Answer Set Programming: From Theory to Practice

Roland Kaminski Javier Romero Torsten Schaub Philipp Wanko

University of Potsdam

Outline

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Introduction: Overview

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Outline

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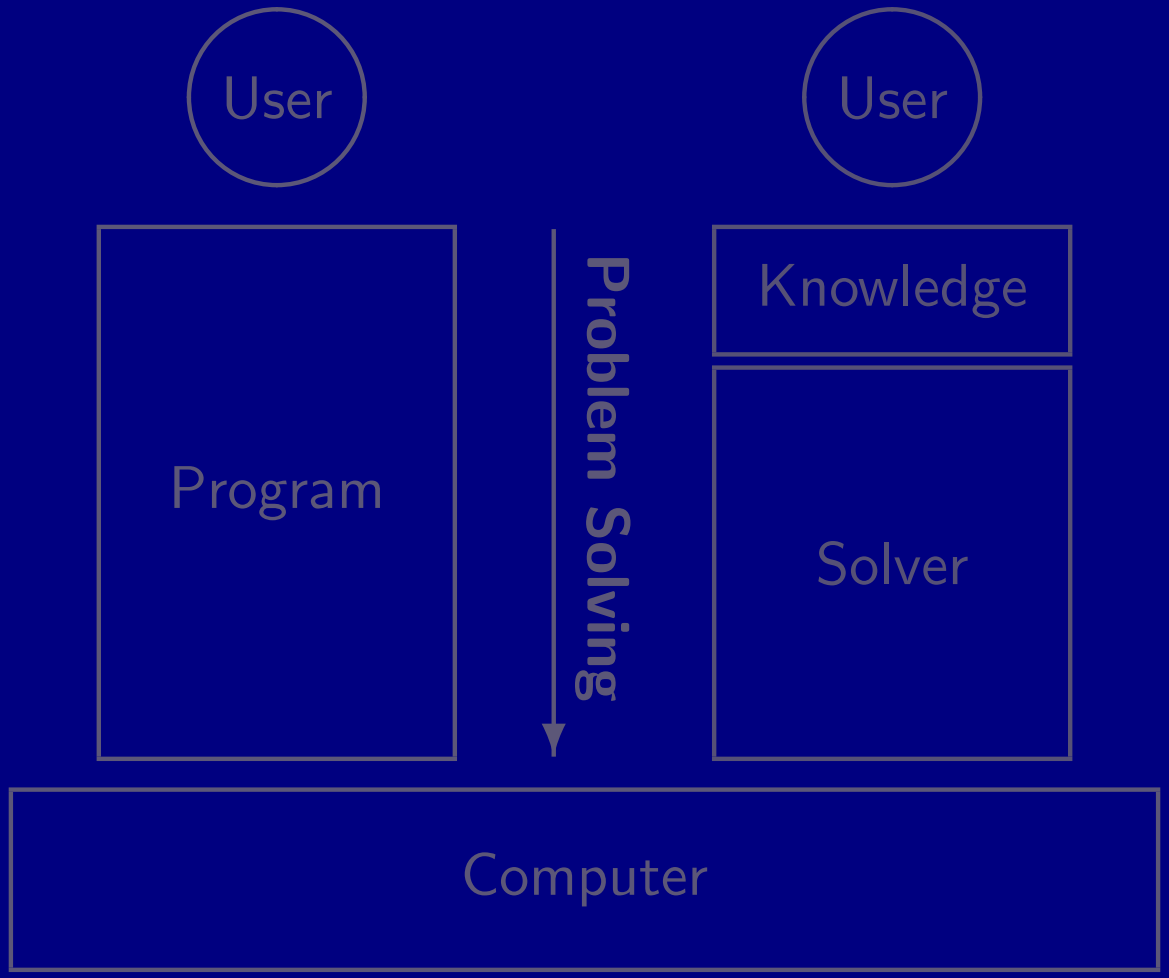
Traditional Software



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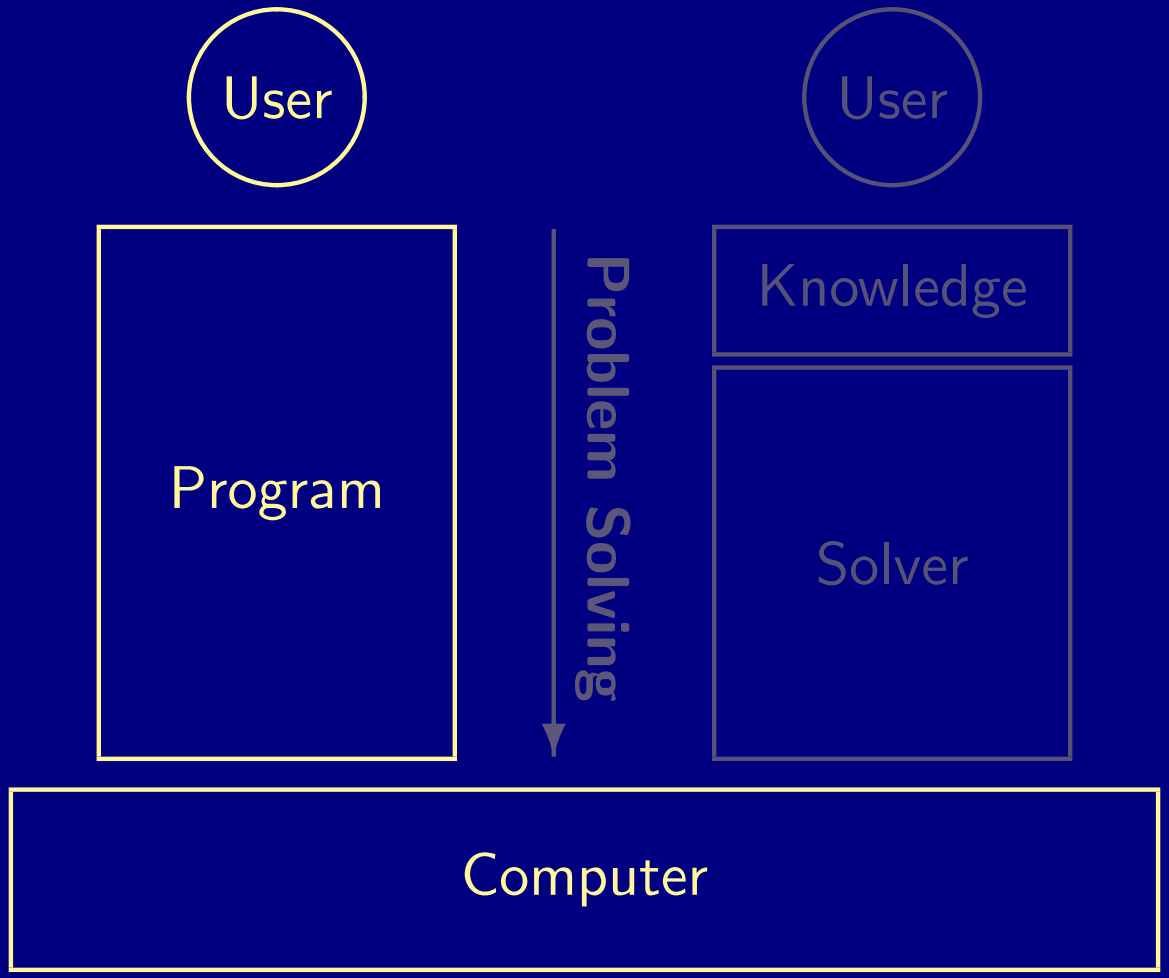

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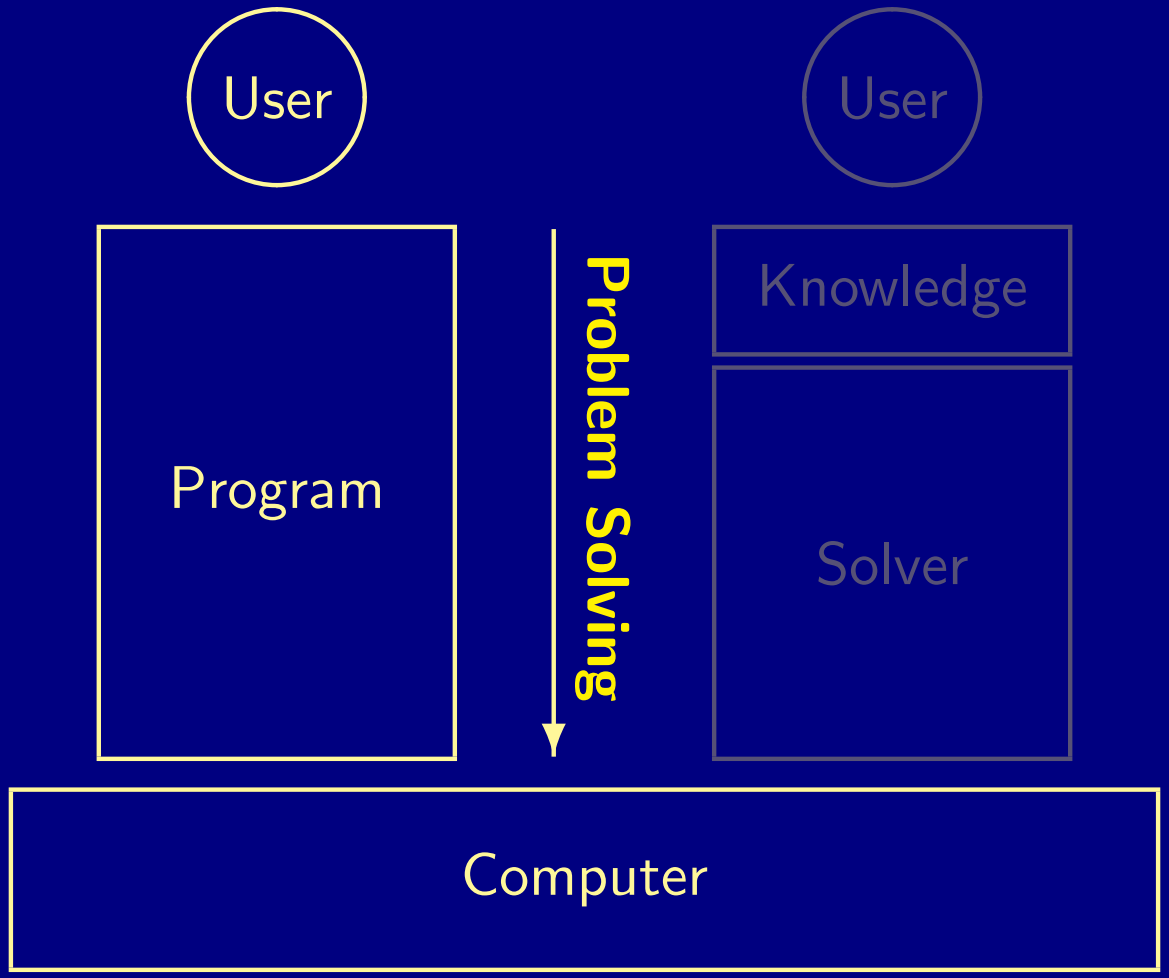

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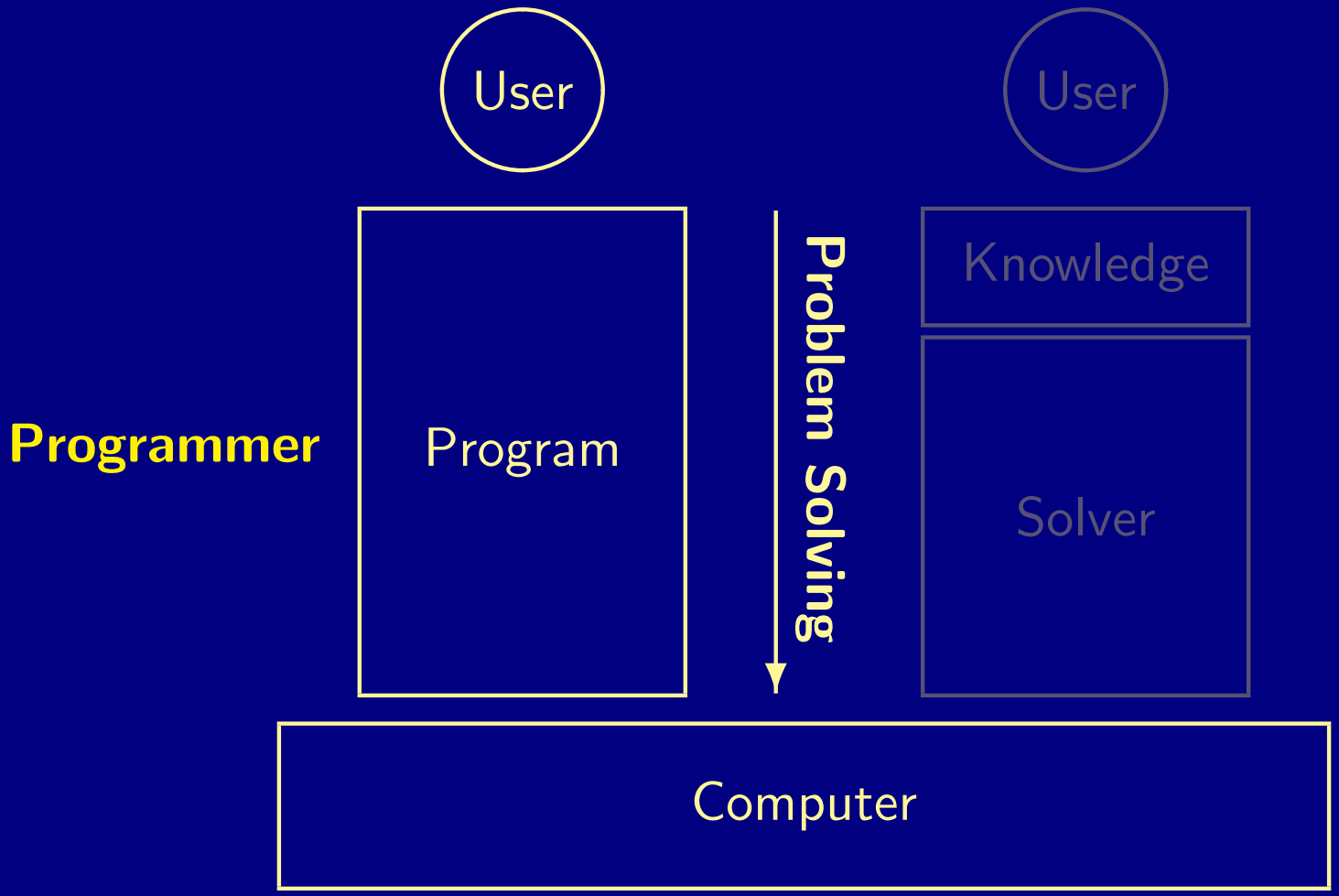

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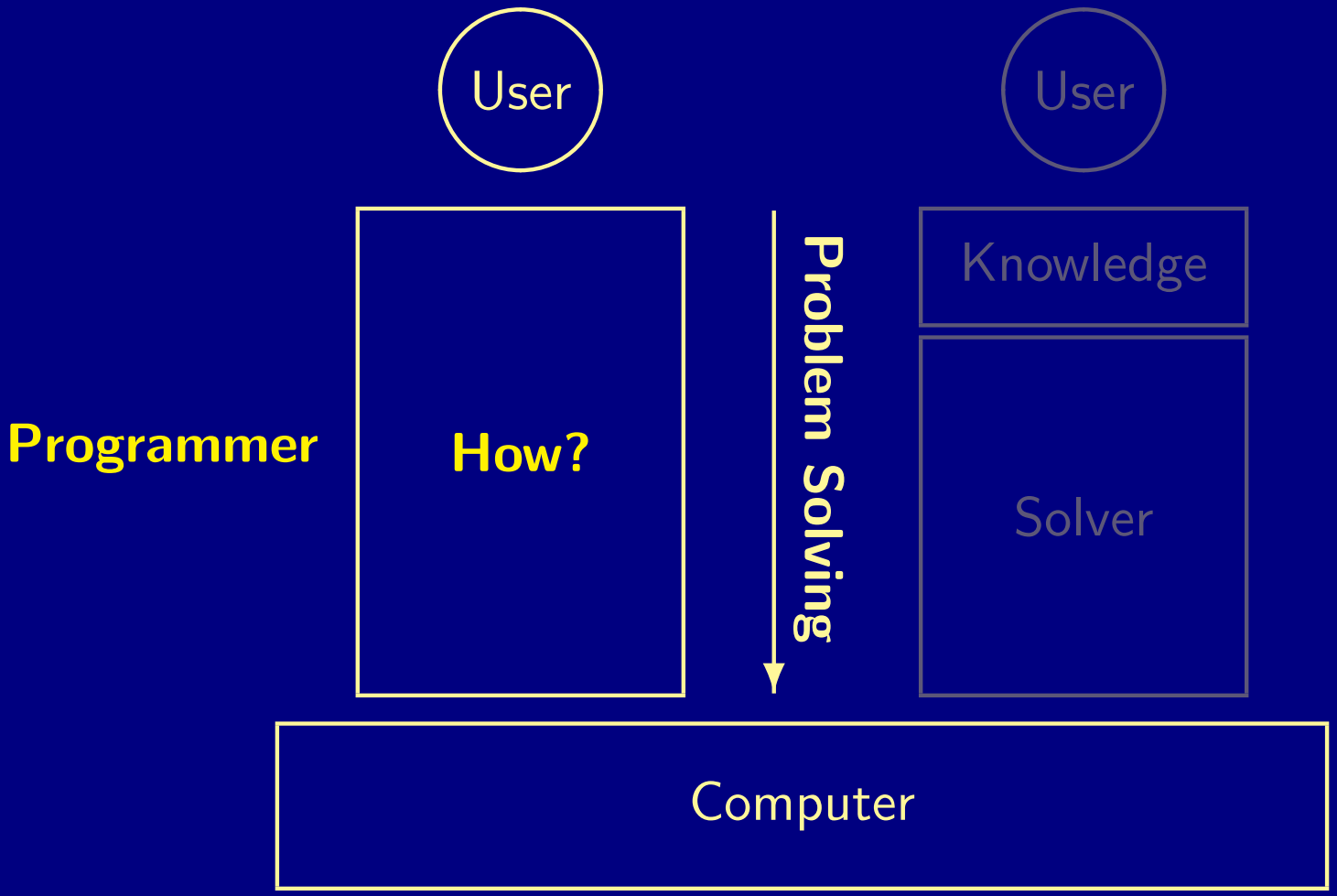

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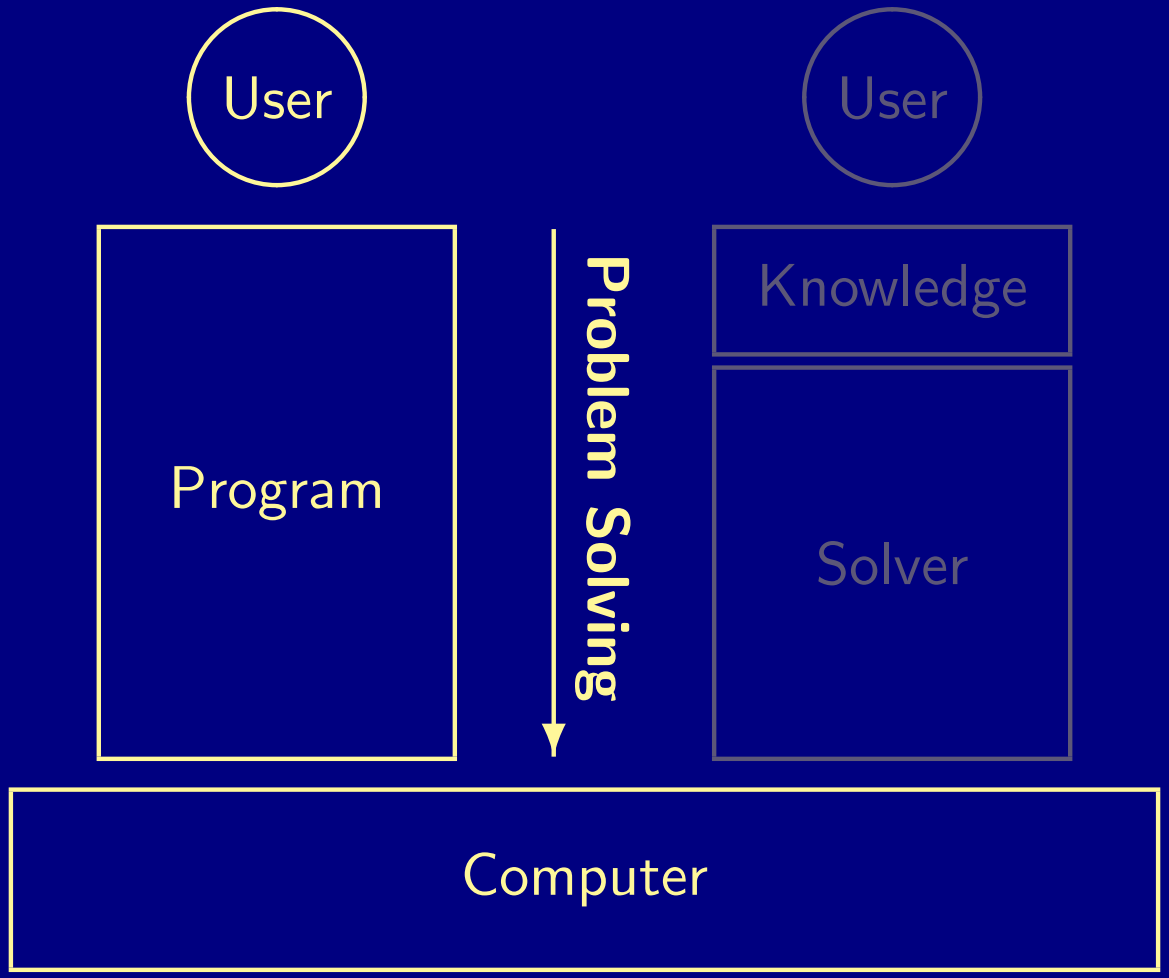
Knowledge-driven Software



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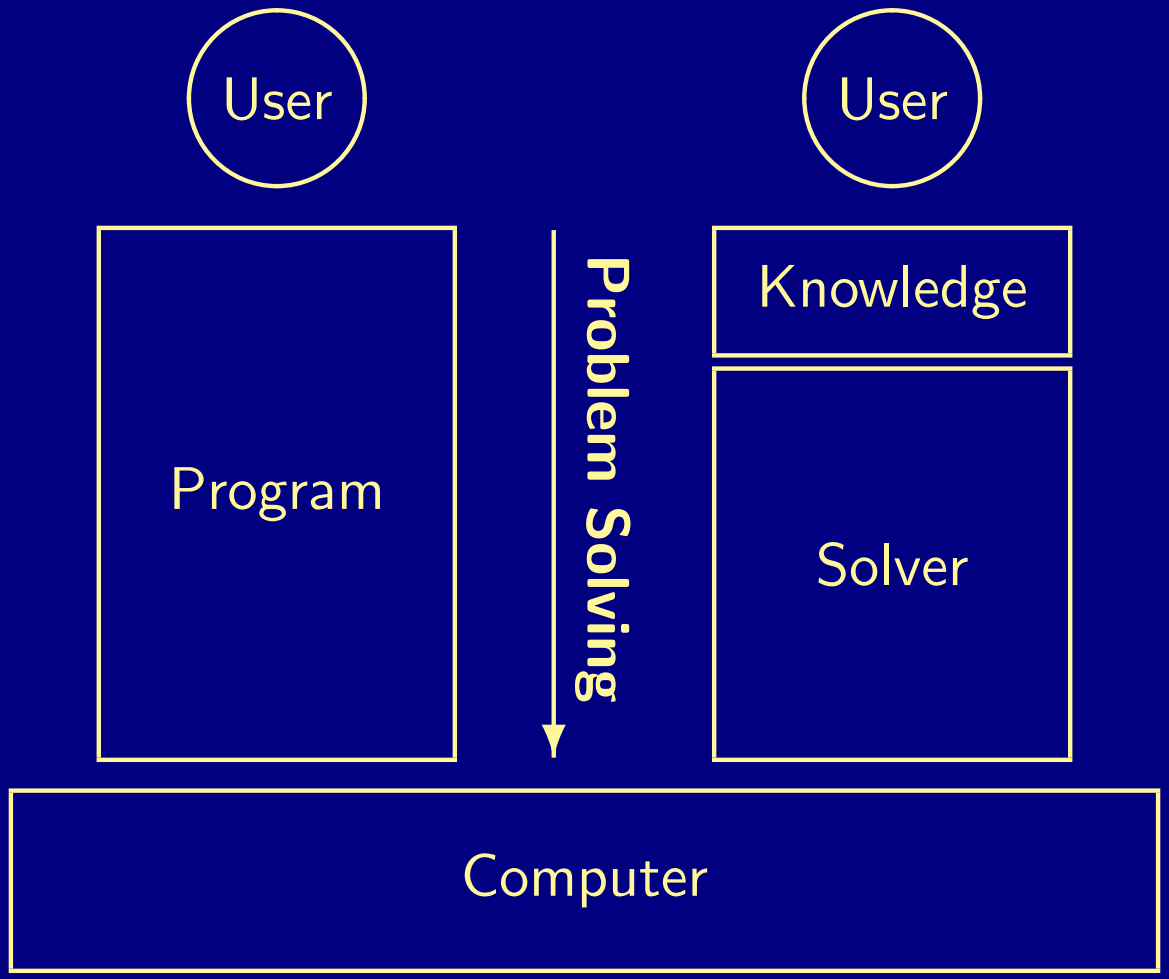

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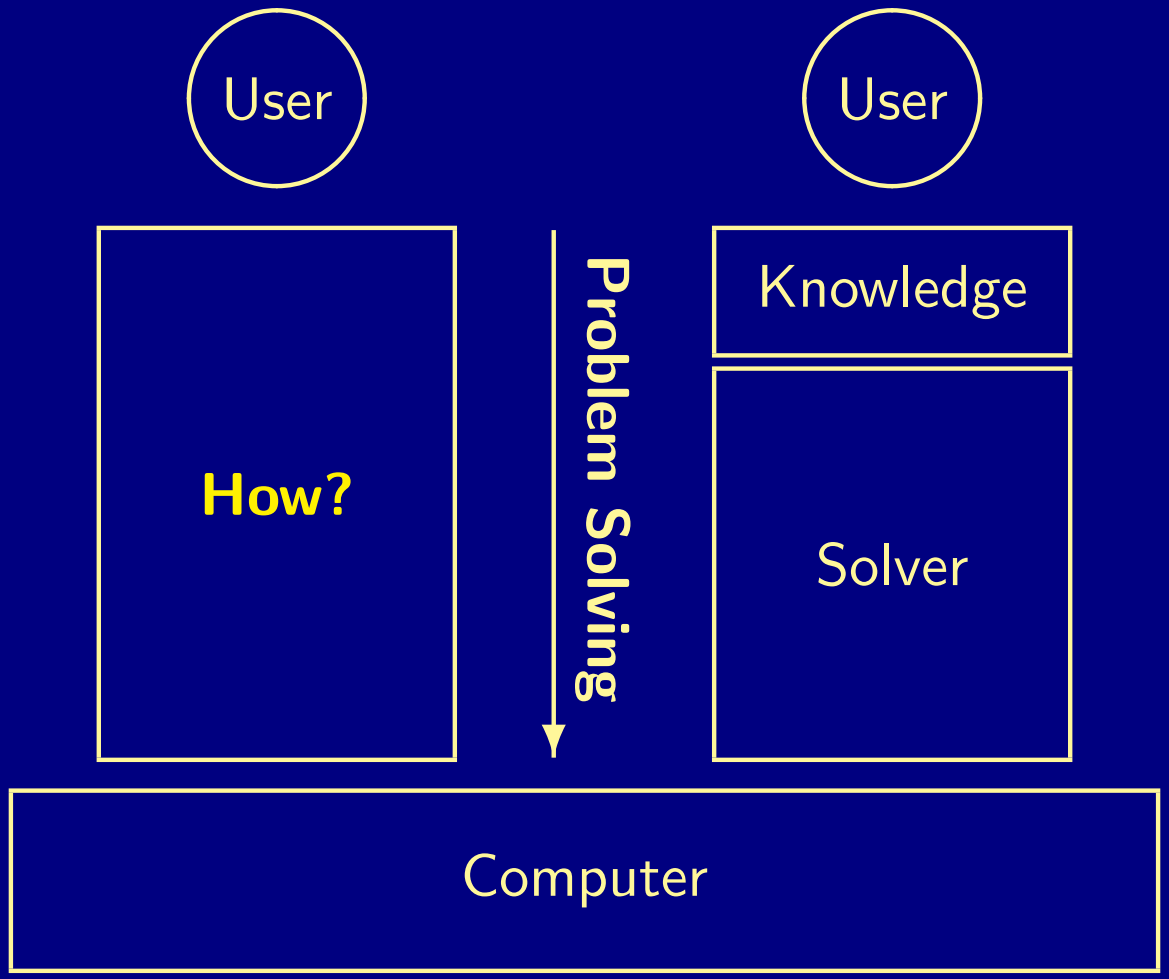

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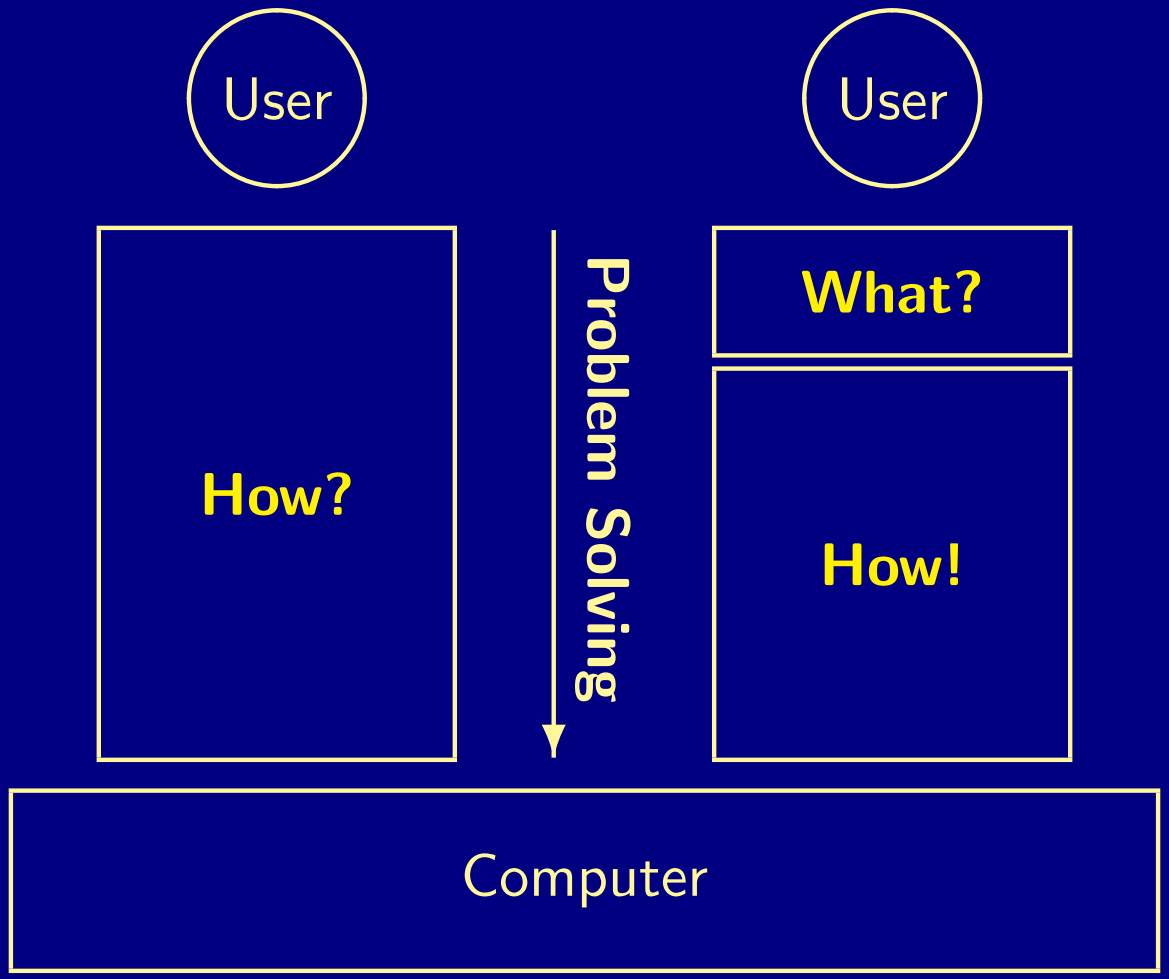

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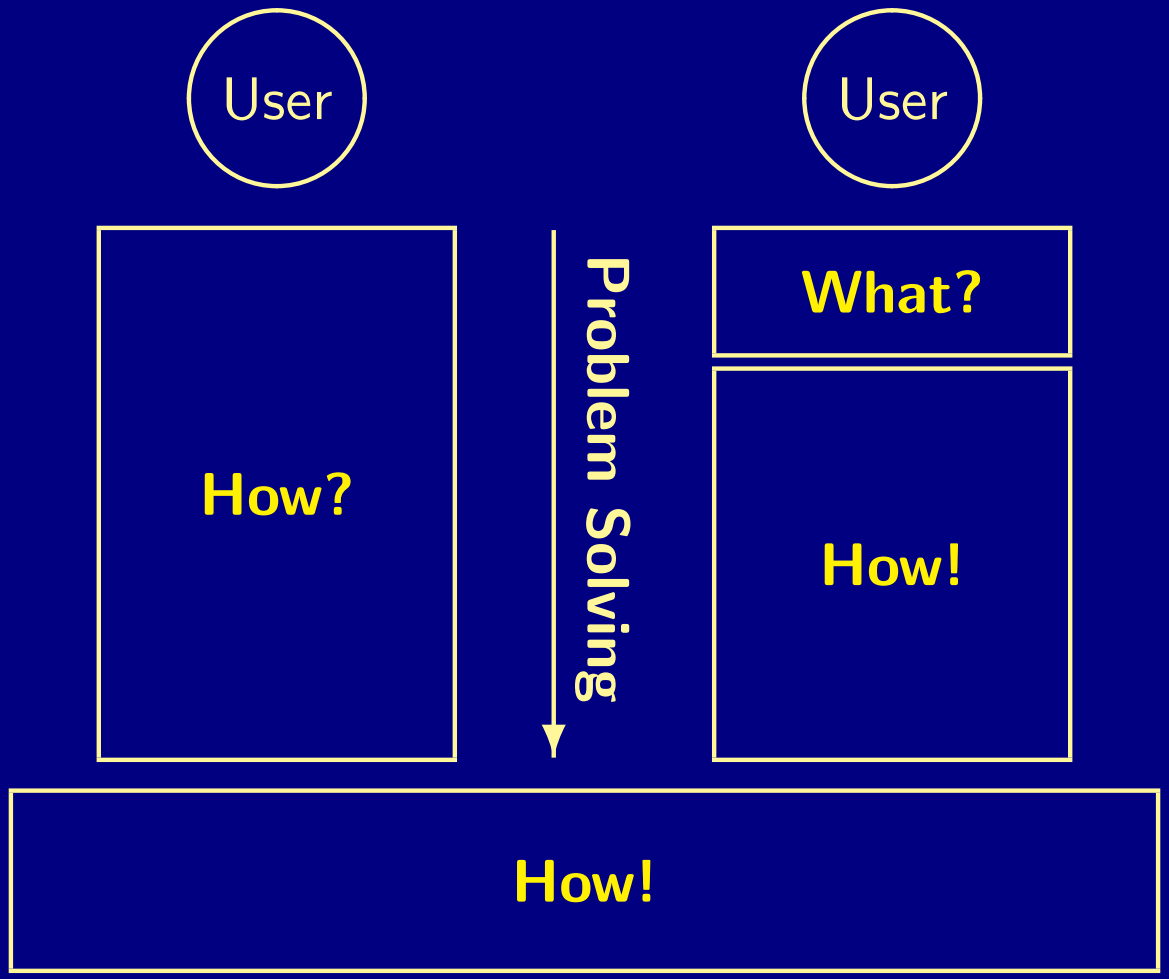

