

# Answer Set Solving in Practice

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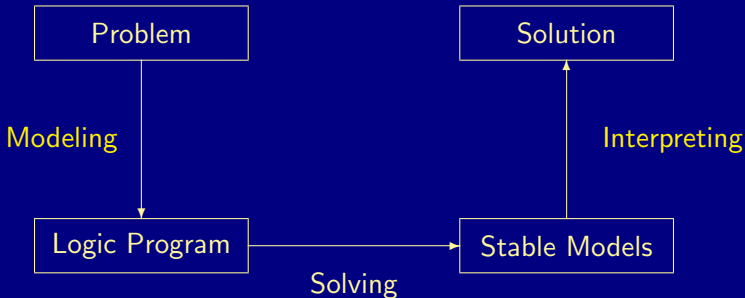


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# Basic Modeling: Overview

- 1 Elaboration tolerance
- 2 ASP solving process
- 3 Methodology
- 4 Case studies

# Modeling and Interpreting



# Outline

- 1 Elaboration tolerance
- 2 ASP solving process
- 3 Methodology
- 4 Case studies

## Guiding principle

- Elaboration Tolerance (McCarthy, 1998)

*“A formalism is **elaboration tolerant** [if] it is convenient to modify a set of facts expressed in the formalism to take into account new phenomena or changed circumstances.”*

- Uniform problem representation

For solving a problem instance  $I$  of a problem class  $C$ ,

- $I$  is represented as a set of facts  $P_I$ ,
- $C$  is represented as a set of rules  $P_C$ , and
- $P_C$  can be used to solve all problem instances in  $C$

## Guiding principle

- Elaboration Tolerance (McCarthy, 1998)

*“A formalism is elaboration tolerant [if] it is convenient to modify a set of facts expressed in the formalism to take into account new phenomena or changed circumstances.”*

- Uniform problem representation

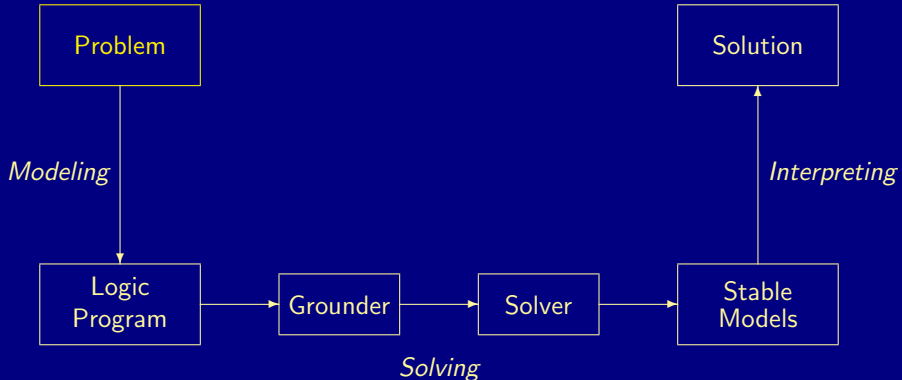
For solving a problem instance **I** of a problem class **C**,

- **I** is represented as a set of facts  $P_I$ ,
- **C** is represented as a set of rules  $P_C$ , and
- $P_C$  can be used to solve all problem instances in **C**

# Outline

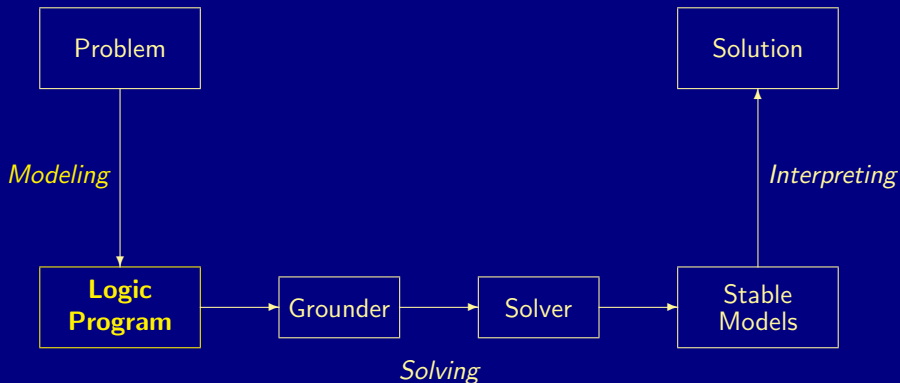
- 1 Elaboration tolerance
- 2 ASP solving process
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## ASP solving process

