# Resources

- Course material
  - `http://www.cs.uni-potsdam.de/wv/lehre`
  - `http://moodle.cs.uni-potsdam.de`
  - `http://potassco.sourceforge.net/teaching.html`
- Systems
  - clasp `http://potassco.sourceforge.net`
  - dlv `http://www.dlvsystem.com`
  - smodels `http://www.tcs.hut.fi/Software/smodels`
  - gringo `http://potassco.sourceforge.net`
  - lparse `http://www.tcs.hut.fi/Software/smodels`
  - clingo `http://potassco.sourceforge.net`
  - iclingo `http://potassco.sourceforge.net`
  - oclingo `http://potassco.sourceforge.net`
  - asparagus `http://asparagus.cs.uni-potsdam.de`

# The Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions




## Resources

- http://potassco.sourceforge.net/book.html
- http://potassco.sourceforge.net/teaching.html

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## Literature

Books [4], [29], [53]

Surveys [50], [2], [39], [21], [11]

Articles [41], [42], [6], [61], [54], [49], [40], etc.

# Language: Overview

1. Motivation
2. Core language
   - Integrity constraint
   - Choice rule
   - Cardinality rule
   - Weight rule
3. Extended language
   - Conditional literal
   - Optimization statement
4. smodels format
5. ASP language standard

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# Outline


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# Basic language extensions

- The expressiveness of a language can be enhanced by introducing new constructs
- To this end, we must address the following issues:
  - What is the syntax of the new language construct?
  - What is the semantics of the new language construct?
  - How to implement the new language construct?
- A way of providing semantics is to furnish a translation removing the new constructs, eg. classical negation
- This translation might also be used for implementing the language extension

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# Integrity constraint

- Idea Eliminate unwanted solution candidates
- Syntax An integrity constraint is of the form

$$\leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n$$

where $0 \leq m \leq n$ and each $a_i$ is an atom for $1 \leq i \leq n$

- Example `:- edge(3,7), color(3,red), color(7,red).`
- Embedding The above integrity constraint can be turned into the normal rule

$$x \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n, \sim x$$

where $x$ is a new symbol, that is, $x \notin \mathcal{A}$.

- Another example $P = \{a \leftarrow \sim b,\ b \leftarrow \sim a\}$
versus $P' = P \cup \{\leftarrow a\}$ and $P'' = P \cup \{\leftarrow \sim a\}$

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- Idea Choices over subsets
- Syntax A choice rule is of the form

$$\{a_1, \dots, a_m\} \leftarrow a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o$$

where $0 \leq m \leq n \leq o$ and each $a_i$ is an atom for $1 \leq i \leq o$

- Informal meaning If the body is satisfied by the stable model at hand, then any subset of $\{a_1, \dots, a_m\}$ can be included in the stable model
- Example `{ buy(pizza), buy(wine), buy(corn) } :- at(grocery).`
- Another Example $P = \{\{a\} \leftarrow b, \ b \leftarrow\}$ has two stable models: $\{b\}$ and $\{a, b\}$

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# Embedding in normal rules

- A choice rule of form

$$\{a_1, \dots, a_m\} \leftarrow a_{m+1}, \dots, a_n, {\sim}a_{n+1}, \dots, {\sim}a_o$$

can be translated into $2m+1$ normal rules

$$\begin{array}{lcl} a' & \leftarrow & a_{m+1}, \dots, a_n, {\sim}a_{n+1}, \dots, {\sim}a_o \\ a_1 \leftarrow a', {\sim}\overline{a_1} & \dots & a_m \leftarrow a', {\sim}\overline{a_m} \\ \overline{a_1} \leftarrow {\sim}a_1 & \dots & \overline{a_m} \leftarrow {\sim}a_m \end{array}$$

by introducing new atoms $a', \overline{a_1}, \dots, \overline{a_m}$.

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$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \}$$

where $0 \leq m \leq n$ and each $a_i$ is an atom for $1 \leq i \leq n$; $l$ is a non-negative integer.