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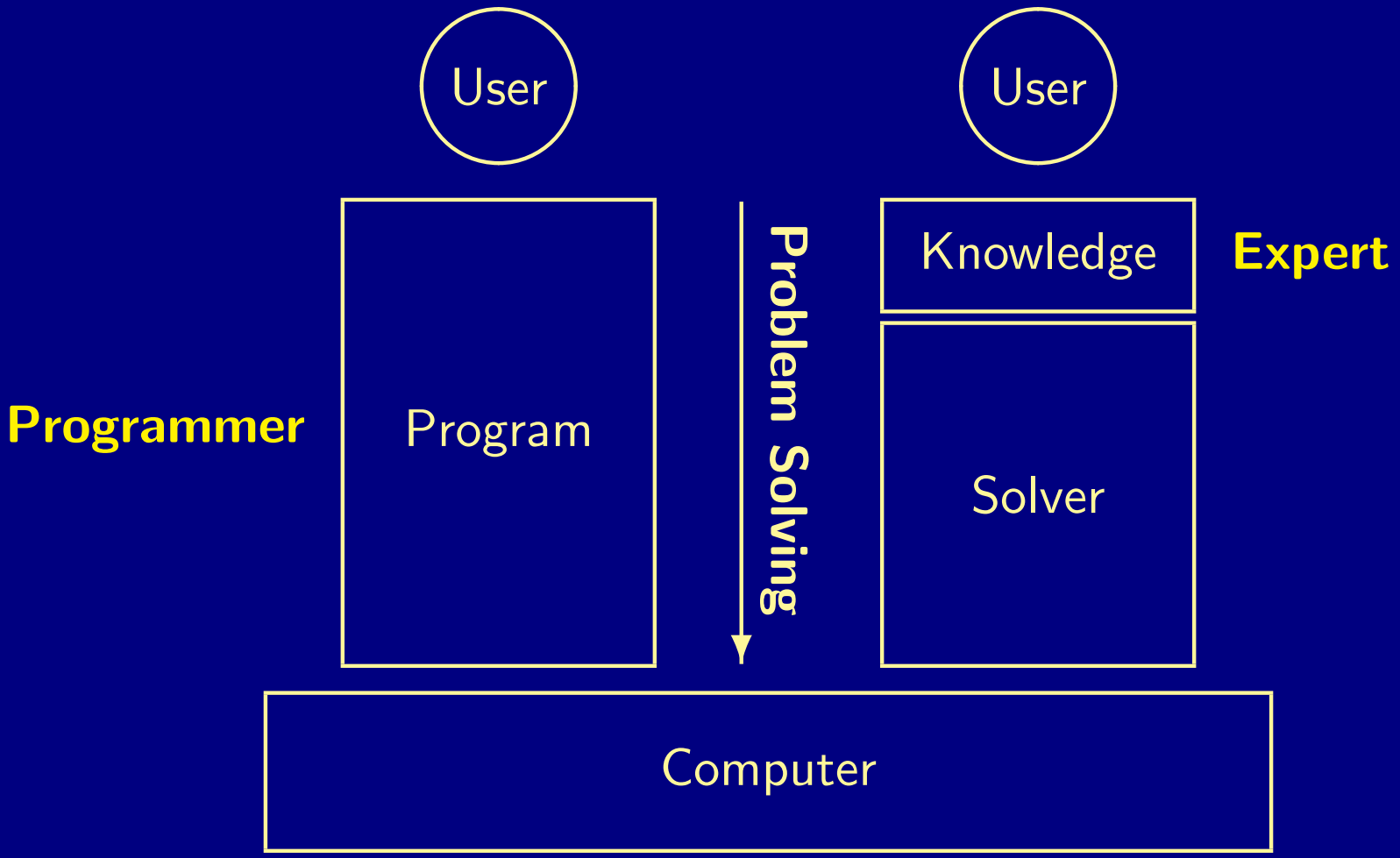

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+ Transparency
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+ Maintainability
+ Reliability

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+ Efficiency
+ Optimality
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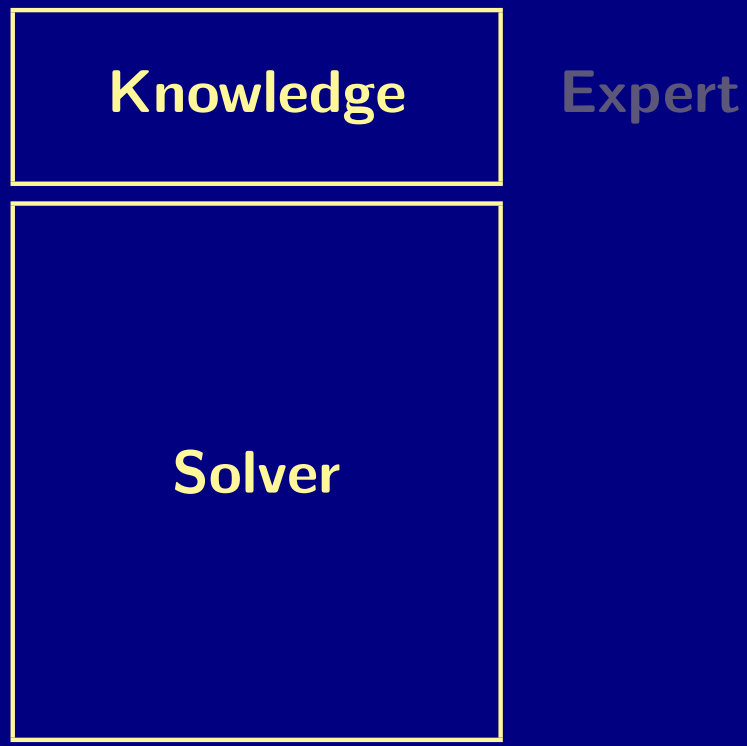

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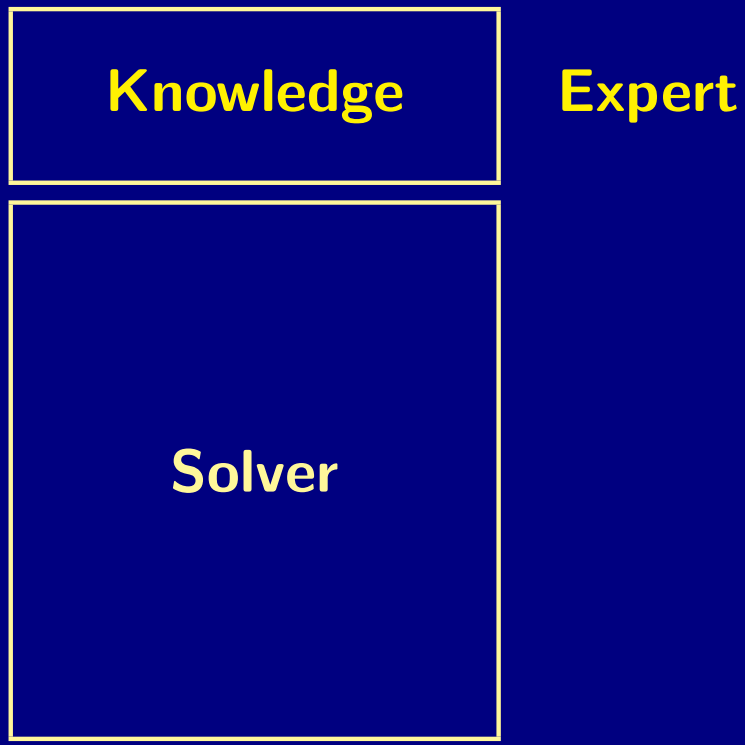

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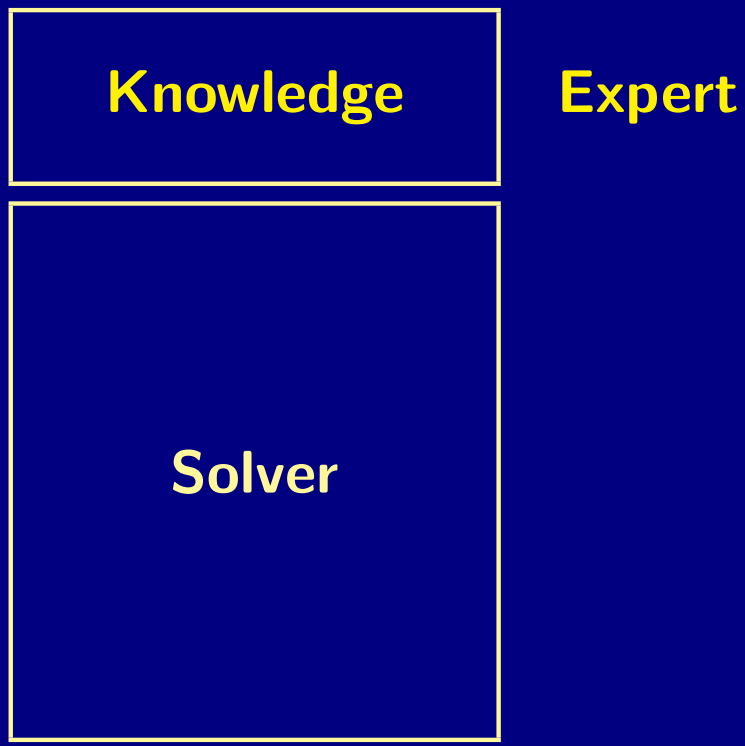

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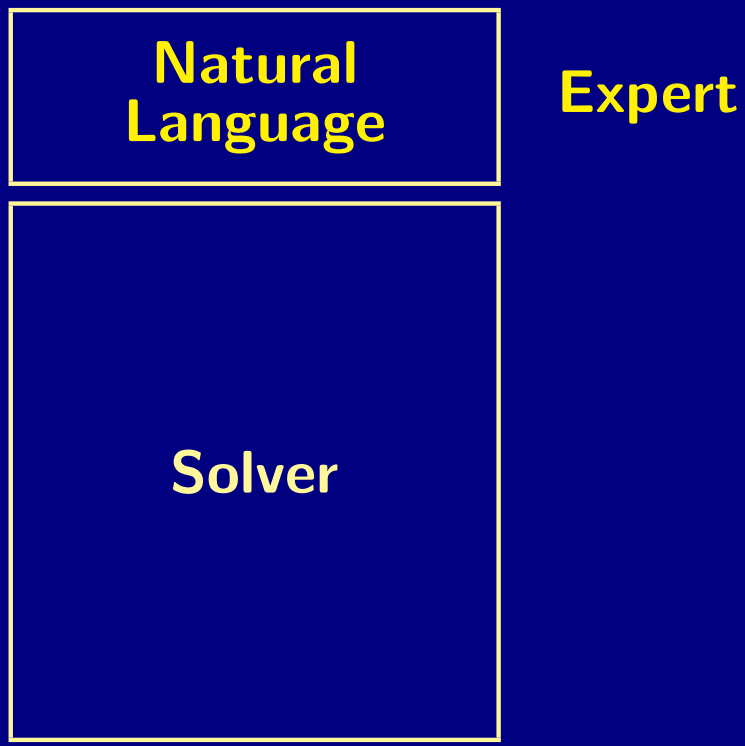

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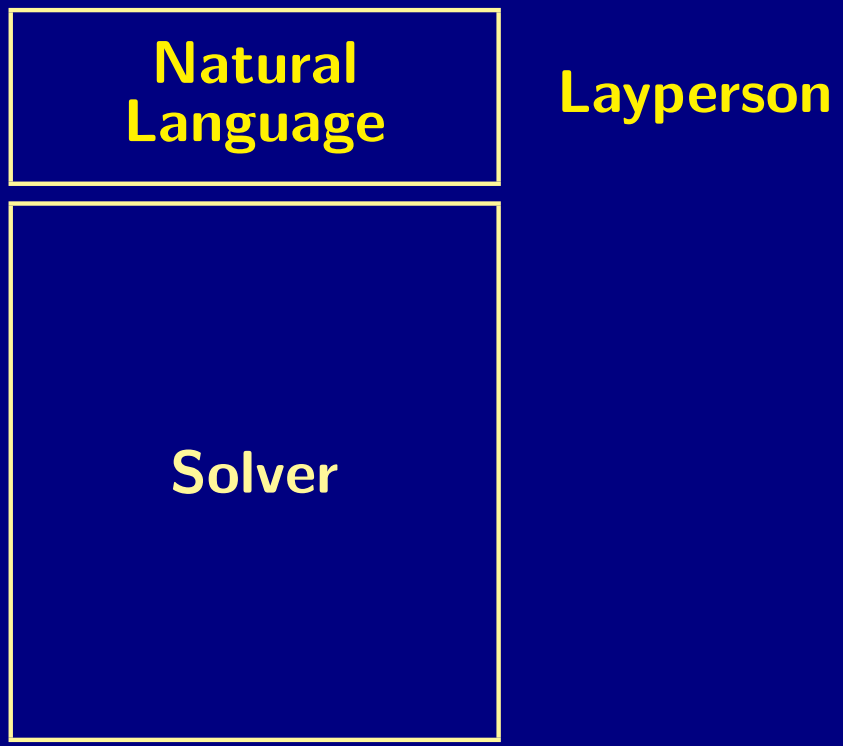

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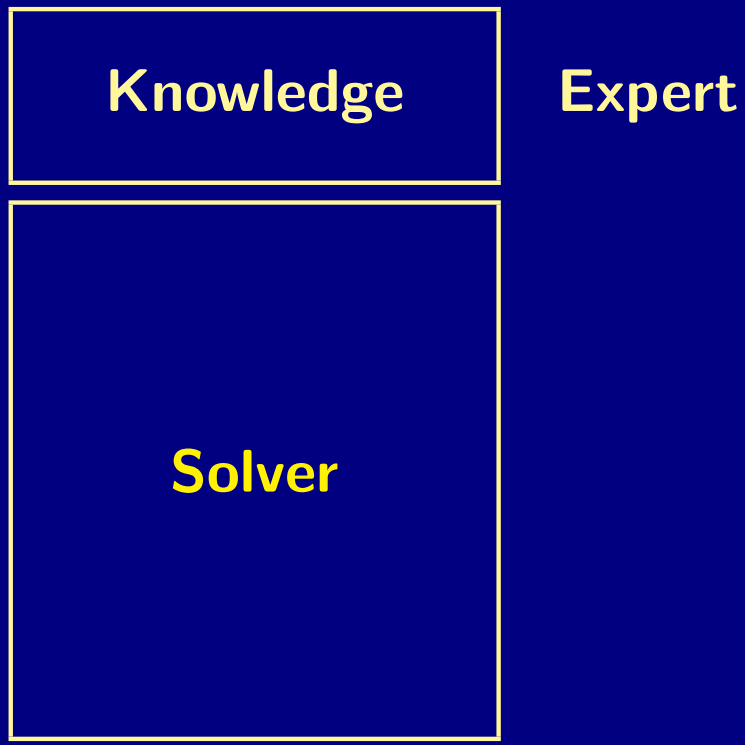

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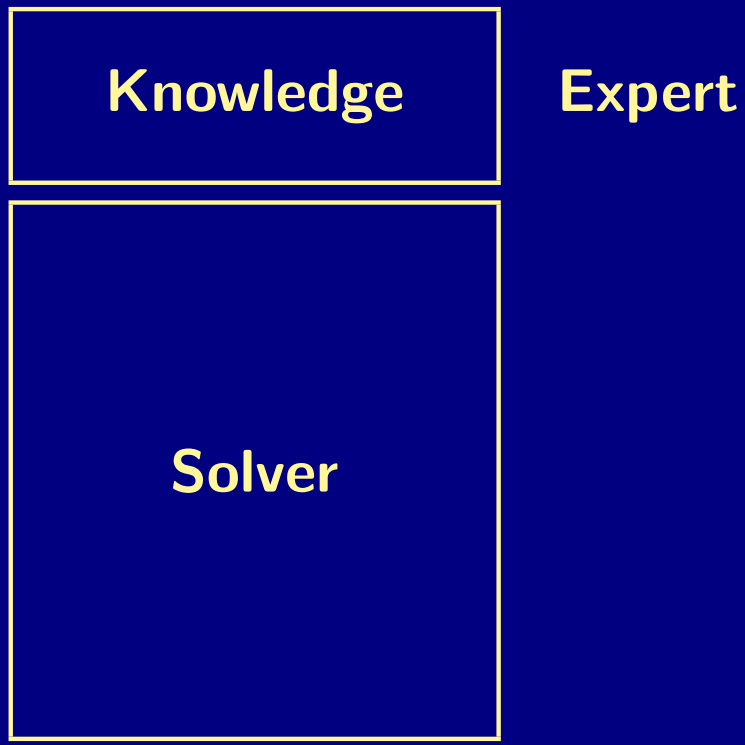

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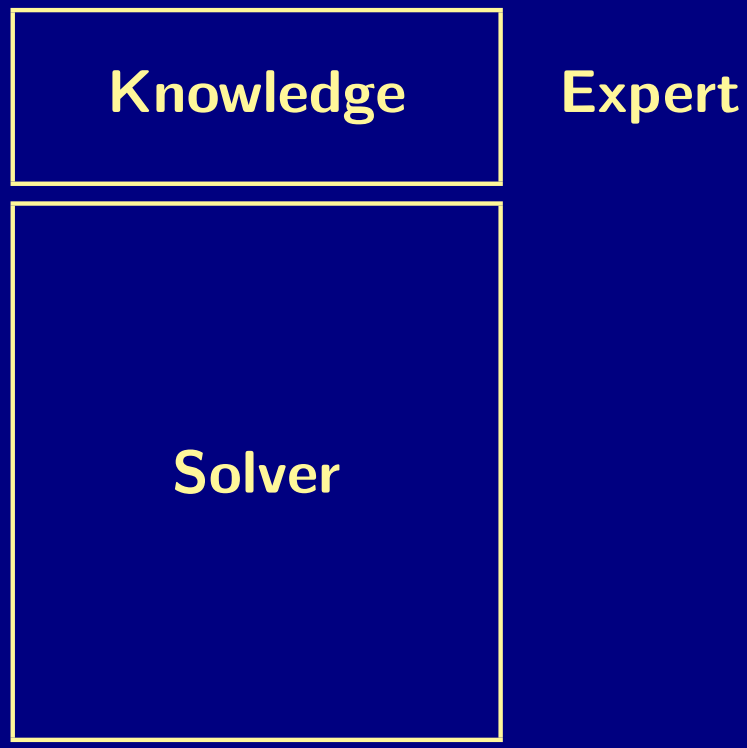

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Answer Set Programming (ASP)

- What is ASP?
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- Where is ASP from?
  - Databases
  - Logic programming
  - Knowledge representation and reasoning
  - Satisfiability solving

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  Examples Sudoku, Configuration, Diagnosis, Music composition, Planning, System design, Time tabling, etc.

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  - Debian, Ubuntu: Linux package configuration
  - Exeura: Call routing
  - Fcc: Radio frequency auction
  - Gioia Tauro: Workforce management
  - Nasa: Decision support for Space Shuttle
  - Siemens: Partner units configuration
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Over 13 months in 2016–17 the US Federal Communications Commission conducted an "incentive auction" to repurpose radio spectrum from broadcast television to wireless internet. In the end, the auction yielded $19.8 billion, $10.05 billion of which was paid to 175 broadcasters for voluntarily relinquishing their licenses across 14 UHF channels. Stations that continued broadcasting were assigned potentially new channels to fit as densely as possible into the channels that remained. The government netted more than $7 billion (used to pay down the national debt) after covering costs. A crucial element of the auction design was the construction of a solver, dubbed SATFC, that determined whether sets of stations could be "repacked" in this way; it needed to run every time a station was given a price quote. This

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  - High performance solvers
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  - Qualitative and quantitative optimization
- Any industrial impact?
  - ASP Tech companies: DLV Systems and **Potassco Solutions**
  - Increasing interest in (large) companies

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Some biased moments in time

- '70/'80 Capturing incomplete information

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- '70/'80 Capturing incomplete information
  - Databases Closed world assumption
  - Logic programming Negation as failure
  - Non-monotonic reasoning
    Auto-epistemic and Default logics, Circumscription

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    - Extensions of first-order logic
    - Modalities, fix-points, second-order logic

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    Well-founded and stable models semantics
  - ASP solving
    "Stable models = Well-founded semantics + Branch"

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    - Modeling — Grounding — Solving
    - Icebreakers: `lparse` and `smodels`

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    - Roots: Logic of Here-and-There , G3

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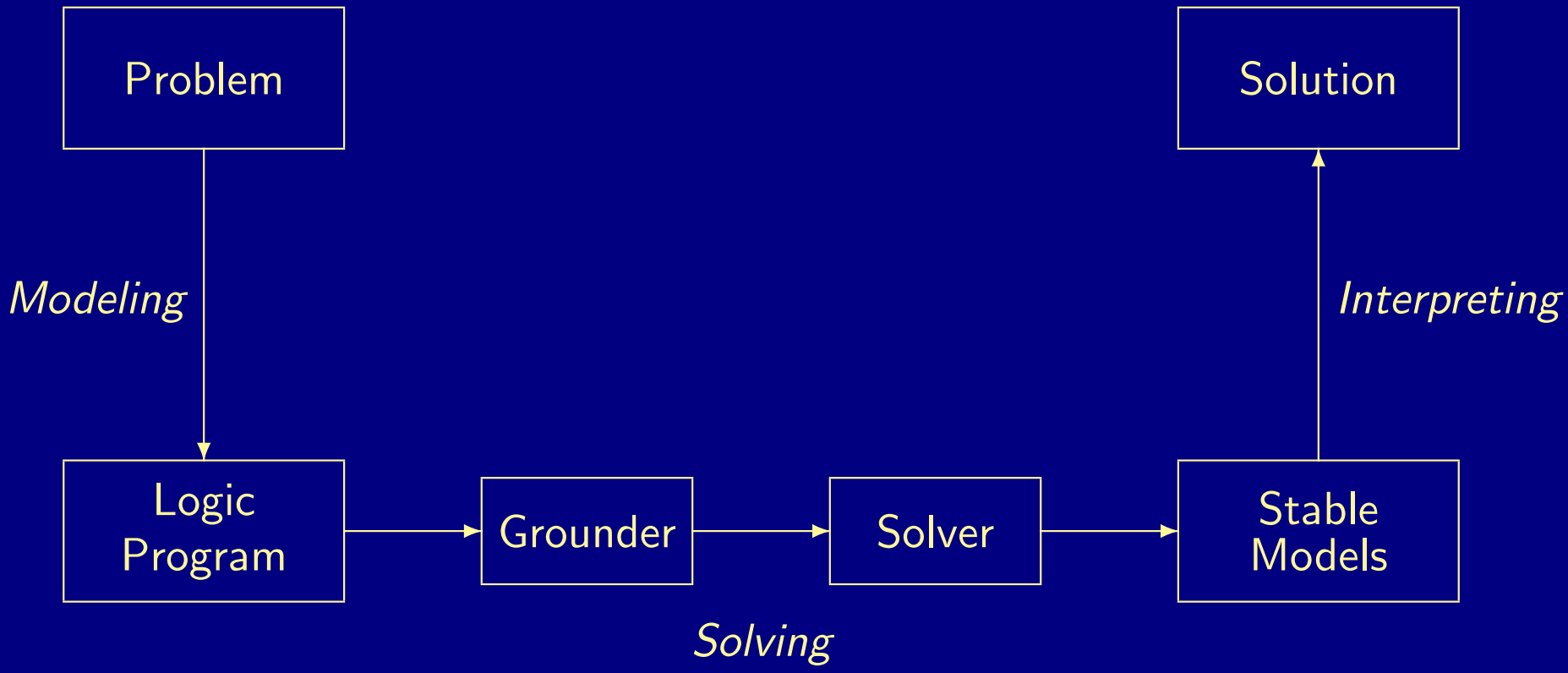
ASP modeling, grounding, and solving



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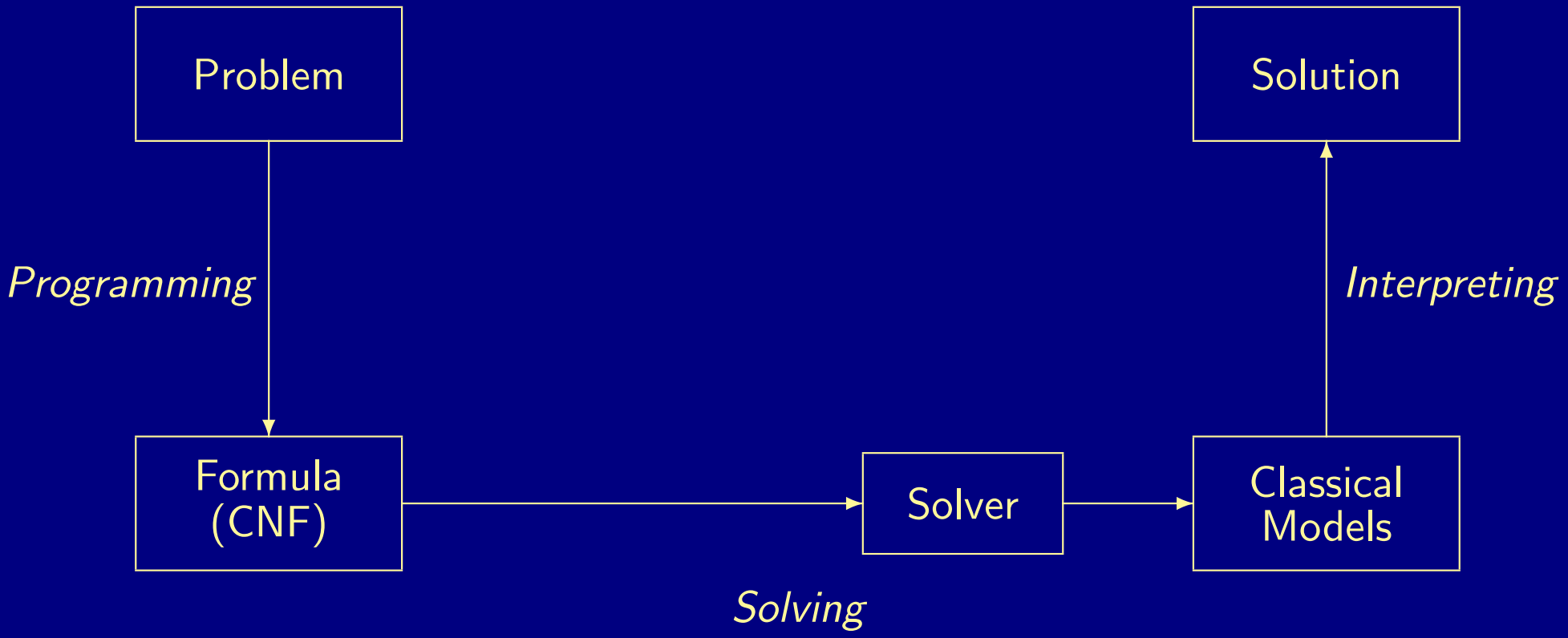
SAT solving



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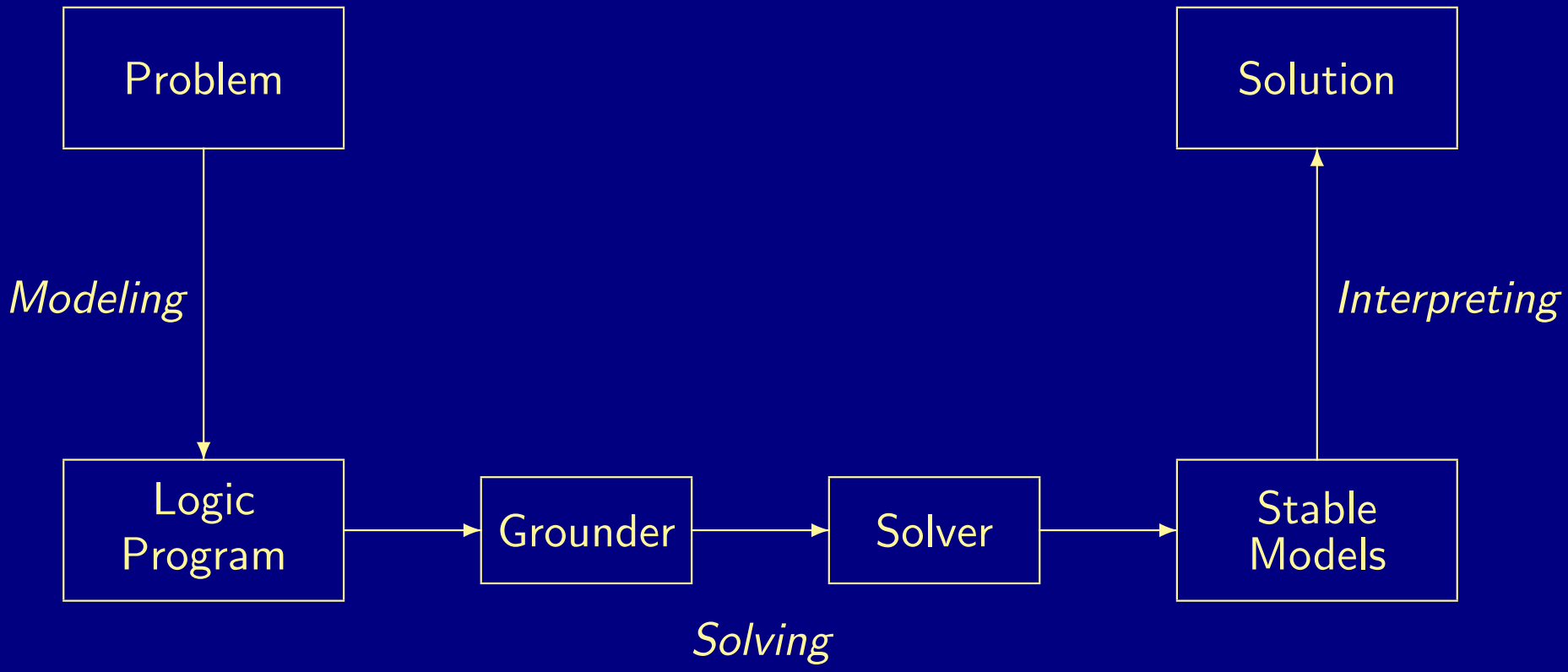
Rooting ASP solving



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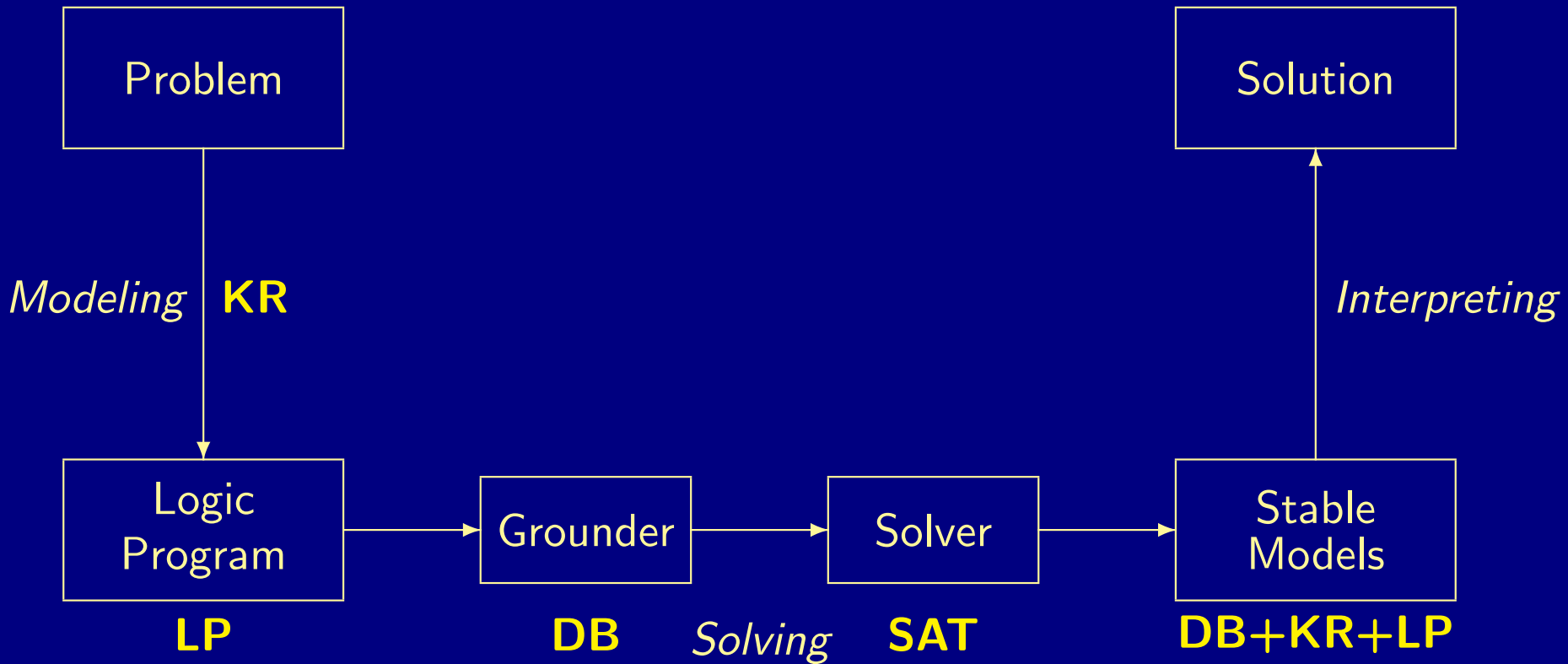

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Two sides of a coin

- ASP as High-level Language
  - Express problem instance as sets of facts
  - Encode problem class as a set of rules
  - Read off solutions from stable models of facts and rules
- ASP as Low-level Language
  - Compile a problem into a set of facts and rules
  - Solve the original problem by solving its compilation
- ASP and Imperative language

  Control continuously changing logic programs

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  - Read off solutions from stable models of facts and rules
- ASP as "Low-level" Language
  - Compile a problem instance into a set of facts
  - Encode problem class as a set of rules
  - Solve the original problem by solving its compilation
- ASP and Imperative language
  - Control continuously changing logic programs

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Two and a half sides of a coin

- ASP as High-level Language
  - Express problem instance as sets of facts
  - Encode problem class as a set of rules
  - Read off solutions from stable models of facts and rules
- ASP as "Low-level" Language
  - Compile a problem instance into a set of facts
  - Encode problem class as a set of rules
  - Solve the original problem by solving its compilation
- ASP and Imperative language
  - Control continuously changing logic programs

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Foundations: Overview

6 Reduct-based characterization

7 Axiomatic characterization

8 Logical characterization

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What is a stable model?