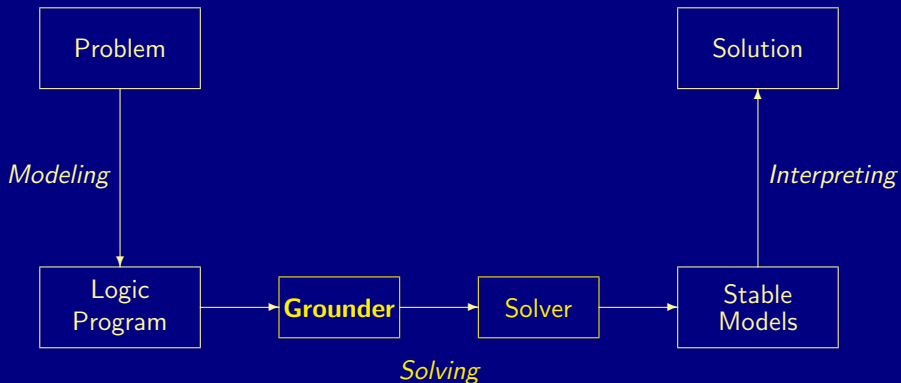




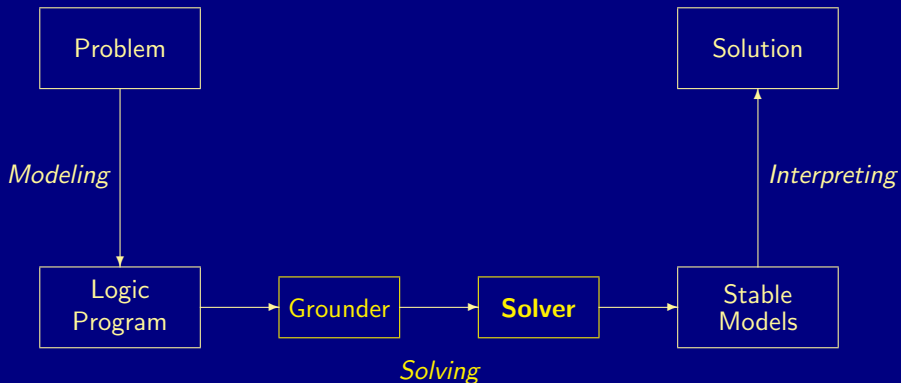
## ASP solving process



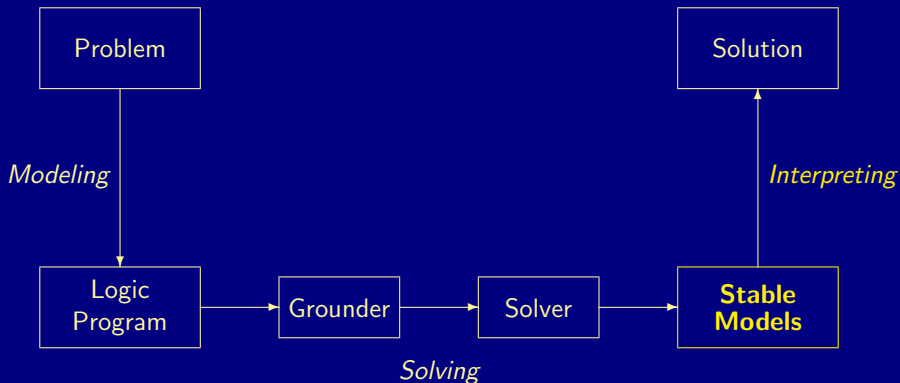
## ASP solving process



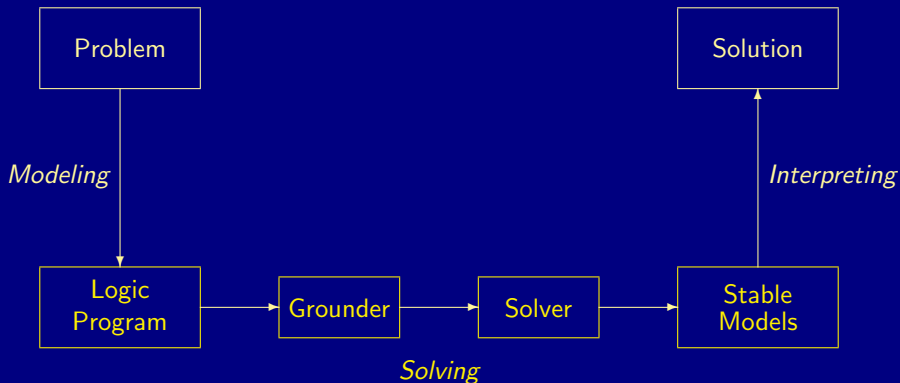
## ASP solving process



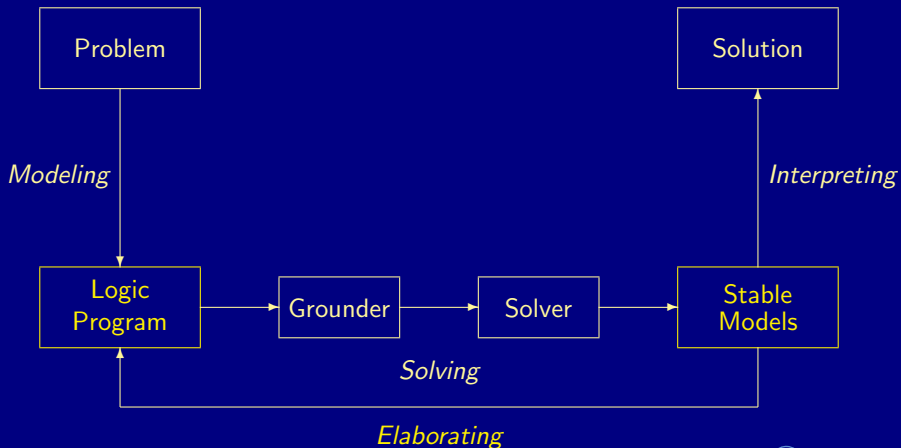
## ASP solving process



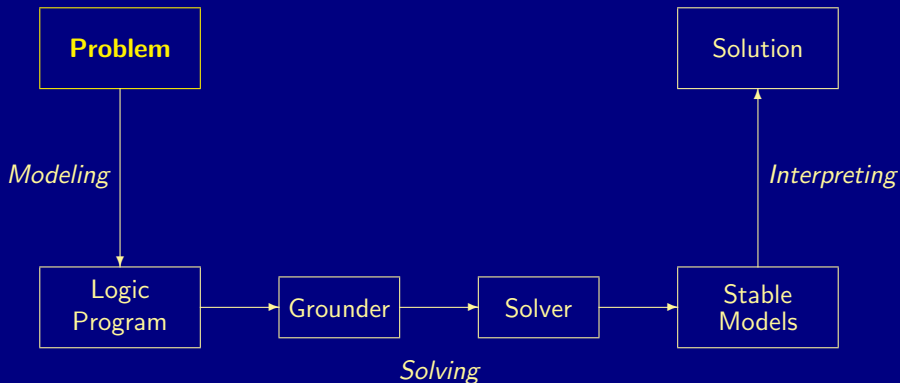
## ASP solving process



## ASP solving process



# A case-study: Graph coloring



# Graph coloring

- Problem instance A graph consisting of nodes and edges

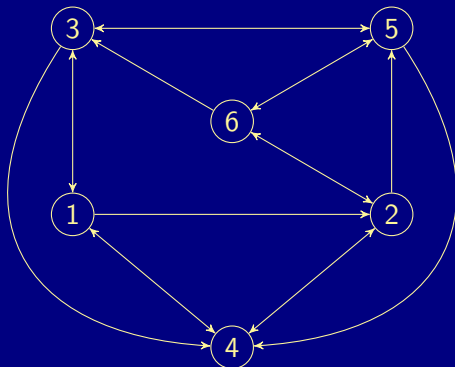


# Graph coloring

- Problem instance A graph consisting of nodes and edges

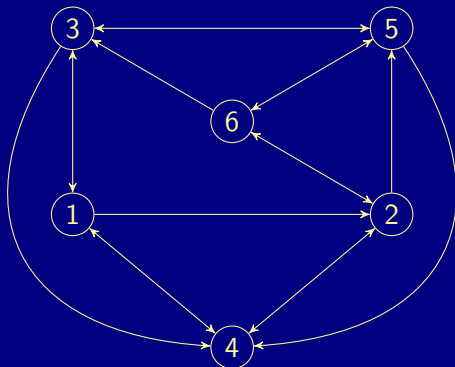
# Graph coloring

- Problem instance A graph consisting of nodes and edges



# Graph coloring

- Problem instance A graph consisting of nodes and edges
  - facts formed by predicates `node/1` and `edge/2`



# Graph coloring

- Problem instance A graph consisting of nodes and edges
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  - facts formed by predicate `color/1`

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- Problem instance A graph consisting of nodes and edges
  - facts formed by predicates `node/1` and `edge/2`
  - facts formed by predicate `color/1`
- Problem class Assign each node one color such that no two nodes connected by an edge have the same color

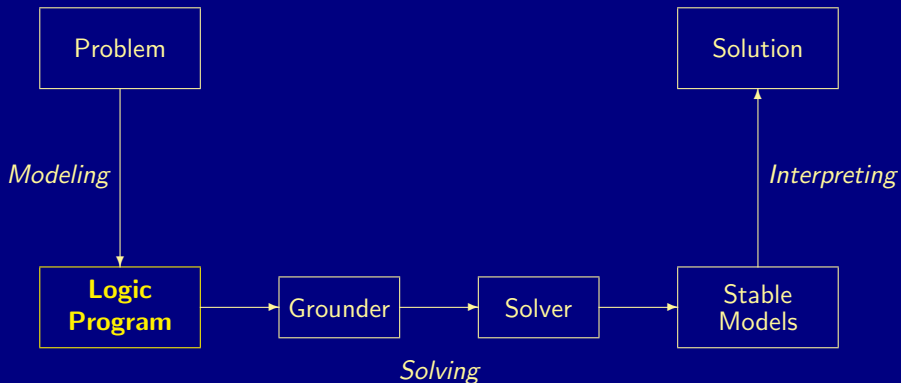
# Graph coloring

- Problem instance A graph consisting of nodes and edges
  - facts formed by predicates `node/1` and `edge/2`
  - facts formed by predicate `color/1`
- Problem class Assign each node one color such that no two nodes connected by an edge have the same color