- Informal meaning The head atom belongs to the stable model, if at least $l$ elements of the body are included in the stable model
- Note $l$ acts as a lower bound on the body
- Example `pass(c42) :- 2 { pass(a1), pass(a2), pass(a3) }.`
- Another Example $P = \{a \leftarrow 1\{b, c\}, \; b \leftarrow\}$ has stable model $\{a, b\}$

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# Embedding in normal rules

- Replace each cardinality rule

$$a_0 \leftarrow l \ \{ \ a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n \ \}$$

by $\quad a_0 \leftarrow ctr(1, l)$

where atom $ctr(i, j)$ represents the fact that at least $j$ of the literals having an equal or greater index than $i$, are in a stable model

- The definition of $ctr/2$ is given for $0 \leq k \leq l$ by the rules

$$\begin{array}{rcll} ctr(i, k{+}1) & \leftarrow & ctr(i+1, k), a_i & \\ ctr(i, k) & \leftarrow & ctr(i+1, k) & \text{for } 1 \leq i \leq m \\ ctr(j, k{+}1) & \leftarrow & ctr(j+1, k), {\sim}a_j & \\ ctr(j, k) & \leftarrow & ctr(j+1, k) & \text{for } m+1 \leq j \leq n \\ ctr(n+1, 0) & \leftarrow & & \end{array}$$

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# An example

- Program $\{a \leftarrow,\ c \leftarrow 1\ \{a, b\}\}$ has the stable model $\{a, c\}$
- Translating the cardinality rule yields the rules

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having stable model $\{a, ctr(3,0), ctr(2,0), ctr(1,0), ctr(1,1), c\}$

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## . . . and vice versa

- A normal rule

$$a_0 \leftarrow a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n,$$

can be represented by the cardinality rule

$$a_0 \leftarrow n\ \{a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n\}$$

## Cardinality rules with upper bounds

- A rule of the form

$$a_0 \leftarrow l \ \{ \ a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n \ \} \ u$$

  where $0 \leq m \leq n$ and each $a_i$ is an atom for $1 \leq i \leq n$; $l$ and $u$ are non-negative integers

  stands for

$$\begin{array}{rcl} a_0 & \leftarrow & b, \sim c \\ b & \leftarrow & l \ \{ \ a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n \ \} \\ c & \leftarrow & u{+}1 \ \{ \ a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n \ \} \end{array}$$

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- The single constraint in the body of the above cardinality rule is referred to as a cardinality constraint

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$$\begin{array}{rcl} a_0 & \leftarrow & b, {\sim}c \\ b & \leftarrow & l \; \{ \; a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n \; \} \\ c & \leftarrow & u{+}1 \; \{ \; a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n \; \} \end{array}$$

where $b$ and $c$ are new symbols

- The single constraint in the body of the above cardinality rule is referred to as a cardinality constraint

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## Cardinality constraints

- Syntax A cardinality constraint is of the form

$$l \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \} \; u$$

  where $0 \leq m \leq n$ and each $a_i$ is an atom for $1 \leq i \leq n$; $l$ and $u$ are non-negative integers

- Informal meaning A cardinality constraint is satisfied by a stable model $X$, if the number of its contained literals satisfied by $X$ is between $l$ and $u$ (inclusive)
- In other words, if

$$l \leq |\,(\{a_1, \ldots, a_m\} \cap X) \cup (\{a_{m+1}, \ldots, a_n\} \setminus X)\,| \leq u$$

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# Cardinality constraints as heads

- A rule of the form

$$l\ \{a_1,\dots,a_m,\sim a_{m+1},\dots,\sim a_n\}\ u \leftarrow a_{n+1},\dots,a_o,\sim a_{o+1},\dots,\sim a_p$$

where $0 \leq m \leq n \leq o \leq p$ and each $a_i$ is an atom for $1 \leq i \leq p$; $l$ and $u$ are non-negative integers

stands for

$$\begin{array}{rcl} b & \leftarrow & a_{n+1},\dots,a_o,\sim a_{o+1},\dots,\sim a_p \\ \{a_1,\dots,a_m\} & \leftarrow & b \\ c & \leftarrow & l\ \{a_1,\dots,a_m,,\sim a_{m+1},\dots,\sim a_n\}\ u \\ & \leftarrow & b,\sim c \end{array}$$

where $b$ and $c$ are new symbols

- Example `1 { color(v42,red),color(v42,green),color(v42,blue) } 1.`

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# Full-fledged cardinality rules