- Vladimir Lifschitz, Twelve Definitions of a Stable Model, [3]

- Reduct-based characterization
- Axiomatic characterization
- Logical characterization

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Outline

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Propositional Normal Logic Programs

- A logic program P is a set of rules of the form

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \ldots, b_m, \neg c_1, \ldots, \neg c_n}_{\text{body}}$$

  - a and all b_i, c_j are atoms (propositional variables)
  - $\leftarrow$, ,, $\neg$ denote if, and, and negation
  - intuitive reading: head must be true if body holds

- Semantics given by stable models, informally,
  models of P justifying each true atom by some rule in P

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Logic Programs

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Normal Logic Programs

- A logic program P is a set of rules of the form

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \ldots, b_m, \neg c_1, \ldots, \neg c_n}_{\text{body}}$$

  - a and all b_i, c_j are atoms (propositional variables)
  - $\leftarrow$, ,, $\neg$ denote if, and, and negation
  - intuitive reading: head must be true if body holds

- Semantics given by stable models, informally,
  models of P justifying each true atom by some rule in P

- Disclaimer The following formalities apply to normal logic programs

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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\neg\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$(\neg\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\neg\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
F	**T**	**T**	$(\neg\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	**F**	**F**	$(\neg\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
T	**F**	**T**	$(\neg\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
T	**T**	**F**	$(\neg\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
T	**T**	**T**	$(\neg\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
F	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
T	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
T	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
T	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

Potassco

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	F	T	$(\mathbf{T} \rightarrow \mathbf{F}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	T	F	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
F	T	T	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	F	F	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
T	F	T	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
T	T	F	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{F})$
T	T	T	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	**F**	**T**	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge \mathbf{F}$
F	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	**F**	**F**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
T	**F**	**T**	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
T	**T**	**F**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge \mathbf{F}$
T	**T**	**T**	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{F})$
F	F	T	$\mathbf{F} \wedge (\mathbf{F} \rightarrow \mathbf{T})$
F	T	F	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge \mathbf{F}$
F	T	T	$(\mathbf{F} \rightarrow \mathbf{F}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$
T	F	F	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{F})$
T	F	T	$(\mathbf{T} \rightarrow \mathbf{T}) \wedge (\mathbf{F} \rightarrow \mathbf{T})$
T	T	F	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge \mathbf{F}$
T	T	T	$(\mathbf{F} \rightarrow \mathbf{T}) \wedge (\mathbf{T} \rightarrow \mathbf{T})$

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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	F $\wedge$ T
F	F	T	F $\wedge$ T
F	T	F	T $\wedge$ F
F	T	T	T $\wedge$ T
T	F	F	T $\wedge$ T
T	F	T	T $\wedge$ T
T	T	F	T $\wedge$ F
T	T	T	T $\wedge$ T

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Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

Potassco

Some truth tabling, back to SAT

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

- We get four models: $\{b, c\}$, $\{a\}$, $\{a, c\}$, and $\{a, b, c\}$

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Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

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Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\neg \mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**T**	$(\neg \mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**F**	$(\neg \mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**T**	$(\neg \mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**F**	$(\neg \mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**T**	$(\neg \mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**F**	$(\neg \mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**T**	$(\neg \mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$

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Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**T**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**F**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**T**	$(\mathbf{T} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$

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Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c)$
F	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
T	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$

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Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c)$
F	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
F	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
T	**T**	**F**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$
T	**T**	**T**	$(\mathbf{F} \rightarrow a) \wedge (b \rightarrow c)$

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a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c)$
F	**T**	**F**	$\mathbf{T} \wedge (b \rightarrow c)$
F	**T**	**T**	$\mathbf{T} \wedge (b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
T	**T**	**F**	$\mathbf{T} \wedge (b \rightarrow c)$
T	**T**	**T**	$\mathbf{T} \wedge (b \rightarrow c)$

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Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$
F	**F**	**F**	$a \wedge (b \rightarrow c)$
F	**F**	**T**	$a \wedge (b \rightarrow c)$
F	**T**	**F**	$(b \rightarrow c)$
F	**T**	**T**	$(b \rightarrow c)$
T	**F**	**F**	$a \wedge (b \rightarrow c)$
T	**F**	**T**	$a \wedge (b \rightarrow c)$
T	**T**	**F**	$(b \rightarrow c)$
T	**T**	**T**	$(b \rightarrow c)$

Reduct

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Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	Reduct
F	F	F	$a \wedge (b \rightarrow c)$	
F	F	T	$a \wedge (b \rightarrow c)$	
F	T	F	$(b \rightarrow c)$	
F	T	T	$(b \rightarrow c)$	
T	F	F	$a \wedge (b \rightarrow c)$	
T	F	T	$a \wedge (b \rightarrow c)$	
T	T	F	$(b \rightarrow c)$	
T	T	T	$(b \rightarrow c)$	

Potassco

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	**F**	**F**	$a \wedge (b \rightarrow c)$	
F	**F**	**T**	$a \wedge (b \rightarrow c)$	
F	**T**	**F**	$(b \rightarrow c)$	
F	**T**	**T**	$(b \rightarrow c)$	$\models$
T	**F**	**F**	$a \wedge (b \rightarrow c)$	$\models a$
T	**F**	**T**	$a \wedge (b \rightarrow c)$	$\models a$
T	**T**	**F**	$(b \rightarrow c)$	
T	**T**	**T**	$(b \rightarrow c)$	$\models$

Reduct

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Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	F	F	$a \wedge (b \rightarrow c)$	
F	F	T	$a \wedge (b \rightarrow c)$	
F	T	F	$(b \rightarrow c)$	
F	T	T	$(b \rightarrow c)$	$\models$
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$
T	F	T	$a \wedge (b \rightarrow c)$	$\models a$
T	T	F	$(b \rightarrow c)$	
T	T	T	$(b \rightarrow c)$	$\models$

Reduct

Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	F	F	$a \wedge (b \rightarrow c)$	$\models a$
F	F	T	$a \wedge (b \rightarrow c)$	$\models a$
F	T	F	$(b \rightarrow c)$	$\models$
F	T	T	$(b \rightarrow c)$	$\models$
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$
T	F	T	$a \wedge (b \rightarrow c)$	$\models a$
T	T	F	$(b \rightarrow c)$	$\models$
T	T	T	$(b \rightarrow c)$	$\models$

Reduct

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Some truth tabling, and now ASP

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F	F	F	$a \wedge (b \rightarrow c)$	
F	F	T	$a \wedge (b \rightarrow c)$	
F	T	F	$(b \rightarrow c)$	
F	T	T	$(b \rightarrow c)$	
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$ Stable model
T	F	T	$a \wedge (b \rightarrow c)$	
T	T	F	$(b \rightarrow c)$	
T	T	T	$(b \rightarrow c)$	

Reduct

- We get one stable model: $\{a\}$

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Some truth tabling, and now ASP

a	b	c	$(\neg b \rightarrow a) \wedge (b \rightarrow c)$	
F	F	F	$a \wedge (b \rightarrow c)$	
F	F	T	$a \wedge (b \rightarrow c)$	
F	T	F	$(b \rightarrow c)$	
F	T	T	$(b \rightarrow c)$	
T	F	F	$a \wedge (b \rightarrow c)$	$\models a$ Stable model
T	F	T	$a \wedge (b \rightarrow c)$	
T	T	F	$(b \rightarrow c)$	
T	T	T	$(b \rightarrow c)$	
			Reduct	

- We get one stable model: $\{a\}$
- Stable models = Smallest models of (respective) reducts

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Outline

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Propositional Normal Logic Programs

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  - intuitive reading: head must be true if body holds

- Semantics given by stable models, informally, sets X of atoms such that
  - X is a (classical) model of P and
  - each atom in X is justified by some rule in P

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Logic Programs

- A logic program P is a set of rules of the form

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Logic Programs

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Logic Programs as Propositional Formulas

$$P = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

$$CF(P) = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow (a \wedge \neg c) \vee y \quad y \leftarrow x \wedge b\} \cup \{c \leftrightarrow \bot\}$$

$$LF(P) = \{(x \vee y) \rightarrow a \wedge \neg c\}$$

Classical models of $CF(P)$:

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- Unsupported atoms
- Unfounded atoms

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Logic Programs as Propositional Formulas

$$P = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

$$RF(P) = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow (a \wedge \neg c) \vee y \quad y \leftarrow x \wedge b\}$$

$$\cup \{c \leftrightarrow \bot\}$$

$$LF(P) = \{(x \vee y) \rightarrow a \wedge \neg c\}$$

Classical models of $RF(P)$: (only true atoms shown)

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- Unsupported atoms
- Unfounded atoms

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Logic Programs as Propositional Formulas

$$P = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

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$$\cup \{c \leftrightarrow \bot\}$$

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Classical models of $RF(P)$:

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- Unsupported atoms
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Potassco

Logic Programs as Propositional Formulas

$$P = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

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Classical models of $RF(P)$:

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- Unsupported atoms
- Unfounded atoms

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Logic Programs as Propositional Formulas

$$P = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

$$CF(P) = \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\} \cup \{c \leftrightarrow \bot\}$$

$$LF(P) = \{(x \vee y) \rightarrow a \wedge \neg c\}$$

Classical models of $CF(P)$:

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- ~~Unsupported atoms~~
- Unfounded atoms

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Logic Programs as Propositional Formulas

$$P = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

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Classical models of $CF(P)$:

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- ~~Unsupported atoms~~
- Unfounded atoms

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Logic Programs as Propositional Formulas

$$P = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

$$CF(P) = \{a \leftrightarrow \neg b \quad b \leftrightarrow \neg a \quad x \leftrightarrow (a \wedge \neg c) \vee y \quad y \leftrightarrow x \wedge b\} \cup \{c \leftrightarrow \bot\}$$

$$LF(P) = \{(x \vee y) \rightarrow a \wedge \neg c\}$$

Classical models of $CF(P) \cup LF(P)$:

$\{b\}$, $\{b, c\}$, $\{b, x, y\}$, $\{b, c, x, y\}$, $\{a, c\}$, $\{a, b, c\}$, $\{a, x\}$, $\{a, c, x\}$, $\{a, x, y\}$, $\{a, c, x, y\}$, $\{a, b, x, y\}$, $\{a, b, c, x, y\}$

- ~~Unsupported atoms~~
- ~~Unfounded atoms~~

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Logic Programs as Propositional Formulas

$$P = \{a \leftarrow \neg b \quad b \leftarrow \neg a \quad x \leftarrow a, \neg c \quad x \leftarrow y \quad y \leftarrow x, b\}$$

$$CF(P) = \{a \leftrightarrow (\bigvee_{(a \leftarrow B) \in P} BF(B)) \mid a \in A(P)\}$$

$$BF(B) = \bigwedge_{b \in B \cap A(P)} b \wedge \bigwedge_{\neg c \in B} \neg c$$

$$LF(P) = \{(\bigvee_{a \in L} a) \rightarrow (\bigvee_{a \in L, (a \leftarrow B) \in P, B \cap L = \emptyset} BF(B)) \mid L \in loop(P)\}$$

Classical models of $CF(P) \cup LF(P)$:

Theorem (Lin and Zhao)

Let P be a normal logic program and $X \subseteq A(P)$.
Then, X is a stable model of P iff $X \models CF(P) \cup LF(P)$.

- Size of $CF(P)$ is linear in the size of P
- Size of $LF(P)$ may be exponential in the size of P

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ASP and SAT

- SAT = ASP + Law of the excluded middle
- ASP = SAT + Completion and Loop formulas

- Note Checking whether a propositional formula has a stable model is Σ_P^2-complete

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[1] For instance, '{a}.' stands for '$a \vee \neg a$'.

ASP and SAT

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Outline

Potassco

The logic of Here-and-There (HT)

- An interpretation is a pair $\langle H, T \rangle$ of sets of atoms with $H \subseteq T$
  - H is called "here" and
  - T is called "there"
- Note $\langle H, T \rangle$ is a simplified Kripke structure
- Intuition
  - H represents provably true atoms
  - T represents possibly true atoms
  - atoms not in T are false
- Idea
  - $\langle H, T \rangle \models \varphi \quad \sim \quad \varphi$ is provably true
  - $\langle T, T \rangle \models \varphi \quad \sim \quad \varphi$ is possibly true, that is, classically true

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Potassco

Satisfaction

- $\langle H, T\rangle \models a$ if $a \in H$ for any atom a
- $\langle H, T\rangle \models \varphi \wedge \psi$ if $\langle H, T\rangle \models \varphi$ and $\langle H, T\rangle \models \psi$
- $\langle H, T\rangle \models \varphi \vee \psi$ if $\langle H, T\rangle \models \varphi$ or $\langle H, T\rangle \models \psi$
- $\langle H, T\rangle \models \varphi \rightarrow \psi$ if $\langle X, T\rangle \models \varphi$ implies $\langle X, T\rangle \models \psi$ for both $X = H, T$
- Note $\langle H, T\rangle \models \neg\varphi$ if $\langle T, T\rangle \not\models \varphi$ since $\neg\varphi = \varphi \rightarrow \bot$
- An interpretation $\langle H, T\rangle$ is a model of φ, if $\langle H, T\rangle \models \varphi$

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Potassco

Classical tautologies

H	T	a	$\neg a$	$a \vee \neg a$	$\neg\neg a$	$a \leftarrow \neg\neg a$
$\{a\}$	$\{a\}$	**T**	**F**	**T**	**T**	**T**
$\emptyset$	$\{a\}$	**F**	**F**	**F**	**T**	**F**
$\emptyset$	$\emptyset$	**F**	**T**	**T**	**F**	**T**

Equilibrium models

- A total interpretation $\langle T, T\rangle$ is an equilibrium model of a formula φ, if
  1. $\langle T, T\rangle \models \varphi$,
  2. $\langle H, T\rangle \not\models \varphi$ for all $H \subset T$
- T is called a stable model of φ
- Note $\langle T, T\rangle$ acts as a classical model
- Note $\langle H, T\rangle \models P$ iff $H \models P^T$ (P^T is the reduct of P by T)

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Grounding: Overview

Potassco

Example

$d(a)$
$d(c)$
$d(d)$

$p(a, b)$
$p(b, c)$
$p(c, d)$
$p(X, Z) \leftarrow p(X, Y), p(Y, Z)$

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Ground instantiation

Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructible from $\mathcal{T}$
- A variable-free atom is also called ground
- Ground instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \to \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground instantiation of P: $ground(P) = \bigcup_{r \in P} ground(r)$

Potassco

Ground instantiation

Let P be a logic program

- Let $\mathcal{T}$ be a set of variable-free terms (also called Herbrand universe)
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructible from $\mathcal{T}$ (also called alphabet or Herbrand base)
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Potassco

An example

$$P = \{\ r(a,b) \leftarrow,\ r(b,c) \leftarrow,\ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a, b, c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

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Grounding aims at reducing the ground instantiation

Potassco

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- Grounding aims at reducing the ground instantiation

Potassco

Stable models of programs with Variables

Let P be a normal logic program with variables

- A set X of (ground) atoms is a stable model of P,
  if $Cn(ground(P)^X) = X$

Potassco

Stable models of programs with Variables

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Safety

- ASP systems require that all rules in a program are safe
- A normal rule is safe, if each of its variables also occurs in some positive body literal
- A normal program is safe, if all of its rules are safe

Potassco

Example

$d(a)$
$d(c)$
$d(d)$

$p(a, b)$
$p(b, c)$
$p(c, d)$
$p(X, Z) \leftarrow p(X, Y), p(Y, Z)$

$q(a)$
$q(b)$
$q(X) \leftarrow \neg r(X)$

$r(X) \leftarrow \neg q(X), d(X)$
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Potassco

Example

Safe ?

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Example

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$d(a)$	✔
$d(c)$	✔
$d(d)$	✔
$p(a,b)$	✔
$p(b,c)$	✔
$p(c,d)$	✔
$p(X,Z) \leftarrow p(X,Y), p(Y,Z)$	
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$q(X) \leftarrow \neg r(X)$	
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Example

	Safe ?
$d(a)$	✔
$d(c)$	✔
$d(d)$	✔
$p(a, b)$	✔
$p(b, c)$	✔
$p(c, d)$	✔
$p(X, Z) \leftarrow p(X, Y), p(Y, Z)$	✔
$q(a)$	✔
$q(b)$	✔
$q(X) \leftarrow \neg r(X), d(X)$	✔
$r(X) \leftarrow \neg q(X), d(X)$	✔
$s(X) \leftarrow \neg r(X), p(X, Y), q(Y)$	

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Example

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Solving: Overview

9 Conflict-driven constraint learning

10 Engine

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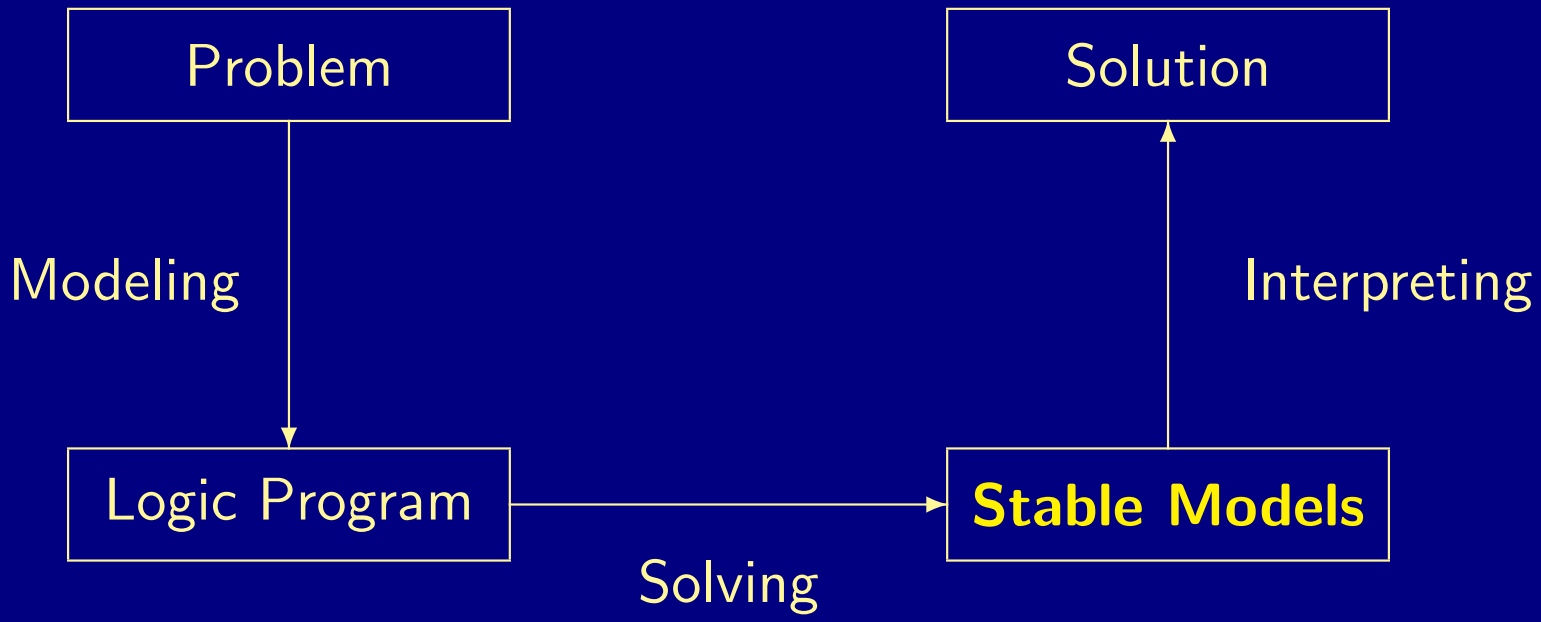
Reasoning modes



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Reasoning modes

- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization
- and combinations of them

† without solution recording

‡ without solution enumeration

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Outline

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Towards conflict-driven search

Boolean constraint solving algorithms pioneered for SAT led to:

- Traditional DPLL-style approach
  (DPLL stands for 'Davis-Putnam-Logemann-Loveland')
  - (Unit) propagation
  - (Chronological) backtracking
  - in ASP, eg *smodels*
- Modern CDCL-style approach
  (CDCL stands for 'Conflict-Driven Constraint Learning')
  - (Unit) propagation
  - Conflict analysis (via resolution)
  - Learning + Backjumping + Assertion
  - in ASP, eg *clasp*

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DPLL-style solving

loop
  propagate // deterministically assign literals
  if no conflict **then**
    if all variables assigned **then return** solution
    else *decide* // non-deterministically assign some literal
  else
    if top-level conflict **then return** unsatisfiable
    else
      backtrack // unassign literals propagated after last decision
      flip // assign complement of last decision literal

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CDCL-style solving

```
loop
    propagate                          // deterministically assign literals
    if no conflict then
        if all variables assigned then return solution
        else decide                    // non-deterministically assign some literal
    else
        if top-level conflict then return unsatisfiable
        else
            analyze                    // analyze conflict and add conflict constraint
            backjump                   // unassign literals until conflict constraint is unit
```

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Outline

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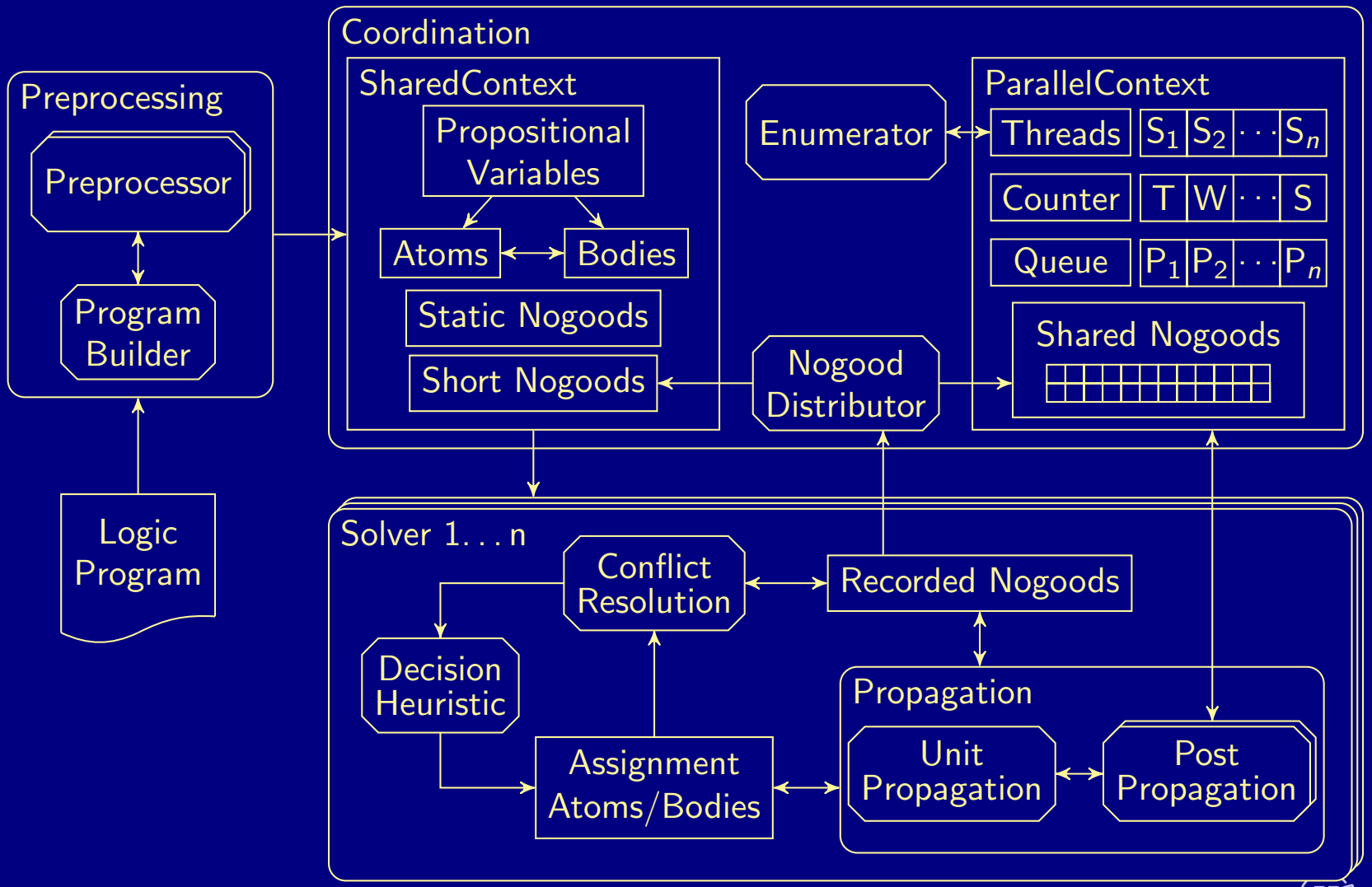
Multi-threaded architecture of *clasp*



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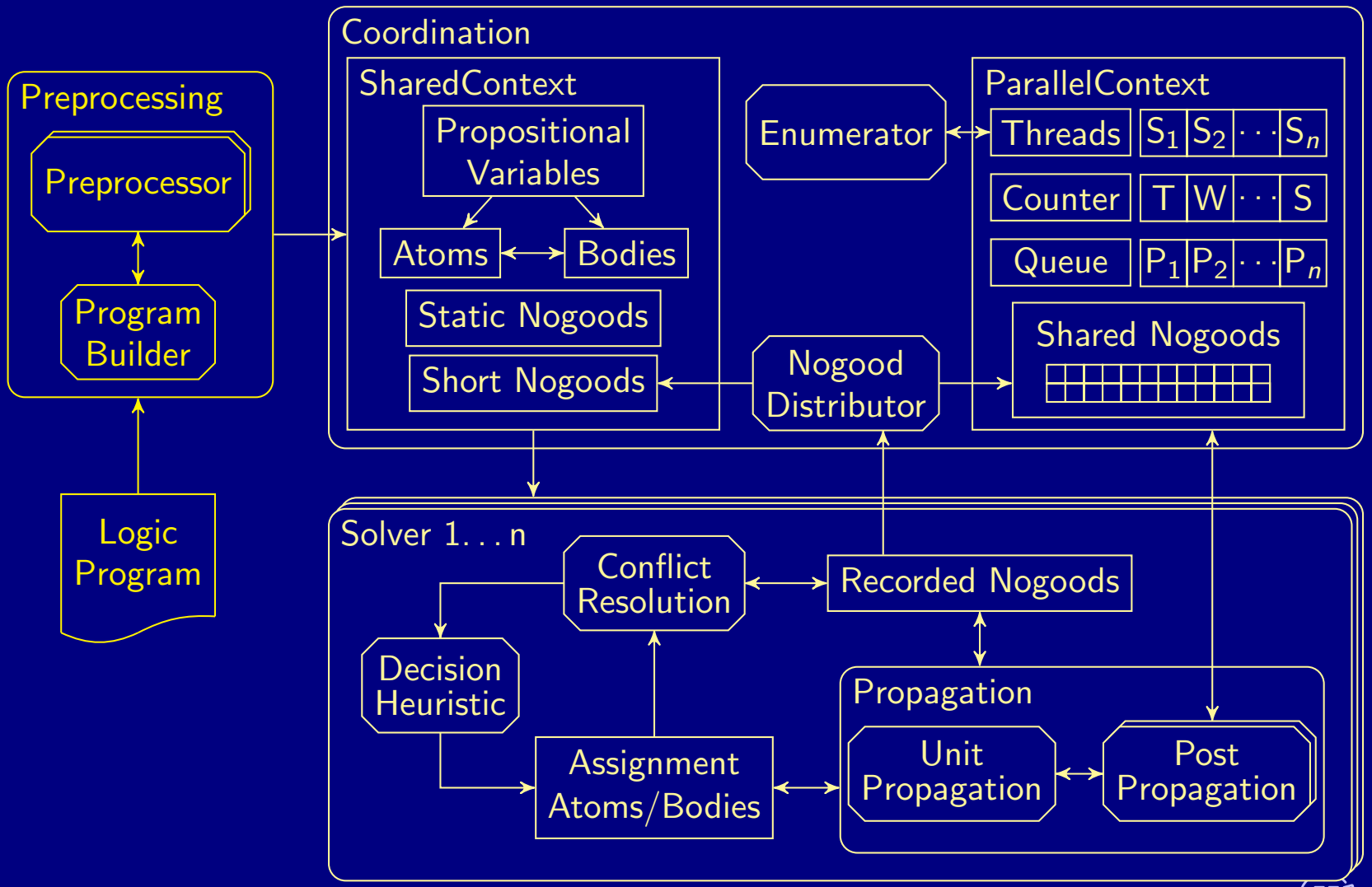

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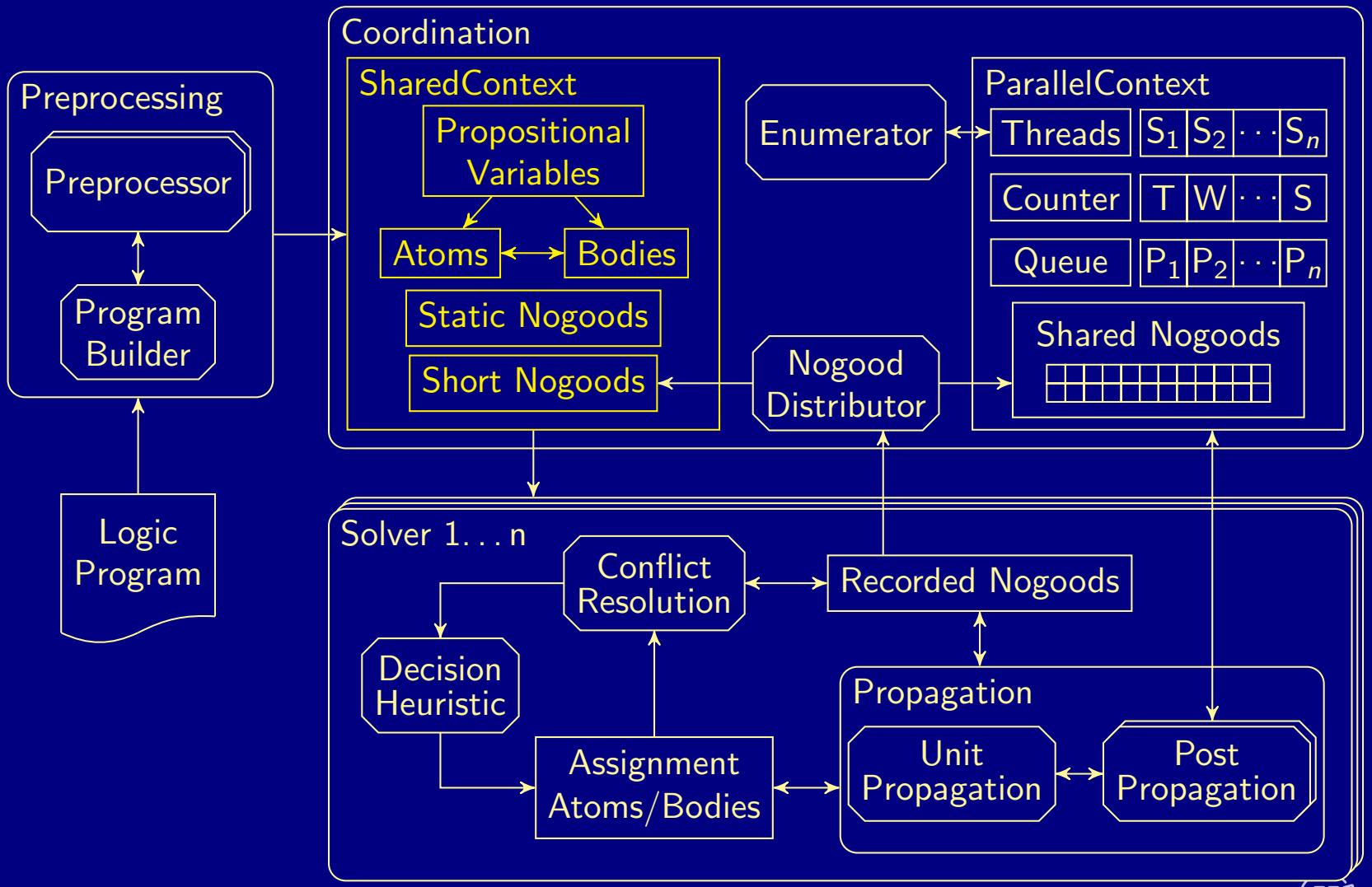

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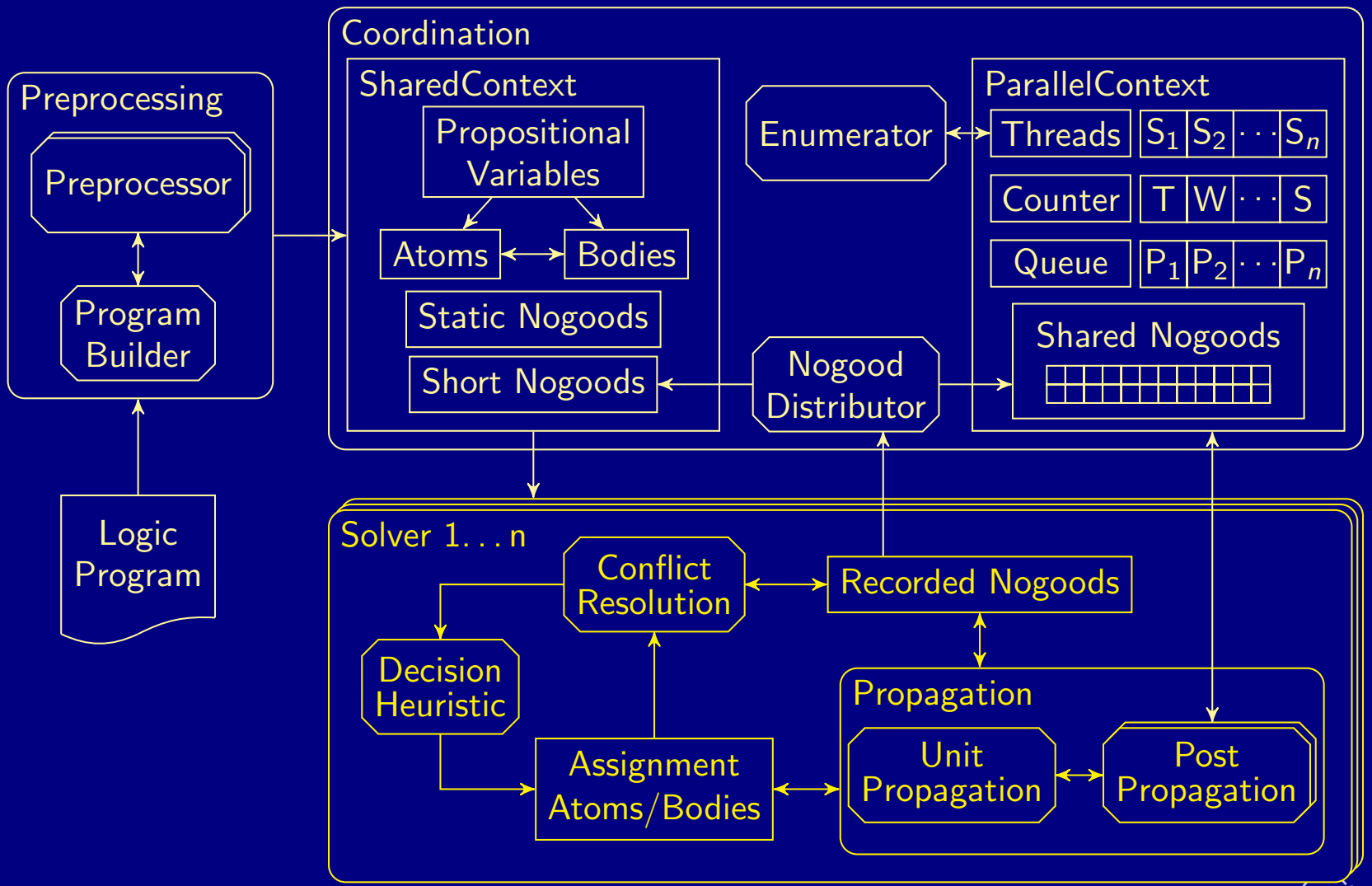