In other words,

- 1 Each node has one color
- 2 Two connected nodes must not have the same color

## ASP solving process



## Graph coloring

```
node(1..6).
```

```
edge(1,2).  edge(1,3).  edge(1,4).
```

```
edge(2,4).  edge(2,5).  edge(2,6).
```

```
edge(3,1).  edge(3,4).  edge(3,5).
```

```
edge(4,1).  edge(4,2).
```

```
edge(5,3).  edge(5,4).  edge(5,6).
```

```
edge(6,2).  edge(6,3).  edge(6,5).
```

```
color(r).  color(b).  color(g).
```

} Problem  
instance

```
{ assign(N,C) : color(C) } = 1 :- node(N).
```

```
:- edge(N,M), assign(N,C), assign(M,C).
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} Problem  
encoding



## Graph coloring

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edge(1,2). edge(1,3). edge(1,4).
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color(r).  color(b).  color(g).
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```
color(r).   color(b).   color(g).
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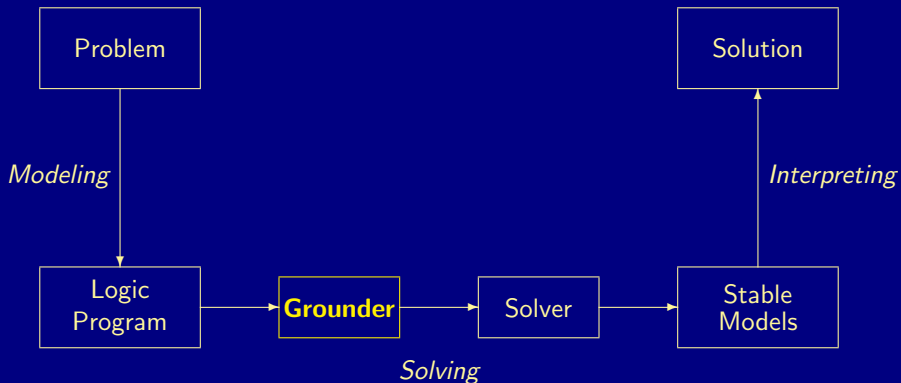
} graph.lp

```
{ assign(N,C) : color(C) } = 1 :- node(N).
```

```
:- edge(N,M), assign(N,C), assign(M,C).
```

} color.lp

## ASP solving process



# Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1). node(2). node(3). node(4). node(5). node(6).
```

```
edge(1,2). edge(2,4). edge(3,1). edge(4,1). edge(5,3). edge(6,2).
edge(1,3). edge(2,5). edge(3,4). edge(4,2). edge(5,4). edge(6,3).
edge(1,4). edge(2,6). edge(3,5). edge(5,6). edge(6,5).
```

```
color(r). color(b). color(g).
```

```
{assign(1,r), assign(1,b), assign(1,g)} = 1. {assign(4,r), assign(4,b), assign(4,g)} = 1.
{assign(2,r), assign(2,b), assign(2,g)} = 1. {assign(5,r), assign(5,b), assign(5,g)} = 1.
{assign(3,r), assign(3,b), assign(3,g)} = 1. {assign(6,r), assign(6,b), assign(6,g)} = 1.
```

```
:- assign(1,r), assign(2,r). :- assign(2,r), assign(4,r). [...] :- assign(6,r), assign(2,r).
:- assign(1,b), assign(2,b). :- assign(2,b), assign(4,b). :- assign(6,b), assign(2,b).
:- assign(1,g), assign(2,g). :- assign(2,g), assign(4,g). :- assign(6,g), assign(2,g).
:- assign(1,r), assign(3,r). :- assign(2,r), assign(5,r). :- assign(6,r), assign(3,r).
:- assign(1,b), assign(3,b). :- assign(2,b), assign(5,b). :- assign(6,b), assign(3,b).
:- assign(1,g), assign(3,g). :- assign(2,g), assign(5,g). :- assign(6,g), assign(3,g).
:- assign(1,r), assign(4,r). :- assign(2,r), assign(6,r). :- assign(6,r), assign(5,r).
:- assign(1,b), assign(4,b). :- assign(2,b), assign(6,b). :- assign(6,b), assign(5,b).
:- assign(1,g), assign(4,g). :- assign(2,g), assign(6,g). :- assign(6,g), assign(5,g).
```

## Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1). node(2). node(3). node(4). node(5). node(6).
```

```
edge(1,2). edge(2,4). edge(3,1). edge(4,1). edge(5,3). edge(6,2).
edge(1,3). edge(2,5). edge(3,4). edge(4,2). edge(5,4). edge(6,3).
edge(1,4). edge(2,6). edge(3,5). edge(5,6). edge(6,5).
```

```
color(r). color(b). color(g).
```

```
{assign(1,r), assign(1,b), assign(1,g)} = 1. {assign(4,r), assign(4,b), assign(4,g)} = 1.
{assign(2,r), assign(2,b), assign(2,g)} = 1. {assign(5,r), assign(5,b), assign(5,g)} = 1.
{assign(3,r), assign(3,b), assign(3,g)} = 1. {assign(6,r), assign(6,b), assign(6,g)} = 1.
```

```
:- assign(1,r), assign(2,r). :- assign(2,r), assign(4,r). [...] :- assign(6,r), assign(2,r).
:- assign(1,b), assign(2,b). :- assign(2,b), assign(4,b). :- assign(6,b), assign(2,b).
:- assign(1,g), assign(2,g). :- assign(2,g), assign(4,g). :- assign(6,g), assign(2,g).
:- assign(1,r), assign(3,r). :- assign(2,r), assign(5,r). :- assign(6,r), assign(3,r).
:- assign(1,b), assign(3,b). :- assign(2,b), assign(5,b). :- assign(6,b), assign(3,b).
:- assign(1,g), assign(3,g). :- assign(2,g), assign(5,g). :- assign(6,g), assign(3,g).
:- assign(1,r), assign(4,r). :- assign(2,r), assign(6,r). :- assign(6,r), assign(5,r).
:- assign(1,b), assign(4,b). :- assign(2,b), assign(6,b). :- assign(6,b), assign(5,b).
:- assign(1,g), assign(4,g). :- assign(2,g), assign(6,g). :- assign(6,g), assign(5,g).
```

## Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1). node(2). node(3). node(4). node(5). node(6).
```

```
edge(1,2). edge(2,4). edge(3,1). edge(4,1). edge(5,3). edge(6,2).
edge(1,3). edge(2,5). edge(3,4). edge(4,2). edge(5,4). edge(6,3).
edge(1,4). edge(2,6). edge(3,5). edge(5,6). edge(6,5).
```

```
color(r). color(b). color(g).
```

```
{ assign(1,r), assign(1,b), assign(1,g) } = 1. { assign(4,r), assign(4,b), assign(4,g) } = 1.
{ assign(2,r), assign(2,b), assign(2,g) } = 1. { assign(5,r), assign(5,b), assign(5,g) } = 1.
{ assign(3,r), assign(3,b), assign(3,g) } = 1. { assign(6,r), assign(6,b), assign(6,g) } = 1.
```

```
:- assign(1,r), assign(2,r). :- assign(2,r), assign(4,r). [...] :- assign(6,r), assign(2,r).
:- assign(1,b), assign(2,b). :- assign(2,b), assign(4,b). :- assign(6,b), assign(2,b).
:- assign(1,g), assign(2,g). :- assign(2,g), assign(4,g). :- assign(6,g), assign(2,g).
:- assign(1,r), assign(3,r). :- assign(2,r), assign(5,r). :- assign(6,r), assign(3,r).
:- assign(1,b), assign(3,b). :- assign(2,b), assign(5,b). :- assign(6,b), assign(3,b).
:- assign(1,g), assign(3,g). :- assign(2,g), assign(5,g). :- assign(6,g), assign(3,g).
:- assign(1,r), assign(4,r). :- assign(2,r), assign(6,r). :- assign(6,r), assign(5,r).
:- assign(1,b), assign(4,b). :- assign(2,b), assign(6,b). :- assign(6,b), assign(5,b).
:- assign(1,g), assign(4,g). :- assign(2,g), assign(6,g). :- assign(6,g), assign(5,g).
```

## Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1). node(2). node(3). node(4). node(5). node(6).
```

```
edge(1,2). edge(2,4). edge(3,1). edge(4,1). edge(5,3). edge(6,2).
edge(1,3). edge(2,5). edge(3,4). edge(4,2). edge(5,4). edge(6,3).
edge(1,4). edge(2,6). edge(3,5). edge(5,6). edge(6,5).
```

```
color(r). color(b). color(g).
```

```
{ assign(1,r), assign(1,b), assign(1,g) } = 1. { assign(4,r), assign(4,b), assign(4,g) } = 1.
{ assign(2,r), assign(2,b), assign(2,g) } = 1. { assign(5,r), assign(5,b), assign(5,g) } = 1.
{ assign(3,r), assign(3,b), assign(3,g) } = 1. { assign(6,r), assign(6,b), assign(6,g) } = 1.
```

```
:- assign(1,r), assign(2,r). :- assign(2,r), assign(4,r). [...] :- assign(6,r), assign(2,r).
:- assign(1,b), assign(2,b). :- assign(2,b), assign(4,b). :- assign(6,b), assign(2,b).
:- assign(1,g), assign(2,g). :- assign(2,g), assign(4,g). :- assign(6,g), assign(2,g).
:- assign(1,r), assign(3,r). :- assign(2,r), assign(5,r). :- assign(6,r), assign(3,r).
:- assign(1,b), assign(3,b). :- assign(2,b), assign(5,b). :- assign(6,b), assign(3,b).
:- assign(1,g), assign(3,g). :- assign(2,g), assign(5,g). :- assign(6,g), assign(3,g).
:- assign(1,r), assign(4,r). :- assign(2,r), assign(6,r). :- assign(6,r), assign(5,r).
:- assign(1,b), assign(4,b). :- assign(2,b), assign(6,b). :- assign(6,b), assign(5,b).
:- assign(1,g), assign(4,g). :- assign(2,g), assign(6,g). :- assign(6,g), assign(5,g).
```

## Graph coloring: Grounding

```
$ clingo --text graph.lp color.lp
```

```
node(1). node(2). node(3). node(4). node(5). node(6).
```

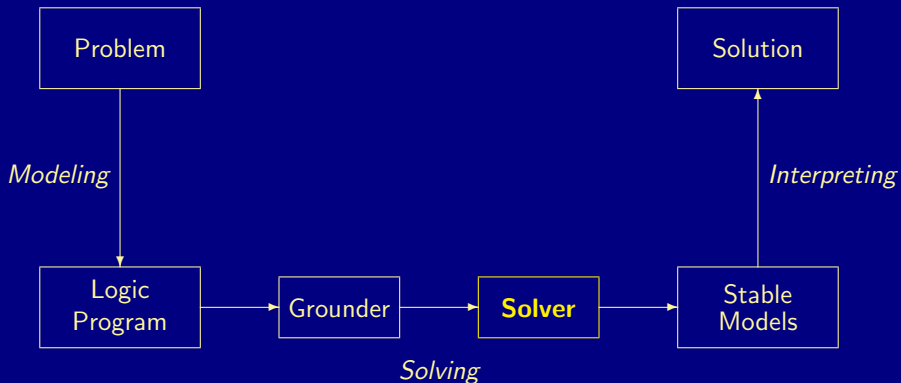
```
edge(1,2). edge(2,4). edge(3,1). edge(4,1). edge(5,3). edge(6,2).
edge(1,3). edge(2,5). edge(3,4). edge(4,2). edge(5,4). edge(6,3).
edge(1,4). edge(2,6). edge(3,5). edge(5,6). edge(6,5).
```

```
color(r). color(b). color(g).
```

```
{ assign(1,r), assign(1,b), assign(1,g) } = 1. { assign(4,r), assign(4,b), assign(4,g) } = 1.
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```
:- assign(1,r), assign(2,r). :- assign(2,r), assign(4,r). [...] :- assign(6,r), assign(2,r).
:- assign(1,b), assign(2,b). :- assign(2,b), assign(4,b). :- assign(6,b), assign(2,b).
:- assign(1,g), assign(2,g). :- assign(2,g), assign(4,g). :- assign(6,g), assign(2,g).
:- assign(1,r), assign(3,r). :- assign(2,r), assign(5,r). :- assign(6,r), assign(3,r).
:- assign(1,b), assign(3,b). :- assign(2,b), assign(5,b). :- assign(6,b), assign(3,b).
:- assign(1,g), assign(3,g). :- assign(2,g), assign(5,g). :- assign(6,g), assign(3,g).
:- assign(1,r), assign(4,r). :- assign(2,r), assign(6,r). :- assign(6,r), assign(5,r).
:- assign(1,b), assign(4,b). :- assign(2,b), assign(6,b). :- assign(6,b), assign(5,b).
:- assign(1,g), assign(4,g). :- assign(2,g), assign(6,g). :- assign(6,g), assign(5,g).
```

## ASP solving process





## Graph coloring: Solving

```
$ gringo graph.lp color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
node(1) [...] assign(6,b) assign(5,g) assign(4,b) assign(3,r) assign(2,r) assign(1,g)
Answer: 2
node(1) [...] assign(6,r) assign(5,g) assign(4,r) assign(3,b) assign(2,b) assign(1,g)
Answer: 3
node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
Answer: 4
node(1) [...] assign(6,r) assign(5,b) assign(4,r) assign(3,g) assign(2,g) assign(1,b)
Answer: 5
node(1) [...] assign(6,g) assign(5,r) assign(4,g) assign(3,b) assign(2,b) assign(1,r)
Answer: 6
node(1) [...] assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
SATISFIABLE

Models      : 6
Time       : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

## Graph coloring: Solving

```
$ gringo graph.lp color.lp | clasp 0
```

```
clasp version 2.1.0
```

```
Reading from stdin
```

```
Solving...
```

```
Answer: 1
```

```
node(1) [...] assign(6,b) assign(5,g) assign(4,b) assign(3,r) assign(2,r) assign(1,g)
```

```
Answer: 2
```

```
node(1) [...] assign(6,r) assign(5,g) assign(4,r) assign(3,b) assign(2,b) assign(1,g)
```

```
Answer: 3
```

```
node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
```

```
Answer: 4
```

```
node(1) [...] assign(6,r) assign(5,b) assign(4,r) assign(3,g) assign(2,g) assign(1,b)
```

```
Answer: 5
```

```
node(1) [...] assign(6,g) assign(5,r) assign(4,g) assign(3,b) assign(2,b) assign(1,r)
```

```
Answer: 6
```

```
node(1) [...] assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
```

```
SATISFIABLE
```

```
Models      : 6
```

```
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

## Graph coloring: Solving