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$$l_0 \; S_0 \; u_0 \leftarrow l_1 \; S_1 \; u_1, \ldots, l_n \; S_n \; u_n$$

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stands for $0 \leq i \leq n$

$$a \leftarrow b_1, \ldots, b_n, \sim c_1, \ldots, \sim c_n$$

$$\begin{array}{lll} {S_0}^+ & \leftarrow & a \\ & \leftarrow & a, \sim b_0 \\ & \leftarrow & a, c_0 \end{array} \qquad \begin{array}{lll} b_i & \leftarrow & l_i \; S_i \\ c_i & \leftarrow & u_i{+}1 \; S_i \end{array}$$

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# Weight rule

- Syntax A weight rule is the form

$$a_0 \leftarrow l \; \{ \; a_1 = w_1, \ldots, a_m = w_m, \sim a_{m+1} = w_{m+1}, \ldots, \sim a_n = w_n \; \}$$

  where $0 \leq m \leq n$ and each $a_i$ is an atom;
  $l$ and $w_i$ are integers for $1 \leq i \leq n$
- A weighted literal, $\ell_i = w_i$, associates each literal $\ell_i$ with a weight $w_i$
- Note A cardinality rule is a weight rule where $w_i = 1$ for $0 \leq i \leq n$

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- Meaning A weight constraint is satisfied by a stable model $X$, if

$$l \leq \left( \sum_{1 \leq i \leq m, a_i \in X} w_i + \sum_{m < i \leq n, a_i \notin X} w_i \right) \leq u$$

- Note (Cardinality and) weight constraints amount to constraints on (count and) sum aggregate functions

- Example `10 [course(db)=6,course(ai)=6,course(project)=8,course(xml)=3] 20`

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# Conditional literals (in *lparse* & *gringo* 3)

- Syntax A conditional literal is of the form

$$\ell : \ell_1 : \cdots : \ell_n$$

  where $\ell$ and $\ell_i$ are literals for $0 \leq i \leq n$

- Informal meaning A conditional literal can be regarded as the list of elements in the set $\{\ell \mid \ell_1, \dots, \ell_n\}$

- Note The expansion of conditional literals is context dependent

- Example Given '`p(1). p(2). p(3). q(2).`'

```
r(X):p(X):not q(X) :- r(X):p(X):not q(X), 1 {r(X):p(X):not q(X)}.
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  is instantiated to

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# Optimization statement

- Idea Express cost functions subject to minimization and/or maximization
- Syntax A minimize statement is of the form

$$minimize\{\ \ell_1 = w_1@p_1, \dots, \ell_n = w_n@p_n\ \}.$$

where each $\ell_i$ is a literal; and $w_i$ and $p_i$ are integers for $1 \leq i \leq n$

Priority levels, $p_i$, allow for representing lexicographically ordered minimization objectives

- Meaning A minimize statement is a directive that instructs the ASP solver to compute optimal stable models by minimizing a weighted sum of elements

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# Optimization statement

- A maximize statement of the form

$$maximize\{\ \ell_1 = w_1@p_1, \dots, \ell_n = w_n@p_n\ \}$$

stands for $minimize\{\ \ell_1 = -w_1@p_1, \dots, \ell_n = -w_n@p_n\ \}$

- Example When configuring a computer, we may want to maximize hard disk capacity, while minimizing price

```
#maximize[ hd(1)=250@1, hd(2)=500@1, hd(3)=750@1, hd(4)=1000@1 ].
#minimize[ hd(1)=30@2, hd(2)=40@2, hd(3)=60@2, hd(4)=80@2 ].
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The priority levels indicate that (minimizing) price is more important than (maximizing) capacity