Potassco

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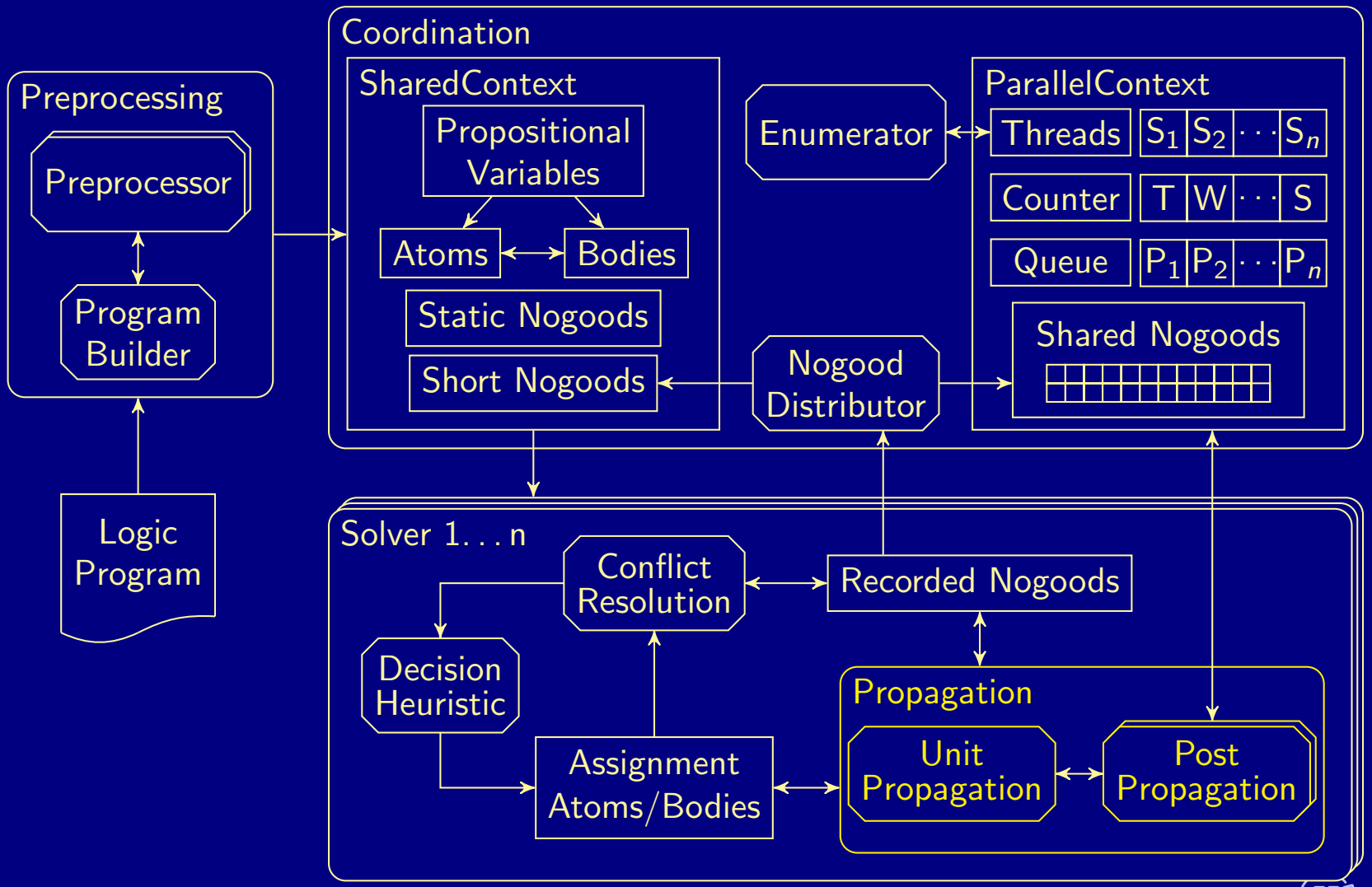

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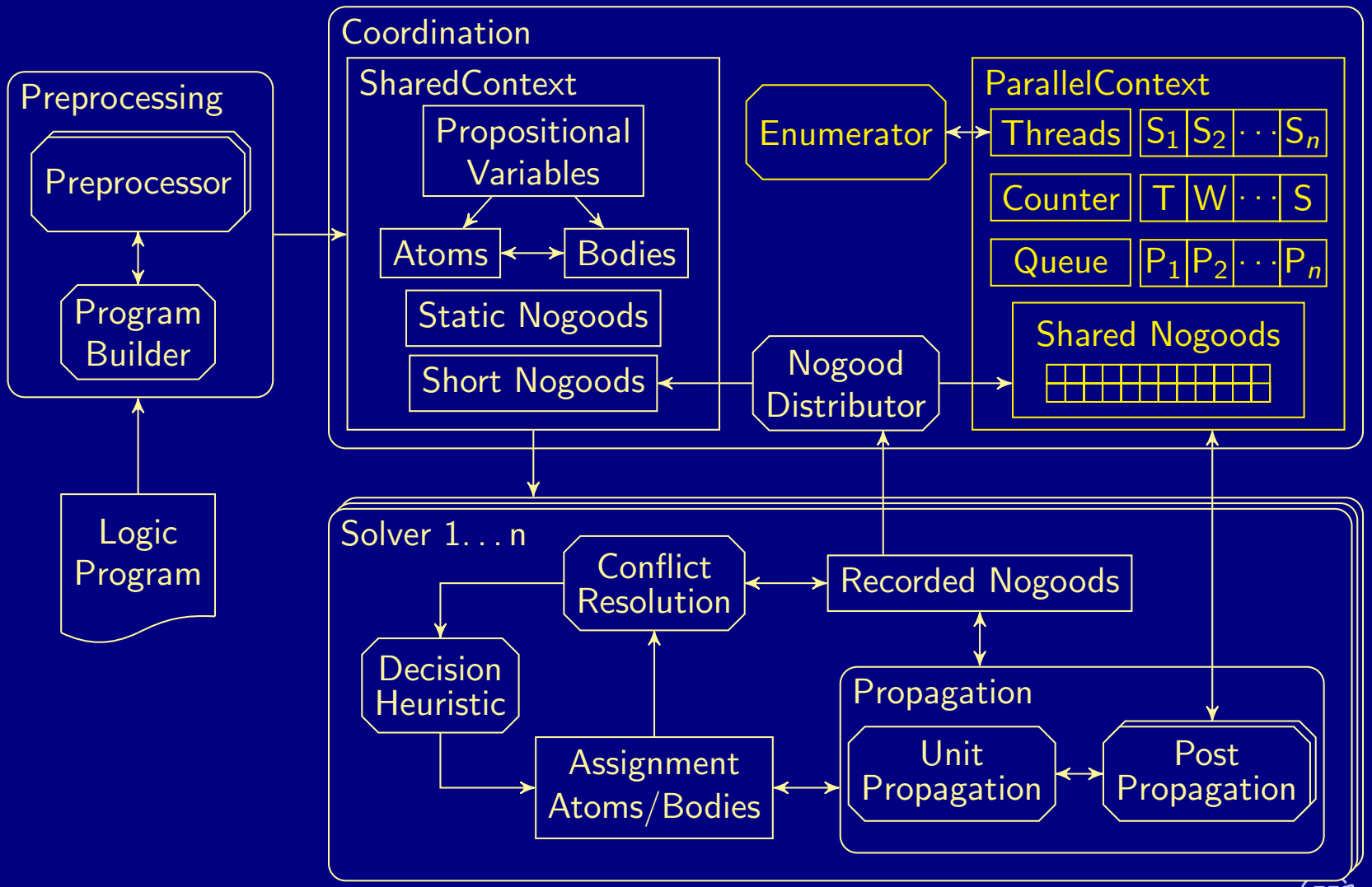

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Modeling: Overview

11 Elaboration tolerance

12 ASP solving process

13 Methodology

14 Case studies

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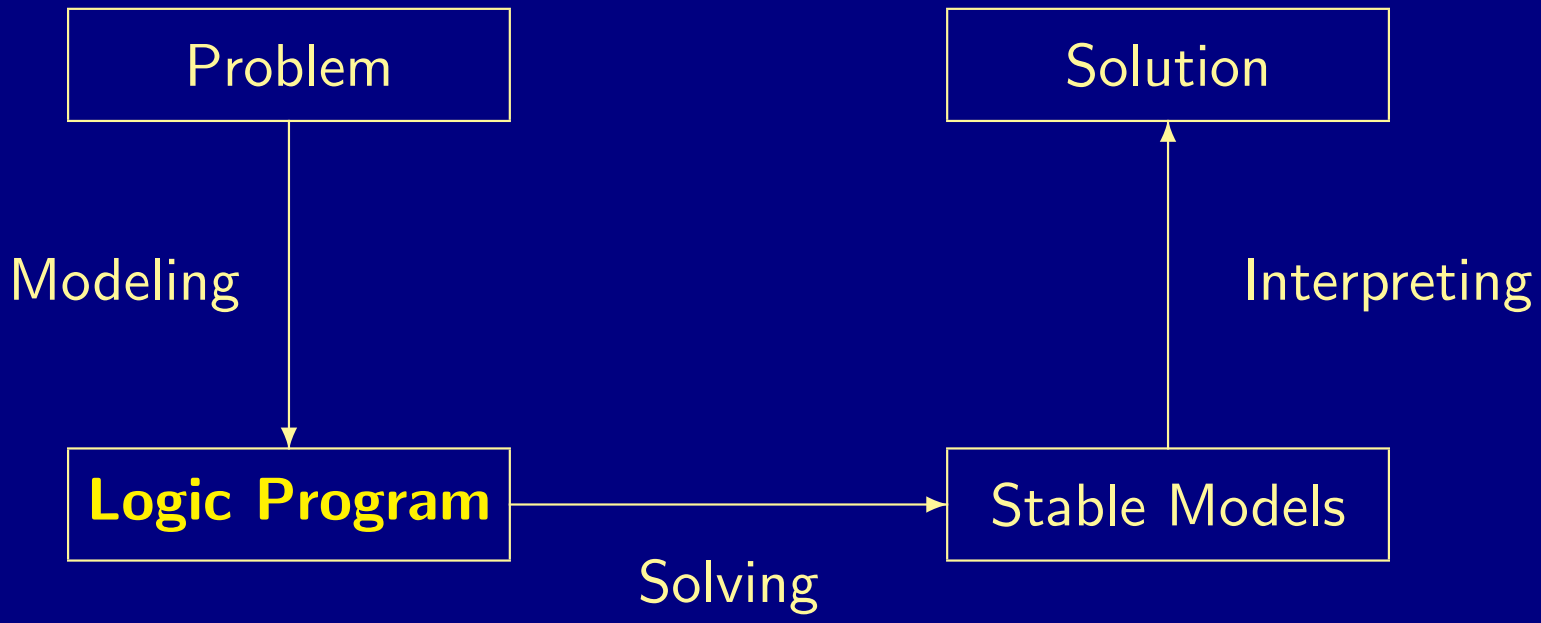
Extended syntax



Potassco

Language constructs

- Facts `q(42).`
- Rules `p(X) :- q(X), not r(X).`
- Conditional literals `p :- q(X) : r(X).`
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- Integrity constraints `:- q(X), p(X).`
- Choice `2 { p(X,Y) : q(X) } 7 :- r(Y).`
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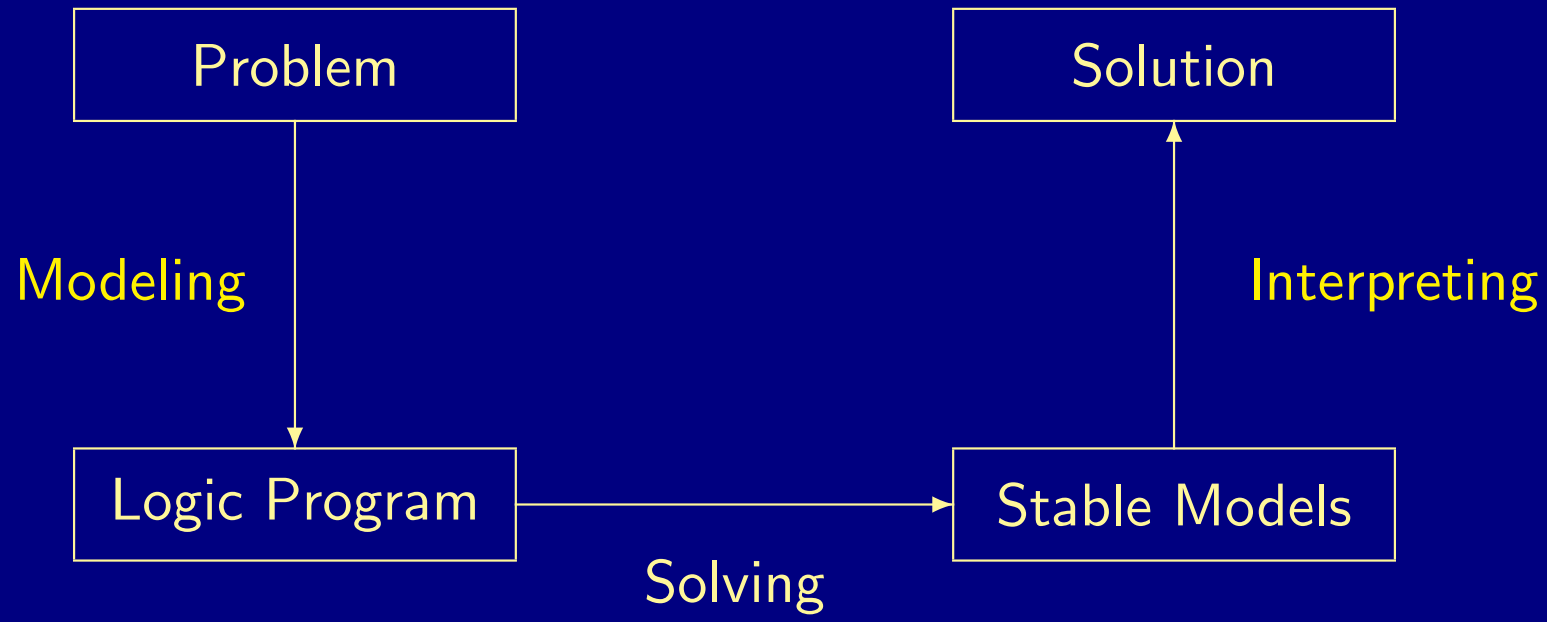
Modeling and Interpreting



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Outline

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Guiding principle

- Elaboration Tolerance (McCarthy, 1998)

  "A formalism is elaboration tolerant [if] *it is convenient to modify a set of facts expressed in the formalism to take into account new phenomena or changed circumstances."*

- Uniform problem representation

  For solving a problem instance **I** of a problem class **C**,
  - **I** is represented as a set of facts $P_{\mathbf{I}}$,
  - **C** is represented as a set of rules $P_{\mathbf{C}}$, and
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Outline

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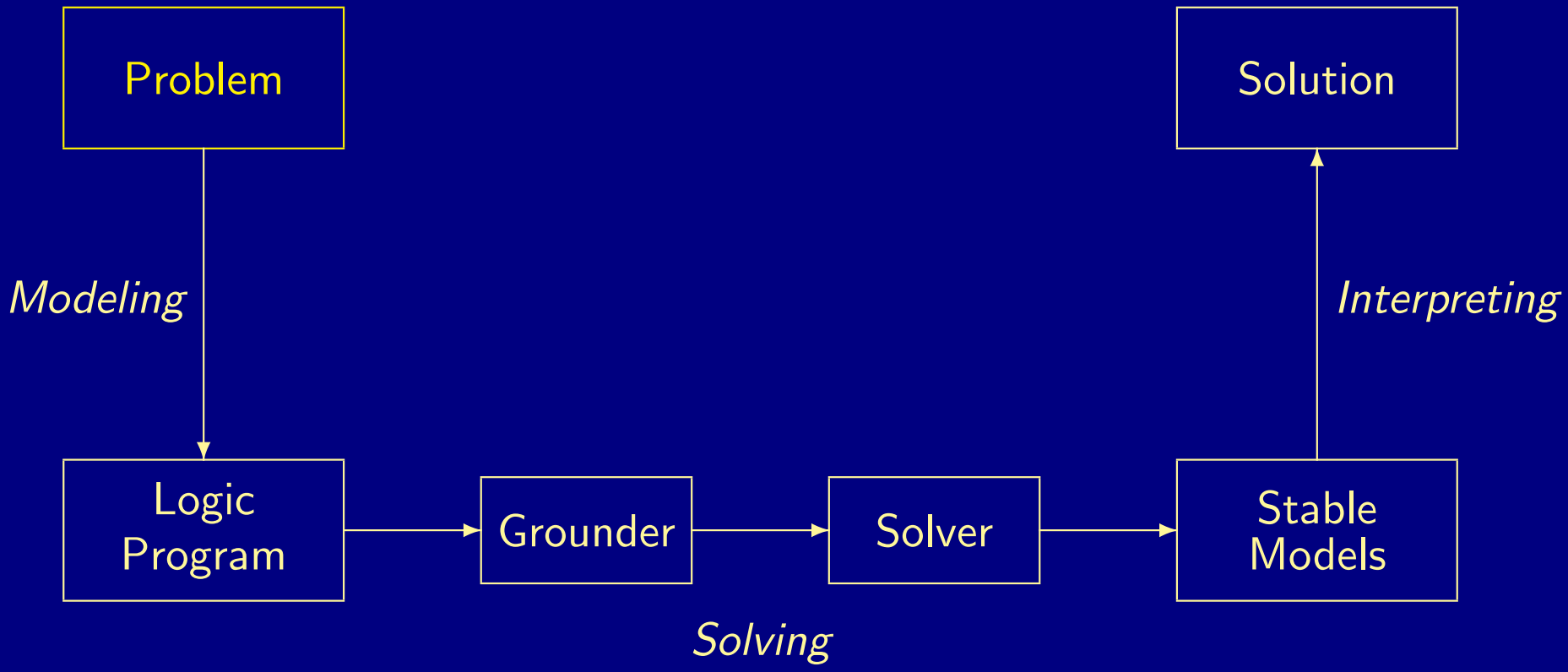
ASP solving process



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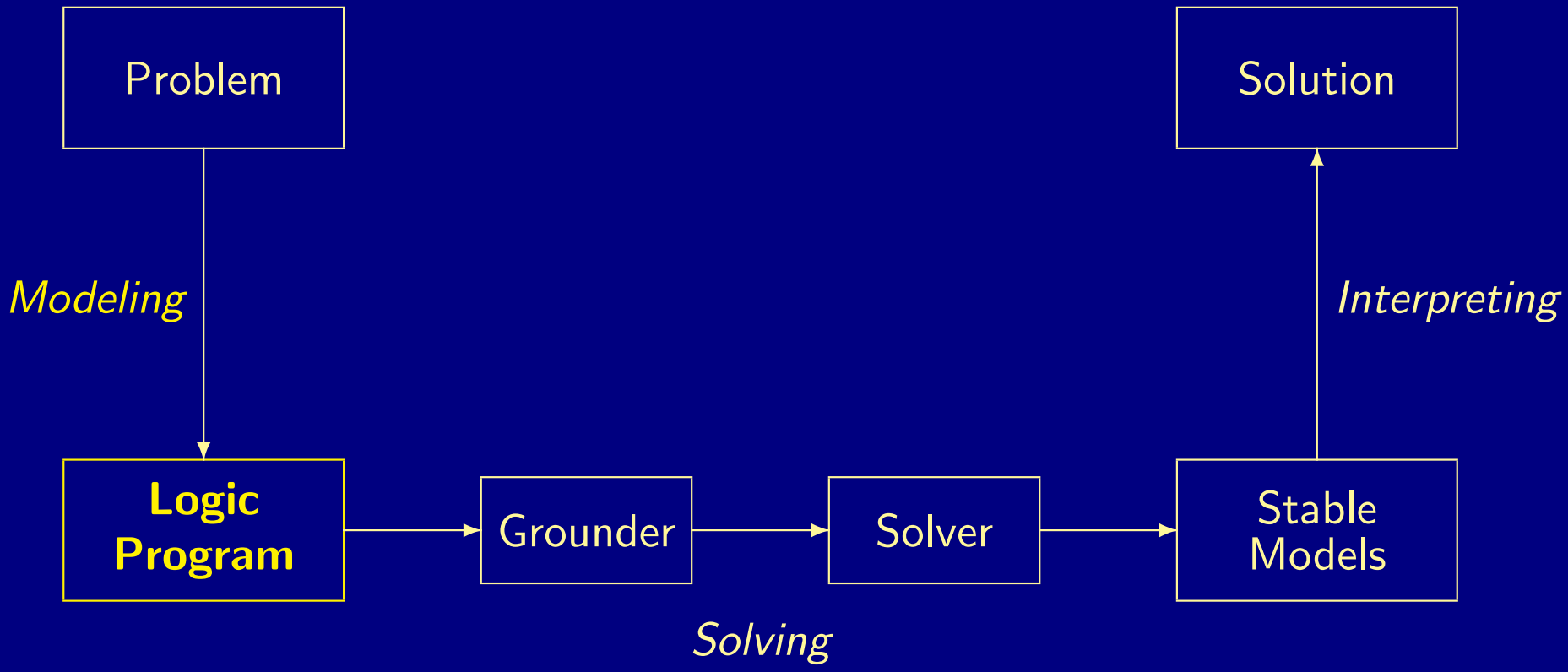

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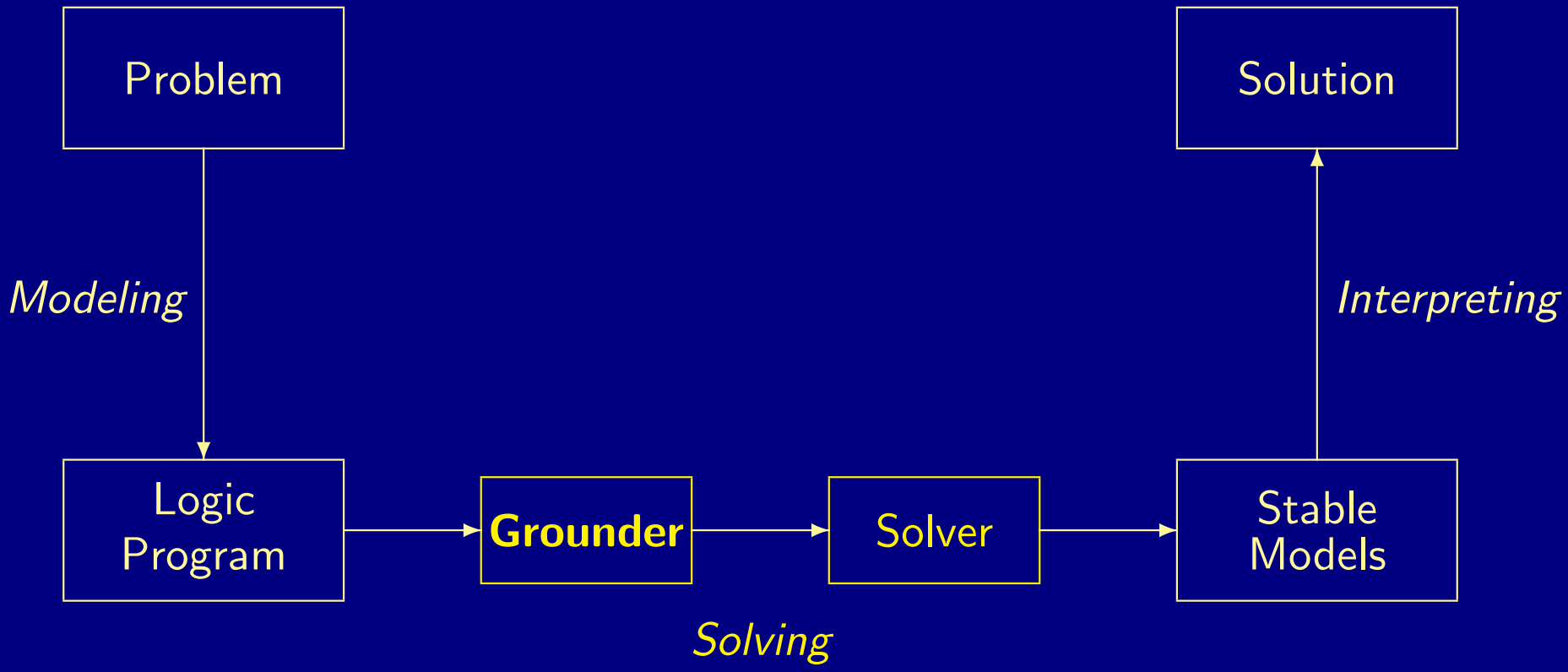

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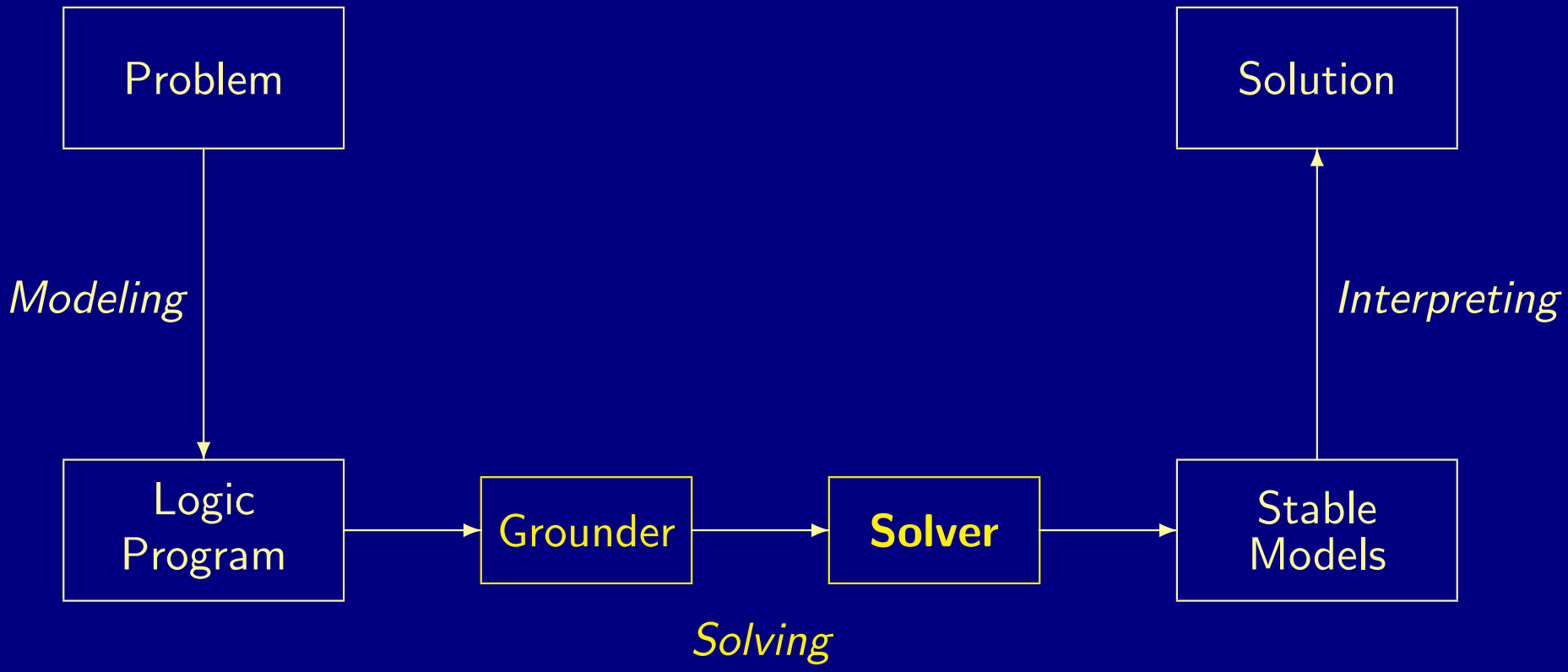

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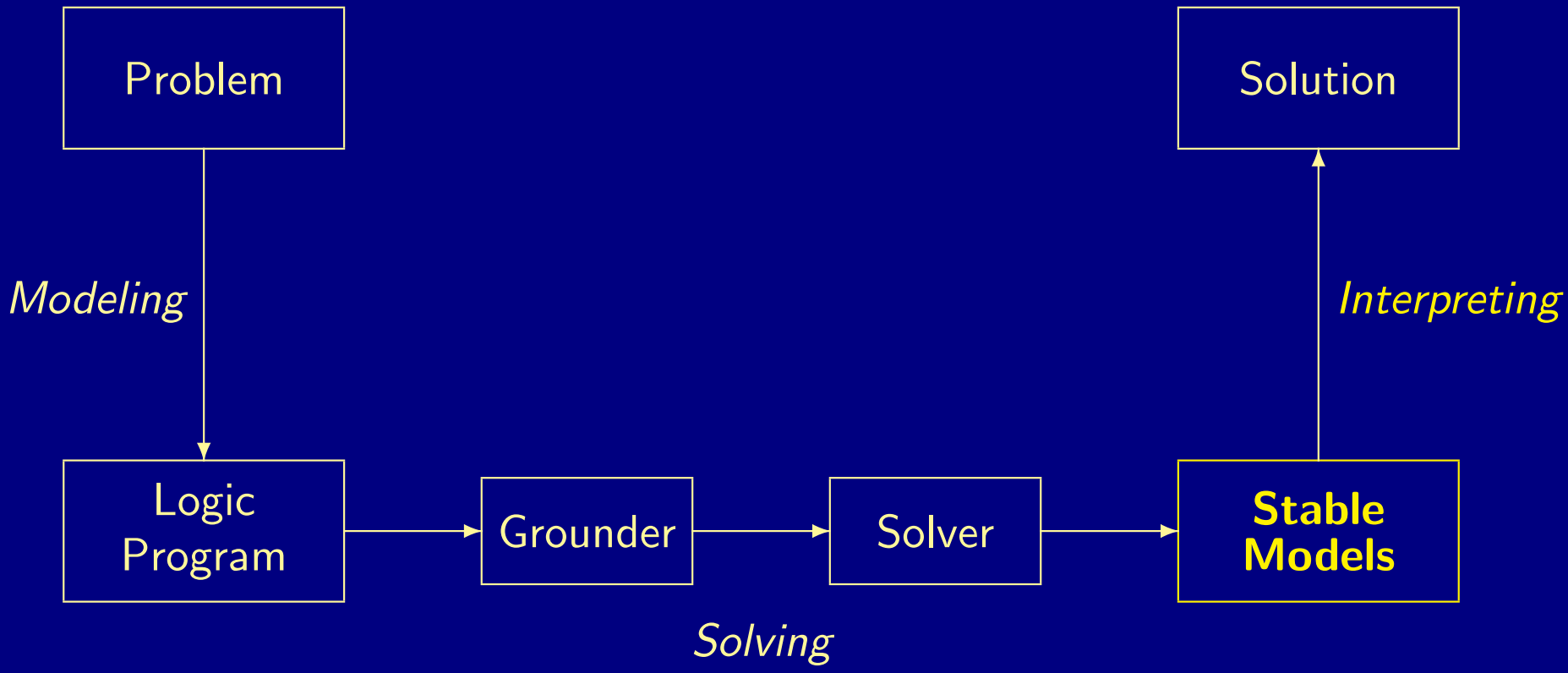

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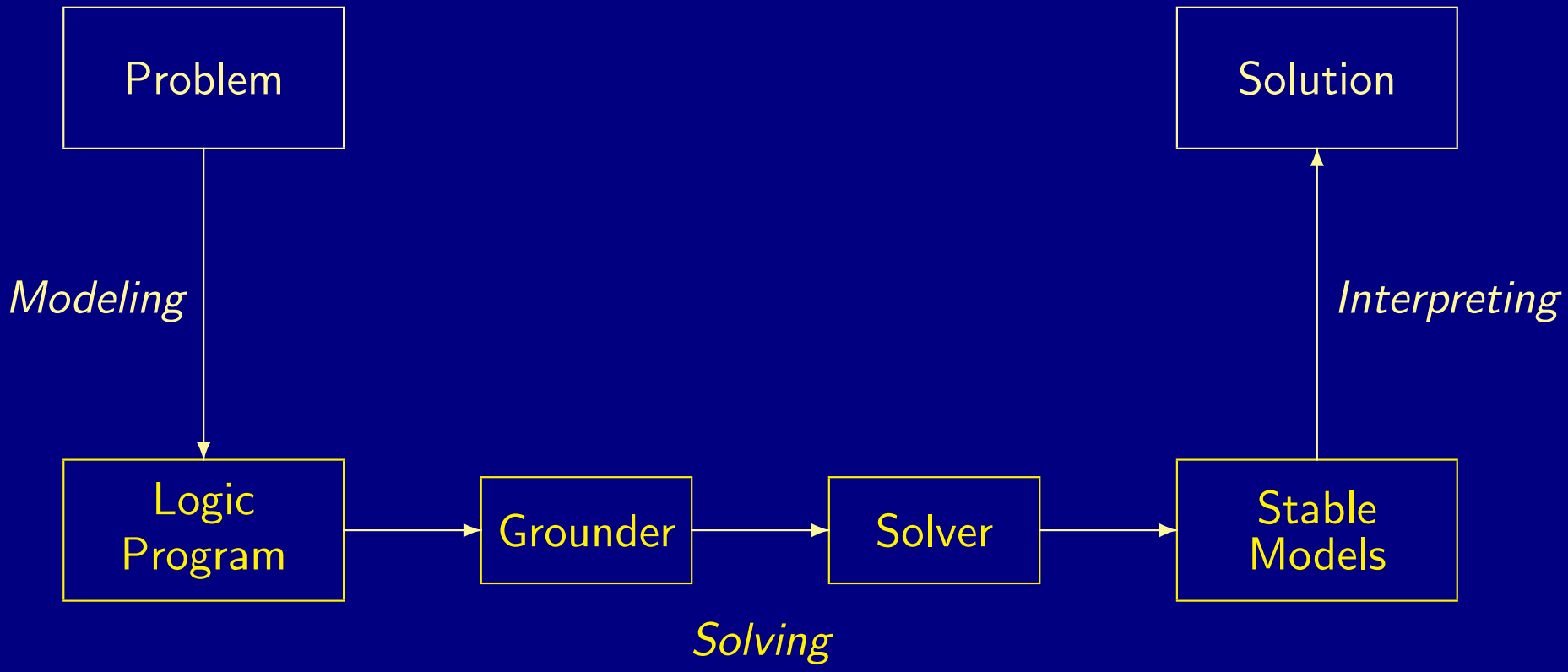

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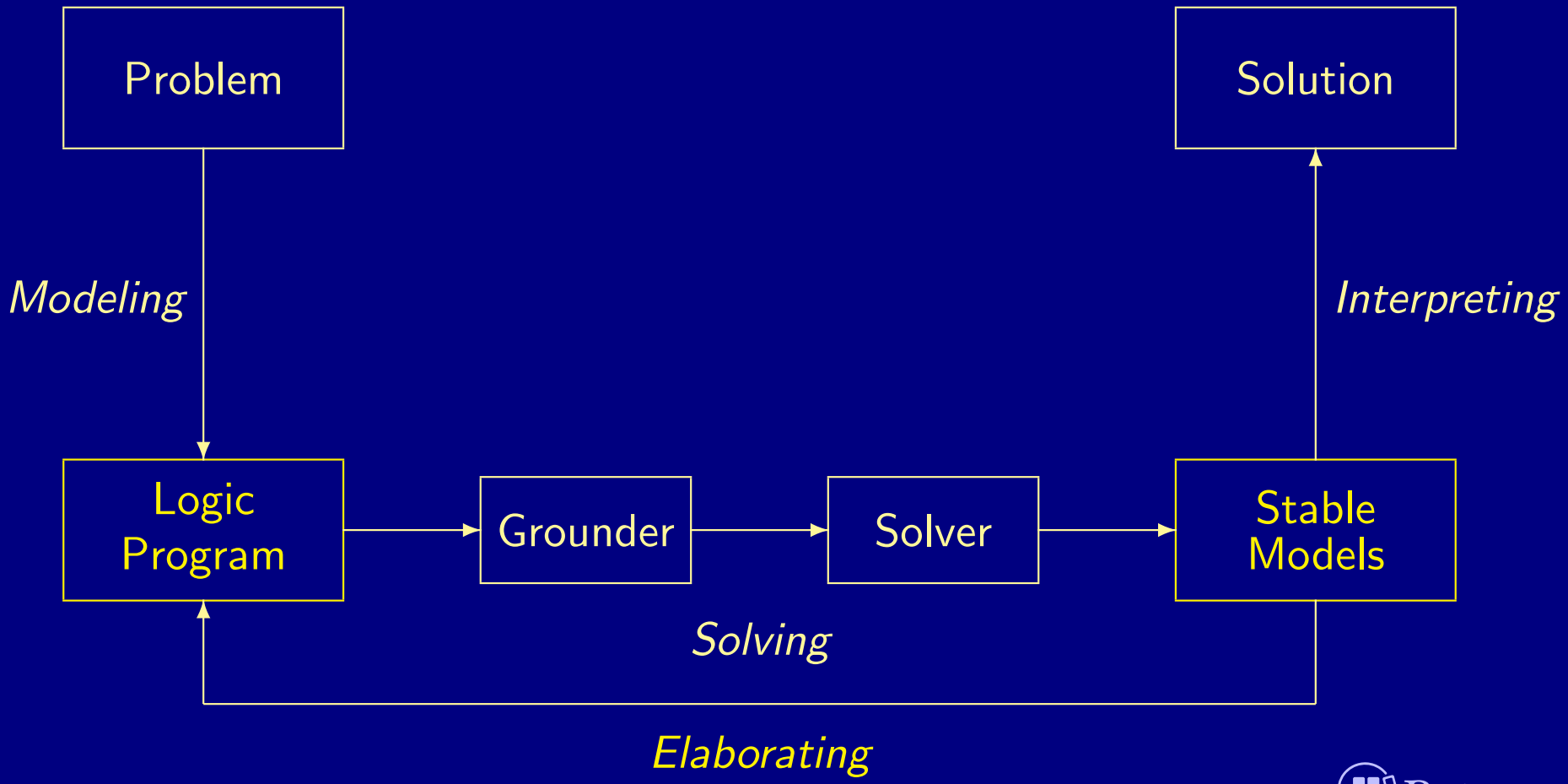