```
$ clingo graph.lp color.lp 0
```

```
clasp version 2.1.0
```

```
Reading from stdin
```

```
Solving...
```

```
Answer: 1
```

```
node(1) [...] assign(6,b) assign(5,g) assign(4,b) assign(3,r) assign(2,r) assign(1,g)
```

```
Answer: 2
```

```
node(1) [...] assign(6,r) assign(5,g) assign(4,r) assign(3,b) assign(2,b) assign(1,g)
```

```
Answer: 3
```

```
node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
```

```
Answer: 4
```

```
node(1) [...] assign(6,r) assign(5,b) assign(4,r) assign(3,g) assign(2,g) assign(1,b)
```

```
Answer: 5
```

```
node(1) [...] assign(6,g) assign(5,r) assign(4,g) assign(3,b) assign(2,b) assign(1,r)
```

```
Answer: 6
```

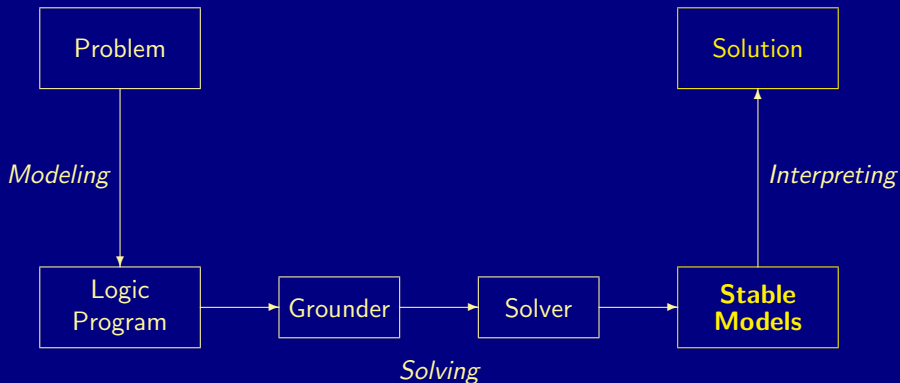
```
node(1) [...] assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
```

```
SATISFIABLE
```

```
Models      : 6
```

```
Time       : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

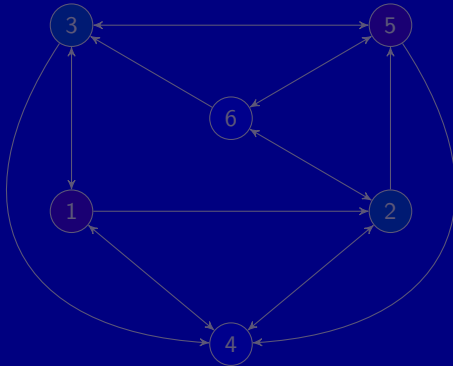
## ASP solving process



## A coloring

Answer: 6

```
node(1) [...] \
assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
```

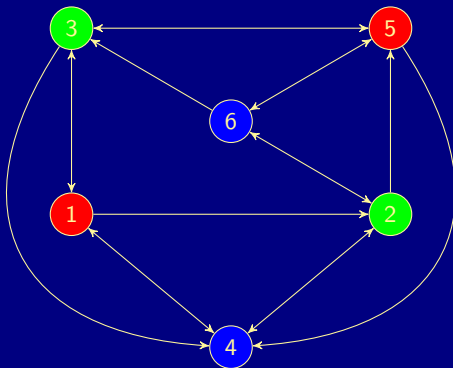


## A coloring

Answer: 6

```
node(1) [...] \
```

```
assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
```



# Outline

- 1 Elaboration tolerance
- 2 ASP solving process
- 3 Methodology**
- 4 Case studies

# Basic methodology

## Methodology

### Generate and Test (or: Guess and Check)

Generator Generate potential stable model candidates  
(typically through non-deterministic constructs)

Tester Eliminate invalid candidates  
(typically through integrity constraints)

## Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)



# Basic methodology

## Methodology

### Generate and Test (or: Guess and Check)

Generator Generate potential stable model candidates  
(typically through non-deterministic constructs)

Tester Eliminate invalid candidates  
(typically through integrity constraints)

## Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

## Graph coloring

```
node(1..6).
```

```
edge(1,2). edge(1,3). edge(1,4).
```

```
edge(2,4). edge(2,5). edge(2,6).
```

```
edge(3,1). edge(3,4). edge(3,5).
```

```
edge(4,1). edge(4,2).
```

```
edge(5,3). edge(5,4). edge(5,6).
```

```
edge(6,2). edge(6,3). edge(6,5).
```

```
color(r). color(b). color(g).
```

**Problem  
instance**

```
{ assign(N,C) : color(C) } = 1 :- node(N).
```

```
:- edge(N,M), assign(N,C), assign(M,C).
```

**Problem  
encoding**

## Graph coloring

```
node(1..6).
```

```
edge(1,2). edge(1,3). edge(1,4).
```

```
edge(2,4). edge(2,5). edge(2,6).
```

```
edge(3,1). edge(3,4). edge(3,5).
```

```
edge(4,1). edge(4,2).
```

```
edge(5,3). edge(5,4). edge(5,6).
```


```
edge(6,2). edge(6,3). edge(6,5).
```

```
color(r). color(b). color(g).
```


**Data**

```
{ assign(N,C) : color(C) } = 1 :- node(N).
```

```
:- edge(N,M), assign(N,C), assign(M,C).
```


**Problem  
encoding**

## Graph coloring

```
node(1..6).
```

```
edge(1,2).  edge(1,3).  edge(1,4).
```

```
edge(2,4).  edge(2,5).  edge(2,6).
```

```
edge(3,1).  edge(3,4).  edge(3,5).
```

```
edge(4,1).  edge(4,2).
```

```
edge(5,3).  edge(5,4).  edge(5,6).
```

```
edge(6,2).  edge(6,3).  edge(6,5).
```

```
color(r).  color(b).  color(g).
```

```
{ assign(N,C) : color(C) } = 1 :- node(N).
```

```
:- edge(N,M), assign(N,C), assign(M,C).
```


**Data**
**Generator**
**Tester**

# Outline

- 1 Elaboration tolerance
- 2 ASP solving process
- 3 Methodology
- 4 Case studies

# Outline

- 1 Elaboration tolerance
- 2 ASP solving process
- 3 Methodology
- 4 Case studies
  - Satisfiability
  - Queens
  - Traveling salesperson
  - Reviewer Assignment
  - Planning

# Satisfiability testing

- Problem Instance A propositional formula  $\phi$  in CNF
- Problem Class Is there an assignment of propositional variables to true and false such that a given formula  $\phi$  is true
- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program

**Generator**

$\{a\} \leftarrow$

$\{b\} \leftarrow$

**Tester**

$\leftarrow \sim a, b$

$\leftarrow a, \sim b$

**Stable models**

$X_1 = \{a, b\}$

$X_2 = \{\}$

# Satisfiability testing

- Problem Instance A propositional formula  $\phi$  in CNF
- Problem Class Is there an assignment of propositional variables to true and false such that a given formula  $\phi$  is true
- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program

## Generator

$$\begin{aligned} \{a\} &\leftarrow \\ \{b\} &\leftarrow \end{aligned}$$

## Tester

$$\begin{aligned} &\leftarrow \sim a, b \\ &\leftarrow a, \sim b \end{aligned}$$

## Stable models

$$\begin{aligned} X_1 &= \{a, b\} \\ X_2 &= \{\} \end{aligned}$$



# Satisfiability testing

- Problem Instance A propositional formula  $\phi$  in CNF
- Problem Class Is there an assignment of propositional variables to true and false such that a given formula  $\phi$  is true
- Example: Consider formula