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# *smodels* format

- Logic programs in *smodels* format consist of
  - normal rules
  - choice rules
  - cardinality rules
  - weight rules
  - optimization statements
- Such a format is obtained by grounders *lparse* and *gringo*

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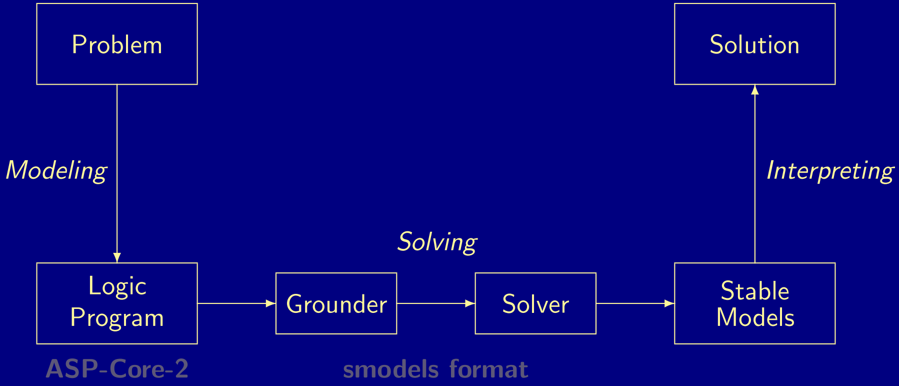
# ASP-Core-2




- smodels format is a machine-oriented standard for ground programs
- ASP-Core-2 is a user-oriented standard for (non-ground) programs, extending the input languages of *dlv* and *gringo* series 3

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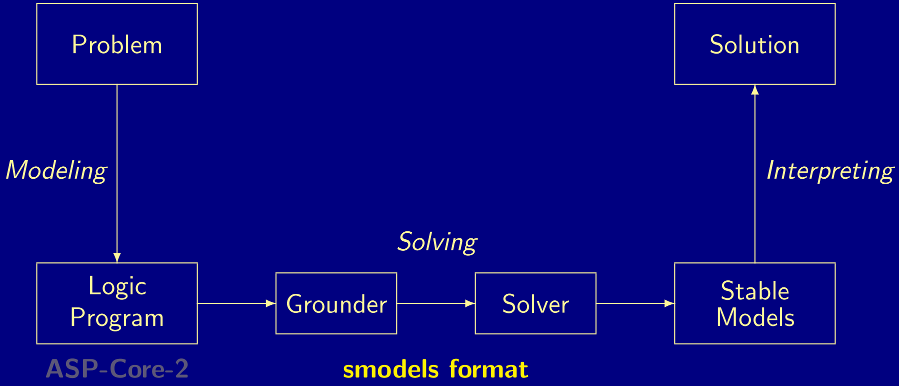


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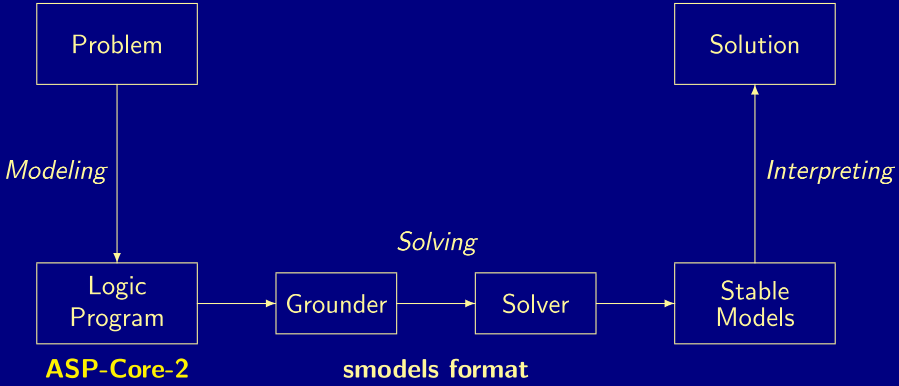