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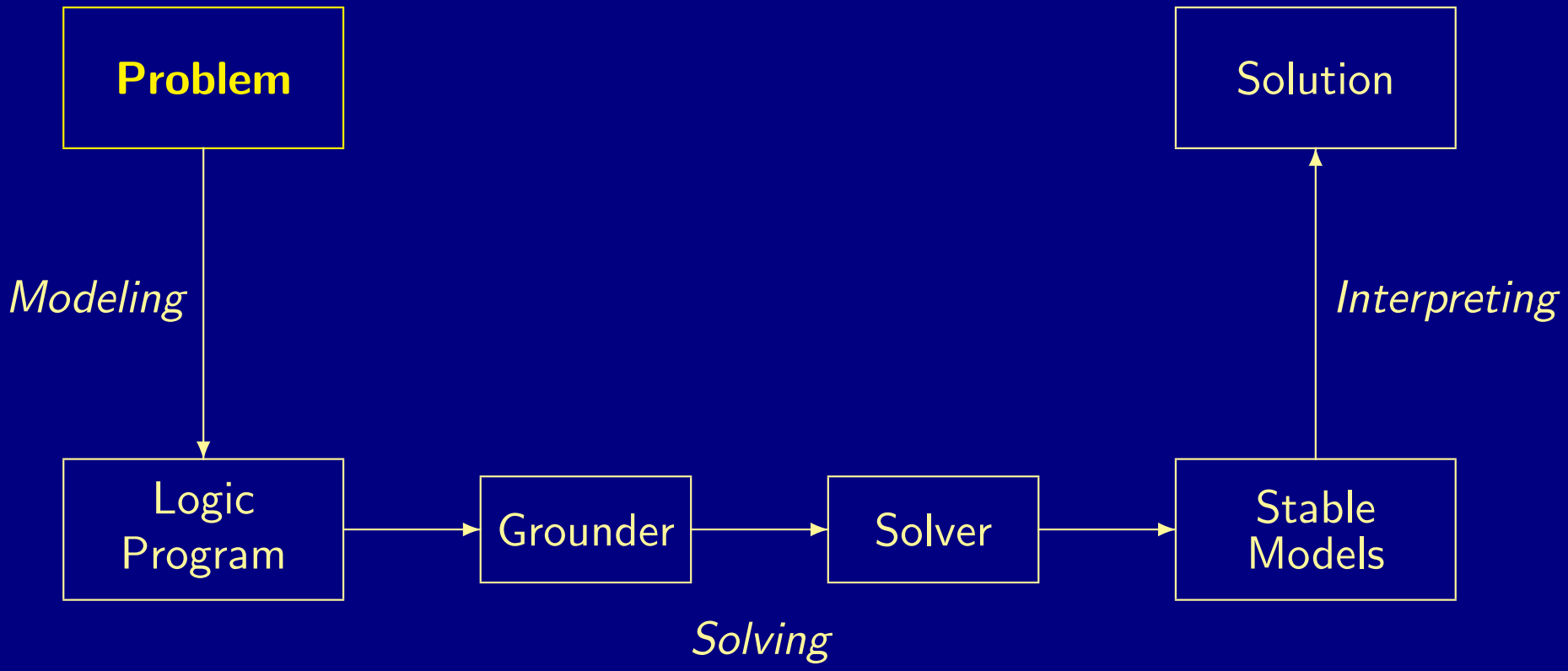
A case-study: Graph coloring



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Graph coloring

- Problem instance A graph consisting of nodes and edges

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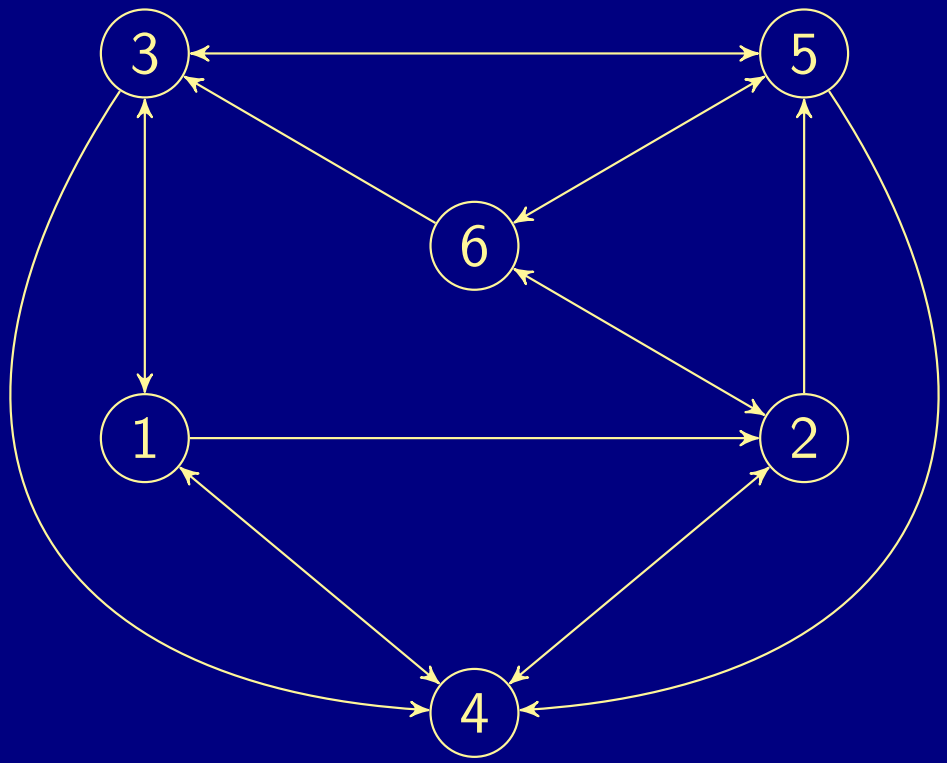

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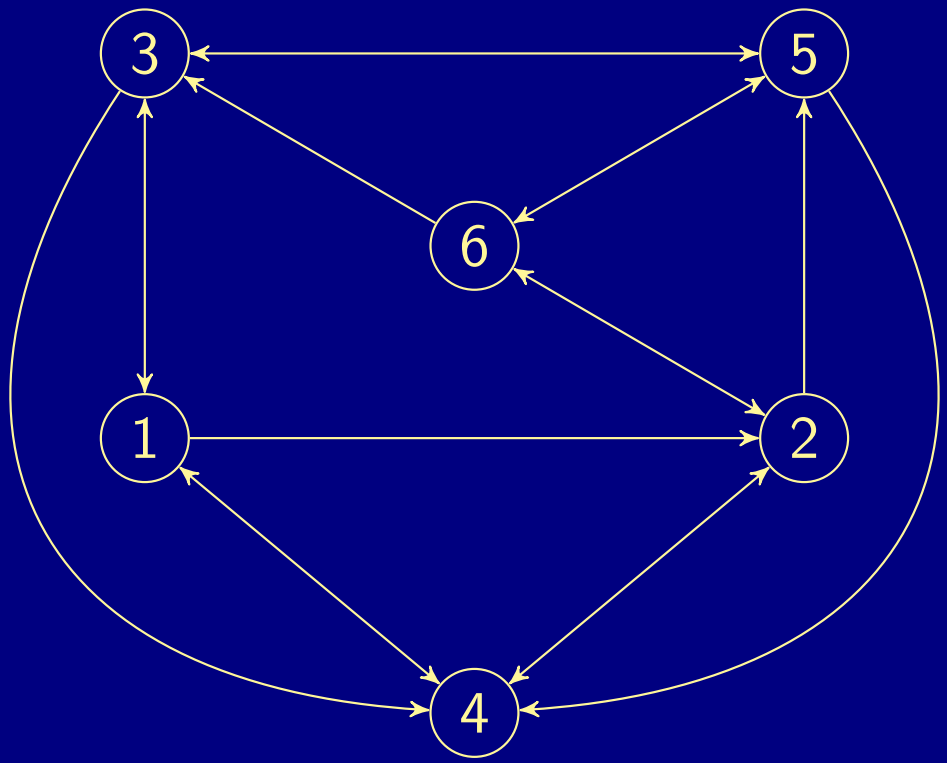
Graph coloring

- Problem instance A graph consisting of nodes and edges
  - facts formed by predicates `node/1` and `edge/2`



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  - facts formed by predicate `color/1`
- Problem class Assign each node one color such that no two nodes connected by an edge have the same color

  In other words,

  1. Each node has one color
  2. Two connected nodes must not have the same color

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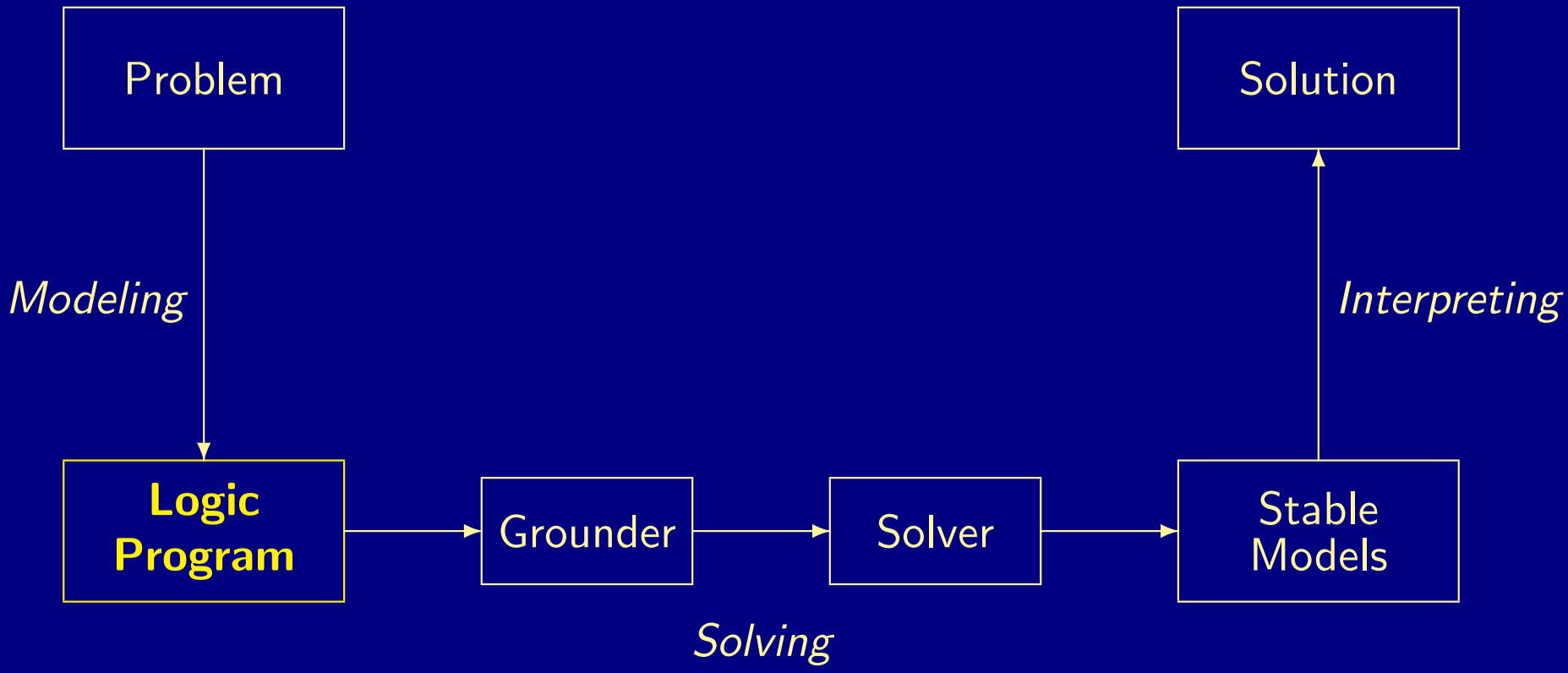
ASP solving process



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Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
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color(r).   color(b).   color(g).
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Problem instance

```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
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Problem encoding

Potassco

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Graph coloring

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node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

Problem instance

```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

Problem encoding

Potassco

Graph coloring

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node(1..6).

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edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
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Potassco

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Potassco

Graph coloring

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edge(3,1).  edge(3,4).  edge(3,5).
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edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

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Problem encoding

Potassco

Graph coloring

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edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

Problem instance

```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

Problem encoding

Potassco

Graph coloring

graph.lp

```
node(1..6).

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edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

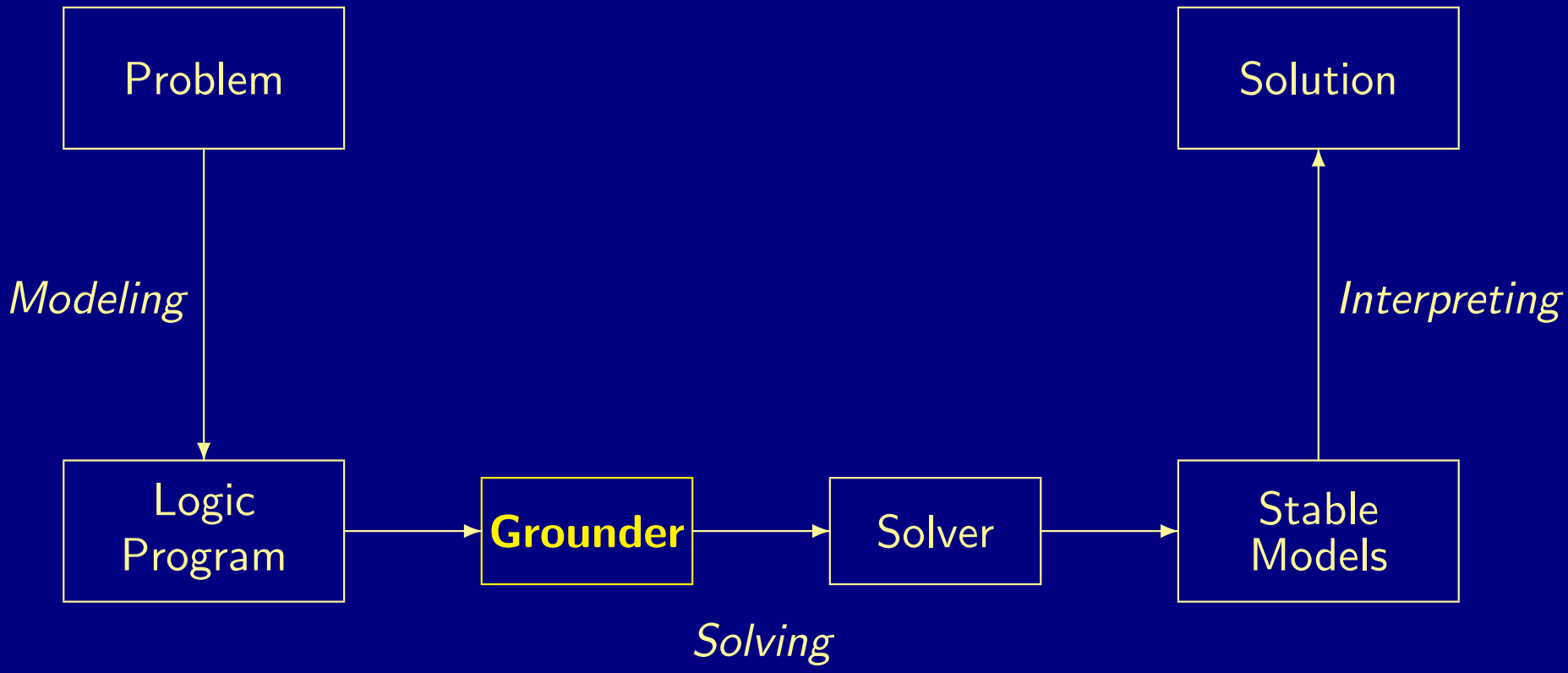
color.lp

```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

Potassco

ASP solving process



Potassco

Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1). node(2). node(3). node(4). node(5). node(6).

edge(1,2). edge(2,4). edge(3,1). edge(4,1). edge(5,3). edge(6,2).
edge(1,3). edge(2,5). edge(3,4). edge(4,2). edge(5,4). edge(6,3).
edge(1,4). edge(2,6). edge(3,5).            edge(5,6). edge(6,5).

color(r). color(b). color(g).

{assign(1,r);assign(1,b);assign(1,g)} = 1. {assign(4,r);assign(4,b);assign(4,g)} = 1.
{assign(2,r);assign(2,b);assign(2,g)} = 1. {assign(5,r);assign(5,b);assign(5,g)} = 1.
{assign(3,r);assign(3,b);assign(3,g)} = 1. {assign(6,r);assign(6,b);assign(6,g)} = 1.

:- assign(1,r),assign(2,r). :- assign(2,r),assign(4,r). [...] :- assign(6,r),assign(2,r).
:- assign(1,b),assign(2,b). :- assign(2,b),assign(4,b).       :- assign(6,b),assign(2,b).
:- assign(1,g),assign(2,g). :- assign(2,g),assign(4,g).       :- assign(6,g),assign(2,g).
:- assign(1,r),assign(3,r). :- assign(2,r),assign(5,r).       :- assign(6,r),assign(3,r).
:- assign(1,b),assign(3,b). :- assign(2,b),assign(5,b).       :- assign(6,b),assign(3,b).
:- assign(1,g),assign(3,g). :- assign(2,g),assign(5,g).       :- assign(6,g),assign(3,g).
:- assign(1,r),assign(4,r). :- assign(2,r),assign(6,r).       :- assign(6,r),assign(5,r).
:- assign(1,b),assign(4,b). :- assign(2,b),assign(6,b).       :- assign(6,b),assign(5,b).
:- assign(1,g),assign(4,g). :- assign(2,g),assign(6,g).       :- assign(6,g),assign(5,g).
```

Potassco

Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1). node(2). node(3). node(4). node(5). node(6).

edge(1,2). edge(2,4). edge(3,1). edge(4,1). edge(5,3). edge(6,2).
edge(1,3). edge(2,5). edge(3,4). edge(4,2). edge(5,4). edge(6,3).
edge(1,4). edge(2,6). edge(3,5).            edge(5,6). edge(6,5).

color(r). color(b). color(g).

{assign(1,r);assign(1,b);assign(1,g)} = 1. {assign(4,r);assign(4,b);assign(4,g)} = 1.
{assign(2,r);assign(2,b);assign(2,g)} = 1. {assign(5,r);assign(5,b);assign(5,g)} = 1.
{assign(3,r);assign(3,b);assign(3,g)} = 1. {assign(6,r);assign(6,b);assign(6,g)} = 1.

:- assign(1,r),assign(2,r). :- assign(2,r),assign(4,r). [...] :- assign(6,r),assign(2,r).
:- assign(1,b),assign(2,b). :- assign(2,b),assign(4,b).       :- assign(6,b),assign(2,b).
:- assign(1,g),assign(2,g). :- assign(2,g),assign(4,g).       :- assign(6,g),assign(2,g).
:- assign(1,r),assign(3,r). :- assign(2,r),assign(5,r).       :- assign(6,r),assign(3,r).
:- assign(1,b),assign(3,b). :- assign(2,b),assign(5,b).       :- assign(6,b),assign(3,b).
:- assign(1,g),assign(3,g). :- assign(2,g),assign(5,g).       :- assign(6,g),assign(3,g).
:- assign(1,r),assign(4,r). :- assign(2,r),assign(6,r).       :- assign(6,r),assign(5,r).
:- assign(1,b),assign(4,b). :- assign(2,b),assign(6,b).       :- assign(6,b),assign(5,b).
:- assign(1,g),assign(4,g). :- assign(2,g),assign(6,g).       :- assign(6,g),assign(5,g).
```

Potassco

Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1). node(2). node(3). node(4). node(5). node(6).

edge(1,2). edge(2,4). edge(3,1). edge(4,1). edge(5,3). edge(6,2).
edge(1,3). edge(2,5). edge(3,4). edge(4,2). edge(5,4). edge(6,3).
edge(1,4). edge(2,6). edge(3,5).            edge(5,6). edge(6,5).

color(r). color(b). color(g).

{assign(1,r);assign(1,b);assign(1,g)} = 1. {assign(4,r);assign(4,b);assign(4,g)} = 1.
{assign(2,r);assign(2,b);assign(2,g)} = 1. {assign(5,r);assign(5,b);assign(5,g)} = 1.
{assign(3,r);assign(3,b);assign(3,g)} = 1. {assign(6,r);assign(6,b);assign(6,g)} = 1.

:- assign(1,r),assign(2,r). :- assign(2,r),assign(4,r). [...] :- assign(6,r),assign(2,r).
:- assign(1,b),assign(2,b). :- assign(2,b),assign(4,b).       :- assign(6,b),assign(2,b).
:- assign(1,g),assign(2,g). :- assign(2,g),assign(4,g).       :- assign(6,g),assign(2,g).
:- assign(1,r),assign(3,r). :- assign(2,r),assign(5,r).       :- assign(6,r),assign(3,r).
:- assign(1,b),assign(3,b). :- assign(2,b),assign(5,b).       :- assign(6,b),assign(3,b).
:- assign(1,g),assign(3,g). :- assign(2,g),assign(5,g).       :- assign(6,g),assign(3,g).
:- assign(1,r),assign(4,r). :- assign(2,r),assign(6,r).       :- assign(6,r),assign(5,r).
:- assign(1,b),assign(4,b). :- assign(2,b),assign(6,b).       :- assign(6,b),assign(5,b).
:- assign(1,g),assign(4,g). :- assign(2,g),assign(6,g).       :- assign(6,g),assign(5,g).
```

Potassco

Graph coloring: Grounding

```
$ gringo --text graph.lp color.lp
```

```
node(1). node(2). node(3). node(4). node(5). node(6).

edge(1,2). edge(2,4). edge(3,1). edge(4,1). edge(5,3). edge(6,2).
edge(1,3). edge(2,5). edge(3,4). edge(4,2). edge(5,4). edge(6,3).
edge(1,4). edge(2,6). edge(3,5).            edge(5,6). edge(6,5).

color(r). color(b). color(g).

{assign(1,r);assign(1,b);assign(1,g)} = 1. {assign(4,r);assign(4,b);assign(4,g)} = 1.
{assign(2,r);assign(2,b);assign(2,g)} = 1. {assign(5,r);assign(5,b);assign(5,g)} = 1.
{assign(3,r);assign(3,b);assign(3,g)} = 1. {assign(6,r);assign(6,b);assign(6,g)} = 1.

:- assign(1,r),assign(2,r).  :- assign(2,r),assign(4,r). [...] :- assign(6,r),assign(2,r).
:- assign(1,b),assign(2,b).  :- assign(2,b),assign(4,b).       :- assign(6,b),assign(2,b).
:- assign(1,g),assign(2,g).  :- assign(2,g),assign(4,g).       :- assign(6,g),assign(2,g).
:- assign(1,r),assign(3,r).  :- assign(2,r),assign(5,r).       :- assign(6,r),assign(3,r).
:- assign(1,b),assign(3,b).  :- assign(2,b),assign(5,b).       :- assign(6,b),assign(3,b).
:- assign(1,g),assign(3,g).  :- assign(2,g),assign(5,g).       :- assign(6,g),assign(3,g).
:- assign(1,r),assign(4,r).  :- assign(2,r),assign(6,r).       :- assign(6,r),assign(5,r).
:- assign(1,b),assign(4,b).  :- assign(2,b),assign(6,b).       :- assign(6,b),assign(5,b).
:- assign(1,g),assign(4,g).  :- assign(2,g),assign(6,g).       :- assign(6,g),assign(5,g).
```

Potassco

Graph coloring: Grounding

```
$ clingo --text graph.lp color.lp
```

```
node(1). node(2). node(3). node(4). node(5). node(6).

edge(1,2). edge(2,4). edge(3,1). edge(4,1). edge(5,3). edge(6,2).
edge(1,3). edge(2,5). edge(3,4). edge(4,2). edge(5,4). edge(6,3).
edge(1,4). edge(2,6). edge(3,5).            edge(5,6). edge(6,5).

color(r). color(b). color(g).

{assign(1,r);assign(1,b);assign(1,g)} = 1. {assign(4,r);assign(4,b);assign(4,g)} = 1.
{assign(2,r);assign(2,b);assign(2,g)} = 1. {assign(5,r);assign(5,b);assign(5,g)} = 1.
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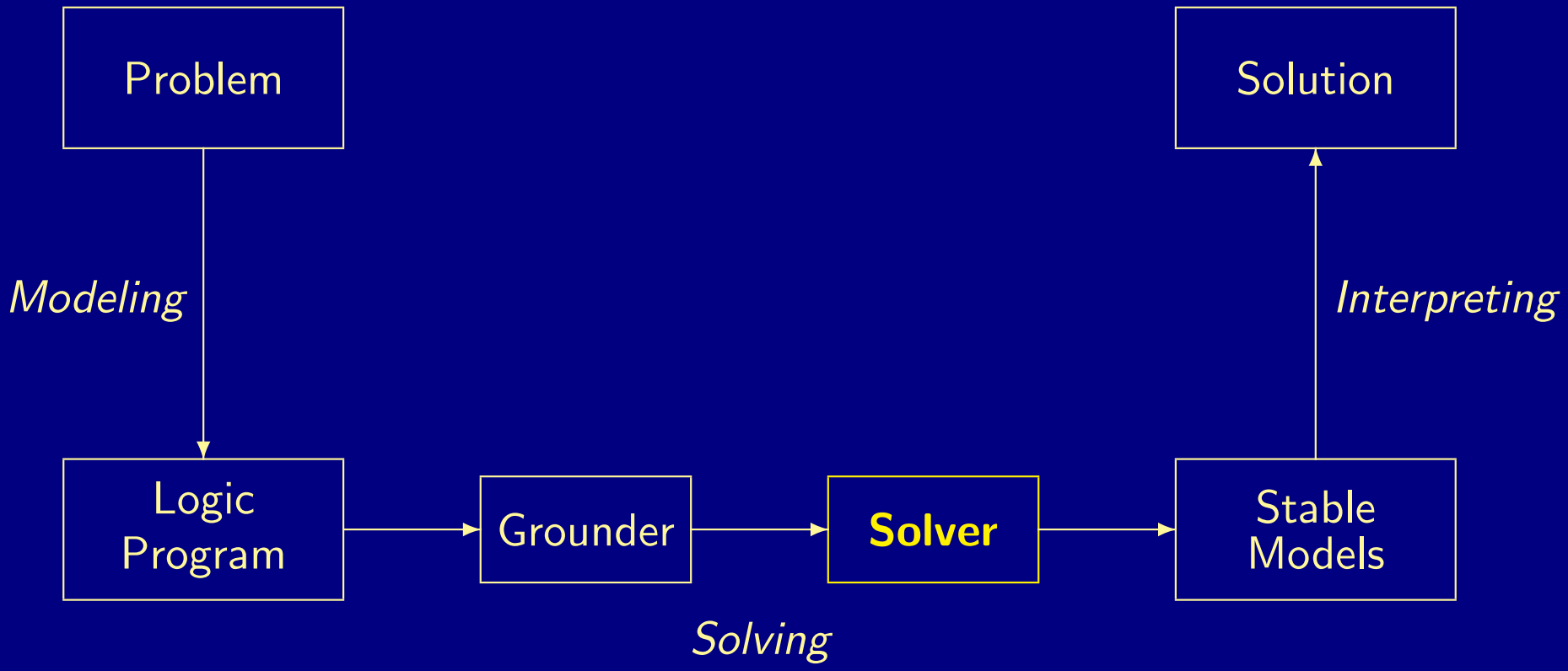
:- assign(1,r),assign(2,r).  :- assign(2,r),assign(4,r). [...] :- assign(6,r),assign(2,r).
:- assign(1,b),assign(2,b).  :- assign(2,b),assign(4,b).       :- assign(6,b),assign(2,b).
:- assign(1,g),assign(2,g).  :- assign(2,g),assign(4,g).       :- assign(6,g),assign(2,g).
:- assign(1,r),assign(3,r).  :- assign(2,r),assign(5,r).       :- assign(6,r),assign(3,r).
:- assign(1,b),assign(3,b).  :- assign(2,b),assign(5,b).       :- assign(6,b),assign(3,b).
:- assign(1,g),assign(3,g).  :- assign(2,g),assign(5,g).       :- assign(6,g),assign(3,g).
:- assign(1,r),assign(4,r).  :- assign(2,r),assign(6,r).       :- assign(6,r),assign(5,r).
:- assign(1,b),assign(4,b).  :- assign(2,b),assign(6,b).       :- assign(6,b),assign(5,b).
:- assign(1,g),assign(4,g).  :- assign(2,g),assign(6,g).       :- assign(6,g),assign(5,g).
```

Potassco

ASP solving process



Potassco

Graph coloring: Solving

```
$ gringo graph.lp color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
node(1) [...] assign(6,b) assign(5,g) assign(4,b) assign(3,r) assign(2,r) assign(1,g)
Answer: 2
node(1) [...] assign(6,r) assign(5,g) assign(4,r) assign(3,b) assign(2,b) assign(1,g)
Answer: 3
node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
Answer: 4
node(1) [...] assign(6,r) assign(5,b) assign(4,r) assign(3,g) assign(2,g) assign(1,b)
Answer: 5
node(1) [...] assign(6,g) assign(5,r) assign(4,g) assign(3,b) assign(2,b) assign(1,r)
Answer: 6
node(1) [...] assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
SATISFIABLE

Models       : 6
Time         : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

Potassco

Graph coloring: Solving

```
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Reading from stdin
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node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
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Answer: 5
node(1) [...] assign(6,g) assign(5,r) assign(4,g) assign(3,b) assign(2,b) assign(1,r)
Answer: 6
node(1) [...] assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
SATISFIABLE

Models       : 6
Time         : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

Potassco

Graph coloring: Solving

```
$ clingo graph.lp color.lp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
node(1) [...] assign(6,b) assign(5,g) assign(4,b) assign(3,r) assign(2,r) assign(1,g)
Answer: 2
node(1) [...] assign(6,r) assign(5,g) assign(4,r) assign(3,b) assign(2,b) assign(1,g)
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node(1) [...] assign(6,g) assign(5,b) assign(4,g) assign(3,r) assign(2,r) assign(1,b)
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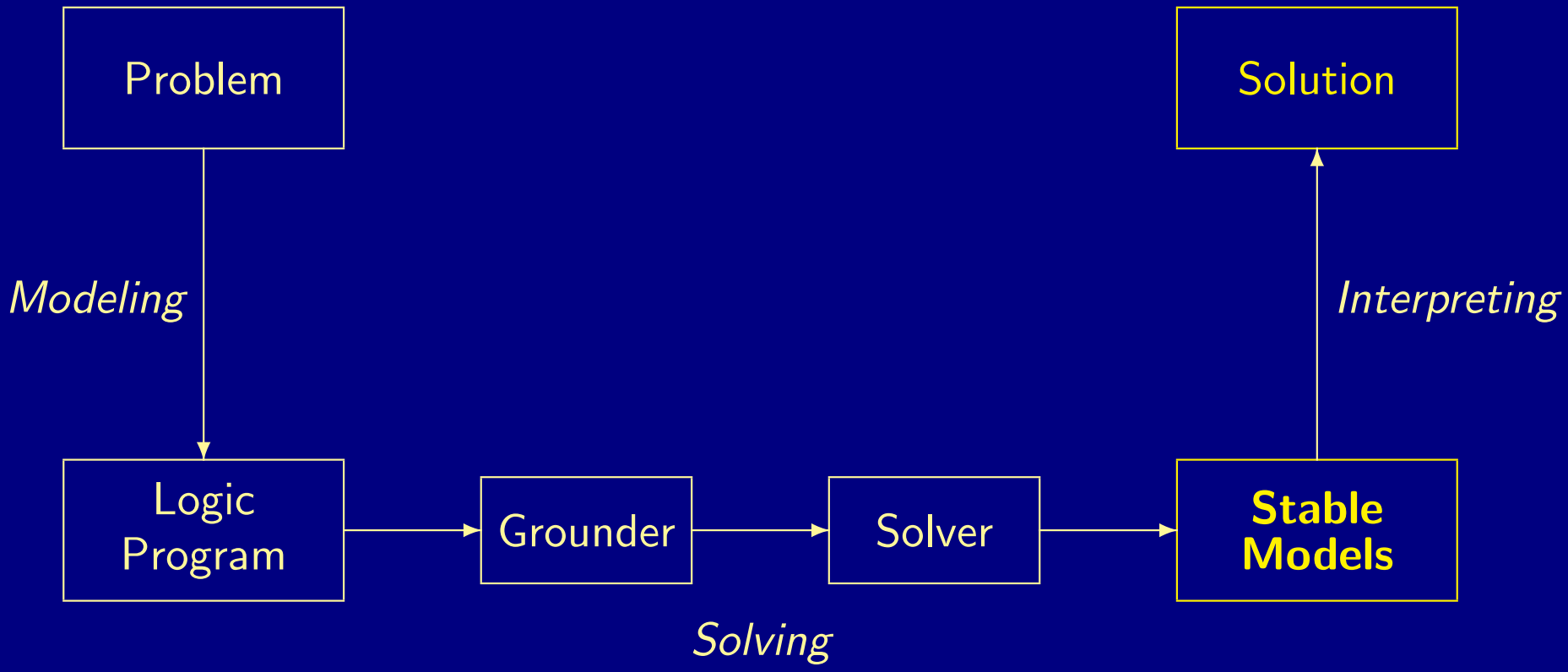
SATISFIABLE

Models       : 6
Time         : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
```

Potassco

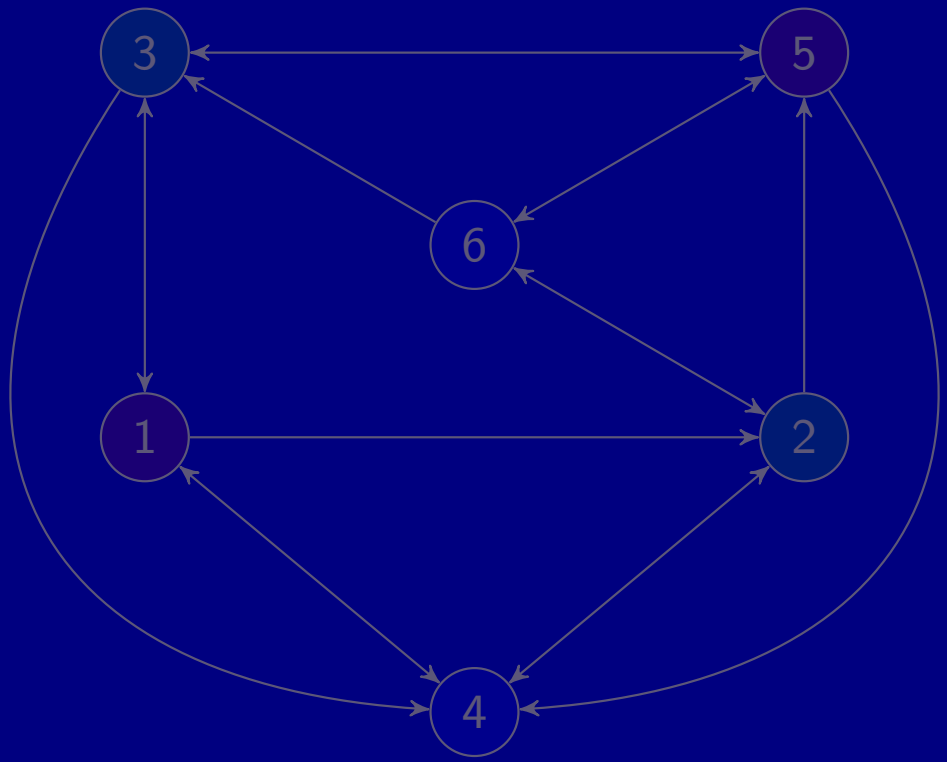
ASP solving process



Potassco

A coloring

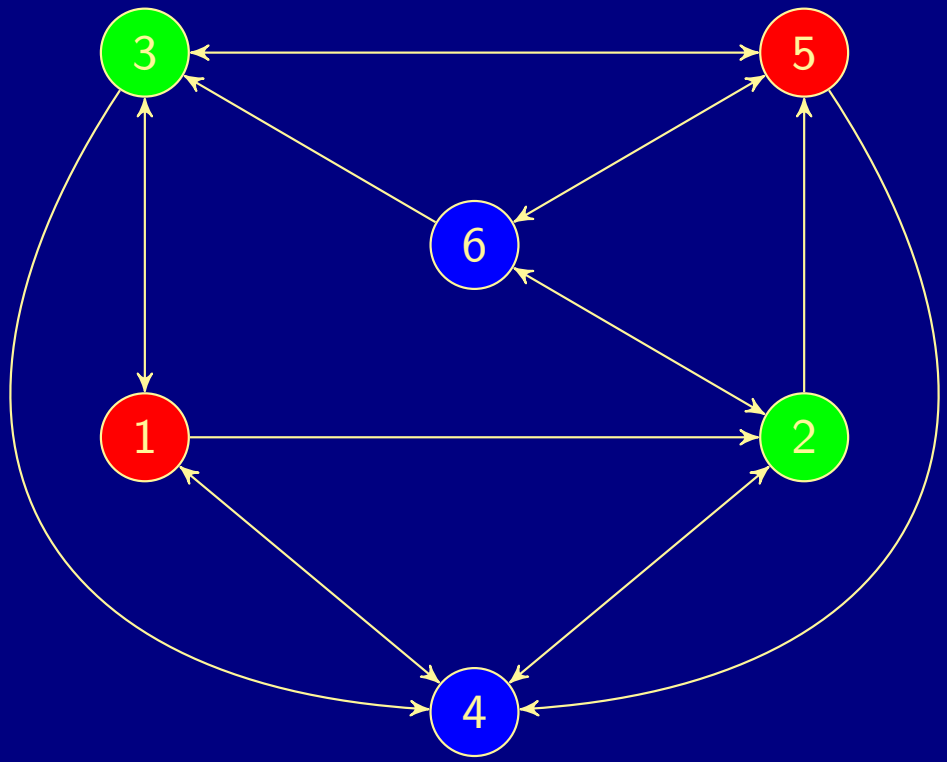
```
Answer: 6
node(1)   [...]    \
assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
```



Potassco

A coloring

```
Answer: 6
node(1)   [...]    \
assign(6,b) assign(5,r) assign(4,b) assign(3,g) assign(2,g) assign(1,r)
```



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Outline

Potassco

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

- **Generator** Generate potential stable model candidates (typically through non-deterministic constructs)
- **Tester** Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Potassco

Basic methodology

Methodology

Generate and **Test** (or: Guess and Check)

Generator: Generate potential stable model candidates (typically through non-deterministic constructs)

Tester: Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Potassco

Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

Problem instance

```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

Problem encoding

Potassco

Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

Data

```
{ assign(N,C) : color(C) } = 1 :- node(N).