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- Logic Program

## Generator

$$\{a\} \leftarrow$$

$$\{b\} \leftarrow$$

## Tester

$$\leftarrow \sim a, b$$

$$\leftarrow a, \sim b$$

## Stable models

$$X_1 = \{a, b\}$$

$$X_2 = \{\}$$

# Satisfiability testing

- Problem Instance A propositional formula  $\phi$  in CNF
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- Logic Program

## Generator

$$\begin{aligned} \{a\} &\leftarrow \\ \{b\} &\leftarrow \end{aligned}$$

## Tester

$$\begin{aligned} &\leftarrow \sim a, b \\ &\leftarrow a, \sim b \end{aligned}$$

## Stable models

$$\begin{aligned} X_1 &= \{a, b\} \\ X_2 &= \{\} \end{aligned}$$

# Satisfiability testing

- Problem Instance A propositional formula  $\phi$  in CNF
- Problem Class Is there an assignment of propositional variables to true and false such that a given formula  $\phi$  is true
- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program

## Generator

$$\begin{aligned} \{a\} &\leftarrow \\ \{b\} &\leftarrow \end{aligned}$$

## Tester

$$\begin{aligned} &\leftarrow \sim a, b \\ &\leftarrow a, \sim b \end{aligned}$$

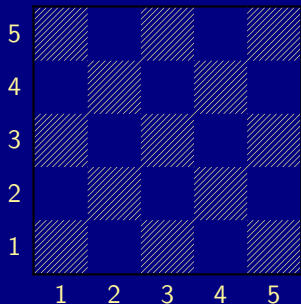
## Stable models

$$\begin{aligned} X_1 &= \{a, b\} \\ X_2 &= \{\} \end{aligned}$$

# Outline

- 1 Elaboration tolerance
- 2 ASP solving process
- 3 Methodology
- 4 Case studies
  - Satisfiability
  - Queens
  - Traveling salesperson
  - Reviewer Assignment
  - Planning

# The $n$ -queens problem



- Place  $n$  queens on an  $n \times n$  chess board
- Queens must not attack one another



## Defining the field

```
queens.lp
```

```
row(1..n).  
col(1..n).
```

- Create file `queens.lp`
- Define the field
  - $n$  rows
  - $n$  columns

## Defining the field

Running ...

```
$ clingo queens.lp --const n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE
```

```
Models      : 1
Time        : 0.000
```

## Placing some queens

```
queens.lp
```

```
row(1..n).  
col(1..n).  
{ queen(I,J) : row(I), col(J) }.
```

- Guess a solution candidate  
by placing some queens on the board



# Placing some queens

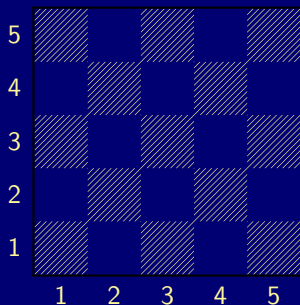
Running ...

```
$ clingo queens.lp --const n=5 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE
```

Models : 3+

# Placing some queens

Answer: 1

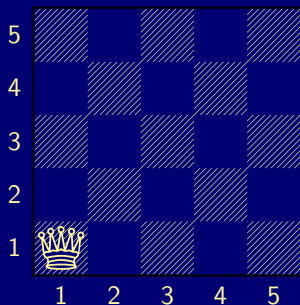


Answer: 1

```
row(1) row(2) row(3) row(4) row(5) \  
col(1) col(2) col(3) col(4) col(5)
```

# Placing some queens

Answer: 2

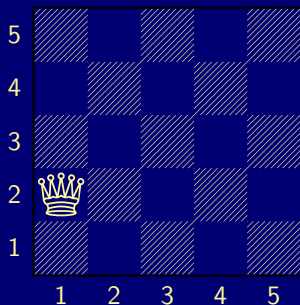


Answer: 2

```
row(1) row(2) row(3) row(4) row(5) \  
col(1) col(2) col(3) col(4) col(5) \  
queen(1,1)
```

# Placing some queens

Answer: 3



Answer: 3

```
row(1) row(2) row(3) row(4) row(5) \  
col(1) col(2) col(3) col(4) col(5) \  
queen(2,1)
```

# Placing $n$ queens

```
queens.lp
```

```
row(1..n).  
col(1..n).  
{ queen(I,J) : row(I), col(J) }.  
:- { queen(I,J) } != n.
```

- Place exactly  $n$  queens on the board

# Placing $n$ queens

```
queens.lp
```

```
row(1..n).  
col(1..n).  
{ queen(I,J) : row(I), col(J) }.  
:- not { queen(I,J) } = n.
```

- Place exactly  $n$  queens on the board

# Placing $n$ queens

Running ...

```
$ clingo queens.lp --const n=5 2
```

```
Answer: 1
```

```
row(1) row(2) row(3) row(4) row(5) \
```

```
col(1) col(2) col(3) col(4) col(5) \
```

```
queen(5,1) queen(4,1) queen(3,1) queen(2,1) queen(1,1)
```

```
Answer: 2
```

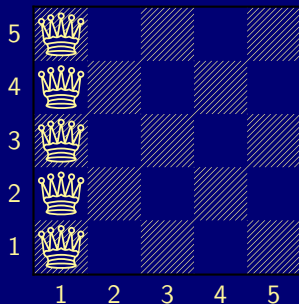
```
row(1) row(2) row(3) row(4) row(5) \
```

```
col(1) col(2) col(3) col(4) col(5) \
```

```
queen(1,2) queen(4,1) queen(3,1) queen(2,1) queen(1,1)
```

# Placing $n$ queens

Answer: 1



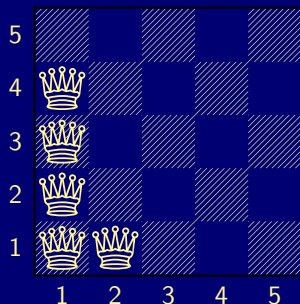
Answer: 1

```
row(1) row(2) row(3) row(4) row(5) \  
col(1) col(2) col(3) col(4) col(5) \  
queen(5,1) queen(4,1) queen(3,1) queen(2,1)  
queen(1,1)
```



# Placing $n$ queens

Answer: 2



Answer: 2

```

row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) queen(2,1)
queen(1,1)

```

# Horizontal and vertical attack

```
queens.lp
```

```
row(1..n).  
col(1..n).  
{ queen(I,J) : row(I), col(J) }.  
:- { queen(I,J) } != n.  
:- queen(I,J), queen(I,J'), J != J'.  
:- queen(I,J), queen(I',J), I != I'.
```

- Forbid horizontal attacks
- Forbid vertical attacks

# Horizontal and vertical attack

```
queens.lp
```

```
row(1..n).  
col(1..n).  
{ queen(I,J) : row(I), col(J) }.  
:- { queen(I,J) } != n.  
:- queen(I,J), queen(I,J'), J != J'.  
:- queen(I,J), queen(I',J), I != I'.
```

- Forbid horizontal attacks
- Forbid vertical attacks

# Horizontal and vertical attack

Running ...

```
$ clingo queens.lp --const n=5
```

```
Answer: 1
```

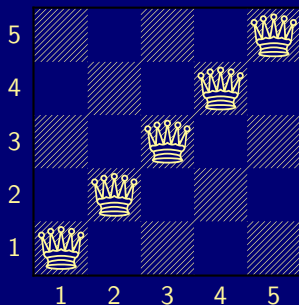
```
row(1) row(2) row(3) row(4) row(5) \
```

```
col(1) col(2) col(3) col(4) col(5) \
```

```
queen(5,5) queen(4,4) queen(3,3) queen(2,2) queen(1,1)
```

# Horizontal and vertical attack

Answer: 1



Answer: 1

```
row(1) row(2) row(3) row(4) row(5) \  
col(1) col(2) col(3) col(4) col(5) \  
queen(5,5) queen(4,4) queen(3,3) queen(2,2)  
queen(1,1)
```

# Diagonal attack

```
queens.lp
```

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I-J == I'-J'.
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I+J == I'+J'.
```

- Forbid diagonal attacks

# Diagonal attack

Running ...