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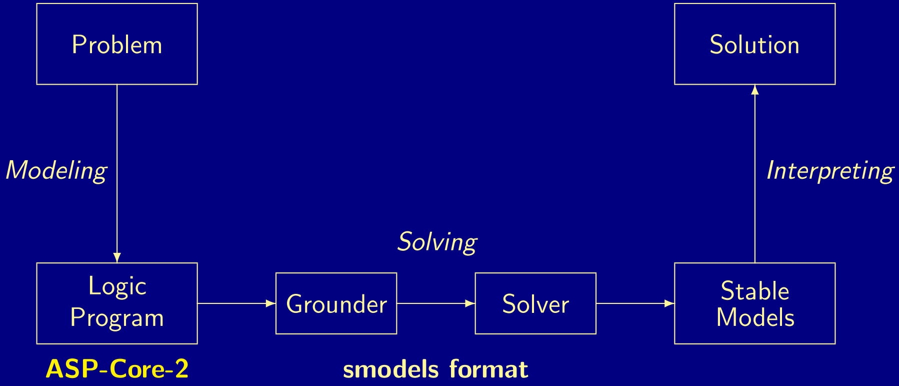
# ASP-Core-2

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# Aggregates

- Syntax ASP-Core-2 aggregates are of the form

$$t_1 \prec_1 \#\mathtt{A}\{t_{1_1},\dots,t_{m_1} : \ell_{1_1},\dots,\ell_{n_1}\} \prec_2 t_2$$

  where
  - $\#\mathtt{A} \in \{\#\mathtt{count}, \#\mathtt{sum}, \#\mathtt{max}, \#\mathtt{min}\}$
  - $\prec_1, \prec_2 \in \{<, \leq, =, \neq, >, \geq\}$
  - $t_{1_1},\dots,t_{m_1}$ and $t_1, t_2$ are terms
  - $\ell_{1_1},\dots,\ell_{n_1}$ are literals
- Example Weight constraint

```
10 [course(db)=6,course(ai)=6,course(project)=8,course(xml)=3] 20
```

  is written as an ASP-Core-2 aggregate as

```
10 ≤ #sum{6,db:course(db); 6,ai:course(ai);
             8,project:course(project); 3,xml:course(xml)} ≤ 20
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# Weak constraints

- Syntax A weak constraint is of the form

$$:\sim a_1,\dots,a_m,\sim a_{m+1},\dots,\sim a_n.\,[w@p,t_1,\dots,t_m]$$

  where
  - $a_1,\dots,a_n$ are atoms
  - $t_1,\dots,t_m$, $w$, and $p$ are terms
- $a_1,\dots,a_n$ may contain ASP-Core-2 aggregates
- $w$ and $p$ stand for a weight and priority level ($p=0$ if '@$p$' is omitted)
- Example Minimize statement

```
#minimize[ hd(1)=30@2, hd(2)=40@2, hd(3)=60@2, hd(4)=80@2 ].
```

  can be written in terms of weak constraints as

```
:~ hd(1). [30@2,1]        :~ hd(3). [60@2,3]
:~ hd(2). [40@2,2]        :~ hd(4). [80@2,4]
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# *gringo* 4

- The input language of *gringo* series 4 comprises
  - ASP-Core-2
  - concepts from *gringo* 3 (conditional literals, `#show` directives, ...)
- Example The *gringo* 3 rule

```
r(X):p(X):not q(X) :- r(X):p(X):not q(X), 1 {r(X):p(X):not q(X)}.
```

can be written as follows in the language of *gringo* 4:

```
r(X):p(X),not q(X) :- r(X):p(X),not q(X);
                      1 <= #count{X:r(X),p(X),not q(X)}.
```

Term-based `#show` directives as in

```
#show. #show hello. #show X : p(X).  1{p(earth);p(mars);p(venus)}1.
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The languages of *gringo* 3 and 4 are not fully compatible

Many example programs given in this tutorial are written for *gringo* 3

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