:- edge(N,M), assign(N,C), assign(M,C).
```

Problem encoding

Potassco

Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

color(r).   color(b).   color(g).
```

Data

```
{ assign(N,C) : color(C) } = 1 :- node(N).
```

Generator

```
:- edge(N,M), assign(N,C), assign(M,C).
```

Tester

Potassco

Outline

Potassco

Outline

Potassco

Satisfiability testing

- Problem Instance A propositional formula ϕ in CNF
- Problem Class Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program

Generator		**Tester**		**Stable models**		
$\{a\}$	$\leftarrow$	$\leftarrow$	$\neg a, b$	X_1	$=$	$\{a, b\}$
$\{b\}$	$\leftarrow$	$\leftarrow$	$a, \neg b$	X_2	$=$	$\{\}$

Potassco

Satisfiability testing

- Problem Instance A propositional formula ϕ in CNF
- Problem Class Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program

Generator		**Tester**		**Stable models**		
$\{a\}$	$\leftarrow$	$\leftarrow$	$\neg a, b$	X_1	$=$	$\{a, b\}$
$\{b\}$	$\leftarrow$	$\leftarrow$	$a, \neg b$	X_2	$=$	$\{\}$

Potassco

Satisfiability testing

- Problem Instance A propositional formula ϕ in CNF
- Problem Class Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program

Generator		**Tester**		**Stable models**		
$\{a\}$	$\leftarrow$	$\leftarrow$	$\neg a, b$	X_1	$=$	$\{a, b\}$
$\{b\}$	$\leftarrow$	$\leftarrow$	$a, \neg b$	X_2	$=$	$\{\}$

Potassco

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Generator		**Tester**		**Stable models**		
$\{a\}$	$\leftarrow$	$\leftarrow$	$\neg a, b$	X_1	$=$	$\{a, b\}$
$\{b\}$	$\leftarrow$	$\leftarrow$	$a, \neg b$	X_2	$=$	$\{\}$

Potassco

Satisfiability testing

- Problem Instance A propositional formula ϕ in CNF
- Problem Class Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

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- Logic Program

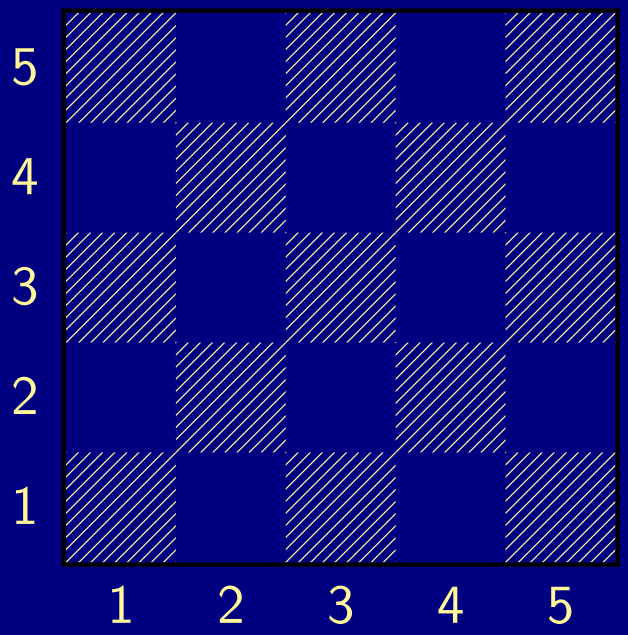
Generator		**Tester**		**Stable models**		
$\{a\}$	$\leftarrow$	$\leftarrow$	$\neg a, b$	X_1	$=$	$\{a, b\}$
$\{b\}$	$\leftarrow$	$\leftarrow$	$a, \neg b$	X_2	$=$	$\{\}$

Potassco

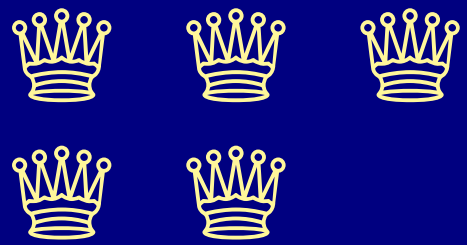
Outline

Potassco

The n-queens problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another



Potassco

Defining the field

queens.lp

```
row(1..n).
col(1..n).
```

- Create file `queens.lp`
- Define the field
  - n rows
  - n columns

Defining the field

Running ...

```
$ clingo queens.lp --const n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models       : 1
Time         : 0.000
```

Potassco

Placing some queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
```

- Guess a solution candidate

  by placing some queens on the board

Placing some queens

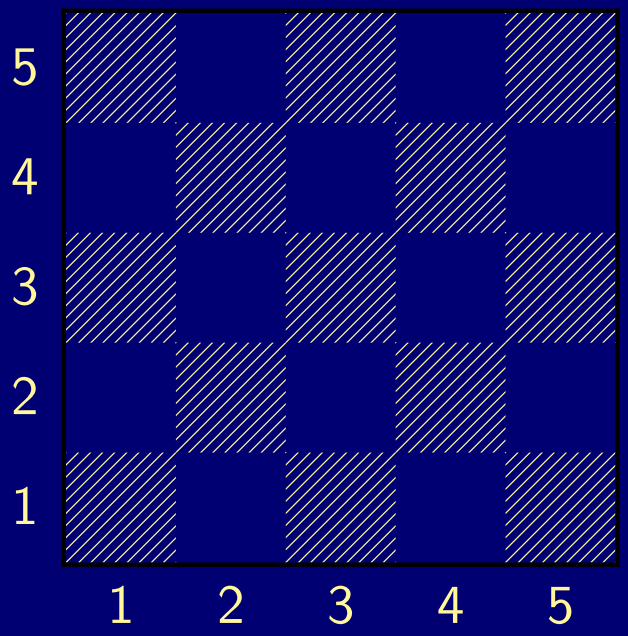
Running ...

```
$ clingo queens.lp --const n=5 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE

Models       : 3+
```

Placing some queens

```
Answer: 1
```

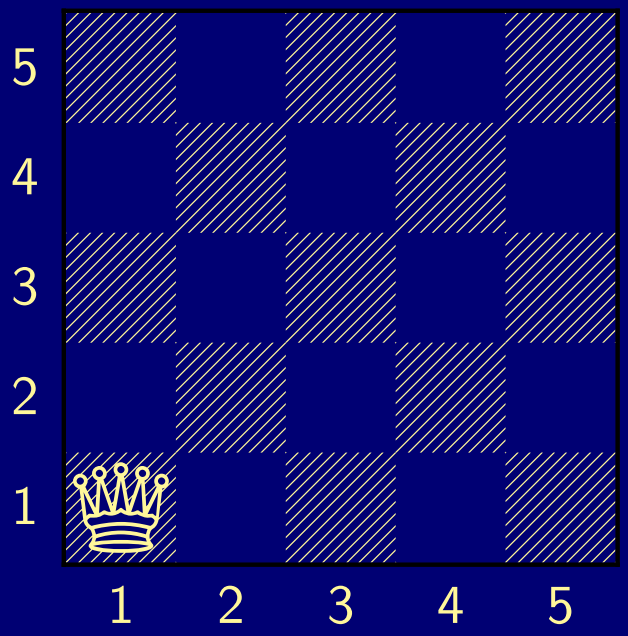



```
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
```

Potassco

Placing some queens

```
Answer: 2
```

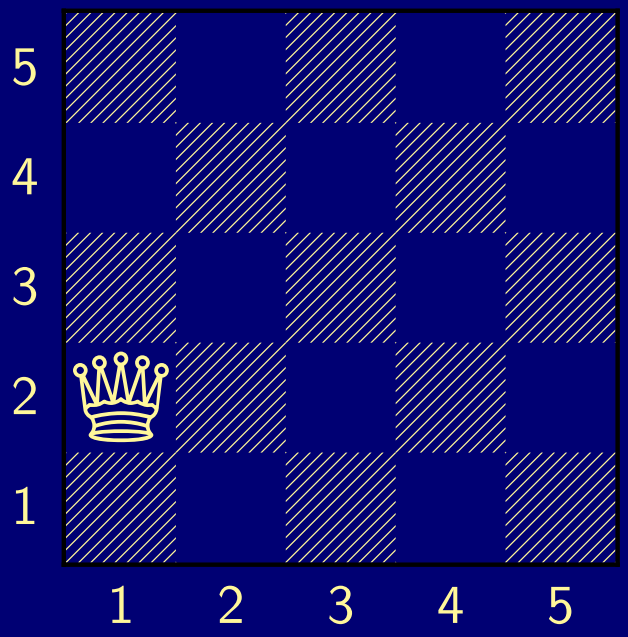



```
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,1)
```

Potassco

Placing some queens

```
Answer: 3
```




```
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(2,1)
```

Potassco

Placing n queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
```

- Place exactly n queens on the board

Potassco

Placing n queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- not { queen(I,J) } = n.
```

- Place exactly n queens on the board

Placing *n* queens

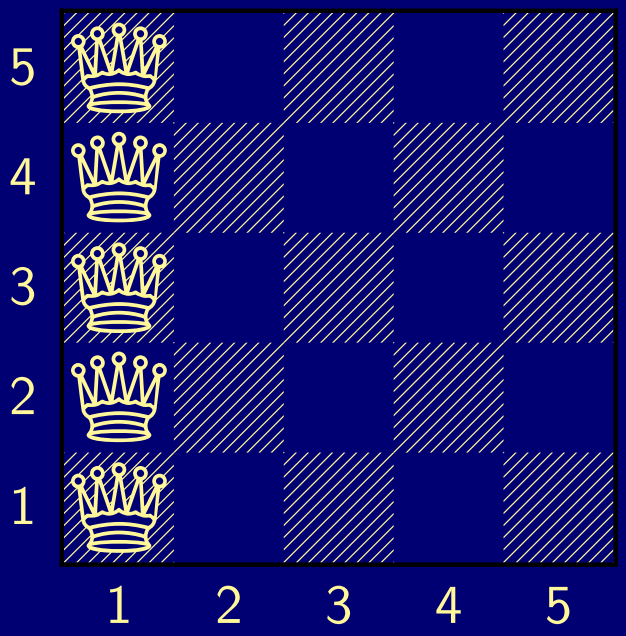
Running ...

```
$ clingo queens.lp --const n=5 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) queen(2,1) queen(1,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) queen(2,1) queen(1,1)
```

Potassco

Placing *n* queens

```
Answer: 1
```

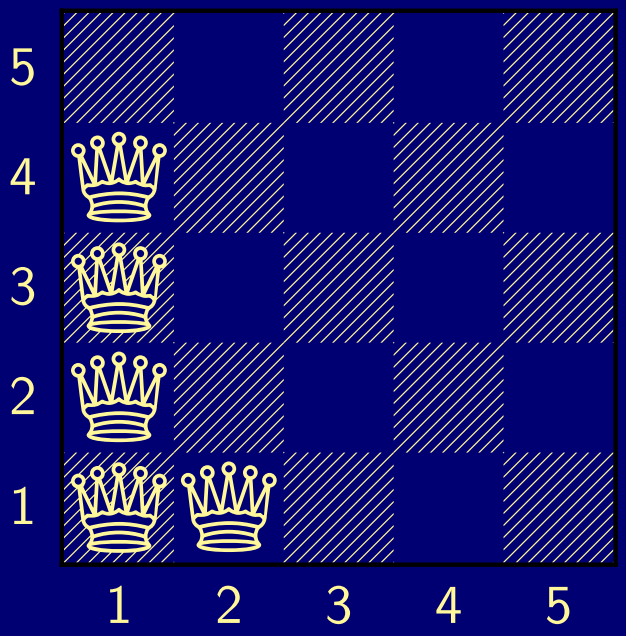



```
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) queen(2,1)
queen(1,1)
```

Potassco

Placing *n* queens

```
Answer: 2
```




```
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) queen(2,1)
queen(1,1)
```

Potassco

Horizontal and vertical attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
```

- Forbid horizontal attacks
- Forbid vertical attacks

Potassco

Horizontal and vertical attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
```

- Forbid horizontal attacks
- Forbid vertical attacks

Potassco

Horizontal and vertical attack

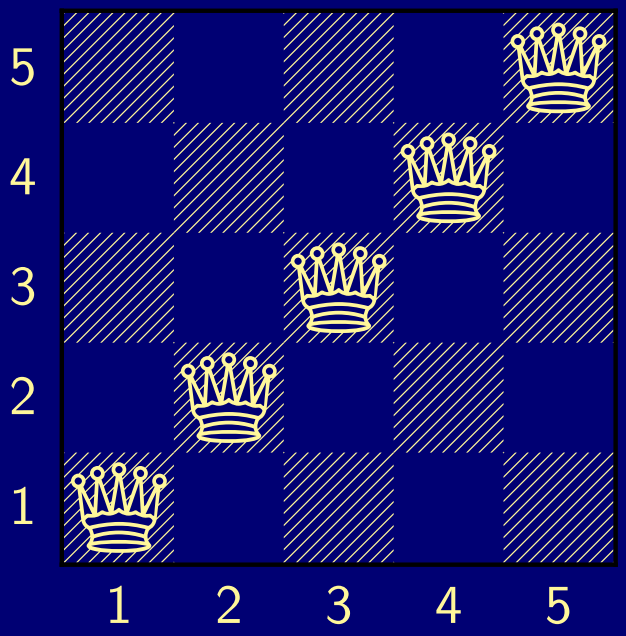
Running ...

```
$ clingo queens.lp --const n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) queen(2,2) queen(1,1)
```

Potassco

Horizontal and vertical attack

```
Answer: 1
```




```
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) queen(2,2)
queen(1,1)
```

Potassco

Diagonal attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I), col(J) }.
:- { queen(I,J) } != n.
:- queen(I,J), queen(I,J'), J != J'.
:- queen(I,J), queen(I',J), I != I'.
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I-J == I'-J'.
:- queen(I,J), queen(I',J'), (I,J) != (I',J'), I+J == I'+J'.
```

- Forbid diagonal attacks

Potassco

Diagonal attack

Running ...

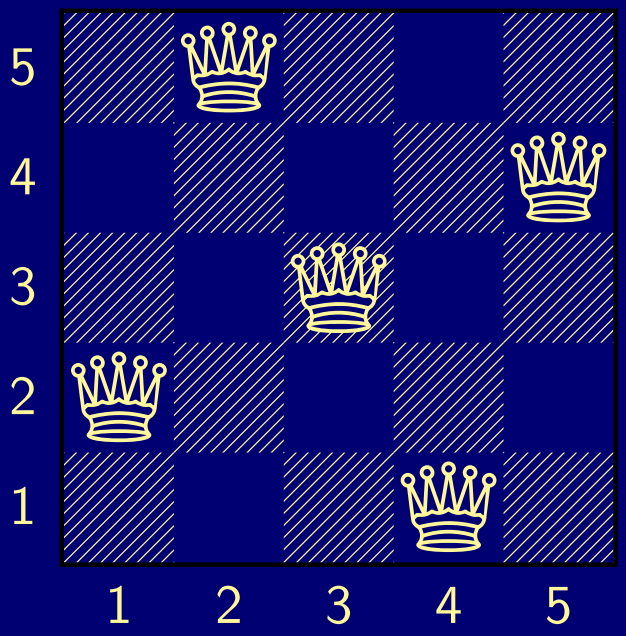
```
$ clingo queens.lp --const n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) queen(5,2) queen(2,1)
SATISFIABLE

Models       : 1+
Time         : 0.000
```

Potassco

Diagonal attack

```
Answer: 1
```




```
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) queen(5,2)
queen(2,1)
```

Potassco

Optimizing

queens-opt.lp

```
{ queen(I,1..n) } = 1 :- I = 1..n.
{ queen(1..n,J) } = 1 :- J = 1..n.
 :- { queen(D-J,J) } > 1, D =   2..2*n.
 :- { queen(D+J,J) } > 1, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

Potassco

And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=2
clingo version 4.1.0
Solving...
SATISFIABLE

Models       : 1+
Time         : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time     : 3758.320s

Choices      : 288594554
Conflicts    : 3442   (Analyzed: 3442)
Restarts     : 17     (Average: 202.47 Last: 3442)
Model-Level  : 7594728.0
Problems     : 1      (Average Length: 0.00 Splits: 0)
Lemmas       : 3442   (Deleted: 0)
  Binary     : 0      (Ratio:   0.00%)
  Ternary    : 0      (Ratio:   0.00%)
  Conflict   : 3442   (Average Length: 229056.5 Ratio: 100.00%)
  Loop       : 0      (Average Length:    0.0 Ratio:   0.00%)
  Other      : 0      (Average Length:    0.0 Ratio:   0.00%)

Atoms        : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules        : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies       : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight        : Yes
Variables    : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664  (Binary: 35.6% Ternary:  0.0% Other: 64.4%)

Backjumps    : 3442   (Average: 681.19 Max: 169512 Sum: 2344658)
  Executed   : 3442   (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
  Bounded    : 0      (Average:  0.00 Max:   0 Sum:      0 Ratio:   0.00%)
```

Potassco

And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=2
clingo version 4.1.0
Solving...
SATISFIABLE

Models       : 1+
Time         : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time     : 3758.320s

Choices      : 288594554
Conflicts    : 3442   (Analyzed: 3442)
Restarts     : 17     (Average: 202.47 Last: 3442)
Model-Level  : 7594728.0
Problems     : 1      (Average Length: 0.00 Splits: 0)
Lemmas       : 3442   (Deleted: 0)
  Binary     : 0      (Ratio:   0.00%)
  Ternary    : 0      (Ratio:   0.00%)
  Conflict   : 3442   (Average Length: 229056.5 Ratio: 100.00%)
  Loop       : 0      (Average Length:    0.0 Ratio:   0.00%)
  Other      : 0      (Average Length:    0.0 Ratio:   0.00%)

Atoms        : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules        : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies       : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight        : Yes
Variables    : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664  (Binary: 35.6% Ternary:  0.0% Other: 64.4%)

Backjumps    : 3442   (Average: 681.19 Max: 169512 Sum: 2344658)
  Executed   : 3442   (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
  Bounded    : 0      (Average:  0.00 Max:   0 Sum:      0 Ratio:   0.00%)
```

Potassco

Outline

Potassco

Traveling salesperson

```
start(a).

city(a). city(b). city(c). city(d).

road(a,b,10). road(b,c,20). road(c,d,25). road(d,a,40).
road(b,d,30). road(d,c,25). road(c,a,35).
```

Traveling salesperson

```
start(a).

city(a). city(b). city(c). city(d).

road(a,b,10). road(b,c,20). road(c,d,25). road(d,a,40).
road(b,d,30). road(d,c,25). road(c,a,35).
```

Potassco

Traveling salesperson

```
{ travel(X,Y) } :- road(X,Y,_).

visited(Y) :- travel(X,Y), start(X).
visited(Y) :- travel(X,Y), visited(X).

:- city(X), not visited(X).

:- city(X), 2 { travel(X,Y) }.
:- city(X), 2 { travel(Y,X) }.

start(a).

city(a). city(b). city(c). city(d).

road(a,b,10). road(b,c,20). road(c,d,25). road(d,a,40).
road(b,d,30). road(d,c,25). road(c,a,35).
```

Potassco

Traveling salesperson

```
{ travel(X,Y) } :- road(X,Y,_).

visited(Y) :- travel(X,Y), start(X).
visited(Y) :- travel(X,Y), visited(X).

:- city(X), not visited(X).

:- city(X), 2 { travel(X,Y) }.
:- city(X), 2 { travel(Y,X) }.

:~ travel(X,Y), road(X,Y,D). [D,X,Y]

start(a).

city(a). city(b). city(c). city(d).

road(a,b,10). road(b,c,20). road(c,d,25). road(d,a,40).
road(b,d,30). road(d,c,25). road(c,a,35).
```

Potassco

Traveling salesperson

```
{ travel(X,Y) } :- road(X,Y,_).

visited(Y) :- travel(X,Y), start(X).
visited(Y) :- travel(X,Y), visited(X).

:- city(X), not visited(X).

:- city(X), 2 { travel(X,Y) }.
:- city(X), 2 { travel(Y,X) }.

:~ travel(X,Y), road(X,Y,D). [D,X,Y]
```

tsp.lp

---

cities.lp

```
start(a).

city(a). city(b). city(c). city(d).

road(a,b,10). road(b,c,20). road(c,d,25). road(d,a,40).
road(b,d,30). road(d,c,25). road(c,a,35).
```

Potassco

Running salesperson

```
$ clingo tsp.lp cities.lp
clingo version 5.3.1
Reading...
Solving...
Answer: 1
start(a) [...] road(c,a,35)
travel(a,b) travel(b,d) travel(d,c) travel(c,a)
visited(b) visited(c) visited(d) visited(a)
Optimization: 100
Answer: 2
start(a) [...] road(c,a,35)
travel(a,b) travel(b,c) travel(c,d) travel(d,a)
visited(b) visited(c) visited(d) visited(a)
Optimization: 95
OPTIMUM FOUND

Models       : 2
  Optimum    : yes
Optimization : 95
Calls        : 1
Time         : 0.005s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.002s
```

Potassco

Outline

Potassco

Reviewer Assignment

- Problem Instance A set of papers and a set of reviewers along with their first and second choices of papers and conflict of interests
- Problem Class A nice assignment of three reviewers to each paper

Potassco

Reviewer Assignment

- Problem Instance A set of papers and a set of reviewers along with their first and second choices of papers and conflict of interests
- Problem Class A "nice" assignment of three reviewers to each paper

Potassco

Reviewer Assignment

by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Potassco

Reviewer Assignment

by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Potassco

Reviewer Assignment

by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Potassco

Reviewer Assignment

by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

Potassco

Reviewer Assignment

by Ilkka Niemelä

```
paper(p1).  reviewer(r1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
paper(p2).  reviewer(r2). classA(r2,p3). classB(r2,p4). coi(r2,p6).
[...]

{ assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

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Reviewer Assignment

by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
[...]

#count { P,R : assigned(P,R) : reviewer(R) } = 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 <= #count { P,R : assigned(P,R), paper(P) } <= 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 <= #count { P,R : assignedB(P,R), paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

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by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
[...]

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 :- 3 <= #count { P,R : assignedB(P,R), paper(P) }, reviewer(R).

#minimize { 1,P,R : assignedB(P,R), paper(P), reviewer(R) }.
```

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Outline

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Simplified STRIPS[1] Planning

- Problem Instance
  - set of fluents
  - initial and goal state
  - set of actions, consisting of pre- and postconditions
  - number k of allowed actions
- Problem Class Find a plan, that is, a sequence of k actions leading from the initial state to the goal state
- Example
  - fluents $\{p, q, r\}$
  - initial state $\{p\}$
  - goal state $\{r\}$
  - actions $a = (\{p\}, \{q, \neg p\})$ and $b = (\{q\}, \{r, \neg q\})$
  - length 2

  plan $\langle a, b \rangle$

[1] Stanford Research Institute Problem Solver, 1971

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  - length 2
  - plan $\langle a, b \rangle$

[1] Stanford Research Institute Problem Solver, 1971

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Simplistic STRIPS Planning

```
time(1..k).

fluent(p).      action(a).      action(b).      init(p).
fluent(q).        pre(a,p).       pre(b,q).
fluent(r).        add(a,q).       add(b,r).     query(r).
                  del(a,p).       del(b,q).

holds(P,0) :- init(P).

{ occ(A,T) : action(A) } = 1 :- time(T).
:- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- occ(A,T), add(A,F).
holds(F,T) :- holds(F,T-1), time(T), not occ(A,T) : del(A,F).

:- query(F), not holds(F,k).
```

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Simplistic STRIPS Planning

```
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Simplistic STRIPS Planning

```
time(1..k).

fluent(p).      action(a).      action(b).        init(p).
fluent(q).        pre(a,p).       pre(b,q).
fluent(r).        add(a,q).       add(b,r).       query(r).
                  del(a,p).       del(b,q).

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{ occ(A,T) : action(A) } = 1 :- time(T).
:- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- occ(A,T), add(A,F).
holds(F,T) :- holds(F,T-1), time(T), not occ(A,T) : del(A,F).

:- query(F), not holds(F,k).
```

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Engineering: Overview

15 Meta programming

16 Controlling

17 Multi-shot solving

18 Theory solving

19 Heuristic-driven solving

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Do it yourself!

- Roland Kaminski, Javier Romero, Torsten Schaub, Philipp Wanko:
  How to build your own ASP-based system?!
  CoRR abs/2008.06692 (2020)

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Outline

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What is this?

- A technique in which programs treat other programs as data
- Workflow
  1. Translate a logic program into facts
  2. Combine the obtained facts with a meta logic program
- The first step is called reification

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What is this?

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Potassco

What is this?

- A technique in which programs treat other programs as data
- Workflow
  1. Translate a logic program into facts
  2. Combine the obtained facts with a meta logic program
- The first step is called reification

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An example

- Logic program `ezy.lp`

```
{a}.
b :- a.
c :- not a.
```

- Implicit statements

```
#show a.
#show b.
#show c.
```

- Reification

```
$ clingo --output=reify ezy.lp
```

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An example

- Logic program `ezy.lp`

```
{a}.
b :- a.
c :- not a.
```

- Implicit statements

```
#show a.
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#show c.
```

- Reification

```
$ clingo --output=reify ezy.lp
```

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An example

- Logic program `ezy.lp`

```
{a}.
b :- a.
c :- not a.
```

- Implicit statements

```
#show a.
#show b.
#show c.
```

- Reification

```
$ clingo --output=reify ezy.lp
```

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An example, reified

```
$ clingo --output=reify ezy.lp
atom_tuple(0).
atom_tuple(0,1).
literal_tuple(0).
rule(choice(0),normal(0)).
atom_tuple(1).
atom_tuple(1,2).
literal_tuple(1).
literal_tuple(1,-1).
rule(disjunction(1),normal(1)).
atom_tuple(2).
atom_tuple(2,3).
literal_tuple(2).
literal_tuple(2,1).
rule(disjunction(2),normal(2)).
output(a,2).
literal_tuple(3).
literal_tuple(3,3).
output(b,3).
literal_tuple(4).
literal_tuple(4,2).
output(c,4).
```

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An example, again

```
{a}.
b :- a.
c :- not a.

#show a.
#show b.
#show c.
```

Potassco

An example, reified again

```
rule(choice(0),normal(0)).

rule(disjunction(1),normal(1)).

rule(disjunction(2),normal(2)).

output(a,2).

output(b,3).

output(c,4).
```

```
atom_tuple(0).
atom_tuple(0,1).

atom_tuple(1).
atom_tuple(1,2).

atom_tuple(2).
atom_tuple(2,3).
```

```
literal_tuple(0).

literal_tuple(1).
literal_tuple(1,-1).

literal_tuple(2).
literal_tuple(2,1).

literal_tuple(3).
literal_tuple(3,3).

literal_tuple(4).
literal_tuple(4,2).
```

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Meta encoding, or ASP in ASP

```
conjunction(B) :- literal_tuple(B),
        hold(L) : literal_tuple(B, L), L > 0;
    not hold(L) : literal_tuple(B,-L), L > 0.

body(normal(B)) :- rule(_,normal(B)), conjunction(B).
body(sum(B,G))  :- rule(_,sum(B,G)),
    #sum { W,L :     hold(L), weighted_literal_tuple(B, L,W), L > 0 ;
           W,L : not hold(L), weighted_literal_tuple(B,-L,W), L > 0 } >= G.

  hold(A) : atom_tuple(H,A)   :- rule(disjunction(H),B), body(B).
{ hold(A) : atom_tuple(H,A) } :- rule(     choice(H),B), body(B).

#show.
#show T : output(T,B), conjunction(B).
```

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An example, running

- Logic program `ezy.lp`

```
{a}.
b :- a.
c :- not a.
```

- Running

```
$ clingo ezy.lp 0
clingo version 5.5.0
Reading from ezy.lp
Solving...
Answer: 1
c
Answer: 2
a b
SATISFIABLE
```

Potassco

An example, running

- Logic program `ezy.lp`

```
{a}.
b :- a.
c :- not a.
```

- Running

```
$ clingo ezy.lp 0
clingo version 5.5.0
Reading from ezy.lp
Solving...
Answer: 1
c
Answer: 2
a b
SATISFIABLE
```

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An example, running reified

- Logic program `ezy.lp`

```
{a}.
b :- a.
c :- not a.
```

- Running reified

```
$ clingo --output=reify ezy.lp | clingo - meta.lp 0
clingo version 5.5.0
Reading from - ...
Solving...
Answer: 1
c
Answer: 2
a b
SATISFIABLE
```

Potassco

An example, running reified

- Logic program `ezy.lp`

```
{a}.
b :- a.
c :- not a.
```

- Running reified

```
$ clingo --output=reify ezy.lp | clingo - meta.lp 0
clingo version 5.5.0
Reading from - ...
Solving...
Answer: 1
c
Answer: 2
a b
SATISFIABLE
```

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Some applications

- Re-interpretation of language constructs
- Interpretation of new language constructs
- Computation of classical models
- Computation of supported models
- Computation of here-and-there models
- Computation of diverse stable models
- Computation of preferred models
- Checking program equivalence
- Guess-and-check programming

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Outline

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Taming the ASP system, imperatively

- Three alternative ways of combining ASP with other languages, either via
  - embedded script
  - module import
  - application class
- We use Python, although other choices exist

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An example

- Input program `example.lp`

```
num(3).
num(6).
div(N,@divisors(N)) :- num(N).
```

- Resulting program

```
num(3).
num(6).
div(3,(1;3)).
div(6,(1;2;3;6)).
```

Potassco

An example

- Input program `example.lp`

```
num(3).
num(6).
div(N,@divisors(N)) :- num(N).
```

- Resulting program

```
num(3).
num(6).
div(3,(1;3)).
div(6,(1;2;3;6)).
```

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Embedded script (embedded.lp)

```
#script (python)
import clingo
def divisors(a):
    a = a.number
    for i in range(1, a+1):
        if a % i == 0:
            yield clingo.Number(i)
#end.
```

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Embedded script, running

```
$ clingo example.lp embedded.lp
clingo version 5.5.0
Reading from example.lp ...
Solving...
Answer: 1
num(3) num(6) div(3,1) div(3,3) \
div(6,1) div(6,2) div(6,3) div(6,6)
SATISFIABLE
```

Potassco

Module import (module.py)

```
import clingo
class ExampleApp:
    @staticmethod
    def divisors(a):
        a = a.number
        for i in range(1, a+1):
            if a % i == 0:
                yield clingo.Number(i)
    def run(self):
        ctl = clingo.Control()
        ctl.load("example.lp")
        ctl.ground([("base", [])], context=self)
        ctl.solve(on_model=print)
if __name__ == "__main__":
    ExampleApp().run()
```

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Embedded script, running

```
$ python module.py
num(3) num(6) div(3,1) div(3,3) \
div(6,1) div(6,2) div(6,3) div(6,6)
```

Application class (app.py)

```
import sys
import clingo

class ExampleApp(clingo.Application):
    program_name = "example"
    version = "1.0"

    @staticmethod
    def divisors(a):
        a = a.number
        for i in range(1, a+1):
            if a % i == 0:
                yield clingo.Number(i)

    def main(self, ctl, files):
        for path in files: ctl.load(path)
        if not files:
            ctl.load("-")
        ctl.ground([("base", [])], context=self)
        ctl.solve()

if __name__ == "__main__":
    clingo.clingo_main(ExampleApp(), sys.argv[1:])
```

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Application class, running

```
$ python app.py example.lp
example version 1.0
Reading from example.lp
Solving...
Answer: 1
num(3) num(6) div(3,1) div(3,3) \
div(6,1) div(6,2) div(6,3) div(6,6)
SATISFIABLE
```

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What to use when. . . ?