```
$ clingo queens.lp --const n=5
```

```
Answer: 1
```

```
row(1) row(2) row(3) row(4) row(5) \
```

```
col(1) col(2) col(3) col(4) col(5) \
```

```
queen(4,5) queen(1,4) queen(3,3) queen(5,2) queen(2,1)
```

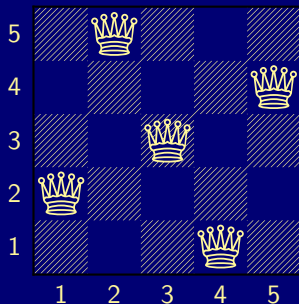
```
SATISFIABLE
```

```
Models      : 1+
```

```
Time       : 0.000
```

# Diagonal attack

Answer: 1



Answer: 1

```
row(1) row(2) row(3) row(4) row(5) \  
col(1) col(2) col(3) col(4) col(5) \  
queen(4,5) queen(1,4) queen(3,3) queen(5,2)  
queen(2,1)
```



# Optimizing

```
queens-opt.lp
```

```
{ queen(I,1..n) } = 1 :- I = 1..n.  
{ queen(1..n,J) } = 1 :- J = 1..n.  
:- { queen(D-J,J) } > 1, D = 2..2*n.  
:- { queen(D+J,J) } > 1, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

# And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=2
```

```
clingo version 4.1.0
Solving...
SATISFIABLE

Models      : 1+
Time        : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time    : 3758.320s

Choices     : 288594554
Conflicts   : 3442 (Analyzed: 3442)
Restarts    : 17 (Average: 202.47 Last: 3442)
Model-Level : 7594728.0
Problems    : 1 (Average Length: 0.00 Splits: 0)
Lemmas      : 3442 (Deleted: 0)
  Binary    : 0 (Ratio: 0.00%)
  Ternary   : 0 (Ratio: 0.00%)
  Conflict   : 3442 (Average Length: 229056.5 Ratio: 100.00%)
  Loop      : 0 (Average Length: 0.0 Ratio: 0.00%)
  Other     : 0 (Average Length: 0.0 Ratio: 0.00%)

Atoms       : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules       : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies      : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight       : Yes
Variables   : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664 (Binary: 35.6% Ternary: 0.0% Other: 64.4%)

Backjumps   : 3442 (Average: 681.19 Max: 169512 Sum: 2344658)
Executed    : 3442 (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
Bounded     : 0 (Average: 0.00 Max: 0 Sum: 0 Ratio: 0.00%)
```

# And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=2
```

```
clingo version 4.1.0
```

```
Solving...
```

```
SATISFIABLE
```

```
Models      : 1+
Time        : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time    : 3758.320s

Choices     : 288594554
Conflicts   : 3442 (Analyzed: 3442)
Restarts    : 17 (Average: 202.47 Last: 3442)
Model-Level : 7594728.0
Problems    : 1 (Average Length: 0.00 Splits: 0)
Lemmas      : 3442 (Deleted: 0)
  Binary    : 0 (Ratio: 0.00%)
  Ternary   : 0 (Ratio: 0.00%)
  Conflict  : 3442 (Average Length: 229056.5 Ratio: 100.00%)
  Loop      : 0 (Average Length: 0.0 Ratio: 0.00%)
  Other     : 0 (Average Length: 0.0 Ratio: 0.00%)

Atoms       : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules       : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies      : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight       : Yes
Variables   : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664 (Binary: 35.6% Ternary: 0.0% Other: 64.4%)

Backjumps   : 3442 (Average: 681.19 Max: 169512 Sum: 2344658)
Executed    : 3442 (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
Bounded     : 0 (Average: 0.00 Max: 0 Sum: 0 Ratio: 0.00%)
```

# Outline

- 1 Elaboration tolerance
- 2 ASP solving process
- 3 Methodology
- 4 Case studies
  - Satisfiability
  - Queens
  - Traveling salesperson
  - Reviewer Assignment
  - Planning

# The traveling salesperson problem (TSP)

- Problem Instance A set of cities and distances among them, or simply a weighted graph
- Problem Class What is the shortest possible route visiting each city and returning to the city of origin?
- Note
  - TSP extends the Hamiltonian cycle problem:  
Is there a cycle in a graph visiting each node exactly once
  - TSP is relevant to applications in logistics, planning, chip design, and the core of the vehicle routing problem

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Is there a cycle in a graph visiting each node exactly once
  - TSP is relevant to applications in logistics, planning, chip design, and the core of the vehicle routing problem

# Traveling salesperson

```
node(1..6).
```

```
edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).  
edge(4,(1;2)).  edge(5,(3;4;6)).  edge(6,(2;3;5)).
```

```
cost(1,2,2).  cost(1,3,3).  cost(1,4,1).  
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).  
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).  
cost(4,1,1).  cost(4,2,2).  
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).  
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

# Traveling salesperson

```
node(1..6).
```

```
edge(1,(2;3;4)).   edge(2,(4;5;6)).   edge(3,(1;4;5)).  
edge(4,(1;2)).     edge(5,(3;4;6)).   edge(6,(2;3;5)).
```

```
cost(1,2,2).   cost(1,3,3).   cost(1,4,1).  
cost(2,4,2).   cost(2,5,2).   cost(2,6,4).  
cost(3,1,3).   cost(3,4,2).   cost(3,5,2).  
cost(4,1,1).   cost(4,2,2).  
cost(5,3,2).   cost(5,4,2).   cost(5,6,1).  
cost(6,2,4).   cost(6,3,3).   cost(6,5,1).
```



# Traveling salesperson

```
node(1..6).
```

```
edge(1,(2;3;4)). edge(2,(4;5;6)). edge(3,(1;4;5)).  
edge(4,(1;2)). edge(5,(3;4;6)). edge(6,(2;3;5)).
```

```
cost(1,2,2). cost(1,3,3). cost(1,4,1).  
cost(2,4,2). cost(2,5,2). cost(2,6,4).  
cost(3,1,3). cost(3,4,2). cost(3,5,2).  
cost(4,1,1). cost(4,2,2).  
cost(5,3,2). cost(5,4,2). cost(5,6,1).  
cost(6,2,4). cost(6,3,3). cost(6,5,1).
```

# Traveling salesperson

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node(1..6).
```

```
edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).  
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).
```

```
cost(1,2,2).  cost(1,3,3).  cost(1,4,1).  
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).  
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).  
cost(4,1,1).  cost(4,2,2).  
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).  
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

```
edge(X,Y) :- cost(X,Y,_).
```

# Traveling salesperson

```
node(1..6).
```

```
edge(1,(2;3;4)).  edge(2,(4;5;6)).  edge(3,(1;4;5)).  
edge(4,(1;2)).    edge(5,(3;4;6)).  edge(6,(2;3;5)).
```

```
cost(1,2,2).  cost(1,3,3).  cost(1,4,1).  
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).  
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).  
cost(4,1,1).  cost(4,2,2).  
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).  
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

```
edge(X,Y) :- cost(X,Y,_).  
node(X) :- cost(X,_,_).  node(Y) :- cost(_,Y,_).
```

# Traveling salesperson

```
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(X).  
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(Y).  
  
reached(Y) :- cycle(1,Y).  
reached(Y) :- cycle(X,Y), reached(X).  
  
:- node(Y), not reached(Y).  
  
#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

# Traveling salesperson

```
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(X).  
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(Y).  
  
reached(Y) :- cycle(1,Y).  
reached(Y) :- cycle(X,Y), reached(X).  
  