- embedded script
  - suitable for small amendments to the logic program, anything on the term level during grounding
  - perform calculations that are hard or inconvenient to express in ASP
- module import
  - convenient way to use *clingo* as part of a larger project
  - provides high level functions to control grounding and solving
  - surrounding application is in charge of the control flow and ASP is used to perform specific computations
- application class
  - aims at building custom systems based on *clingo*
  - similar to module import but with more customization capabilities

☞ constitutes the cornerstone of recent *clingo*-based systems such as *clingcon*, *clingo*[DL], *eclingo*, and *telingo*

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Outline

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Motivation

- Multi-shot solving allows for solving continuously changing logic programs in an operative way
  - Single-shot solving: *ground* | *solve*
  - Multi-shot solving: *ground* | *solve*

Agents, Assisted Living, Robotics, Planning, Query-answering, etc

clingo = ASP + Control

Extend ASP with dedicated directives

Provide powerful API (here: Python)

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  - Multi-shot solving: (*input* | *ground** | *solve**)*
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  - Single-shot solving: *ground* | *solve*
  - Multi-shot solving: (*input* | *ground** | *solve**| *theory*)*
- Application areas

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  - Single-shot solving: *ground* | *solve*
  - Multi-shot solving: (*input* | *ground** | *solve**| *theory* | . . .)*
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  - Single-shot solving: *ground* | *solve*
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  - Multi-shot solving: (*input* | *ground** | *solve**| *theory* | . . .)*
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- Idea *clingo* = ASP + Control
  - Extend ASP with dedicated directives
  - Provide powerful API (here: Python)

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Structuring logic programs

- Program directive

  ```
  #program <name> [ (<parameters>) ]
  ```

  where
  - `<name>` is a term
  - `(<parameters>)` is a tuple of terms

- Example `#program play(p,t).`
- Default `#program base.`

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Structuring logic programs

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  ```

  where
  - `<name>` is a term
  - `(<parameters>)` is a tuple of terms

- Example `#program play(p,t).`
- Default `#program base.`

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An example (chemistry.lp)

```
a(1).
#program acid(k).
b(k).
c(X,k) :- a(X).
#program base.
a(2).
```

The example, processing (control-base.py)

```
import clingo
ctl = clingo.Control()
ctl.load("chemistry.lp")
ctl.ground([("base", [])])
ctl.solve(on_model=print)
```

```
a(1).
#program acid(k).
b(k).
c(X,k) :- a(X).
#program base.
a(2).
```

```
$ python control-base.py
a(1) a(2)
```

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```

```
a(1).
#program acid(k).
b(k).
c(X,k) :- a(X).
#program base.
a(2).
```

```
$ python control-base.py
a(1) a(2)
```

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```
import clingo
ctl = clingo.Control()
ctl.load("chemistry.lp")
ctl.ground([("base", [])])
ctl.solve(on_model=print)
```

```
a(1).
#program acid(k).
b(k).
c(X,k) :- a(X).
#program base.
a(2).
```

```
$ python control-base.py
a(1) a(2)
```

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The example, processing (control-acid.py)

```
import clingo
ctl = clingo.Control()
ctl.load("chemistry.lp")
ctl.ground([("acid",[42])])
ctl.solve(on_model=print)
```

```
a(1).
#program acid(k).
b(k).
c(X,k) :- a(X).
#program base.
a(2).
```

```
$ python control-acid.py
b(42)
```

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The example, processing (control-acid.py)

```
import clingo
ctl = clingo.Control()
ctl.load("chemistry.lp")
ctl.ground([("acid",[42])])
ctl.solve(on_model=print)
```

```
a(1).
#program acid(k).
b(k).
c(X,k) :- a(X).
#program base.
a(2).
```

```
$ python control-acid.py
b(42)
```

Potassco

The example, processing (control-acid.py)

```
import clingo
ctl = clingo.Control()
ctl.load("chemistry.lp")
ctl.ground([("acid",[42])])
ctl.solve(on_model=print)
```

```
a(1).
#program acid(k).
b(k).
c(X,k) :- a(X).
#program base.
a(2).
```

```
$ python control-acid.py
b(42)
```

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External atoms

- External directive

  ```
  #external <atom> [ :  <body> ]
  ```

  where
  - `<atom> [ : <body> ]` is a (conditional) literal

- Example `#external mark(X,Y,p,t) : field(X,Y).`
- Note External atoms are
  - protected from program simplifications
  - assigned truth values via API (default: false)

  and can be
  - overwritten by adding rules defining the atom
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An example (chemistry-external.lp)

```
a(1).
#program acid(k).
b(k).
c(X,k) :- a(X).
#program base.
a(2).

#program acid(k).
#external d(X,k) : c(X,k).
e(X,k) :- d(X,k).
```

Note Grounding both base and acid(42) yields two externals

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e(X,k) :- d(X,k).
```

Note Grounding both `base` and `acid(42)` yields two externals

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The example, processing (control-external.py)

```
ctl = clingo.Control()
ctl.load("chemistry-external.lp")
ctl.ground([("base", []),("acid",[42])])
ctl.solve(on_model=print)
ctl.assign_external(Function("d", [2,42]), True)
ctl.solve(on_model=print)
```

```
$ python control-external.py
a(1) a(2) c(1,42) c(2,42) b(42)
a(1) a(2) c(1,42) c(2,42) b(42) d(2,42) e(2,42)
```

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ctl.load("chemistry-external.lp")
ctl.ground([("base", []),("acid",[42])])
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ctl.assign_external(Function("d", [2,42]), True)
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```

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$ python control-external.py
a(1) a(2) c(1,42) c(2,42) b(42)
a(1) a(2) c(1,42) c(2,42) b(42) d(2,42) e(2,42)
```

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An example of incremental solving

```
#program base.
p(0).
#program step (t).
p(t) :- p(t-1).
#program check (t).
#external query(t).        % added in python below
:- not p(42), query(t).
```

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#external query(t).          % added in python below
:- not p(42), query(t).
```

Incremental solving

```
'''
Example implementing an iclingo-like application.
'''
import sys
from typing import Optional, Any, Callable, Iterable, cast
from clingo import Application, ApplicationOptions, Control, SolveResult
from clingo import Function, Number, clingo_main

class IncConfig:
    '''
    Configuration object for incremental solving.
    '''
    imin: int
    imax: Optional[int]
    istop: str
    def __init__(self):
        self.imin = 1
        self.imax = None
        self.istop = "SAT"

def parse_int(conf: Any,
              attr: str,
              min_value: Optional[int] = None,
              optional: bool = False) -> Callable[[str], bool]:
    '''
    Returns a parser for integers.
    The parser stores its result in the `attr` attribute (given as string) of
    the `conf` object. The parser can be configured to only accept integers
    having a minimum value and also to treat value `"none"` as `None`.
    '''
    def parse(sval: str) -> bool:
        if optional and sval == "none":
            value = None
        else:
            value = int(sval)
            if min_value is not None and value < min_value:
                raise RuntimeError("value too small")
        setattr(conf, attr, value)
        return True
    return parse

def parse_stop(conf: Any, attr: str) -> Callable[[str], bool]:
    '''
    Returns a parser for `istop` values.
    '''
    def parse(sval: str) -> bool:
        if sval not in ("SAT", "UNSAT", "UNKNOWN"):
            raise RuntimeError("invalid value")
        setattr(conf, attr, sval)
        return True
    return parse

class IncApp(Application):
    '''
    The example application implemeting incremental solving.
    '''
    program_name: str = "inc-example"
    version: str = "1.0"
    _conf: IncConfig
    def __init__(self):
        self._conf = IncConfig()
    def register_options(self, options: ApplicationOptions):
        '''
        Register program options.
        '''
        group = "Inc-Example Options"
        options.add(
            group, "imin",
            "Minimum number of steps [{}]".format(self._conf.imin),
            parse_int(self._conf, "imin", min_value=0),
            argument="<n>")
        options.add(
            group, "imax",
            "Maximum number of steps [{}]".format(self._conf.imax),
            parse_int(self._conf, "imax", min_value=0, optional=True),
            argument="<n>")
        options.add(
            group, "istop",
            "Stop criterion {{}}".format(self._conf.istop),
            parse_stop(self._conf, "istop"))
    def main(self, ctl: Control, files: Iterable[str]):
        '''
        The main function implementing incremental solving.
        '''
        if not files:
            files = ["-"]
        for file_ in files:
            ctl.load(file_)
        ctl.add("check", ["t"], "#external query(t).")
        conf = self._conf
        step = 0
        ret: Optional[SolveResult] = None
        while ((conf.imax is None or step < conf.imax) and
               (ret is None or step < conf.imin or (
                   (conf.istop == "SAT" and not ret.satisfiable) or
                   (conf.istop == "UNSAT" and not ret.unsatisfiable) or
                   (conf.istop == "UNKNOWN" and not ret.unknown)))):
            parts = []
            parts.append(("check", [Number(step)]))
            if step > 0:
                ctl.release_external(Function("query", [Number(step - 1)]))
                parts.append(("step", [Number(step)]))
            else:
                parts.append(("base", []))
            ctl.ground(parts)
            ctl.assign_external(Function("query", [Number(step)]), True)
            ret, step = cast(SolveResult, ctl.solve()), step + 1

clingo_main(IncApp(), sys.argv[1:])
```

Incremental solving, zoom on register_options

```
def register_options(self, options: ApplicationOptions):
    '''
    Register program options.
    '''
    group = "Inc-Example Options"
    options.add(
        group, "imin",
        "Minimum number of steps [{}]".format(self._conf.imin),
        parse_int(self._conf, "imin", min_value=0),
        argument="<n>")
    options.add(
        group, "imax",
        "Maximum number of steps [{}]".format(self._conf.imax),
        parse_int(self._conf, "imax", min_value=0, optional=True),
        argument="<n>")
    options.add(
        group, "istop",
        "Stop criterion [{}]".format(self._conf.istop),
        parse_stop(self._conf, "istop"))
```

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Incremental solving, zoom on `main`

```
def main(self, ctl: Control, files: Iterable[str]):
    '''
    The main function implementing incremental solving.
    '''
    if not files:
        files = ["-"]
    for file_ in files:
        ctl.load(file_)
    ctl.add("check", ["t"], "#external query(t).")
    conf = self._conf
    step = 0
    ret: Optional[SolveResult] = None
    while ((conf.imax is None or step < conf.imax) and
           (ret is None or step < conf.imin or (
               (conf.istop == "SAT" and not ret.satisfiable) or
               (conf.istop == "UNSAT" and not ret.unsatisfiable) or
               (conf.istop == "UNKNOWN" and not ret.unknown)))):
        parts = []
        parts.append(("check", [Number(step)]))
        if step > 0:
            ctl.release_external(Function("query", [Number(step - 1)]))
            parts.append(("step", [Number(step)]))
        else:
            parts.append(("base", []))
        ctl.ground(parts)
        ctl.assign_external(Function("query", [Number(step)]), True)
        ret, step = cast(SolveResult, ctl.solve()), step + 1
```

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Optimization

- Imagine some Blocksworld planning problem ...
- Code snippet

```
ngoal(T) :- on(B,L,T), not goal_on(B,L).
:- ngoal(n).
```

  where `n` is a fixed horizon

- Optimization

```
_minimize(1,T) :- ngoal(T).
```

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Optimization

- Imagine some Blocksworld planning problem ...
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ngoal(T) :- on(B,L,T), not goal_on(B,L).
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  where `n` is a fixed horizon

- Optimization

```
_minimize(1,T) :- ngoal(T).
```

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Optimization

```
'''
Exmaple to show branch and bound based optimization using multi-shot solving.
'''

import sys
from typing import Optional, Iterable, cast
from clingo import Model, Control, SolveResult, SymbolType, Application, Number, clingo_main

class OptApp(Application):
    '''
    Example application.
    '''
    program_name: str = "opt-example"
    version: str = "1.0"
    _bound: Optional[int]

    def __init__(self):
        self._bound = None

    def _on_model(self, model: Model):
        self._bound = 0
        for atom in model.symbols(atoms=True):
            if (atom.match("_minimize", 2) and
                    atom.arguments[0].type is SymbolType.Number):
                self._bound += atom.arguments[0].number

    def main(self, ctl: Control, files: Iterable[str]):
        '''
        Main function implementing branch and bound optimization.
        '''
        if not files:
            files = ["-"]
        for file_ in files:
            ctl.load(file_)
        ctl.add("bound", ["b"],
                ":- #sum { V,I: _minimize(V,I) } >= b.")
        ctl.ground([("base", [])])
        while cast(SolveResult, ctl.solve(on_model=self._on_model)).satisfiable:
            print("Found new bound: {}".format(self._bound))
            ctl.ground([("bound", [Number(cast(int, self._bound))])])
        if self._bound is not None:
            print("Optimum found")

clingo_main(OptApp(), sys.argv[1:])
```

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Optimization, zoom on `main`

```
def main(self, ctl: Control, files: Iterable[str]):
    '''
    Main function implementing branch and bound optimization.
    '''
    if not files:
        files = ["-"]
    for file_ in files:
        ctl.load(file_)
    ctl.add("bound", ["b"],
            ":- #sum { V,I: _minimize(V,I) } >= b.")
    ctl.ground([("base", [])])
    while cast(SolveResult, ctl.solve(on_model=self._on_model)).satisfiable:
        print("Found new bound: {}".format(self._bound))
        ctl.ground([("bound", [Number(cast(int, self._bound))])])
    if self._bound is not None:
        print("Optimum found")
```

Optimization, zoom on _on_model

```
def _on_model(self, model: Model):
    self._bound = 0
    for atom in model.symbols(atoms=True):
        if (atom.match("_minimize", 2) and
                atom.arguments[0].type is SymbolType.Number):
            self._bound += atom.arguments[0].number
```

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Outline

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Motivation

- Input ASP = DB+KRR+LP+SAT
- Output ASPmT = DB+KRR+LP+S
- ASP solving: *ground* | *solve*
- Challenge Logic programs with elusive theory atoms
- Application areas

  Agents, Assisted Living, Robotics, Planning, Scheduling, Bio- and Cheminformatics, etc

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Motivation

- Input ASP = DB+KRR+LP+SAT
- Output ASPmT = DB+KRR+LP+SMT — **NO!**
- ASP solving: *ground* | *solve*
- Challenge Logic programs with elusive theory atoms
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- Input ASP = DB+KRR+LP+SAT
- Output ASPmT = (DB+KRR+LP+SAT)mT
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Motivation

- Input ASP = DB+KRR+LP+SAT
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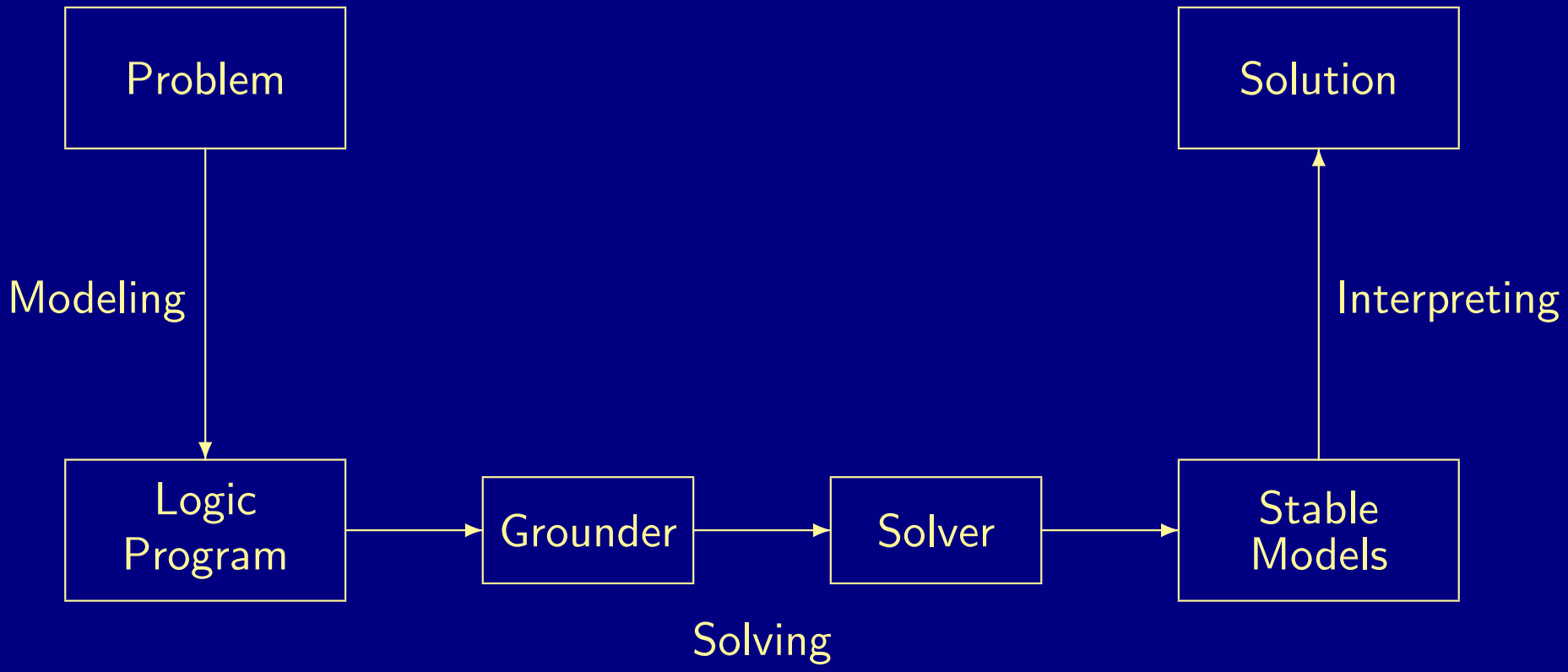
ASP solving process



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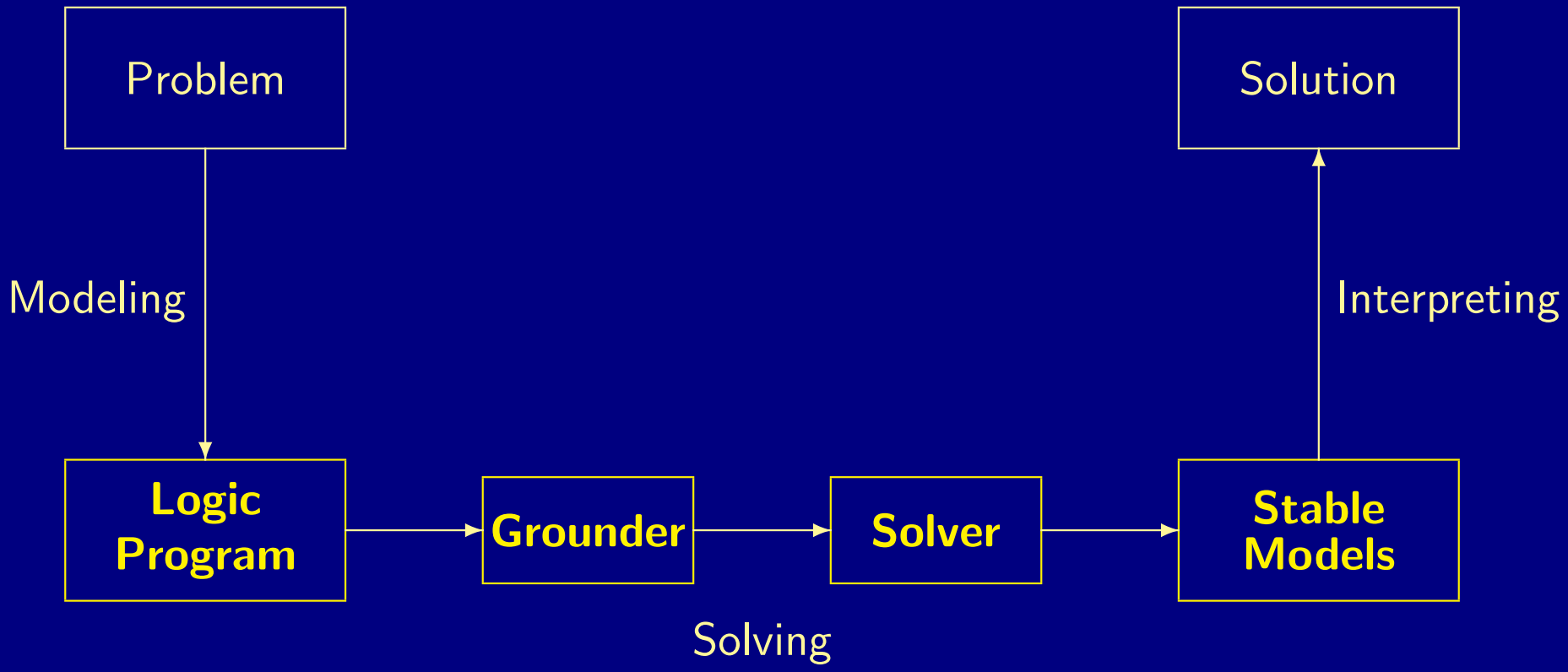
ASP solving process modulo theories



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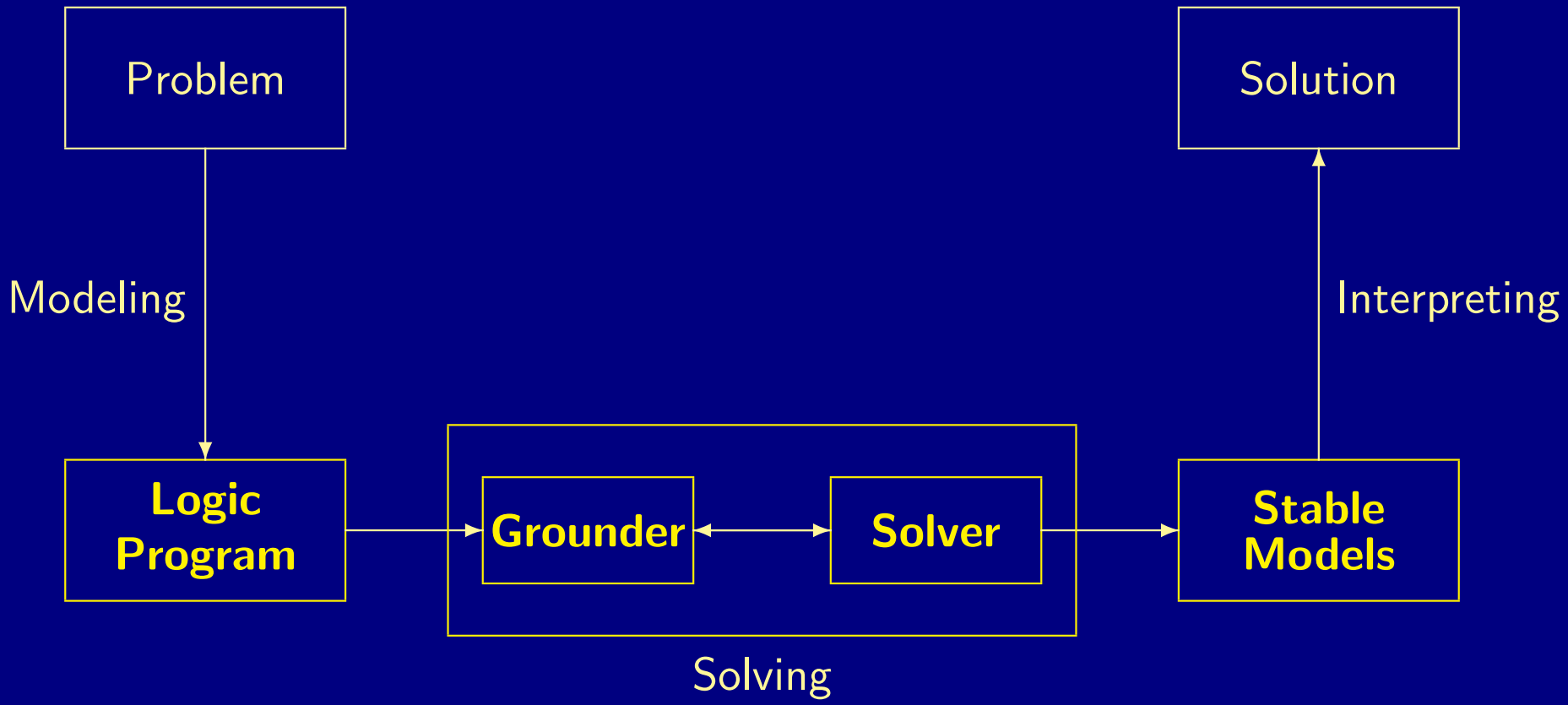
ASP solving process modulo theories



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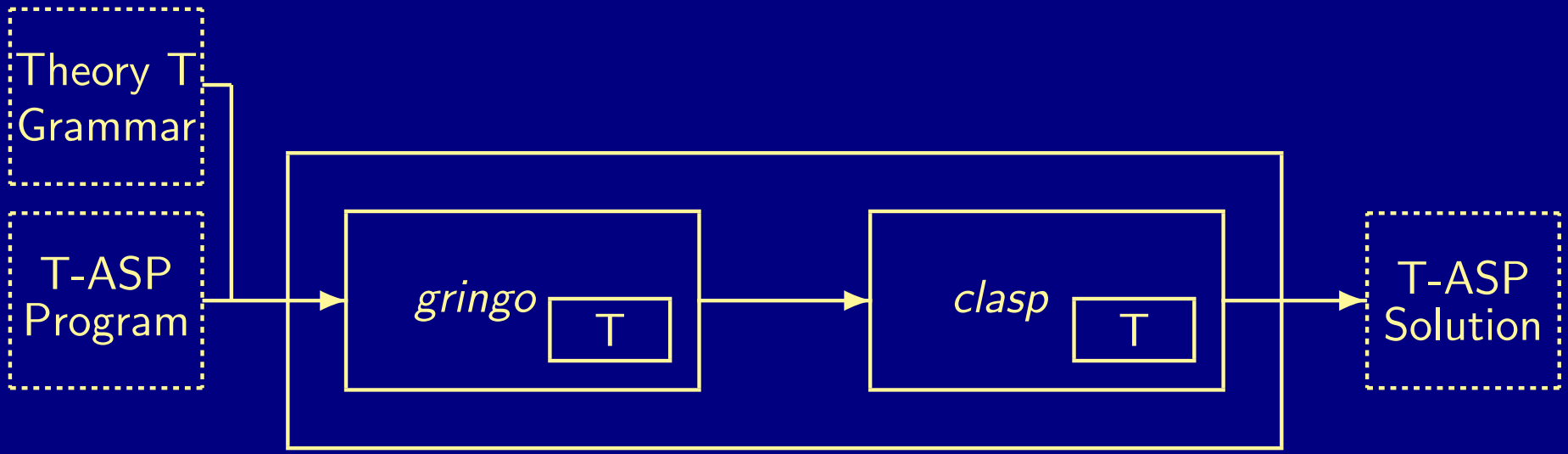
clingo's approach



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Outline

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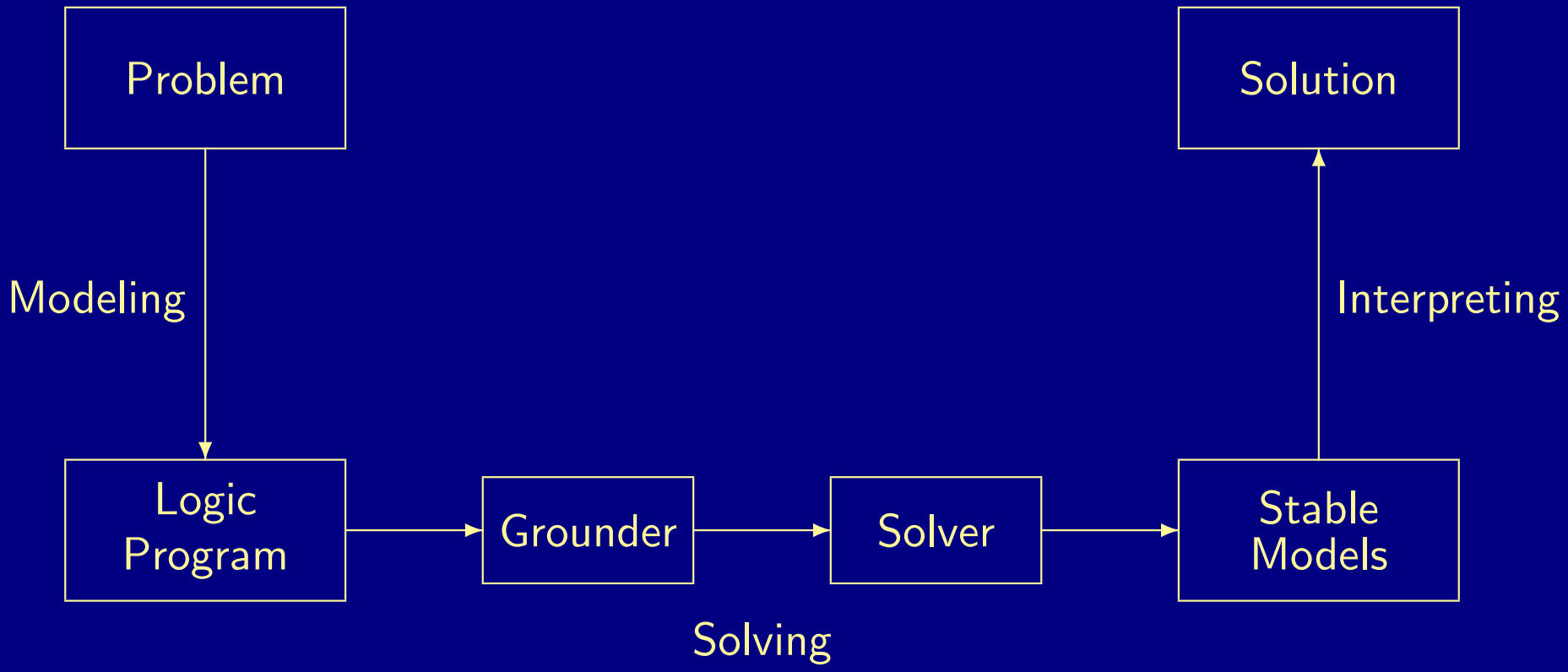
ASP solving process modulo theories



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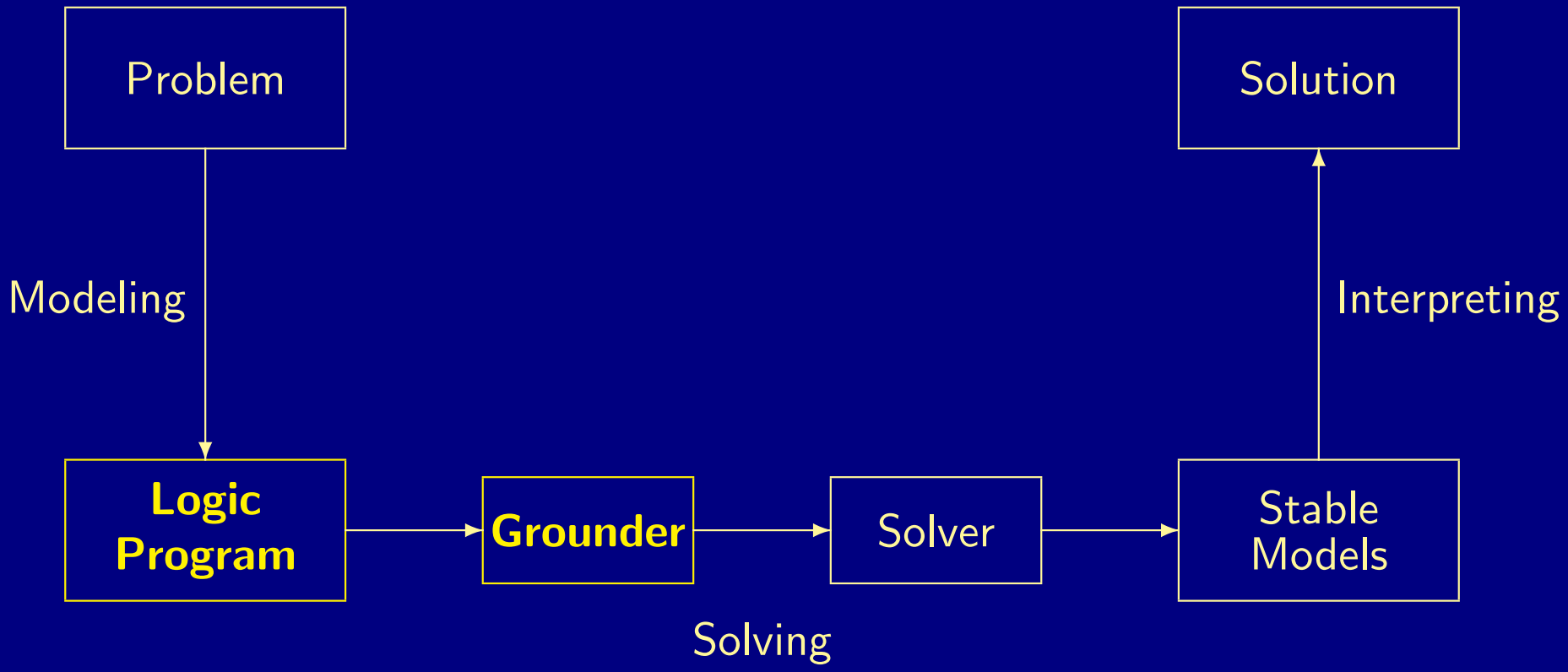
ASP solving process modulo theories



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Linear constraints

```
#theory csp {
    linear_term {
      + : 5, unary;
      - : 5, unary;
      * : 4, binary, left;
      + : 3, binary, left;
      - : 3, binary, left
    };

    dom_term {
      + : 5, unary;
      - : 5, unary;
      .. : 1, binary, left
    };

    show_term {
      / : 1, binary, left
    };

    minimize_term {
      + : 5, unary;
      - : 5, unary;
      * : 4, binary, left;
      + : 3, binary, left;
      - : 3, binary, left;
      @ : 0, binary, left
    };

    &dom/0 : dom_term, {=}, linear_term, any;
    &sum/0 : linear_term, {<=,=,>=,<,>,!=}, linear_term, any;
    &show/0 : show_term, directive;
    &distinct/0 : linear_term, any;
    &minimize/0 : minimize_term, directive
}.
```

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	s	e	n	d
+	m	o	r	e
m	o	n	e	y

Each letter corresponds exactly to one digit and all variables have to be pairwisely distinct

	9	5	6	7
+	1	0	8	5
1	0	6	5	2

The example has exactly one solution

$$\{ s \mapsto 9, e \mapsto 5, n \mapsto 6, d \mapsto 7, m \mapsto 1, o \mapsto 0, r \mapsto 8, y \mapsto 2 \}$$

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	s	e	n	d
+	m	o	r	e
m	o	n	e	y

Each letter corresponds exactly to one digit and all variables have to be pairwisely distinct

	9	5	6	7
+	1	0	8	5
1	0	6	5	2

The example has exactly one solution

$$\{\ s \mapsto 9, e \mapsto 5, n \mapsto 6, d \mapsto 7, m \mapsto 1, o \mapsto 0, r \mapsto 8, y \mapsto 2\ \}$$

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```
#include "csp.lp".

digit(1,3,s).   digit(2,3,m).   digit(sum,4,m).
digit(1,2,e).   digit(2,2,o).   digit(sum,3,o).
digit(1,1,n).   digit(2,1,r).   digit(sum,2,n).
digit(1,0,d).   digit(2,0,e).   digit(sum,1,e).
                                digit(sum,0,y).

base(10).
exp(E) :- digit(_,E,_).

power(1,0).
power(B*P,E) :- base(B), power(P,E-1), exp(E), E>0.

number(N) :- digit(N,_,_), N!= sum.
high(D) :- digit(N,E,D), not digit(N,E+1,_).

&dom {0..9} = X :- digit(_,_,X).

&sum { M*D : digit(N,E,D),   power(M,E), number(N);
      -M*D : digit(sum,E,D), power(M,E)           } = 0.

&sum { D } > 0 :- high(D).

&distinct { D : digit(_,_,D) }.

&show { D : digit(_,_,D) }.
```

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```
#include "csp.lp".

digit(1,3,s).   digit(2,3,m).   digit(sum,4,m).
digit(1,2,e).   digit(2,2,o).   digit(sum,3,o).
digit(1,1,n).   digit(2,1,r).   digit(sum,2,n).
digit(1,0,d).   digit(2,0,e).   digit(sum,1,e).
                                digit(sum,0,y).

base(10).
exp(E) :- digit(_,E,_).

power(1,0).
power(B*P,E) :- base(B), power(P,E-1), exp(E), E>0.

number(N) :- digit(N,_,_), N!= sum.
high(D) :- digit(N,E,D), not digit(N,E+1,_).

&dom {0..9} = X :- digit(_,_,X).

&sum { M*D : digit(N,E,D),   power(M,E), number(N);
      -M*D : digit(sum,E,D), power(M,E)            } = 0.

&sum { D } > 0 :- high(D).

&distinct { D : digit(_,_,D) }.

&show { D : digit(_,_,D) }.
```

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```
#include "csp.lp".

digit(1,3,s).   digit(2,3,m).   digit(sum,4,m).
digit(1,2,e).   digit(2,2,o).   digit(sum,3,o).
digit(1,1,n).   digit(2,1,r).   digit(sum,2,n).
digit(1,0,d).   digit(2,0,e).   digit(sum,1,e).
                                digit(sum,0,y).

base(10).
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      -M*D : digit(sum,E,D), power(M,E)           } = 0.

&sum { D } > 0 :- high(D).

&distinct { D : digit(_,_,D) }.