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# Traveling salesperson

```
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(X).  
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(Y).  
  
reached(Y) :- cycle(1,Y).  
reached(Y) :- cycle(X,Y), reached(X).  
  
:- node(Y), not reached(Y).  
  
#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
```

# Traveling salesperson

```
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(X).  
{ cycle(X,Y) : edge(X,Y) } = 1 :- node(Y).  
  
reached(Y) :- cycle(1,Y).  
reached(Y) :- cycle(X,Y), reached(X).  
  
:- node(Y), not reached(Y).  
  
#minimize { C,X,Y : cycle(X,Y), cost(X,Y,C) }.
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# Outline

- 1 Elaboration tolerance
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# Reviewer Assignment

- Problem Instance A set of papers and a set of reviewers along with their first and second choices of papers and conflict of interests
- Problem Class A nice assignment of three reviewers to each paper

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- Problem Instance A set of papers and a set of reviewers along with their first and second choices of papers and conflict of interests
- Problem Class A “nice” assignment of three reviewers to each paper

# Reviewer Assignment

by Ilkka Niemelä

```
paper(p1).  reviewer(r1).  classA(r1,p1).  classB(r1,p2).  coi(r1,p3).  
paper(p2).  reviewer(r2).  classA(r1,p3).  classB(r1,p4).  coi(r1,p6).  
[...]
```

```
{ assigned(P,R) : reviewer(R) } = 3 :- paper(P).
```

```
:- assigned(P,R), coi(R,P).
```

```
:- assigned(P,R), not classA(R,P), not classB(R,P).
```

```
:- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).
```

```
assignedB(P,R) :- classB(R,P), assigned(P,R).
```

```
:- 3 { assignedB(P,R) : paper(P) }, reviewer(R).
```

```
#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

# Reviewer Assignment

by Ilkka Niemelä

```
paper(p1).  reviewer(r1).  classA(r1,p1).  classB(r1,p2).  coi(r1,p3).  
paper(p2).  reviewer(r2).  classA(r1,p3).  classB(r1,p4).  coi(r1,p6).  
[...]
```

```
{ assigned(P,R) : reviewer(R) } = 3 :- paper(P).
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```

```
assignedB(P,R) :- classB(R,P), assigned(P,R).
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paper(p1).  reviewer(r1).  classA(r1,p1).  classB(r1,p2).  coi(r1,p3).  
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[...]
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assignedB(P,R) :- classB(R,P), assigned(P,R).
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paper(p1).  reviewer(r1).  classA(r1,p1).  classB(r1,p2).  coi(r1,p3).  
paper(p2).  reviewer(r2).  classA(r1,p3).  classB(r1,p4).  coi(r1,p6).  
[...]
```

```
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```

```
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```

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assignedB(P,R) :- classB(R,P), assigned(P,R).
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#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
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# Reviewer Assignment

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paper(p1).  reviewer(r1).  classA(r1,p1).  classB(r1,p2).  coi(r1,p3).  
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[...]
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```
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```

```
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```

```
assignedB(P,R) :- classB(R,P), assigned(P,R).
```

```
:- 3 { assignedB(P,R) : paper(P) }, reviewer(R).
```

```
#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

# Reviewer Assignment

by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
[...]

#count { P,R : assigned(P,R) : reviewer(R) } = 3 :- paper(P).

:- assigned(P,R), coi(R,P).
:- assigned(P,R), not classA(R,P), not classB(R,P).
:- not 6 <= #count { P,R : assigned(P,R), paper(P) } <= 9, reviewer(R).

assignedB(P,R) :- classB(R,P), assigned(P,R).
:- 3 <= #count { P,R : assignedB(P,R), paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```



# Reviewer Assignment

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```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
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# Outline

- 1 Elaboration tolerance
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# Simplified STRIPS<sup>1</sup> Planning

- Problem Instance
  - set of fluents
  - initial and goal state
  - set of actions, consisting of pre- and postconditions
  - number  $k$  of allowed actions
- Problem Class Find a plan, that is, a sequence of  $k$  actions leading from the initial state to the goal state
- Example
  - fluents  $\{p, q, r\}$
  - initial state  $\{p\}$
  - goal state  $\{r\}$
  - actions  $a = (\{p\}, \{q, \neg p\})$  and  $b = (\{q\}, \{r, \neg q\})$
  - length 2
  - plan  $\langle a, b \rangle$

---

<sup>1</sup>Stanford Research Institute Problem Solver, 1971

# Simplified STRIPS<sup>1</sup> Planning

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---

<sup>1</sup>Stanford Research Institute Problem Solver, 1971

# Simplistic STRIPS Planning

```
time(1..k).
```

```
fluent(p).      action(a).      action(b).      init(p).
fluent(q).      pre(a,p).        pre(b,q).
fluent(r).      add(a,q).         add(b,r).       query(r).
                del(a,p).        del(b,q).
```

```
holds(P,0) :- init(P).
```

```
{ occ(A,T) : action(A) } = 1 :- time(T).
:- occ(A,T), pre(A,F), not holds(F,T-1).
```

```
holds(F,T) :- occ(A,T), add(A,F).
```

```
holds(F,T) :- holds(F,T-1), time(T), not occ(A,T) : del(A,F).
```

```
:- query(F), not holds(F,k).
```

# Simplistic STRIPS Planning

```
time(1..k).
```

```

fluent(p).      action(a).      action(b).      init(p).
fluent(q).      pre(a,p).      pre(b,q).
fluent(r).      add(a,q).      add(b,r).      query(r).
                del(a,p).      del(b,q).
```

```
holds(P,0) :- init(P).
```

```

{ occ(A,T) : action(A) } = 1 :- time(T).
:- occ(A,T), pre(A,F), not holds(F,T-1).
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```
holds(F,T) :- occ(A,T), add(A,F).
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holds(F,T) :- holds(F,T-1), time(T), not occ(A,T) : del(A,F).
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:- query(F), not holds(F,k).
```



# Simplistic STRIPS Planning

```
time(1..k).
```

```
fluent(p).      action(a).      action(b).      init(p).
fluent(q).      pre(a,p).        pre(b,q).
fluent(r).      add(a,q).         add(b,r).        query(r).
                del(a,p).         del(b,q).
```

```
holds(P,0) :- init(P).
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```
{ occ(A,T) : action(A) } = 1 :- time(T).
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```
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# Simplistic STRIPS Planning

```
time(1..k).
```

```

fluent(p).      action(a).      action(b).      init(p).
fluent(q).      pre(a,p).      pre(b,q).
fluent(r).      add(a,q).      add(b,r).      query(r).
                del(a,p).      del(b,q).
```

```
holds(P,0) :- init(P).
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```

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```

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```

```
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```

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