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```

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```
digit(1,3,s).   digit(2,3,m).   digit(sum,4,m).
digit(1,2,e).   digit(2,2,o).   digit(sum,3,o).
digit(1,1,n).   digit(2,1,r).   digit(sum,2,n).
digit(1,0,d).   digit(2,0,e).   digit(sum,1,e).
                                digit(sum,0,y).

base(10).
exp(0).  exp(1).  exp(2).  exp(3).  exp(4).

power(1,0).
power(10,1).  power(100,2).  power(1000,3).  power(10000,4).

number(1).  number(2).
high(s).  high(m).

&dom{0..9}=s. &dom{0..9}=m. &dom{0..9}=e. &dom{0..9}=o. &dom{0..9}=n. &d

&sum{  1000*s;   100*e;   10*n;   1*d;
       1000*m;   100*o;   10*r;   1*e;
     -10000*m; -1000*o; -100*n; -10*e; -1*y } = 0.

&sum{s} > 0.  &sum{m} > 0.

&distinct{s; m; e; o; n; r; d; y}.

&show{s; m; e; o; n; r; d; y}.
```

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Outline

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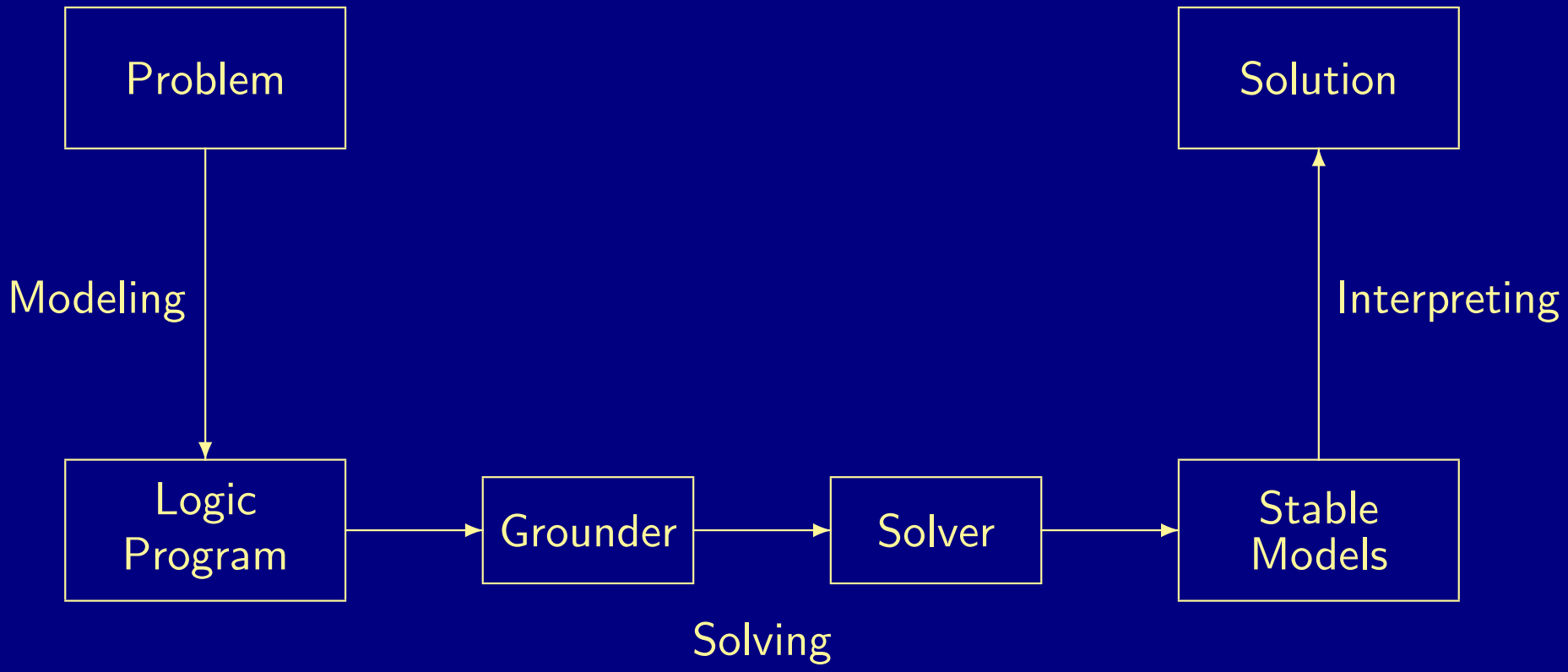
ASP solving process modulo theories



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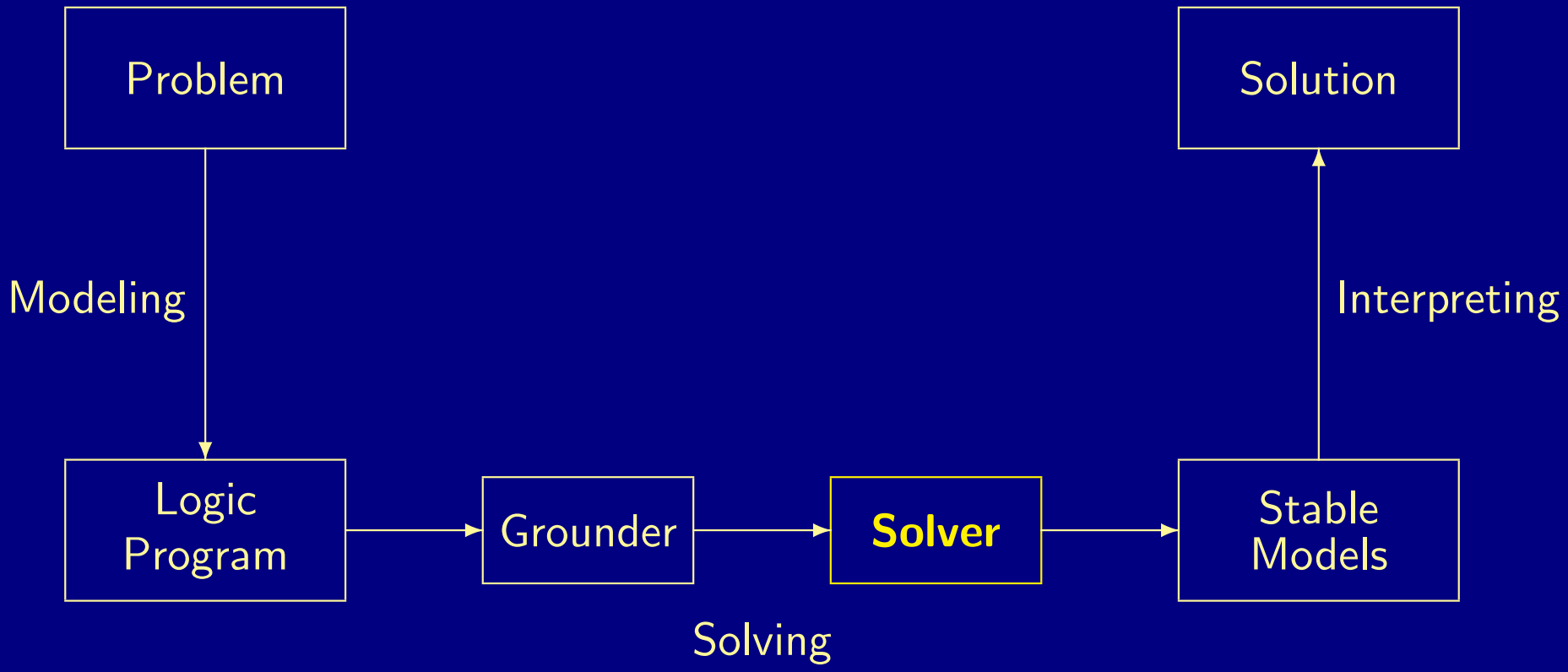
ASP solving process modulo theories



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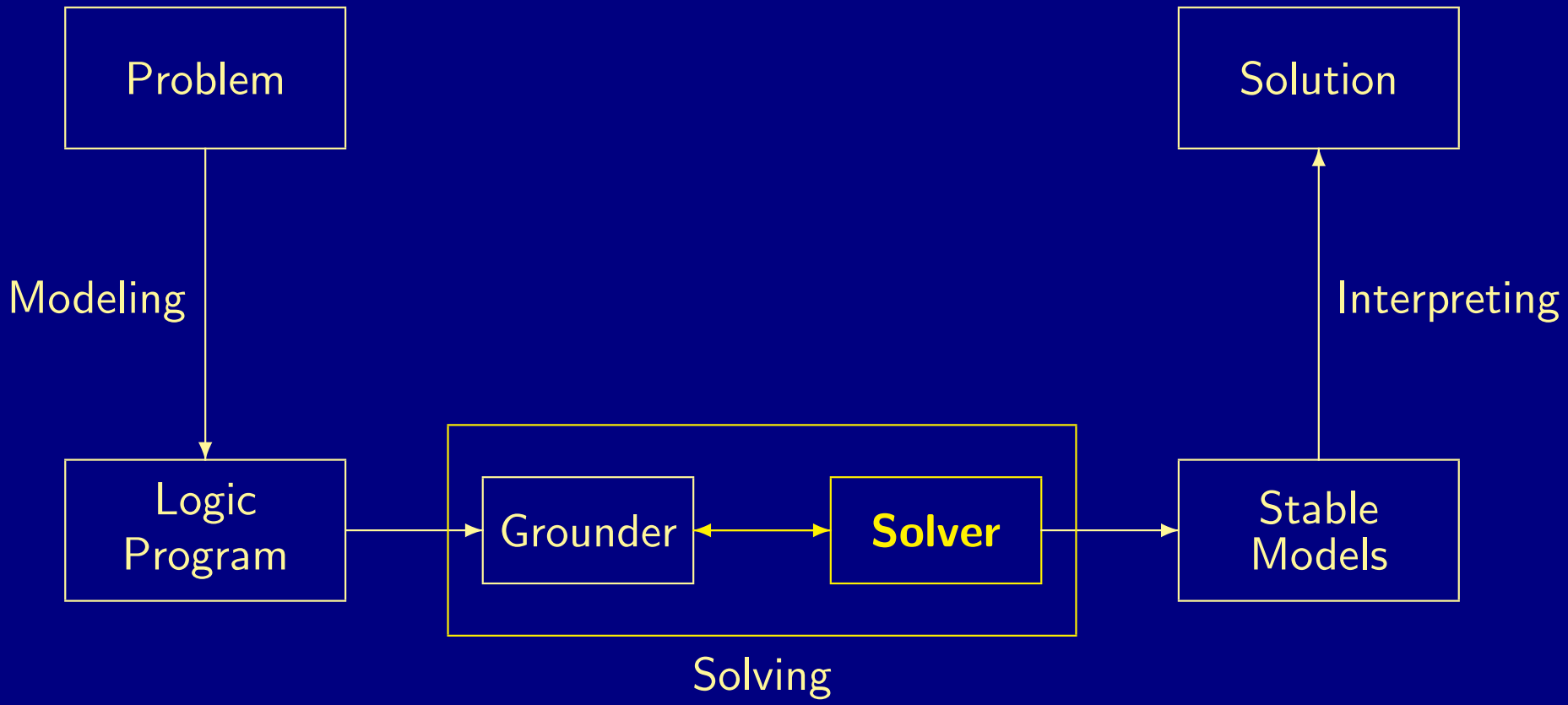

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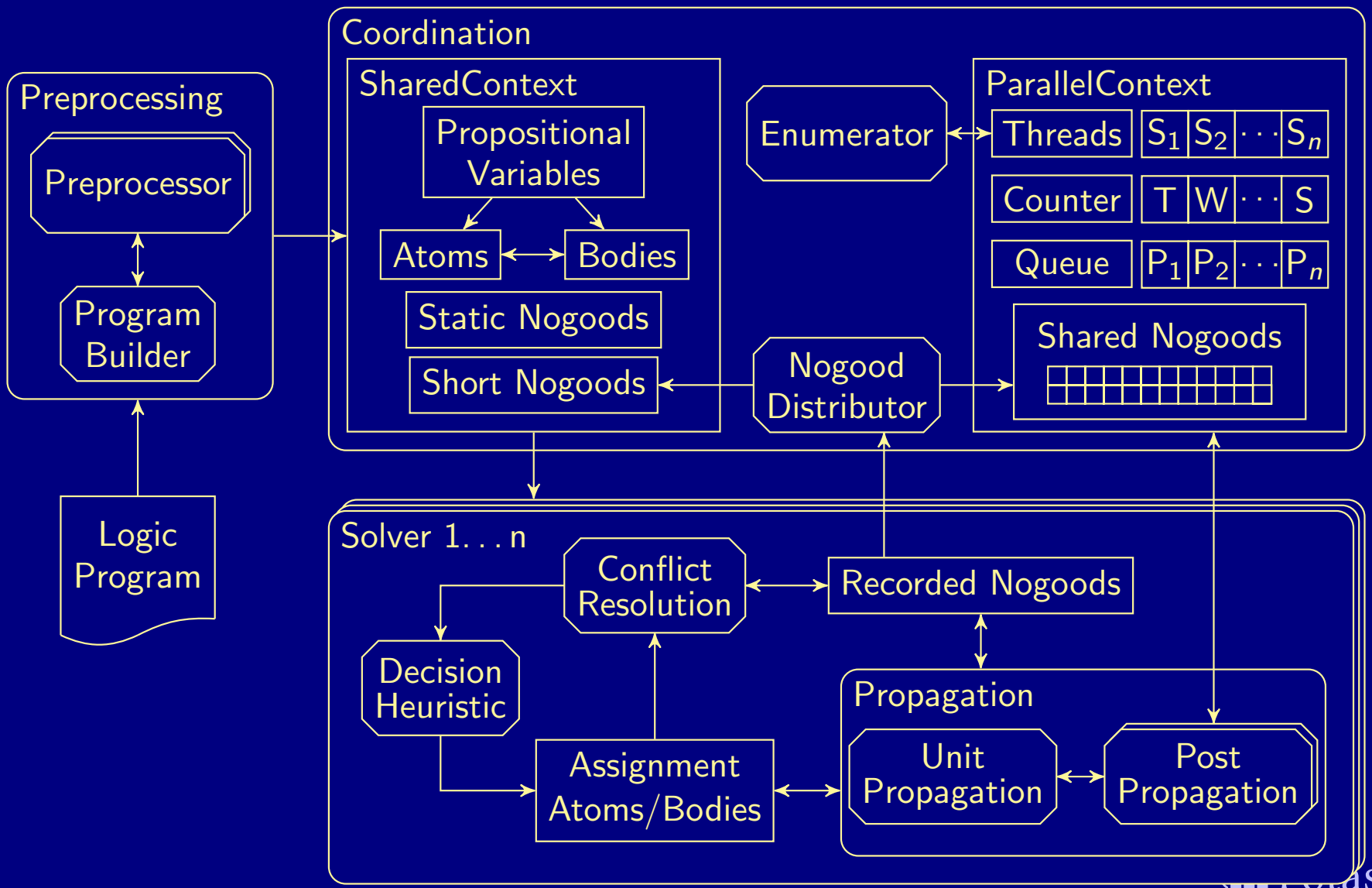
Architecture of *clasp*



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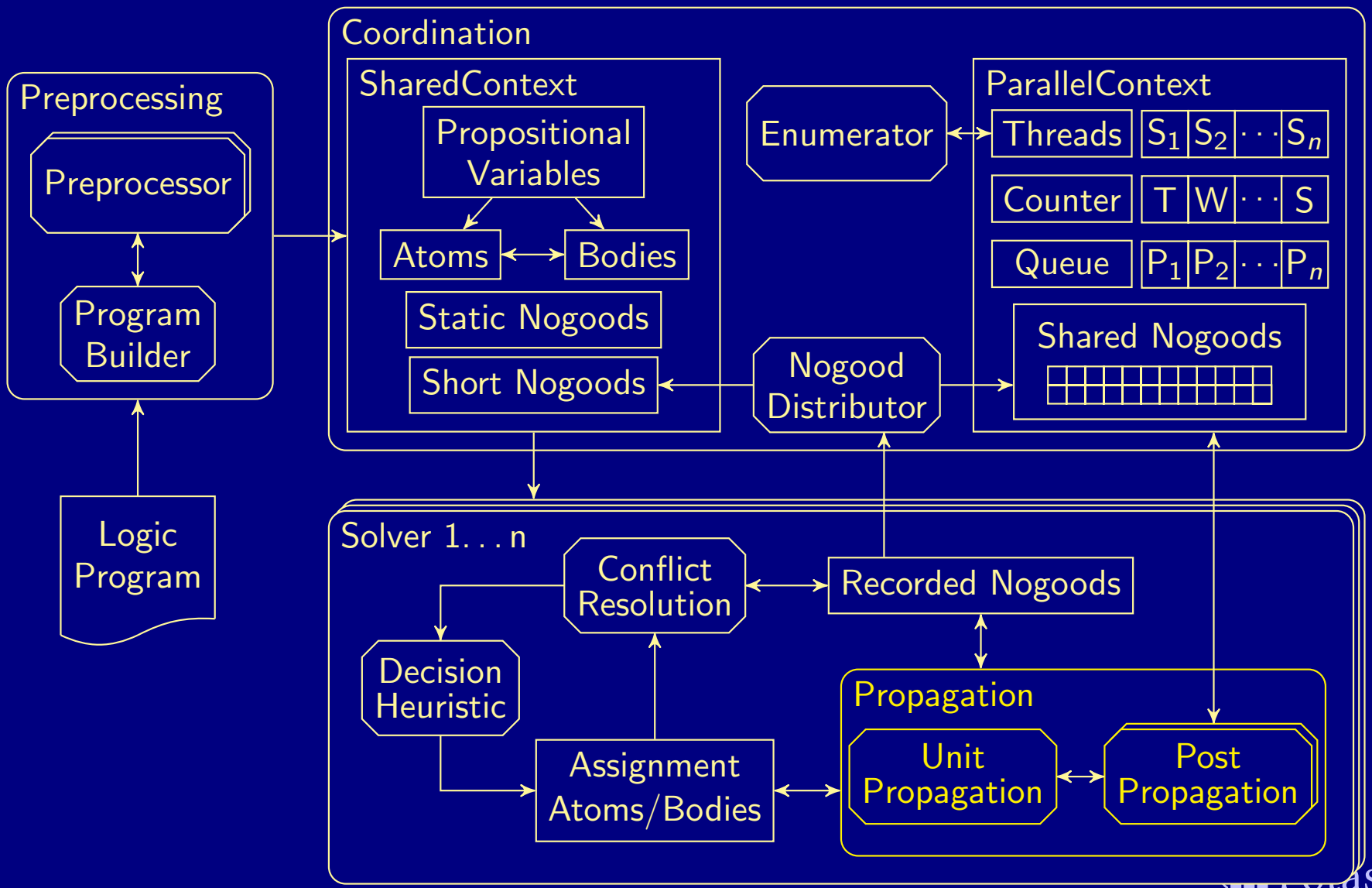

Conflict-driven constraint learning modulo theories

(I) *initialize* // register theory propagators and initialize watches
loop
  propagate completion, loop, and recorded nogoods // deterministically assign literals
  if no conflict **then**
    if all variables assigned **then**
(C) **if** some $\delta \in \Delta_T$ is violated for $T \in \mathbb{T}$ **then** record δ // theory propagator's check
      else return variable assignment // $\mathbb{T}$-stable model found
    else
(P) *propagate* theories $T \in \mathbb{T}$ // theory propagators may record theory nogoods
      if no nogood recorded **then** *decide* // non-deterministically assign some literal
  else
    if top-level conflict **then return** unsatisfiable
    else
      analyze // resolve conflict and record a conflict constraint
(U) *backjump* // undo assignments until conflict constraint is unit

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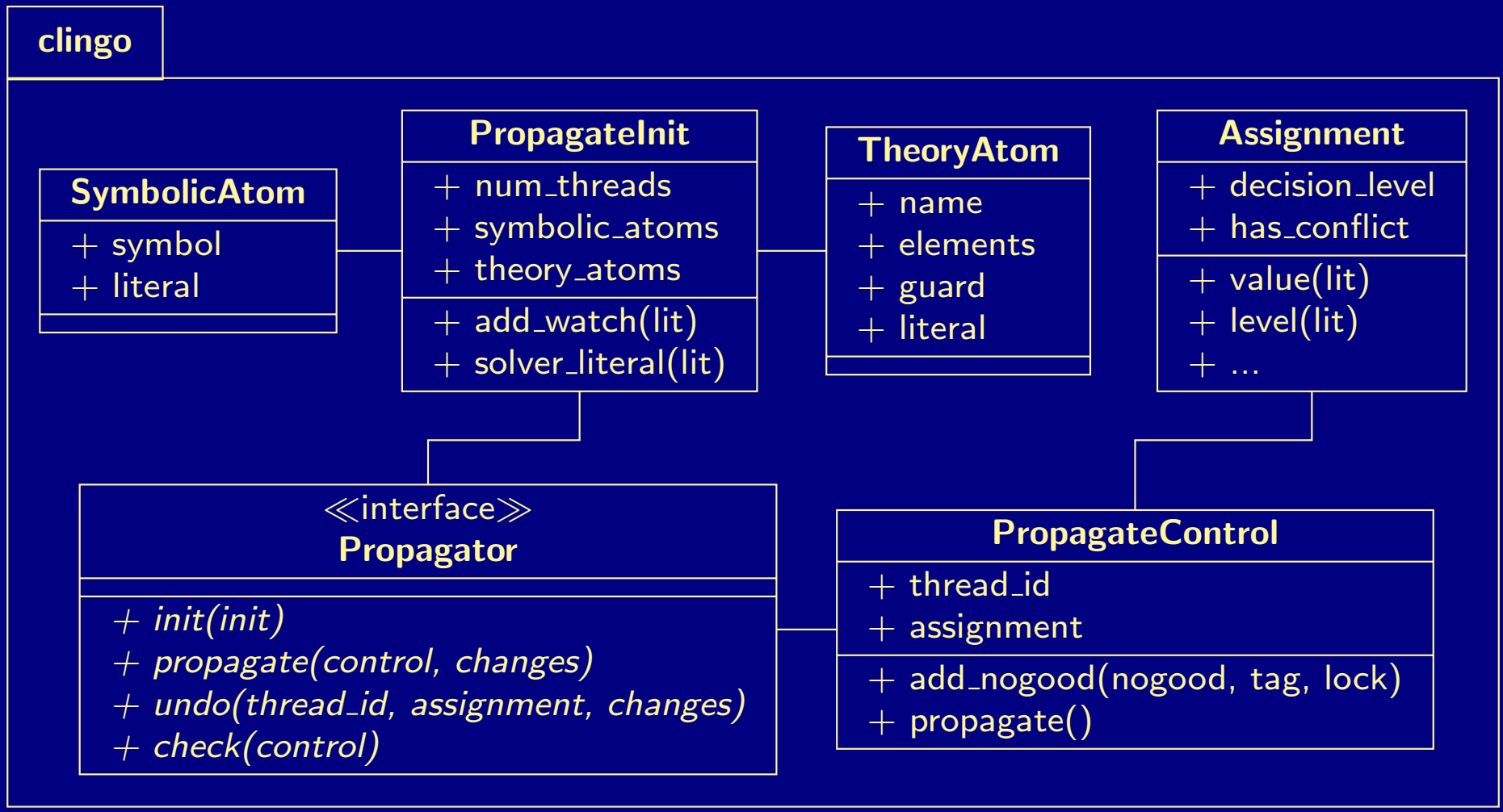
Propagator interface



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The *dot* propagator

```
#script (python)

import sys
import time

class Propagator:
    def init(self, init):
        self.sleep = .1
        for atom in init.symbolic_atoms:
            init.add_watch(init.solver_literal(atom.literal))

    def propagate(self, ctl, changes):
        for l in changes:
            sys.stdout.write(".")
            sys.stdout.flush()
            time.sleep(self.sleep)
        return True

    def undo(self, solver_id, assign, undo):
        for l in undo:
            sys.stdout.write("\b \b")
            sys.stdout.flush()
            time.sleep(self.sleep)

def main(prg):
    prg.register_propagator(Propagator())
    prg.ground([("base", [])])
    prg.solve()
    sys.stdout.write("\n")

#end.
```

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Outline

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Motivation

- Observation Sometimes it is advantageous to take a more application-oriented approach by including domain-specific information
  - domain-specific knowledge can be added for improving propagation
  - domain-specific heuristics can be used for making better choices
- Idea Incorporation of domain-specific heuristics by extending
  - input language and/or solver options for expressing domain-specific heuristics
  - solving capacities for integrating domain-specific heuristics

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Basic CDCL decision algorithm

loop
  propagate // compute deterministic consequences
  if no conflict **then**
    if all variables assigned **then return** variable assignment
    else decide // non-deterministically assign some literal
  else
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Heuristic language

- Heuristic directive

  $\texttt{\#heuristic}\ a\ :\ l_1, \ldots, l_n.\ [k@p, m]$

  where
  - a is an atom, and $l_1, \ldots, l_n$ are literals
  - k and p are integers
  - m is a heuristic modifier
- Heuristic modifiers

`init`	for initializing the heuristic value of a with k
`factor`	for amplifying the heuristic value of a by factor k
`level`	for ranking all atoms; the rank of a is k
`sign`	for attributing the sign of k as truth value to a
- Example

```
#heuristic occurs(A,T) : action(A), time(T). [T, factor]
```

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`true/false`	combine `level` and `sign`

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- Example

```
#heuristic occurs(mv,5) : action(mv), time(5). [5, factor]
```

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Simple STRIPS planning

```
time(1..k).

holds(P,0) :- init(P).

{ occ(A,T) : action(A) } = 1 :- time(T).
:- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- occ(A,T), add(A,F).
holds(F,T) :- holds(F,T-1), time(T), not occ(A,T) : del(A,F).

:- query(F), not holds(F,k).
```

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:- query(F), not holds(F,k).

#heuristic occurs(A,T) : action(A), time(T). [2, factor]
```

Potassco

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:- query(F), not holds(F,k).

#heuristic holds(F,T-1) :     holds(F,T). [t-T+1, true]
#heuristic holds(F,T-1) : not holds(F,T)  [t-T+1, false]
                          fluent(F), time(T).
```

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Heuristic options

- Alternative for specifying structure-oriented heuristics in *clasp*

```
--dom-mod=<arg> : Default modification for
                  domain heuristic
  <arg>: <mod>[,<pick>]
    <mod>  : Modifier
             {1=level|2=pos|3=true|4=neg|
              5=false|6=init|7=factor}
    <pick> : Apply <mod> to
             {0=all|1=scc|2=hcc|4=disj|
              8=min|16=show} atoms
```

Engage heuristic modifications (in both settings!)

```
--heuristic=Domain
```

Potassco

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Inclusion-minimal stable models

- Consider a logic program containing a mimimize statement of form
  - `#minimize{`$a_1, \ldots, a_n$`}`
- Computing one inclusion-minimal stable model can be done either via
  - `#heuristic` a_i `[1,false].` for $i = 1, \ldots, n$, or
  - `--dom-mod=5,16`
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  - by adding `--enum-mod=domRec` to the two options

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Applications: Overview

20 Train scheduling

21 Robotic intra-logistics

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Outline

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Motivation

- Increasing railway traffic demands global and flexible ways for scheduling trains in order to use railway networks to capacity
- Difficulty arises from dependencies among trains induced by connections and shared resources
- Train scheduling combines three distinct tasks
  - Routing
  - Conflict detection and resolution
  - Scheduling
- Solution operational at Swiss Federal Railway using *clingo*[DL]
  - ASP
  - Difference constraints
  - (Hybrid) Optimization
  - Heuristic directives
  - Multi-shot solving

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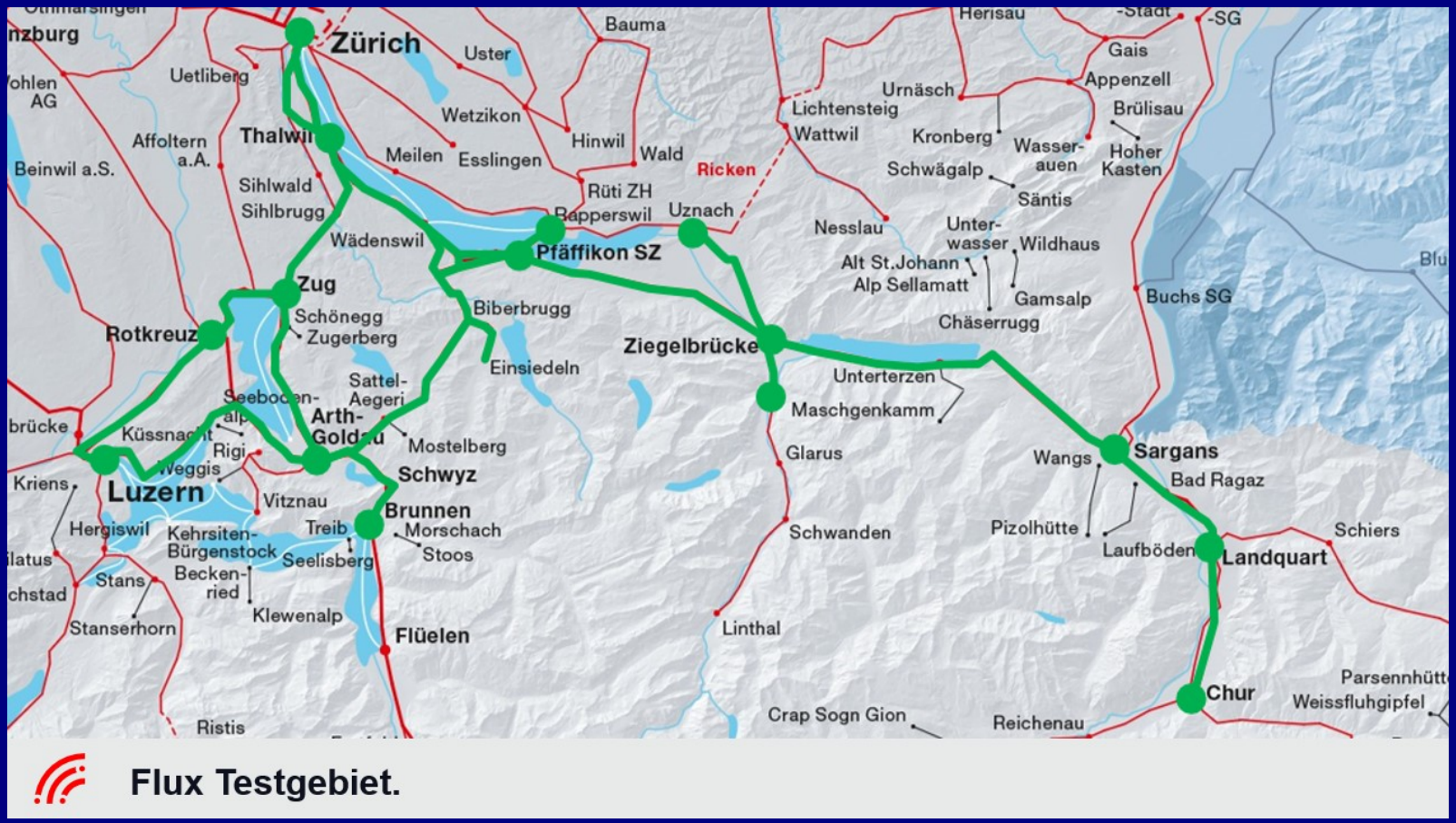
Benchmark



We optimally solved the train scheduling problem on real-world railway networks spanning about 150 km with up to 467 trains within 5 minutes.

Outline

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Motivation

- Objective How to develop robust and scalable AI technology for dealing with complex dynamic application scenarios?
- What's needed? — a fruit fly!

  Robotic intra-logistics
- Why?
  - rich multi-faceted, full of variations
  - scalable layout, objects, granularity
  - measurable makespan, energy, quality of service
  - integrative mapf, data, constraints, decisions
  - relevant industry 4.0
- What for? — enabling research and teaching

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Robotic intra-logistics

- Robotics systems for logistics and warehouse automation based on many
  - mobile robots
  - movable shelves
- Main tasks: order fulfillment, i.e.
  - routing
  - order picking
  - replenishment
- Many competing industry solutions:
  - Amazon, Dematic, Genzebach, Gray Orange, Swisslog

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Robotic intra-logistics at Amazon

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Robotic intra-logistics at Swisslog

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What's (not) in the picture?

- Objects
  floor, robots, shelves, products, people, etc.
- Relations
  positions, carries/d, capacity, orientation, durations, etc.
- Actions
  move, pickup, putdown, pick, charge, restock, etc.
- Objectives
  deadlines, throughput, exploitation, energy management, human machine interaction, etc.

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Outline

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- Potassco Systems — `http://potassco.org`
  - Academic branch
  - Freely available systems
  - Open source license (MIT)
- Potassco Solutions — `http://potassco.com`
  - Service branch
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Sites Germany (HQ@Potsdam), Australia, Austria, China, Cyprus, Finland, France, Japan, Portugal, Spain, Turkey

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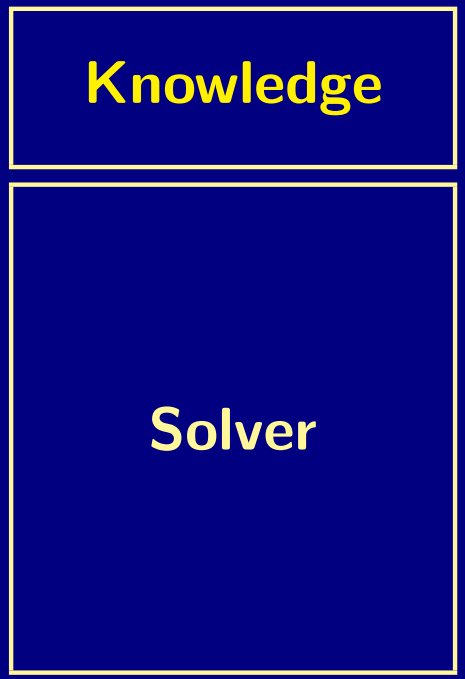
The benefits of ASP

It's yours!

+ Transparency
+ Flexibility
+ Maintainability
+ Reliability

+ Generality
+ Effectiveness
+ Optimality
+ Availability



ASP is a technology,
products emerge from co-operations

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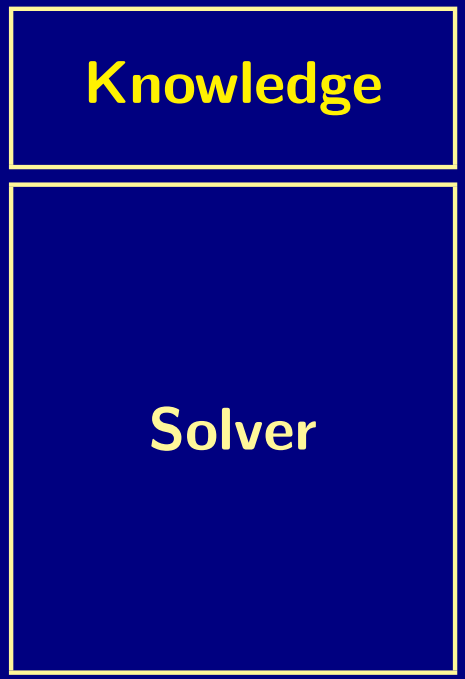

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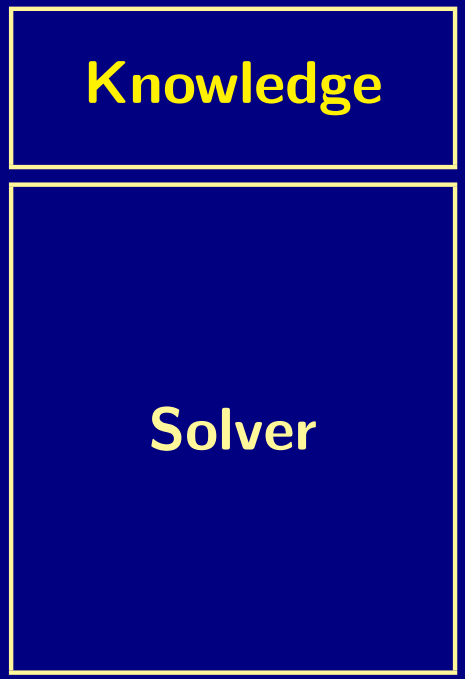

ASP is a technology,
products emerge from co-operations

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[1] K. Gödel.
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[2] A. Heyting.
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